

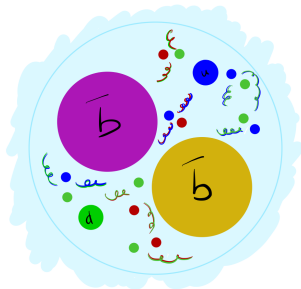
RULING OUT SOME PREDICTIONS OF DEEPLY-BOUND LIGHT-HEAVY TETRAQUARKS USING LATTICE QCD

Brian Colquhoun

w/ R. J. Hudspith, A. Francis,
R. Lewis, K. Maltman



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SCHEMATIC MODEL OF BARYONS AND MESONS

M. GELL-MANN

California Institute of Technology, Pasadena, California

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The existence of tetraquarks
(and other exotic states) has
long been suspected!

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon \mathbf{b} if we assign to the triplet \mathbf{t} the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" \mathbf{q} and the members of the anti-triplet as anti-quarks $\bar{\mathbf{q}}$. Baryons can now be constructed from quarks by using the combinations $(\mathbf{q}\mathbf{q}\mathbf{q})$, $(\mathbf{q}\mathbf{q}\mathbf{q}\bar{\mathbf{q}})$, etc., while mesons are made out of $(\mathbf{q}\bar{\mathbf{q}})$, $(\mathbf{q}\mathbf{q}\bar{\mathbf{q}}\bar{\mathbf{q}})$, etc. It is assumed that the lowest baryon configuration $(\mathbf{q}\mathbf{q}\mathbf{q})$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(\mathbf{q}\bar{\mathbf{q}})$ similarly gives just **1** and **8**.

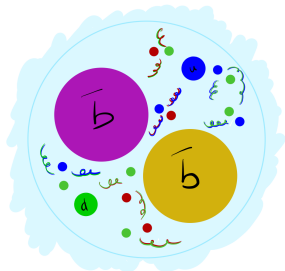
Diquarks

- ★ We are interested in:
 - ▶ light diquarks in a colour $\bar{3}_c$, flavour $\bar{3}_f$ and spin 0 configuration
 - “good light diquark”
 - ▶ heavy diquarks in a colour 3_c , relative s-wave configuration

The term “good diquark” is of Jaffe's invention, for a nice review: [\[hep-ph/0409065\]](#)

A. Francis talk on “good” and “bad” diquarks @ 16:20 Wed, “QCD and hadron structure” session

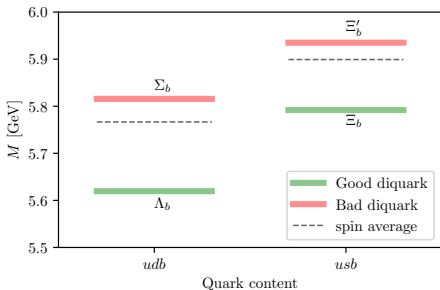
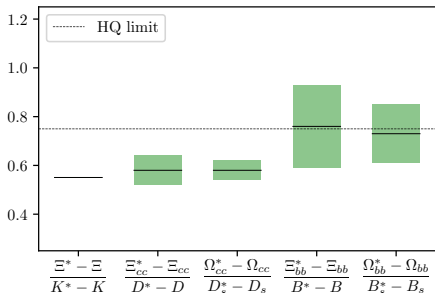
- ★ Degenerate heavy quarks: $J^P = 1^+$
- ★ Otherwise: we have access to $J^P = 0^+$ or $J^P = 1^+$
- ★ Light diquark $\implies I = 0$ or $I = 1/2$



Information from baryons and mesons

- ★ Ordinary baryon and meson spectra can provide constraints for models
- ★ $3_c \bar{Q}\bar{Q}$ serves as near-static colour source, like a single Q in a baryon (plus attractive colour Coulomb interaction)

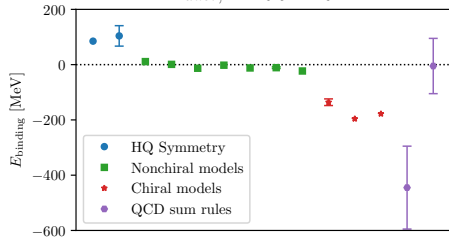
Numbers from PDG & [1409.0497]



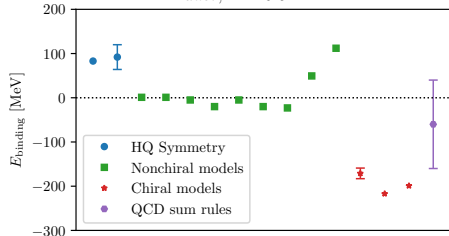
- ★ Baryon spectrum suggests “good” light diquarks result in strong attraction.
- ★ Lighter quark mass \rightarrow stronger attraction

Example of model and other predictions

$ud\bar{c}\bar{b}$, $I = 0$ $J^P = 0^+$



$ud\bar{c}\bar{b}$, $I = 0$ $J^P = 1^+$

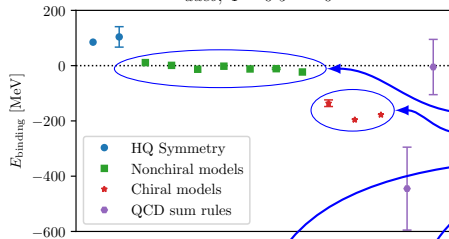


Examples of various $ud\bar{c}\bar{b}$ masses relative to the lowest two-meson threshold in the $I = 0$, $J^P = 0^+$ and $I = 0$, $J^P = 1^+$ channels.

We discuss model results completely for all channels in R.J. Hudspith, BC, A. Francis, R. Lewis and K. Maltman *Phys. Rev. D* 102, 114506 (2020), [2006.14294].

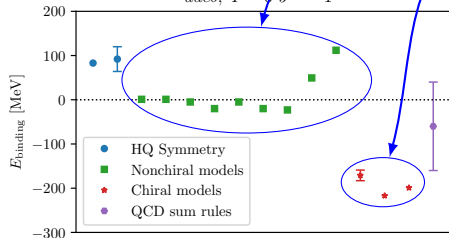
Example of model and other predictions

$ud\bar{c}\bar{b}$, $I = 0$ $J^P = 0^+$



All fit to the ordinary meson/baryon spectrum; clear tetraquark difference

$ud\bar{c}\bar{b}$, $I = 0$ $J^P = 1^+$



Examples of various $ud\bar{c}\bar{b}$ masses relative to the lowest two-meson threshold in the $I = 0$, $J^P = 0^+$ and $I = 0$, $J^P = 1^+$ channels.

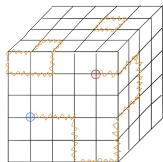
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SOME LATTICE DETAILS

Ensembles

$N_f = 2 + 1$ Wilson-clover ensembles – includes and extends PACS-CS ensembles

$L^3 \times T$	m_π [MeV]	N_{cnfg}	a^{-1} [GeV]
$32^3 \times 64$	700	399	2.194(10)
	575	400	
	415	400	
	299	800	
	182*	121	
$48^3 \times 64$	192	122	
	165	88	



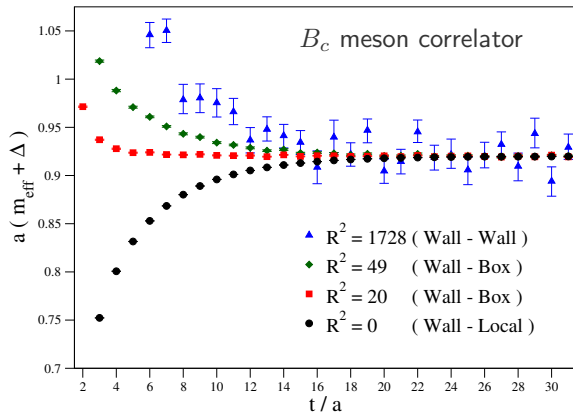
- ★ Coulomb gauge-fixed wall sources
- ★ c quarks: Relativistic Heavy Quark action
- ★ b quarks: Nonrelativistic QCD (NRQCD) action

Recent update: Box-Sinks

R. J. Hudspith, BC, A. Francis, R. L Lewis, K. Maltman [2006.14294]

Improvement: box-sinks for better overlap with ground states.

$$S^B(x, t) = \frac{1}{N} \sum_{r^2 \leq R^2} S(x + r, t)$$



TETRAQUARKS ON THE LATTICE

Fitting our tetraquarks

Construct correlators, $C_{\mathcal{O}_1\mathcal{O}_2}(t) = \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2E_n} e^{-E_n t}$ from:

$$\begin{aligned} D(\Gamma_1, \Gamma_2) &= (\psi_a^T C \Gamma_1 \phi_b)(\bar{\theta}_a C \Gamma_2 \bar{\omega}_b^T), \\ E(\Gamma_1, \Gamma_2) &= (\psi_a^T C \Gamma_1 \phi_b)(\bar{\theta}_a C \Gamma_2 \bar{\omega}_b^T - \bar{\theta}_b C \Gamma_2 \bar{\omega}_a^T), \\ M(\Gamma_1, \Gamma_2) &= (\bar{\theta} \Gamma_1 \psi)(\bar{\omega} \Gamma_2 \phi), & N(\Gamma_1, \Gamma_2) &= (\bar{\theta} \Gamma_1 \phi)(\bar{\omega} \Gamma_2 \psi), \\ O(\Gamma_1, \Gamma_2) &= (\bar{\omega} \Gamma_1 \psi)(\bar{\theta} \Gamma_2 \phi), & P(\Gamma_1, \Gamma_2) &= (\bar{\omega} \Gamma_1 \phi)(\bar{\theta} \Gamma_2 \psi). \end{aligned}$$

We want to solve a GEVP to get energy levels, “principle correlators”:

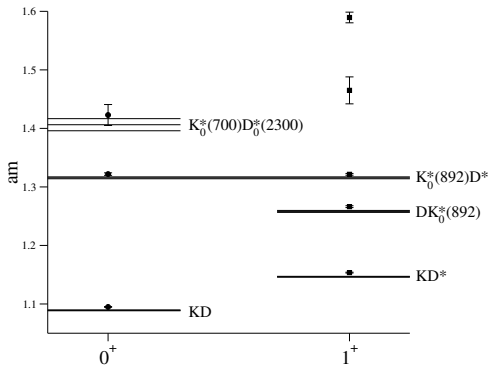
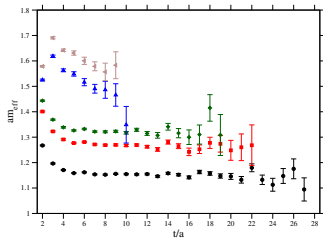
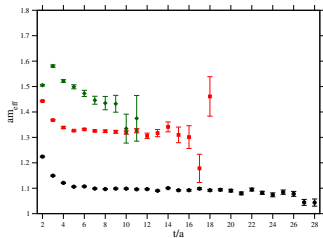
$$C_i(t) = \sum_{j,k} V_{ij}(\tau)^\dagger C_{jk}(t) V_{ki}(\tau)$$

where V is made from columns of the eigenvector solution to:

$$C_{ij}(t)v_j(t) = \lambda_i C_{ij}(t+t_0)v_j(t).$$

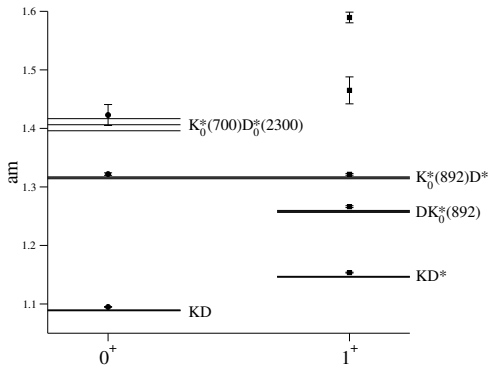
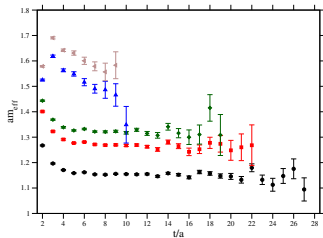
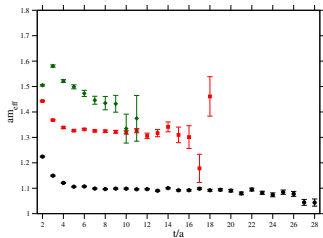
$ud\bar{s}\bar{c}$ tetraquarks

R. J. Hudspith, BC, A. Francis, R. Lewis, K. Maltman [2006.14294]



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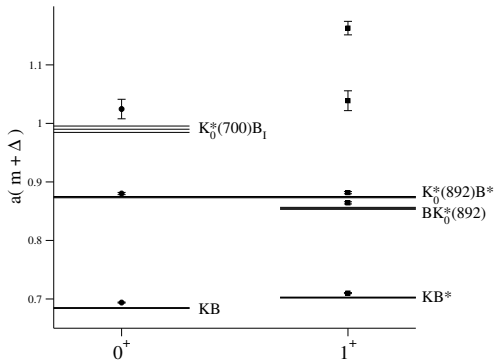
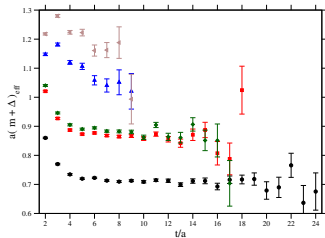
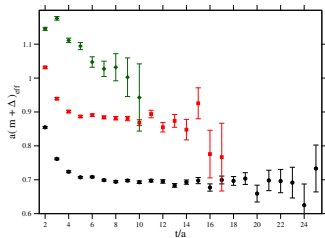
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★ No evidence of deep binding in 0^+ or 1^+ channels

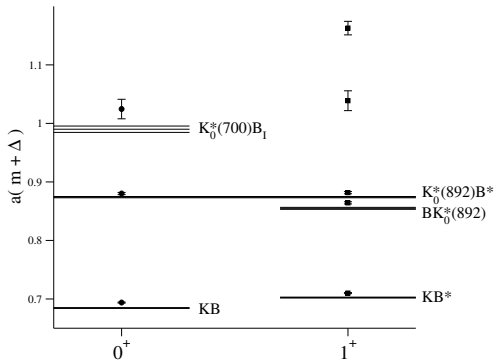
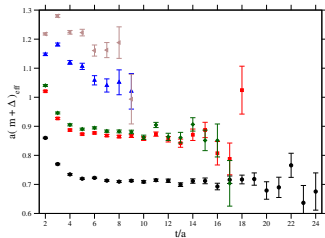
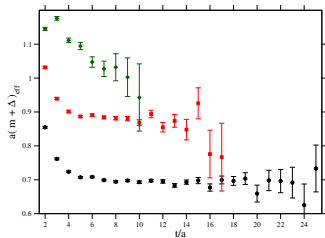
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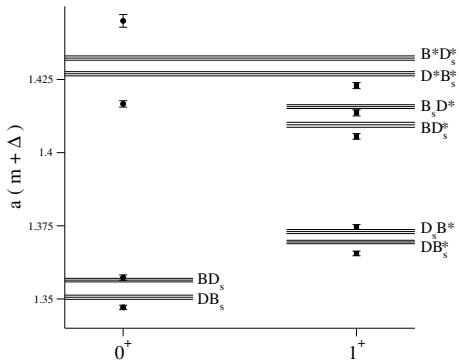
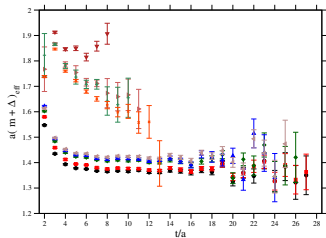
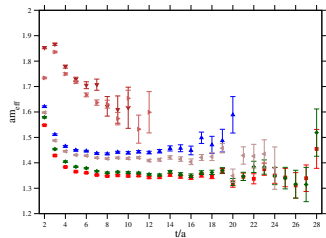
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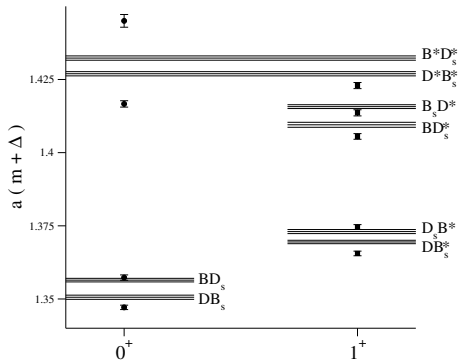
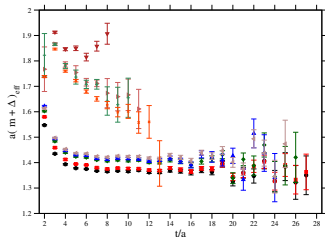
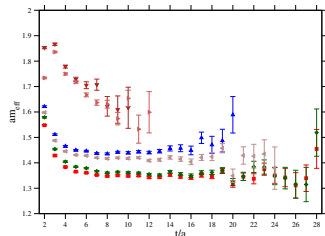
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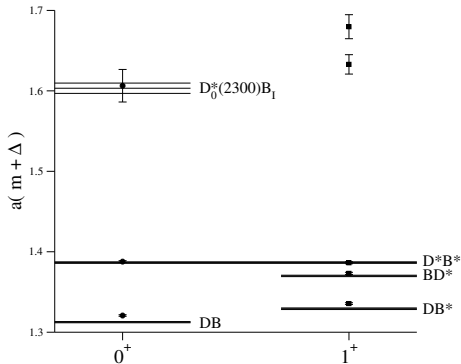
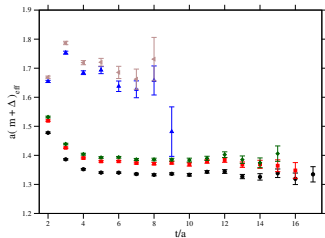
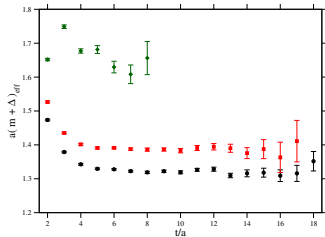
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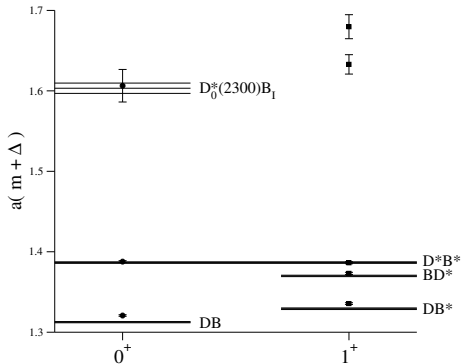
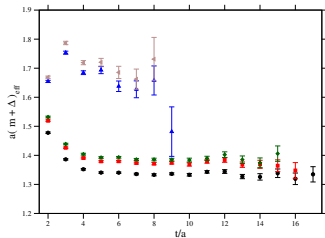
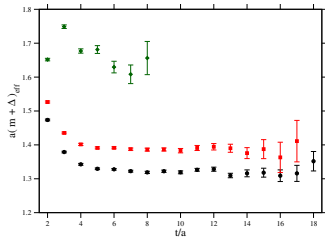


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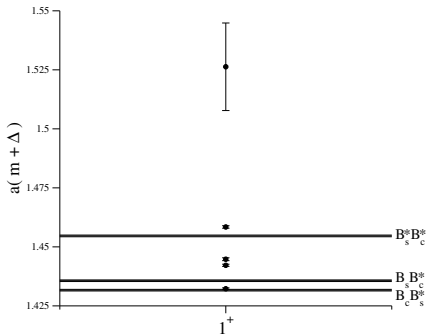
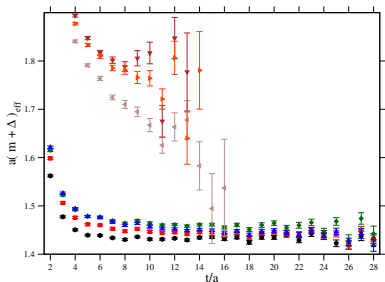


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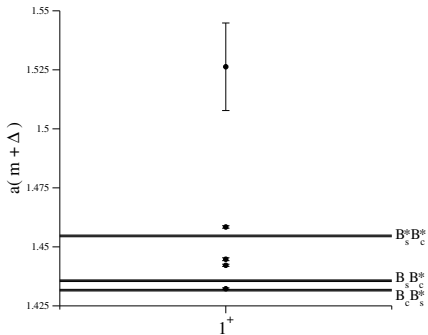
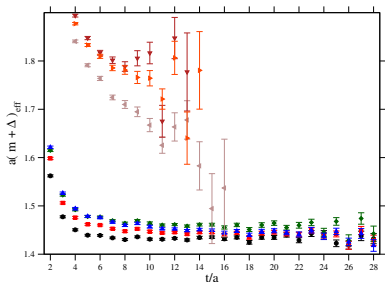


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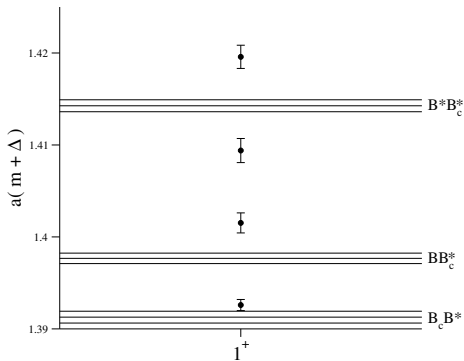
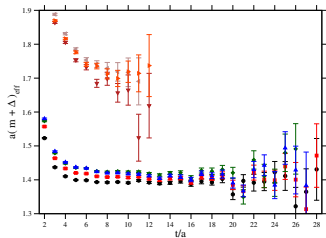


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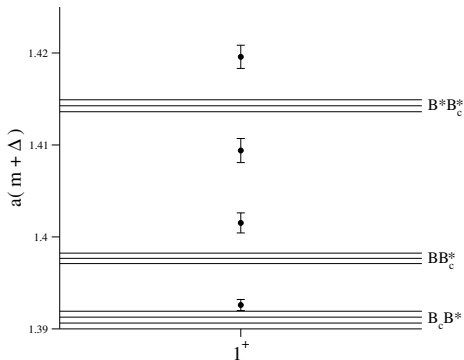
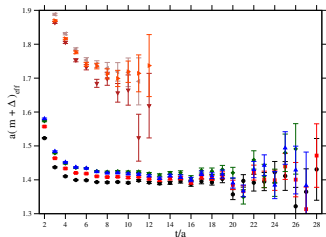


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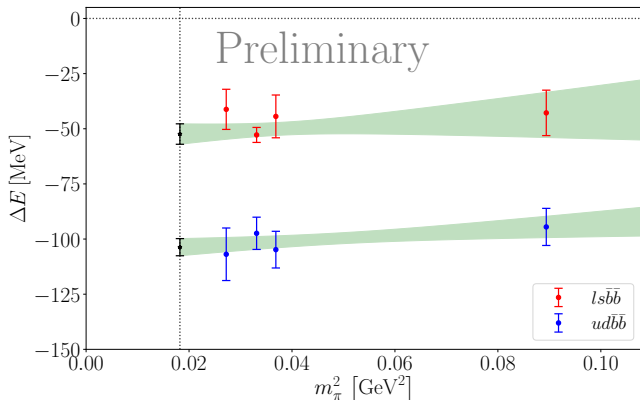


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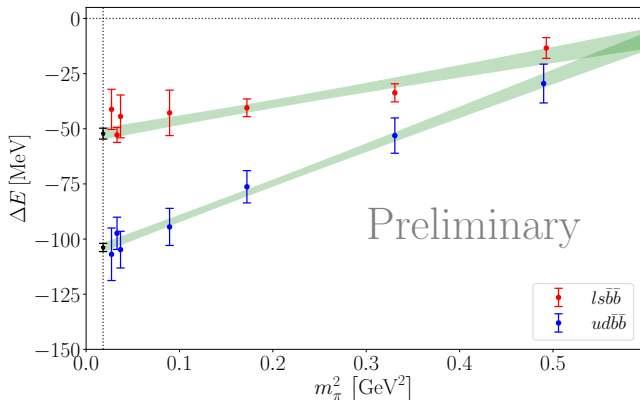
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Doubly-bottom tetraquarks



- ★ $I = 0, J^P = 1^+$ $ud\bar{b}\bar{b}$ and $I = 1/2, J^P = 1^+$ $ls\bar{b}\bar{b}$ strong-interaction stable.
- ★ Consistent binding of $ud\bar{b}\bar{b}$ found by lattice groups + preliminary $ls\bar{b}\bar{b}$.

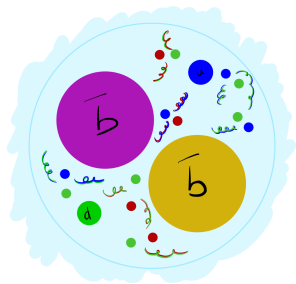
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Summary

- ★ $I = 0, J^P = 1^+$ $ud\bar{b}\bar{b}$ state consistently found stable on lattice:
 $\Delta E_{ud\bar{b}\bar{b}} \gtrsim 100$ MeV
- ★ Evidence for stable $I = 1/2, J^P = 1^+$ $\ell s\bar{b}\bar{b}$:
 $\Delta E_{\ell s\bar{b}\bar{b}} \sim 50$ MeV
- ★ No evidence for deep binding in any other channel considered.
- ★ Chiral model deep binding predictions incompatible with lattice results; nonchiral colour magnetic spin-spin interaction models OK within current lattice errors.



Thank you!

THANK YOU

EXTRAS

Operators

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$ $E(\gamma_5, \gamma_i), E(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i) - N(\gamma_5, \gamma_i)$ $M(I, \gamma_i\gamma_5) - N(I, \gamma_i\gamma_5)$ $O(\gamma_5, \gamma_i) - P(\gamma_5, \gamma_i)$ $O(I, \gamma_i\gamma_5) - P(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$
	$0(0)^+$	$E(\gamma_5, \gamma_5), E(\gamma_t\gamma_5, \gamma_t\gamma_5)$	$M(\gamma_5, \gamma_5) - N(\gamma_5, \gamma_5)$ $M(I, I) - N(I, I)$ $M(\gamma_i, \gamma_i)$

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udbb$	$0(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i) - N(\gamma_5, \gamma_i)$ $M(I, \gamma_i\gamma_5) - N(I, \gamma_i\gamma_5)$
$lsbb/ucbb/scbb$	$\frac{1}{2}(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i), M(I, \gamma_i\gamma_5)$ $N(\gamma_5, \gamma_i), N(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$uscb$	$\frac{1}{2}(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$ $E(\gamma_5, \gamma_i), E(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i), M(I, \gamma_i\gamma_5)$ $N(\gamma_5, \gamma_i), N(I, \gamma_i\gamma_5)$ $O(\gamma_5, \gamma_i), O(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$
	$\frac{1}{2}(0)^+$	$E(\gamma_5, \gamma_5), E(\gamma_t\gamma_5, \gamma_t\gamma_5)$	$M(\gamma_5, \gamma_5), M(I, I)$ $N(\gamma_5, \gamma_5), N(I, I)$ $M(\gamma_i, \gamma_i)$

NRQCD tuning

