Will the relativistic kinematics increase the number of stable multiquarks?

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Contents

First one statement

Non relativistic constituent quark model predict a very small number of bound states unless strong approximations are in place.

- ✓ There are no compact non-exotic bound states: $c\underline{c}n\underline{n}$
  - ✓ *J.P.A., J.-M.R.,P.T., Phys. Rev. D25, 2370 (1982), J.V., A.V. et al., Phys. Rev. D76, 094022 (2007)*
- ✓ There is a molecular non-exotic bound state: $c\underline{c}n\underline{n}$ with $(I)J^{PC}=(0)1^{++} \Rightarrow$ X(3872)
  - ✓ *E.B. M.L. J.L. Phys. Rev. D76, 054010 (2007), T. F.-C., A.V., J.V., Phys. Rev. Lett. 103, 222001 (2009)*
- ✓ There is one compact exotic bound state: $cc\underline{nn}$ or $bb\underline{nn}$ with $J^P=1^+$
  - ✓ *E. E., C. Q. Phys. Rev. Lett. 119, 202002 (2017), J.P.A., J.-M.R.,P.T., Phys. Rev. D25, 2370 (1982), J.V., A.V., N.B., Phys. Rev. D79, 074010 (2009)*
- ✓ There are no fully-heavy four quark states $cc\underline{cc}$ or $bb\underline{bb}$
  - ✓ *C. H., E. E., C. T. H. D., Phys. Rev. D 97, 054505 (2018), J.-M. R, A.V., J.V., Phys. Rev. D. 95, 035211 (2018)*

A detailed list of references can be found in H.-X. C., W. C., X. L., S.-L. Z. Phys Rep. 639, 1 (2016)

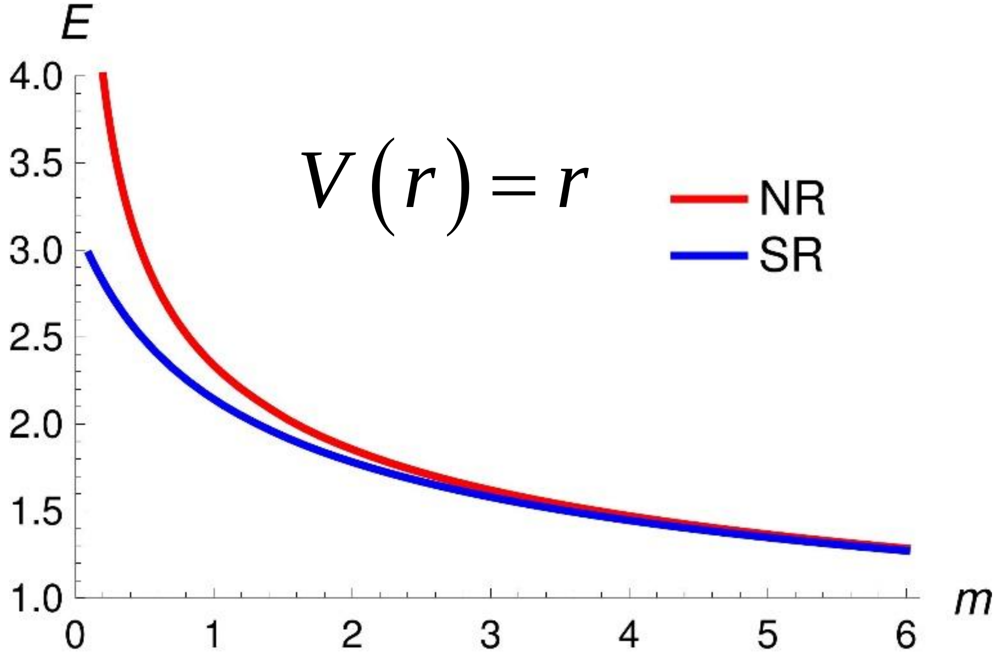
And now some questions

- Including relativistic effects in the kinetic energyimplies $\frac{p^2}{2m} \rightarrow \sqrt{p^2+m^2}-m$
- In the meson or baryon sector this only reflects on their energies, however multiquarks may "break apart" if above the two meson threshold, hence
  - Relativistic corrections are expected to be larger in the qq system than in the qQ or QQ systems. Aren't they?
  - How will relativistic corrections affect a "bigger" system involving different sectors, $qqQQ$ or $QQQQ$?
  - In any case, how this impact the binding energy? Will the binding energy increase or decrease?
- Therefore, summarizing, our question will be

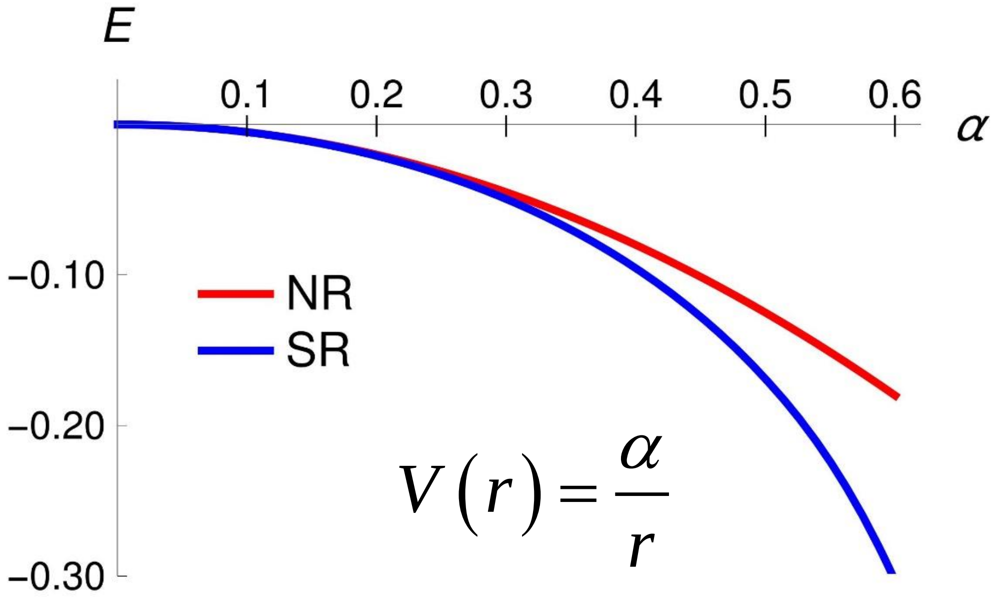
Will the bound nature of multiquark states in quark models benefit from relativistic effects on the kinetic energy operator?

Some examples of the role played by relativistic corrections

Mesons



$$V(r) = r$$

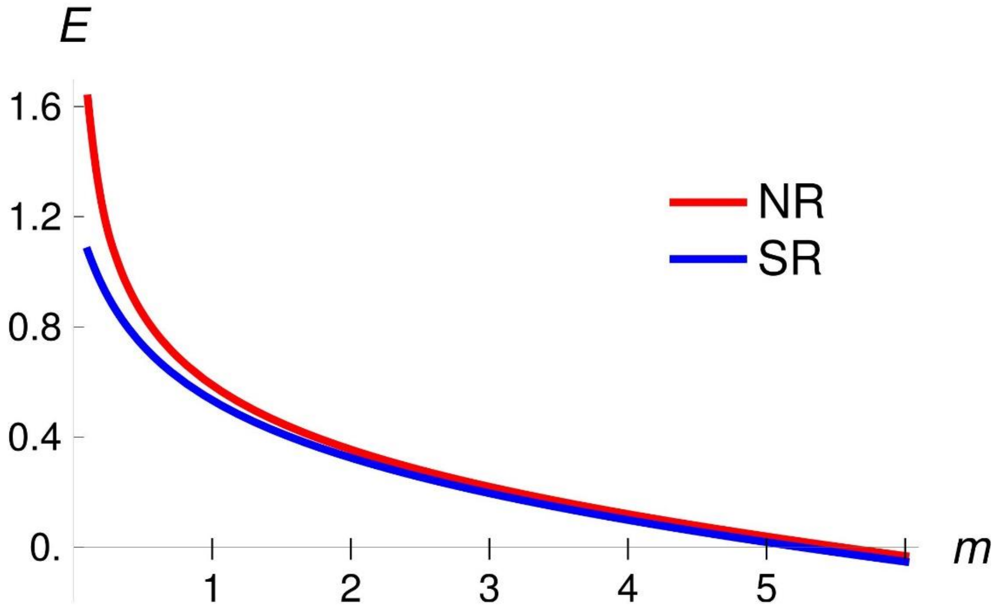


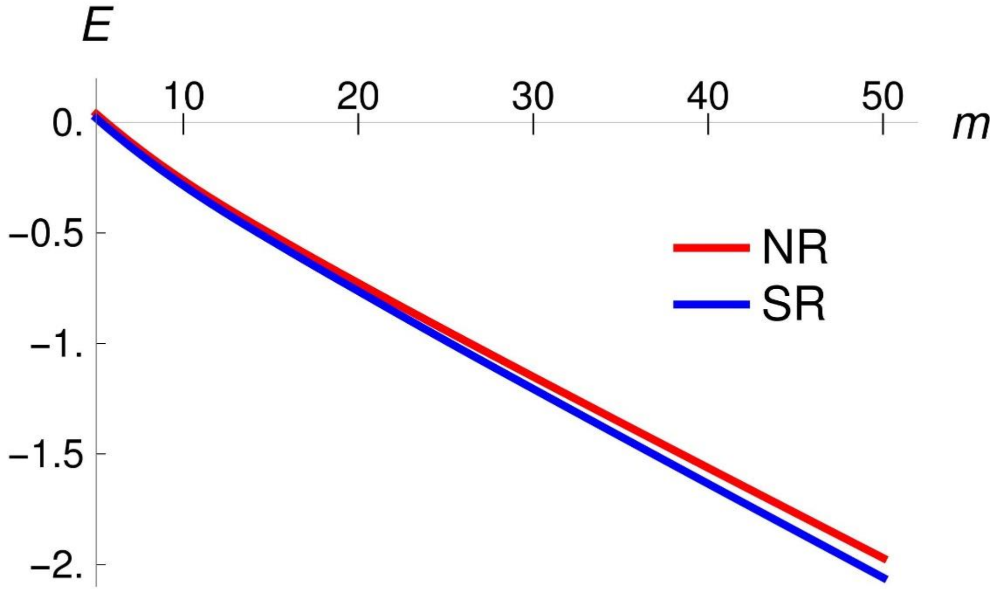
$$V(r) = \frac{\alpha}{r}$$

Some examples of the role played by relativistic corrections

Mesons (a more realistic potential)

$$V(r) = 0.2\,r - \frac{0.4}{r}$$





Some examples of the role played by relativistic corrections

$$V(r) = \frac{1}{2}\sum_{i<j} r_{ij}$$

m_1	m_2	m_3	NR	SR	Diff(%)
1	1	1	3.863	3.522	9
1	4	4	2.985	2.800	6
1	4	8	2.860	2.671	7
1	10	10	2.644	2.454	7
1	10	15	2.591	2.398	7
1	10	20	2.561	2.366	7
1	20	30	2.430	2.222	8
1	30	40	2.353	2.149	8
10	30	40	1.419	1.413	0.5
20	30	40	1.272	1.270	0.2
30	30	40	1.207	1.206	0.1

$NR > SR$

Baryons

$$V(r) = 0.2\, r - \frac{0.4}{r}$$

$(M,M,m) = (5,5,0.5)$

NR	SR
0.780	0.652

NR	SR
0.746	0.473

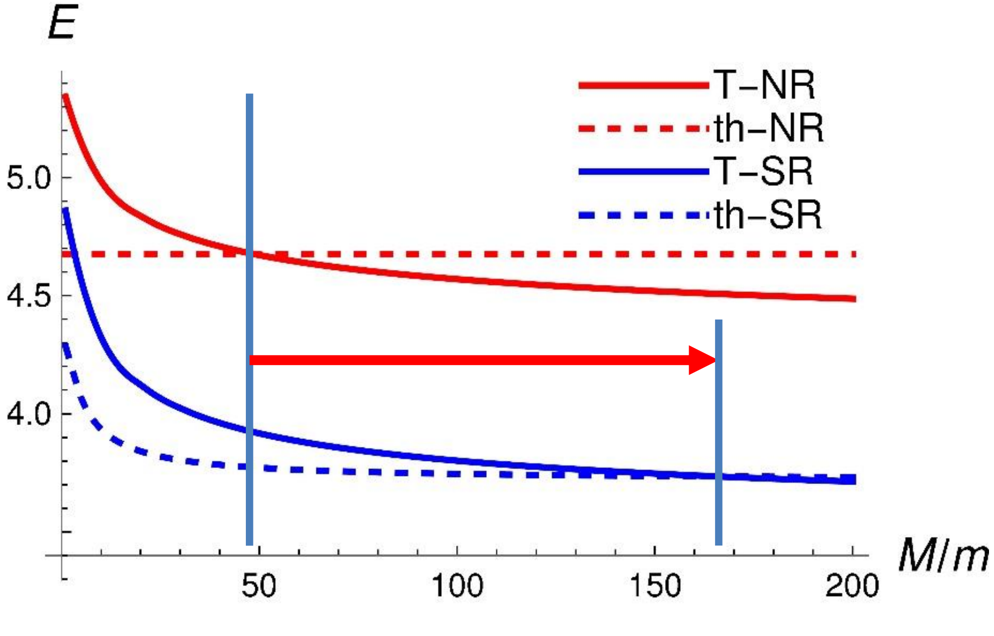
Diquark approximation!

Doubly-Heavy tetraquarks

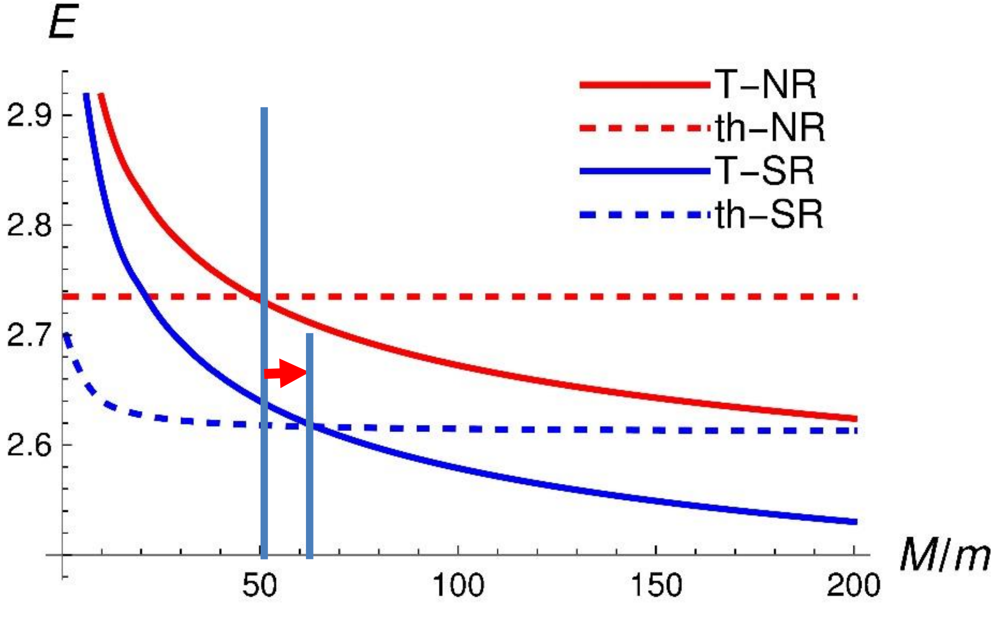
$$V = -\frac{3}{16}\sum_{i<j}^{4}\left(\tilde{\lambda}_i \cdot \tilde{\lambda}_j\right) r_{ij}$$

Only $\bar{3}3$ colour coupling

$$M^{-1} + m^{-1} = 2$$



$$M^{-1} + m^{-1} = 0.4$$

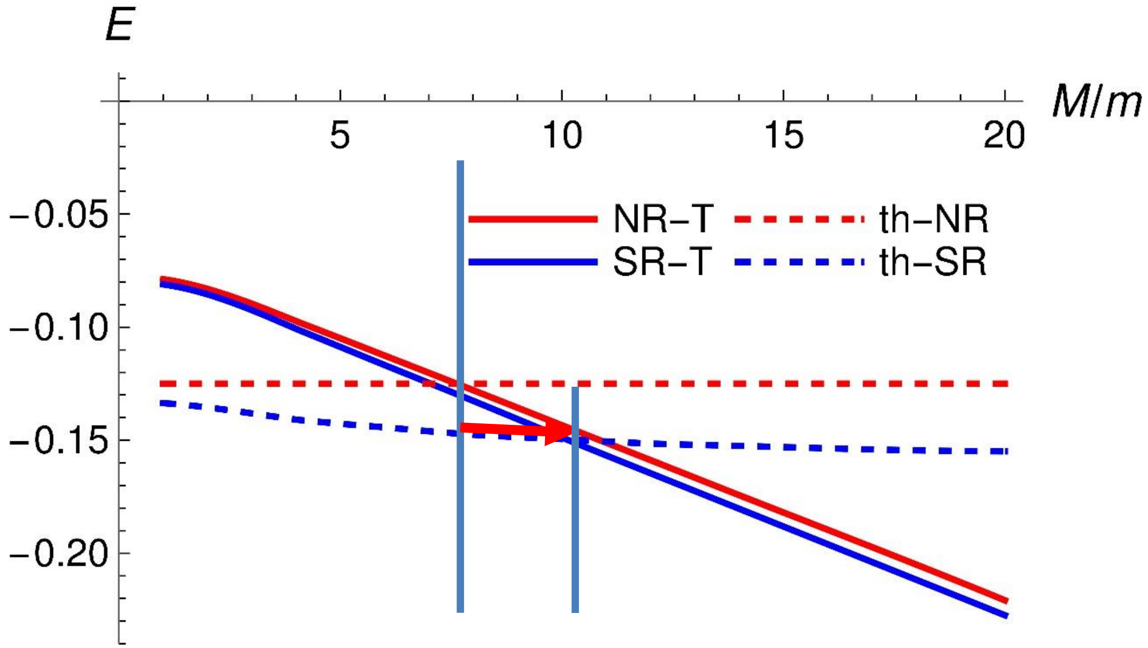


Doubly-Heavy tetraquarks

$$V = -\frac{3}{16}\sum_{i<j}^{4}\left(\tilde{\lambda}_i \cdot \tilde{\lambda}_j\right)\frac{-g}{r_{ij}}$$

Only $\bar{3}3$ colour coupling

$$M^{-1} + m^{-1} = 2$$

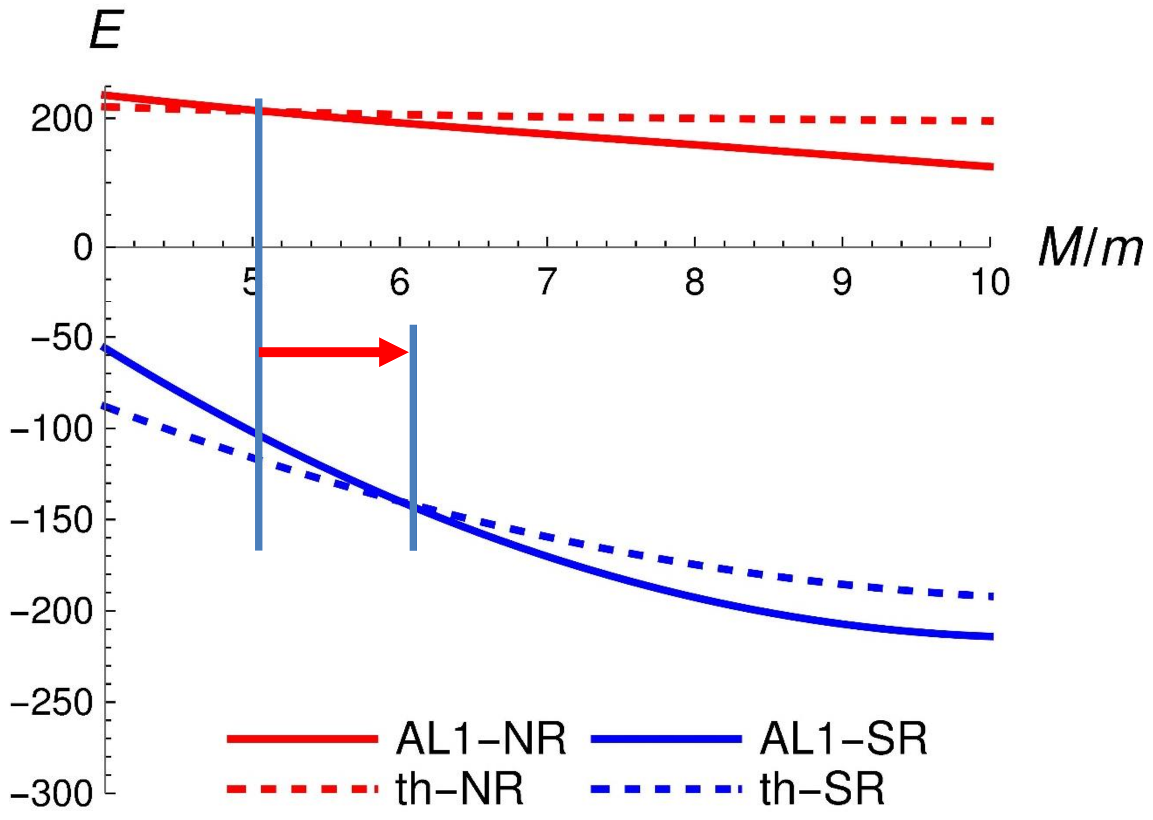


- Bound states appear at larger M/m ratio
- For a given M/m, the Binding energy diminishes

Doubly-Heavy tetraquarks

$$V = -\frac{3}{16}\sum_{i<j}^{4}\left(\tilde{\lambda}_i \cdot \tilde{\lambda}_j\right)\left[-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda + \frac{2\pi}{3}\frac{\alpha}{m_i m_j}\frac{Exp\left(-r^2/r_0^2\right)}{\pi^{3/2} r_0^3}\left(\sigma_i \cdot \sigma_j\right)\right]$$

Full colour coupling



The model needs to be reparametrized to maintain the description of the meson and baryon sectors.

- Bound states appear at larger M/m ratio

- For a given M/m, the Binding energy diminishes

All-Heavy tetraquarks

$$V = -\frac{3}{16}\sum_{i<j}^{4}\left(\tilde{\lambda}_i \cdot \tilde{\lambda}_j\right)\left[-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda + \frac{2\pi}{3}\frac{\alpha}{m_i m_j}\frac{Exp\left(-r^2/r_0^2\right)}{\pi^{3/2} r_0^3}\left(\sigma_i \cdot \sigma_j\right)\right]$$

Full colour coupling

No bound states appear either for nonrelativistic or relativistic kinematics in either the $cc\underline{cc}$ or the $bb\underline{bb}$ sectors.

Conclusions

- Sophisticated numerical tools are required to explore the multiquark sector.
- In the meson and baryon sectors most relativistic effects can be absorbed by a tuning of the model parameters.
- This is not the case for tetraquarks → The two meson threshold benefits more from the relativistic corrections than the collective configuration.
- Therefore relativistic corrections penalize binding → the existence of four-quark bound states is a rare ocurrence within the constituent quark model.