Non-linear Regge trajectories in the context of bottom-up holography

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Phenomenological Motivation

Regge Trajectories

Regge trajectories define a taxonomy of hadrons in terms of their masses and quantum numbers. In the particular case of radial trajectories, we have

$$M^2 = a(n+b)^\nu, \;\; \nu \text{ linearity deviation.}$$

These sort of trajectories come naturally in potential models, Bohr-Sommerfeld approach or Bethe-Salpeter equation analysis.

These objects are also provide a tool to test confinement in effective models for hadrons.
Linearity is connected with constituent mass (Chen 2018): light-(un)flavored mesons will have $\nu = 1$, and heavier ones will exhibit deviations from linearity $\nu \neq 1$, see Gershtein et al. 2006, Chen 2018 and 2021.

Isovector family $I^G J^{PC} = 0^- (1^{--})$

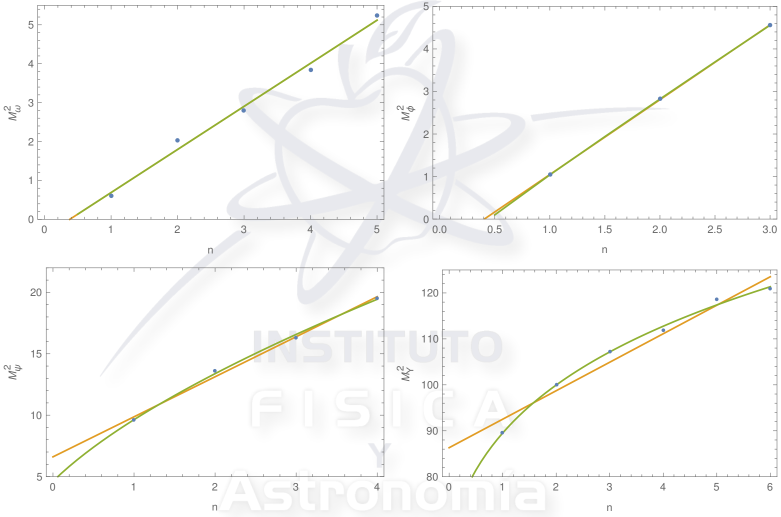


Figure: M^2 vs n for isovector mesons (ω, ϕ, ψ and Υ). Dots represent experimental data (PDG).

Isovector family $I^G J^{PC} = 0^- (1^{--})$

TABLE II. Summary of linear and nonlinear fits for isovector meson Regge trajectories drawn in Fig. 1. We expose parameters for each parametrization considered, altogether with the correlation coefficient R^2. Observe that linear fits bring good description of the trajectories, but R^2 decrease from unity when we increase the quark constituent mass. Also notice that the nonlinear fit is more precise since R^2 is bigger than the linear one in each case.

	Linear Regge Trajectory: $M^2 = a(n+b)$			Nonlinear Regge Trajectory ($M^2 = a(n+b)^\nu$)			
Meson	a	b	R^2	a	b	ν	R^2
ω	1.1074	−0.3781	0.9978	1.1078	−0.3784	0.9998	0.9978
ϕ	1.7595	−0.4048	0.9999	1.8545	−0.4524	0.9617	1.000
ψ	3.2607	2.0259	0.9997	7.6516	0.4460	0.6249	0.9999
Υ	6.2015	13.9182	0.9996	85.3116	0.2849	0.1917	0.9999

Non-linearity Hypothesis

Hypothesis

Linearity is connected with the hadron constituent mass: when constituent mass raises, linearity ceases. The linear case appears when constituent quark masses are supposed to be zero, i.e., they are small enough compared with the meson mass.

From Bethe-Salpeter (Afonin and Pusenkov, 2014; Chen, 2018) we can write the trajectory as

$$(M_n - m_{q_1} - m_{q_2})^2 = a(n + b).$$

When the limit $m_{q_{1,2} \to \infty}$ (as in heavy quarkonium), the trajectory acquires a generic non-linear form:

$$M_n^2 \propto n^{2/3}.$$

Also in heavy-light systems, non-linearity is expected (J. K. Chen, 2018).

Our goal is to translate this hypothesis into a holographic bottom-up language.

AdS and Confinement

Talks by E. Capossouli and M. Rinaldi gave an excellent introduction to AdS/CFT and confinement. Thus, we can summarize the holographic confinement idea as follows:

AdS/QCD spectroscopy hypothesis

By properly breaking conformal invariance in the AdS bulk, we can generate confined states dual to hadrons at the boundary.

Top-down

Fix the boundary theory and look out for the bulk phenomenology.

Bottom-up

Fix the bulk physics and try to match it with the boundary phenomenology.

Both approximations play with the idea of mimicking non-perturbative QCD with holographic tools.

General Bottom-up Algorithm

AdS-like Background

$$dS^2 = \frac{R^2}{z^2}\, e^{h(z)} \left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]$$

with R defined as the AdS radius and $h(z)$ a geometric deformation.

General Action with minimal coupling

$$I_{\text{SWM}} = \int d^5x\, \sqrt{-g}\, e^{-\Phi(z)}\, \mathcal{L}_{\text{Hadron}},$$

with $\Phi(z)$ defined as a static dilaton field. Both $h(z)$ and $\Phi(z)$ are responsible for inducing confinement. The lagrangian $\mathcal{L}_{\text{Hadron}}$ has the information about the bulk field dual to hadrons.

Hadrons are characterized by the scaling dimension Δ of the boundary operator $\hat{\mathcal{O}}$ creating them. This information is encoded into the bulk field mass M_5:

$$M_5^2\, R^2 = (\Delta - S)(\Delta + S - 4)$$

General Bottom-up Algorithm

Holographic Potential

The action written above defines a set of equation of motion for the bulk fields that, in general, has a Schrödinge-like form:

$$-\psi''(z) + V(z)\,\psi(z) = M_n^2\,\psi(z),$$

where $V(z)$ is the holographic potential written in terms of the deformation and the dilaton. In the case of $p-$form bulk fields as follows:

$$V(z) = \frac{1}{4}B'(z)^2 - \frac{1}{2}\,B''(z) + \frac{M_5^2\,R^2}{z^2}\,e^{h(z)}, \tag{1}$$

with $B(z) = \Phi(z) + \beta\,[\log\frac{R}{z} + \frac{1}{2}h(z)]$, M_5 is the bulk mass associated to $\psi(z)$ and $\beta = -(3-2\,p)$. Latter we will connect β with the hadronic (integer) spin.

General Bottom-up Algorithm

Holographic Regge Trajectories

Regge trajectories will emerge as the eigenvalue spectrum associated to the Sturm-Liouville problem defined by $V(z)$:

$$M_n^2 = A\,(n + B)^{\nu},$$

where A is an energy scale defined by the dilaton and/or deformation, B carries information about the hadronic angular momentum, and ν measures linearity. If the deformations and dilatons are quadratic at the high-z limit, the out-coming trajectory will be linear.

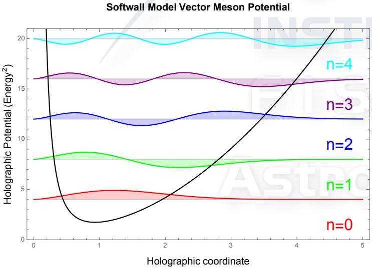
Example: Vector Soft Wall model

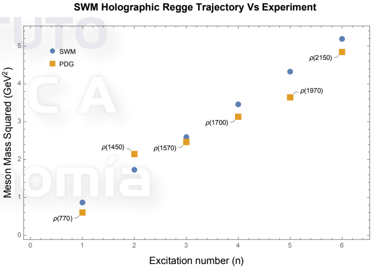
In the KKSS model (Karch et al. 2005), $\Delta = 3$, $\beta = -1$ and $M_5^2\,R^2 = 0$ for vector mesons, and fix $\Phi(z) = \kappa^2\,z^2$ and $h(z) = 0$. The potential is

$$V(z) = \frac{3}{4\,z^2} + \kappa^2\,z^4.$$

Radial vector mesons Regge trajectories in this case are linear

$$M_n^2 = 4\,\kappa^2\,(n+1).$$





Non-quadratic Dilaton

Our proposal

In order to induce non-linear Regge Trajectories we define

$$\Phi(z) = (\kappa\, z)^{2-\alpha}$$

as a deformation of softwall model (quadratic) dilaton field.

Holographic potential for vector hadrons ($\beta = -1$)

This dilaton defines the following potential:

$$\begin{aligned} V(z,\kappa,\alpha) = {} & \frac{3}{4z^2} - \frac{1}{2}\alpha^2\,\kappa^2\,(\kappa\, z)^{-\alpha} + \frac{1}{4}\alpha^2\,\kappa^2(\kappa\, z)^{2-2\alpha} \\ & + \frac{3}{2}\alpha\,\kappa^2\,(\kappa\, z)^{-\alpha} - \kappa^2(\kappa\, z)^{-\alpha} - \alpha\,\kappa^2\,(\kappa\, z)^{2-2\alpha} \\ & + \kappa^2\,(\kappa\, z)^{2-2\alpha} + \frac{\kappa}{z}(\kappa\, z)^{1-\alpha} - \frac{\alpha\,\kappa}{2\,z}(\kappa\, z)^{1-\alpha} + \frac{M_5^2(\Delta)\,R^2}{z^2} \end{aligned}$$

Light unflavored and flavored Isovector Fitting

ω **with** $\alpha = 0.04$ **and** $\kappa = 498$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	782.65 ± 0.12	981.43	25.4
2	$1400 - 1450$	1374	3.6
3	1670 ± 30	1674	0.25
4	1960 ± 25	1967	1.7
5	2290 ± 20	2149	6.2
$M_n^2 = 0.9514(0.012 + n)^{0.9798}$ with $R^2 = 0.999$			

ϕ **with** $\alpha = 0.07$ **and** $\kappa = 585$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	1019.461 ± 0.016	1139.43	11.8
2	1698 ± 20	1583	5.8
3	$2135 \pm 8 \pm 9$	1921	10
$M_n^2 = 1.268(0.0244 + n)^{0.9650}$ with $R^2 = 0.999$			

Table: Summary of results for heavy isovector radial mesonic states considered in this work. Experimental results are read from PDG.

Heavy Isovector Fitting

ψ **with** $\alpha = 0.54$ **and** $\kappa = 2150$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	3096.916 ± 0.011	3077.09	0.61
2	3686.109 ± 0.012	3689.62	0.1
3	4039 ± 1	4137.5	2.44
4	4421 ± 4	4499.4	1.77
$M_n^2 = 8.07(0.287 + n)^{0.6315}$ with $R^2 = 0.999$			

Υ **with** $\alpha = 0.863$ **and** $\kappa = 11209$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	9460.3 ± 0.26	9438.5	0.23
2	10023.26 ± 0.32	9923.32	0.78
3	10355 ± 0.5	10277.2	0.75
4	10579.4 ± 1.2	10558.6	0.19
5	$10889.9^{+3.2}_{-2.6}$	10793.5	0.88
6	$10992.9^{+10.0}_{-3.1}$	10995.7	0.03
$M_n^2 = 76.511(0.901 + n)^{0.2369}$ with $R^2 = 0.999$			

Table: Summary of results for light isovector radial mesonic states considered in this work. Experimental results are read from PDG.

Non-linear fitting: Running of α and κ with $\bar{m}$

The non-quadratic approach induces a running of the parameters κ and α in terms of the inner mesonic structure, parametrized by the average constituent mass $\bar{m}$.

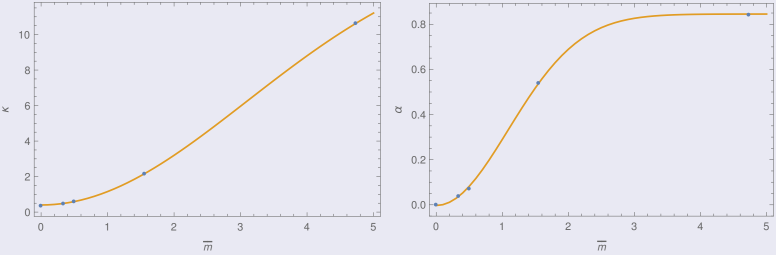


Figure: Running of κ and α in terms of $\bar{m}$.

where for mesons we have

$$\bar{m}(q_1, q_2) = \frac{1}{2}\left(m_{q_1} + m_{q_2}\right).$$

Vector Kaons

Vector kaons are mesonic states labeled by $I(J^P) = 1/2(1^-)$, with $S = \pm 1$ and $C = B = 0$ and $\Delta = 3$. Also we define

$$\bar{m}_{K^*} = \frac{m_s + m_d}{2}, \tag{2}$$

with $m_s = 0.486$ and $m_d = m_u = 0.336$ GeV. The numerical results are summarized in the following table

K^* **with** $\bar{m} = 413$ **MeV,** $\alpha = 0.055$**, and** $\kappa = 531.24$ MeV				
n	**State**	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	$K^*(892)$	895.55 ± 0.8	1038.4	16.2
2	$K^*(1410)$	1414 ± 15	1451.0	2.6
3	$K^*(1680)$	1718 ± 18	1754.5	2.1
Experimental Linear R. T.:	$M^2 = 1.075(-0.2157 + n)$ with $R^2 = 0.9992$.			
Experimental Non-Linear R. T.:	$M^2 = 1.157(-0.6102 + n)^{0.718}$ with $R^2 = 1$.			
Theoretical Non-Linear R. T.:	$M^2 = 1.175(-0.0911 + n)^{0.902}$ with $R^2 = 1$.			

Table: Summary of results for the vector kaon K^* radial states. The last column is the relative error per state. Experimental results are read from PDG

Vector Heavy-light mesons

These mesons are labeled as $I(J^P) = 1/2(1^-)$, with $\Delta = 3$. The average constituent mass is

$$\bar{m}_{qQ} = \frac{m_q + m_Q}{2},$$

with $m_u = m_d = 336$ MeV, $m_s = 0.486$ MeV, $m_c = 1550$ MeV, and $m_b = 4730$ MeV. The numerical results are summarized as:

State	$q_1\, q_2$	$\bar{m}$ (MeV)	κ (MeV)	α	M_{Exp} (MeV)	M_{Th} (MeV)	R. E. (%)
$K^*(782)$	$d\,\bar{s}$	413	531.24	0.055	895.55 ± 0.8	1038.4	16.2
$D^{*0}(2007)$	$c\,\bar{u}$	943	1070.8	0.261	2006.85 ± 0.05	1902.5	5.20
$D^{+0}(2010)$	$c\,\bar{d}$	945	1073.6	0.262	2010.26 ± 0.05	1906.4	5.16
D_s^{*+}	$c\,\bar{s}$	1018	1179.1	0.296	2112.2 ± 0.4	2051.7	2.86
B^{*+}	$u\,\bar{b}$	2533	4681.2	0.800	5324.70 ± 0.22	4561.2	14.3
B^{*0}	$d\,\bar{b}$	2535	4687.3	0.801	5324.70 ± 0.22	4564.4	14.27
B_s^{*0}	$s\,\bar{b}$	2608	4901.2	0.809	$5415^{+1.8}_{-1.5}$	4683.0	13.52

Table: Summary of results for vector heavy-light mesonic states contrasting our theoretical results with the available experimental data. The last column is the relative error per state. Experimental results are read from PDG. Although D_s^{*+} has not been fully identified, their decay modes are consistent with $J^P = 1^-$. See PDG.

Non-$q\,\bar{q}$ states

Another interesting test

We also can study non-$q\,\bar{q}$ states and test which structure works better from holographic grounds. We will focus on the description of tetraquarks in the context of multiquark and gluonic excitation models. (See Brambilla et al. nice review, 2020).

Average constituent mass parametrizations

Gluonic excitations and multiquark states can be summarized in a single parametrization in terms of the number of constituents N per state:

$$\begin{aligned} \bar{m}_{\text{non-}q\bar{q}} &= \sum_{i=1}^{N}\left(P_i^{\text{quark}}\,\bar{m}_{q_i} + P_i^{\text{meson}}\,m_{\text{meson}_i} + P_i^{\text{gluon}}\,m_{\text{gluon}_i}\right) \\ 1 &= \sum_{i=1}^{N}\left(P_i^{\text{quark}} + P_i^{\text{meson}} + P_i^{\text{gluon}}\right), \end{aligned}$$

To switch between models consider the following:

- Multiquark states: take $P_i^{\text{gluon}} = 0$.
- Gluonic excitations: take $P_i^{\text{gluon}} \neq 0$.

Non-$q\,\bar{q}$ states

Non-$q\,\bar{q}$ taxonomy

- Multi-quark states:
  - Diquarks.
  - Hadroquarkonium.
  - Hadronic molecule.
- Gluonic excitations:
  - Hybrid vector mesons.

Non-$q\,\bar{q}$ candidates (PDG)

We will analyze tetraquark candidates split into:

- Multiquark states:
  - Heavy sector: Z_C, Z_B, and ψ states.
- Gluonic excitations:
  - Light sector: π_1 states.
  - Heavy sector: Z_C and Z_B states.

Methodology

We will test each non-$q\,\bar{q}$ state with our model by using $\bar{m}$ as entry to the κ and α curves, and then computing the corresponding mass spectrum. After that, we will compare with the experimental data.

Tetraquarks as Multi-quark states

Holographic spectrum		Non-$q\,\bar{q}$ states			
$\Delta = 6$ and $\bar{m}_{\text{diquark-antidiquark}}$		Multiquark state			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} (MeV)	n	State	M_{Exp} (MeV)	ΔM (%)
1	4004.8	1	$Z_c(3900)$	3887.2 ± 2.3	3.0
2	4384.9	2	$Z_c(4200)$	4196^{+35}_{-32}	4.5
3	4706.6	3	$Z_c(4430)$	4478^{+15}_{-18}	5.1

$\Delta = 6$ and $\bar{m}_{\text{hadronic molecule}}$		Multiquark state			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} (MeV)	n	State	M_{Exp} (MeV)	ΔM (%)
1	3816.3	1	$Z_c(3900)$	3887.2 ± 2.3	1.82
2	4213.9	2	$Z_c(4200)$	4196^{+35}_{-32}	0.43
3	4551.4	3	$Z_c(4430)$	4478^{+15}_{-18}	1.64

$\Delta = 6$ and $\bar{m}_{\text{Hadrocharmonium}}$		Multiquark state			
$\alpha = 0.604$ and $\kappa = 2523$ MeV		$I^G(J^{CP}) = 0^+(1^{--})$ Y or ψ mesons			
n	M_{Th} (MeV)	n	State	M_{Exp} (MeV)	ΔM (%)
1	4228.3	1	$\psi(4260)$	4230 ± 8	0.25
2	4577.3	2	$\psi(4360)$	4368 ± 13	4.8
3	4871.8	3	$\psi(4660)$	4643 ± 9	4.9

$\Delta = 6$ and $\bar{m}_{\text{Hadronic Molecule}}$		Multiquark state			
$\alpha = 0.538$ and $\kappa = 1548.7$ MeV		$I^G(J^{CP}) = 0^+(1^{--})$ Y or ψ mesons			
n	M_{Th} (MeV)	n	State	M_{Exp} (MeV)	ΔM (%)
1	40027.8	1	$\psi(4260)$	4230 ± 8	5.37
2	4383.1	2	$\psi(4360)$	4368 ± 13	0.35
3	4705.1	2	$\psi(4360)$	4643 ± 9	1.34

Tetraquarks as Multi-quark states

Holographic spectrum		Non-$q\bar{q}$ states			
$\Delta = 6$ and $\bar{m}_{\text{hadronic molecule}}$		Multiquark state			
$\alpha = 0.863$ and $\kappa = 11649$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_B mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	10410.9	1	$Z_B(10610)$	10607.2 ± 2	1.85
2	10669.3	2	$Z_B(10650)$	10652.2 ± 1.5	0.16

Table: Summary of results for the set of non-$q\bar{q}$ states considered in this work. Experimental results are read from PDG.

Where we have used:

$$
\begin{aligned}
\bar{m}_{\text{diquark-Antidiquark}} &= \bar{m}_c \\
\bar{m}_{\text{Hadrocharmonium}} &= \frac{1}{2} m_{J/\psi} + \frac{1}{4}\left(\bar{m}_u + \bar{m}_d\right) \\
\bar{m}_{\text{hadronic molecule}} &= \frac{1}{3} m_{J/\psi} + \frac{2}{3} m_\rho \text{ for } \psi \\
\bar{m}_{\text{hadronic molecule}} &= 0.283\, m_{J/\psi} + 0.717\, m_\rho \text{ for } Z_C \\
\bar{m}_{\text{hadronic molecule}} &= 0.458\, m_{\Upsilon(1S)} + 0.542\, m_\rho \text{ for } Z_B
\end{aligned}
$$

$m_{J/\psi} = 3077.9\,\text{MeV}$, $m_{\Upsilon(1S)} = 9460.3$ MeV, and $m_\rho = 770\,\text{MeV}$.

Gluonic Excitations States: Hybrid mesons

Holographic spectrum		Non-$q\,\bar{q}$ states			
$\Delta = 5$ and $\bar{m}_{\text{Hybrid Meson}}$		Gluonic excitation state			
$\alpha = 0.0367$ and $\kappa = 488$ MeV		$I^G(J^{CP}) = 0^-(1^{+-})$ π_1 mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	1351.7	1	$\pi_1(1400)$	1354 ± 25	0.16
2	1646.6	2	$\pi_1(1600)$	1660^{+15}_{-11}	0.8
3	1901.7	3	$\pi_1(2015)$	$2014 \pm 20 \pm 16$	5.58
$\Delta = 5$ and $\bar{m}_{\text{Hybrid meson}}$		Gluonic Excitation			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	3721.9	1	$Z_c(3900)$	3887.2 ± 2.3	4.24
2	4156.4	2	$Z_c(4200)$	4196^{+35}_{-32}	0.94
3	4513.2	3	$Z_c(4430)$	4478^{+15}_{-18}	0.78
$\Delta = 7$ and $\bar{m}_{\text{Hybrid Meson}}$		Gluonic excitation state			
$\alpha = 0.863$ and $\kappa = 11649$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_B mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	10346.7	1	$Z_B(10610)$	10607.2 ± 2	2.52
2	10696.6	2	$Z_B(10650)$	10652.2 ± 1.5	0.42

Table: Summary of results for the set of non-$q\,\bar{q}$ states considered in this work. Experimental results are read from PDG.

Gluonic Excitations States: Hybrid mesons

Where we have used:

$$\bar{m}_{\text{hybrid meson}} = P_q\, m_q + P_{\bar{q}}\, m_{\bar{q}} + P_G\, m_G, \tag{3}$$

with the following probabilities:

Vector hybrid meson	P_q	$P_{\bar{q}}$	P_G
π_1	0.497	0.497	6×10^{-3}
Z_c	0.49	0.49	0.02
Z_b	0.495	0.495	0.01

Table: Summary of coefficients fixed for each hybrid meson candidate.

Important Remark:

In the case of Z_b we are considering two flux tubes instead one.

Our results and conclusions

By looking the tables, and based on the small RMS criterion, we can conclude that:

- Constituent gluons are not so relevant in order to define non-q *stātes*.
- Z_C, Z_B and ψ states are better described as hadronic molecules.
- The hybrid meson descriptions fits well the π_1 spectrum (RMS less than 5%).

General Conclusions

Conclusions

- The family of isovector mesons were fitted as non-linear Regge Trajectories.
- This approach allows us to extend the model by extrapolation to other hadronic vector species.
- Holographic non-linear trajectories provide a good tool to describe mesonic systems. The RMS error for fitting 27 mesonic states was near 13%, with 15 parameters organized as:
    - Two parameters, κ and α, for each isovector meson family, i.e., ω, ϕ, J/Ψ and Υ, implying eight in total.
    - One $\bar{m}$ for the vector kaon K^* system, giving three in total.
    - Six $\bar{m}$ for each heavy-light vector meson considered, i.e., D^{*0}, D^{+0}, D_s^{*0}, B^*, B^{*0} and B_s^{0*}.
- Therefore an RMS error around 13% is reasonable for this model, considering the simplicity of the proposal done and the complexity of the QCD physics at strong regime.

Thank you!