Two-state picture for the $D^*_0(2400)$

Miguel Albaladejo (IFIC)

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Outline

1. Introduction
2. Amplitudes
3. Comparisons:
   - With LQCD
   - With experiments
4. $SU(3)$ study. Other predictions
5. Summary and outlook
Quark model in the singly heavy sector

- Quark model $c\bar{n}$ is still our baseline: “In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons—from the pion to the upsilon—can be described in a unified framework.” [Godfrey, Isgur, PR,D32,189(‘85)]

![Diagram showing the mass spectrum of charm, strange and charm, non-strange states.](image)

- The discovery of $D_{s0}^*(2317)$ in 2003 (and $D_{s1}^*(2460)$ later on) is “equivalent” to the discovery of $X(3872)$ in charmonium-like system.

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  [CLEO, PR,D68,032002(‘03)]
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<table>
<thead>
<tr>
<th>Theoretical interpretations</th>
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<tbody>
<tr>
<td><strong>c\bar{c} q states</strong></td>
<td><strong>c\bar{c}+ tetraquarks or meson–meson</strong></td>
</tr>
<tr>
<td><strong>Pure tetraquarks</strong></td>
<td><strong>Heavy-light meson–meson molecules</strong></td>
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Some attempts to explain $D_{s0}^*(2317)$ as a $c\bar{s}$ state

- Problem: original Quark Model prediction mass is $\sim 150$ MeV above experimental one.
- 1-loop correction to OGE potential ($\mathcal{O}(\alpha_s^2)$) reduces the mass to 2383 MeV, much closer to the experimental one.

$Lakhina, Swanson, PL,B650,159('07)$

- $^3P_0$ mechanism to couple $c\bar{s}$ states to $DK$ meson-pairs, $P_{DK} \sim 30\%$.
- Much better situation, but:
  - Still above $DK$ threshold
  - This mechanism only affects the $0^+$ sector, still problems with $1^+$
  - Coupling to $DK$ is included, but no $DK$ “dynamics”
Meanwhile, in the lattice...

- Masses larger than the physical ones if using $c\bar{s}$ interpolators only.  
  \[ \text{Bali, Phys. Rev. D 68, 071501 (2003)} \]

- Masses consistent with $D_0^*(2400)$ and $D_{s0}^*(2317)$ obtained when “meson-meson” interpolators are employed.  
  \[ \text{Mohler, Prelovsek, Woloshyn, Phys. Rev. D 87, 034501 (2013)} \]
  \[ \text{Mohler et al., Phys. Rev. Lett. 111, 222001 (2013)} \]

- Close to the physical point:  
  \[ \text{RQCD Collab., Phys. Rev. D 96, 074501 (2017)} \]

- More complete studies from the HadSpec collaboration:
  - $D_\pi$, $D_\eta$ and $D_s\bar{K}$ coupled-channel scattering. A bound state with large coupling to $D_\pi$ is identified with $D_0^*(2400)$.  
    \[ \text{HadSpec Collab., JHEP 1610, 011 (2016)} \]
  - $D_{s0}^*(2317)$: A bound state is found in the $DK$ channel, with:
    - $\Delta E = 25(3)$ MeV ($m_\pi = 391$ MeV)
    - $\Delta E = 57(3)$ MeV ($m_\pi = 239$ MeV)
    - Compare with experimental, $\Delta E \approx 45$ MeV (the dependence on $m_\pi$ does not need to be monotonic!)
    \[ \text{HadSpec Collab., 2008.06432} \]
Lightest $0^+$ open-charm situation and puzzles

- $D_{s0}^*(2317)$: $(S, I) = (1, 0)$, $M_{D_{s0}^*(2317)} = 2317.8 \pm 0.5$ MeV (PDG)
- $D_0^*(2400)$: $(S, I) = (0, 1/2)$, not so well established:

<table>
<thead>
<tr>
<th>Collab.</th>
<th>$M$ (MeV)</th>
<th>$\Gamma/2$ (MeV)</th>
<th>Ref.</th>
</tr>
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<tbody>
<tr>
<td>LHCb</td>
<td>2360 ± 33</td>
<td>128 ± 29</td>
<td>Phys. Rev. D 92, 032012 (2015) ((B^0 \to \bar{D}^0 K^+ \pi^-))</td>
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</tr>
</tbody>
</table>

- PDG averages:
  - $D_0^{*0}(2400)$: $M = 2349 \pm 7$ MeV
  - $D_0^{*+}(2400)$: $M = 2300 \pm 19$ MeV

Three puzzles

- Mass problem: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ masses much lower than the CQM expectations?
- Splittings: Why $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D$ (within a few MeV)?
- Hierarchy: Why $M_{D_0^{*+}(2400)} > M_{D_{s0}^*(2317)}$, i.e., why $c\bar{u}$, $c\bar{d}$ heavier than $c\bar{s}$?
**$D\pi$, $D\eta$, $D_s\bar{K}$ scattering amplitudes**

- **Coupled channel $T$-matrix:** $D\pi$, $D\eta$, $D_s\bar{K}$ scattering [$J^P = 0^+, (S, I) = (0, \frac{1}{2})$].

- **Unitarity:** $T^{-1}(s) = V^{-1}(s) - G(s)$

- **Chiral symmetry** used to compute the $\mathcal{O}(p^2)$ potential:

  $$f^2 V_{ij}(s, t, u) = C_{LO}^{ij} \frac{s - u}{4} + \sum_{a=0}^{5} h_a C_a^{ij}(s, t, u)$$


- **Free parameters** previously fixed, not fitted (predictions!):

- **Fitted to reproduce scattering lengths obtained in a LQCD simulation**
Comparison with LQCD energy levels

- \( E_n(L) \) are provided for \( D_\pi, D_\eta, D_s\bar{K} \) in a recent LQCD simulation.
  [G. Moir et al., JHEP 1610, 011 (2016)]

- **Red Bands**: Our amplitude in a finite volume.

- Recall, no fit is performed.

- \( E > 2.7 \) GeV is beyond the range of validity for our \( T \)-matrix.

- Level **below threshold**, associated with a bound state.

- **Second level** has large shifts w. r. t. thresholds, non-interacting energy levels:
  - Strong movement of the amplitude.
  - Check if there is another state (resonance).

<table>
<thead>
<tr>
<th>( M ) (MeV)</th>
<th>Latt.</th>
<th>Phys.</th>
</tr>
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<tbody>
<tr>
<td>( \pi )</td>
<td>391</td>
<td>138</td>
</tr>
<tr>
<td>( K )</td>
<td>550</td>
<td>496</td>
</tr>
<tr>
<td>( \eta )</td>
<td>588</td>
<td>548</td>
</tr>
<tr>
<td>( D )</td>
<td>1886</td>
<td>1867</td>
</tr>
<tr>
<td>( D_s )</td>
<td>1952</td>
<td>1968</td>
</tr>
</tbody>
</table>
Spectroscopy: two-states for $D_0^*(2400)$

We also study $DK$, $D_s\eta$, $(S, I) = (1, 0)$

$D^*_0(2317): M = 2315_{-28}^{+18}$ MeV.

- For lattice masses, we find a bound state (000) and a resonance (110).
- For physical masses:
  - The bound state evolves into a resonance (100) above $D_\pi$ threshold.
  - The resonance varies very little, and is still a resonance (110).
  - For both states, the coupling pattern is similar.
- PDG includes only one resonance, “suspiciously” lying between both.

| Meson Masses | $M$ (MeV) | $\Gamma/2$ (MeV) | RS | $|g_{D\pi}|$ | $|g_{D\eta}|$ | $g_{D_s\bar{K}}$ |
|--------------|-----------|------------------|----|------------|------------|-------------|
| lattice      | 2264$^{+8}_{-14}$ | 0   | (000) | 7.7$^{+1.2}_{-1.1}$ | 0.3$^{+0.5}_{-0.3}$ | 4.2$^{+1.1}_{-1.0}$ |
|              | 2468$^{+32}_{-25}$ | 113$^{+18}_{-16}$ | (110) | 5.2$^{+0.6}_{-0.4}$ | 6.7$^{+0.6}_{-0.4}$ | 13.2$^{+0.6}_{-0.5}$ |
|              | 2105$^{+6}_{-8}$ | 102$^{+10}_{-12}$ | (100) | 9.4$^{+0.2}_{-0.2}$ | 1.8$^{+0.7}_{-0.7}$ | 4.4$^{+0.5}_{-0.5}$ |
|              | 2451$^{+36}_{-26}$ | 134$^{+7}_{-8}$ | (110) | 5.0$^{+0.7}_{-0.4}$ | 6.3$^{+0.8}_{-0.5}$ | 12.8$^{+0.8}_{-0.6}$ |

For lattice masses, we find a bound state (000) and a resonance (110).
Comparison with experimental data: \( B^- \rightarrow D^+ \pi^- \pi^- \)

- \( A(s, z) = A_0(s) + \sqrt{3}A_1(s)P_1(z) + \sqrt{5}A_2(s)P_2(z) + \ldots \)

- \( P,D \)-wave as in LHCb paper

- \( S \)-wave parameterization:

\[
A_0(s) = \frac{B^-}{D^+} \rightarrow \pi^- + \pi^- + A, B, D_s, \pi, \eta, \bar{K}, D^+, \pi^+ \pi^- \pi^-
\]

\[
A_0(s) = A \left\{ E_\pi \left[ 2 + G_1(s) \left( \frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T_{3/2}^3(s) \right) \right] \right. \\
+ \frac{1}{3} E_\eta G_2(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_3(s) T_{31}^{1/2}(s) \left\} + B E_\eta G_2(s) T_{21}^{1/2}(s),
\]

- Angular moments: \( \langle P_\ell \rangle(s) = \int dz |A(s, z)|^2 P_\ell(z) \)

\[
\langle P_0 \rangle \propto |A_0|^2 + |A_1|^2 + |A_2|^2 , \quad \langle P_2 \rangle \propto \frac{2}{5} |A_1|^2 + \frac{2}{7} |A_2|^2 + \frac{2}{\sqrt{5}} |A_0| |A_2| \cos(\delta_0 - \delta_2) , \\
\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9} \langle P_3 \rangle \propto \frac{2}{\sqrt{3}} |A_0| |A_1| \cos(\delta_0 - \delta_1) .
\]
Comparison with experimental data: $B^- \rightarrow D^+ \pi^- \pi^-$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

Parameters: $B/A = -3.6 \pm 0.1$, $a_A = 1.0 \pm 0.1$, $\chi^2$/d.o.f. = 1.7

- This work. - - - LHCb. Bands: fit uncertainty

Very good agreement with data & with LHCb fit

Rapid movement in $\langle P_{13} \rangle$ [no $D_2(2460)$] between 2.4 and 2.5 GeV. Related to $D\eta$ and $D_s\bar{K}$ openings.

Recall: these are the amplitudes with two states in the $D_0^*(2400)$ region, and no fit of the $T$-matrix parameters is done.
**SU(3) light–flavor limit**

- **SU(3) flavor limit:** $m_i \rightarrow m = 0.49$ GeV, $M_i \rightarrow M = 1.95$ GeV.

- Irrep decomposition: $\bar{3} \otimes 8 = \mathbf{15} \oplus \mathbf{6} \oplus \mathbf{3}$. $T$ and $V$ can be diagonalized:

  $$V_d(s) = D^\dagger V(s)D = \text{diag} (V_{15}(s), V_6(s), V_3(s)) = A(s) \text{diag} (1, -1, -3),$$

- $\mathbf{15}$ is repulsive. $\mathbf{6}$ and $\mathbf{3}$ are attractive. “Curiously”, $\mathbf{3}$ admits a $c\bar{q}$ interpretation.

<table>
<thead>
<tr>
<th>State</th>
<th>Channels</th>
<th>$(S, I)$</th>
<th>$\mathbf{15}$</th>
<th>$\mathbf{6}$</th>
<th>$\mathbf{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_0^*$</td>
<td>$D_\pi, D_\eta, D_s\bar{K}$</td>
<td>$(0, \frac{1}{2})$</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>$D_{s0}^*(2317)$</td>
<td>$D_K, D_s\eta$</td>
<td>$(1, 0)$</td>
<td>✔</td>
<td>X</td>
<td>✔</td>
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\]

- \( \bar{15} \) is repulsive. \( 6 \) and \( \bar{3} \) are attractive. “Curiously”, \( \bar{3} \) admits a \( c\bar{q} \) interpretation.

\[
\begin{array}{cccc}
S &=& 2 \\
S &=& 1 \\
S &=& 0 \\
S &=& -1 \\
\end{array}
\]

<table>
<thead>
<tr>
<th>State</th>
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<th>((S, I))</th>
<th>(15)</th>
<th>(6)</th>
<th>(\bar{3})</th>
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<td>(D_0^*)</td>
<td>(D\pi, D\eta, D_s\bar{K})</td>
<td>((0, \frac{1}{2}))</td>
<td>✔️</td>
<td>✔️</td>
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<td>(D_{s0}^*(2317))</td>
<td>(DK, D_s\eta)</td>
<td>((1, 0))</td>
<td>✔️</td>
<td>❌</td>
<td>✔️</td>
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[Hadron Spectrum Collab., 2008.06432]
### Predictions for other sectors: charm

<table>
<thead>
<tr>
<th>$(S, l)$</th>
<th>Channels</th>
<th>$0^+$</th>
<th>$1^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$\Gamma/2$</td>
<td>$M$</td>
</tr>
<tr>
<td>$(0, \frac{1}{2})$</td>
<td>$D(<em>)\pi, D(</em>)\eta, D_s^(*)\bar{K}$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1, 0)$</td>
<td>$D(<em>)K, D_s^(</em>)\eta$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-1, 0)$</td>
<td>$D(*)\bar{K}$</td>
<td>$\times$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$(1, 1)$</td>
<td>$D_s^(<em>)\pi, D(</em>)K$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

- **HQSS** relates $0^+$ ($D_s(P)$) and $1^+$ ($D_s^*(P)$) sectors: similar resonance pattern.
- Two pole structure: higher $D_1$ pole probably affected by $\rho$ channels.
- $D\bar{K} [0^+, (-1, 0)]$: this virtual state (from 6) has a large impact on the scattering length, $a_{D\bar{K}(-1,0)}^{D_s^*} \approx 0.8$ fm. (Rest of scattering lengths are $|a| \approx 0.1$ fm.)
Heavy flavour symmetry relates charm (D) and bottom (B) sectors.

(0, 1/2): \(B_0^*\), two-pole pattern also observed.

(−1, 0): \([B(\ast)\bar{K}]\): very close to threshold. Relevant prediction. Can be either bound or virtual (6) within our errors.

(1, 1): \([\bar{B}_s\pi, \bar{B}K, 0^+]\), \(X(5568)\) channel. No state is found: \(15\) and 6. If it exists, it is not generated with these \(B_s\pi, B\bar{K}\) interactions.


(1, 0): Our results for \(B_{s0}^*\) and \(B_{s1}\) agree with other results from LQCD:


Comparison of 0\(^+\), 1\(^+\) beauty states by Colangelo et al., Phys. Rev. D 86 054024 (2012): agreement in (1, 0) \([b\bar{s}]\), but not in (0, 1/2) \([b\bar{q}]\).
Conclusions about $D_0^*(2400), D_{s0}^*(2317)$

- The $D_0^*(2400)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds.

- This two-state structure for $D_0^*(2400)$ was previously reported:

- The amplitudes containing these two-poles are compatible with available LQCD simulations and experimental data.

- This picture for $D_0^*(2400)$ and $D^*(2317)$ nicely solves simultaneously all the puzzles.
Open questions for the community

- Need of more collaboration between (and simultaneous use of!) different “subcommunities”: LQCD, molecular/tetraquarks/QM models...

- **Spectroscopy, mixing:** Specific example of $D_{s0}^*(2317)$, take for granted the presence of a CQM $c\bar{s}$ state. 

  **Theoretical possibilities:**
  - Genuine $c\bar{s}$, (very) renormalized by $DK$ threshold. Or renormalized by $DK$ interactions themselves?
  - Or, there is a $S = 1, I = 0$ state coming from $DK$ interactions in addition to the $c\bar{s}$ state. If so, where are those two poles? Which is which?

- **Nature/size:**
  - Can we address the question of $4q, q\bar{q}$, molecule based on the size of the object?

- For $\pi\pi$ scattering, $\sigma$ meson: MA, Oller, PR,D86,034003(’12)
  - $\sqrt{\langle r^2 \rangle_\sigma} \simeq 0.44$ fm
  - $\sqrt{\langle r^2 \rangle_\pi} \simeq 0.81$ fm
  - Perhaps only theoretical? Future lattice QCD calculations?

  Briceño et al., PR,D100,034511(’19); PR,D100,114505(’19), …
Two-state picture for the $D^*_0(2400)$
Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$G_{ii}(s) \rightarrow G_{ii}(s) + i \frac{p_i(s)}{4\pi \sqrt{s}} \xi_i$$

$SU(3)$ limit:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}),$$

$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}).$$

- Physical case ($x = 0$): RS specified by $(\xi_1 \xi_2 \xi_3)$, $\xi_i = 0$ or 1.
- $SU(3)$ symmetric case ($x = 1$): all channels have the same threshold, so there are only two RS (000) and (111).

To connect the lower pole with the $T_6$ virtual state,

$$\xi_3 = x \quad (1, 1, 0) \rightarrow (1, 1, x)$$

To connect the lower pole with the $T_3$ bound state,

$$\xi_1 = 1 - x \quad (1, 0, 0) \rightarrow (1 - x, 0, 0)$$
Connecting physical \((x = 0)\) and flavor \(SU(3)\) \((x = 1)\) limits:

\[
m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}), \quad (m = 0.49 \text{ GeV}) ,
\]
\[
M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}), \quad (M = 1.95 \text{ GeV}) .
\]

- The high \(D_0^*\) connects with a \(\mathbf{6}\) virtual state (unph. RS, below threshold).
- The low \(D_0^*\) connects with a \(\mathbf{\bar{3}}\) bound state (ph. RS, below threshold).
- The \(D_{s0}^*(2317)\) also connects with the \(\mathbf{\bar{3}}\) bound state.

The low \(D_0^*\) and the \(D_{s0}^*(2317)\) are \(SU(3)\) flavor partners.

This solves the “puzzle” of \(D_{s0}^*(2317)\) being lighter than \(D_0^*(2400)\): it is not, the lower \(D_0^*\) pole \((M = 2105 \text{ MeV})\) is lighter.
Form factors in semileptonic $D \rightarrow \pi \bar{\ell} \nu_{\ell}$


- General definitions:
  \[
  \frac{d\Gamma(D \rightarrow \pi \bar{\ell} \nu_{\ell})}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_\pi|^3 |V_{cd}|^2 |f_+(q^2)| . \quad [q^2 = 0 : f_+(0) = f_0(0)]
  \]

  \[
  \langle \pi(p')|\bar{q}\gamma^\mu Q|D(p)\rangle = f_+(q^2) \left[ \sum^\mu - \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu ,
  \]

- “Isospin” form factors, related to $D\pi$, $D\eta$, $Ds\bar{K}$ scattering:
  \[
  \mathcal{F}(0,1/2)(s) \equiv \begin{pmatrix}
    -\sqrt{\frac{3}{2}} f_0^{D_0^0 \rightarrow \pi^-} (s) \\
    -f_0^{D^+ \rightarrow \eta} (s) \\
    -f_0^{D_s^+ \rightarrow \kappa^0} (s)
  \end{pmatrix}, \quad \text{Im}\mathcal{F}(s) = T^*(s)\Sigma(s)\mathcal{F}(s)
  \]

- Write form factors as Omnés matrix times polynomials
  \[
  \mathcal{F}(s) = \Omega(s) \cdot \mathcal{P}(s)
  \]

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:
  \[
  \mathcal{L}_0 = f_\mathcal{P} \left( m\mathcal{P}_\mu^* - \partial_\mu \mathcal{P} \right) u^\dagger J^\mu ,
  \]
  \[
  \mathcal{L}_0 = \beta_1 \mathcal{P} u (\partial_\mu U^\dagger) J^\mu + \beta_2 (\partial_\mu \partial_\nu \mathcal{P}) u (\partial^\nu U^\dagger) J^\mu .
  \]
- Points mostly from LQCD
- Also LCSR for $q^2 \to 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

<table>
<thead>
<tr>
<th>This work</th>
<th>Exp.</th>
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<tbody>
<tr>
<td>$10^3</td>
<td>V_{ub}</td>
</tr>
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<td></td>
<td></td>
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<tr>
<td>$</td>
<td>V_{cd}</td>
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<tr>
<td>$</td>
<td>V_{cs}</td>
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</tbody>
</table>
Why is $D^*_0(2400)$ interesting?

- Lightest systems to test ChPT with heavy mesons, besides $D^* \to D\pi$.
- $D\pi$ interactions (where it shows up) are relevant, since $D\pi$ appears as a final state in many reactions that are being considered now (i.e., $Z_c(3900)$ and $\bar{D}^*D\pi$).
- $D^*_0(2400)$ is important in weak interactions and CKM parameters:
  
  
  - It determines the shape of the scalar form factor $f_0(q^2)$ in semileptonic $D \to \pi$ decays.
  - Relation to $|V_{cd}|$: $f_+(0) = f_0(0)$ and $d\Gamma \propto |V_{cd}f_+(q^2)|^2$.
  - Even more interesting: the bottom analogue $|V_{ub}|$. 
Periodic boundary conditions imposes momentum quantization

Lüscher formalism:

<table>
<thead>
<tr>
<th>infinite volume</th>
<th>finite volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{q} \in \mathbb{R}^3$</td>
<td>$\vec{q} = \frac{2\pi}{L}\vec{n}$, $\vec{n} \in \mathbb{Z}^3$</td>
</tr>
<tr>
<td>$\int_{\mathbb{R}^3} \frac{d^3 q}{(2\pi)^3}$</td>
<td>$\frac{1}{L^3} \sum_{\vec{n} \in \mathbb{Z}^3}$</td>
</tr>
</tbody>
</table>

In practice, changes in the $T$-matrix: $T(s) \rightarrow \tilde{T}(s, L)$:

$G_{ii}(s) \rightarrow \tilde{G}_{ii}(s, L) = G_{ii}(s) + \lim_{\Lambda \rightarrow \infty} \left( \frac{1}{L^3} \sum_{\vec{n}} l_i(\vec{q}) - \int_0^{\Lambda} \frac{q^2 dq}{2\pi^2} l_i(\vec{q}) \right)$,

$V(s) \rightarrow \tilde{V}(s, L) = V(s)$,

$T^{-1}(s) \rightarrow \tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{G}(s, L)$,

Free energy levels: $E_{n,\text{free}}^{(i)}(L) = \omega_{i1} \left( (2\pi n/L)^2 \right) + \omega_{i2} \left( (2\pi n/L)^2 \right)$

Interacting energy levels $E_n(L)$: $\tilde{T}^{-1}(F^2(L), L) = 0$ (poles of the $\tilde{T}$-matrix)
Normalization: 

\[ -ip_{ii}(s)T_{ii}(s) = 4\pi \sqrt{s} \left( \eta_i(s)e^{2i\delta_i(s)} - 1 \right) \]

\[ G_{ii}(s) = G(s, m_i, M_i), \text{ regularized with a subtraction constant } a(\mu) (\mu = 1 \text{ GeV}) \]

Riemann sheets (RS) denoted as \((\xi_1\xi_2\xi_3)\):

\[ G_{ii}(s) \rightarrow G_{ii}(s) + i \frac{p_{i}(s)}{4\pi \sqrt{s}} \xi_i \]
Chiral dynamics and two-state structure(s)

- Other famous two-poles structures rooted in chiral dynamics:
  
  \[ \Lambda(1405) \ [\Sigma\pi, N\bar{K}] \quad \text{and} \quad K_1(1270) \]


- Chiral dynamics:
  - Incorporates the \( SU(3) \) light-flavor structure,
  - Determines the strength of the interaction,
  - Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.
Conclusions of $D_0^*(2400)$ work

- We have studied $D\pi$, $D\eta$, $D_s\bar{K}$ scattering $[0^+, (S, I) = (0, \frac{1}{2})]$

- So far only one pole reported experimentally, but we have presented a strong support for the existence of two $D_0^*(2400)$ states (different poles):
  - Successful, no-fitting comparison of our $T$-matrix with the energy levels of a recent LQCD simulation.
  - We are also able to reproduce the LHCb experimental information for $B^- \rightarrow D^+\pi^-\pi^-$, also without fitting any of the $T$-matrix parameters.
  - The lower pole ($M = 2105^{+6}_{-8}$ MeV) is lighter than $D_{s0}^*(2317)$, solving this (apparent) puzzle.
  - A $SU(3)$ study shows that $D_{s0}^*(2317)$ and the lower $D_0^*(2400)$ are flavour partners: they complete a $\bar{3}$ multiplet.

- Predictions for other sectors (heavy vectors, bottom sector) have been also given. In particular:
  - The two-pole structure is also seen in the bottom sector.
  - A very near-threshold state (bound or virtual) is predicted for $BK$ ($\bar{B}\bar{K}$).