Isospin-1/2 $D\pi$ scattering and the lightest D_0^* resonance from lattice QCD

Based on [arXiv:2102.04973]

Hadron 2021

Speaker: Nicolas Lang[a]
Authors: L. Gayer, N. Lang, S. M. Ryan, D. Tims, C. E. Thomas, D. J. Wilson
for the Hadron Spectrum Collaboration

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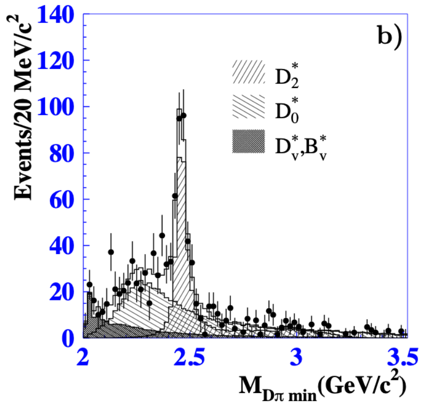
[a]Trinity College Dublin

Introduction: D_0^* - the experimental puzzle

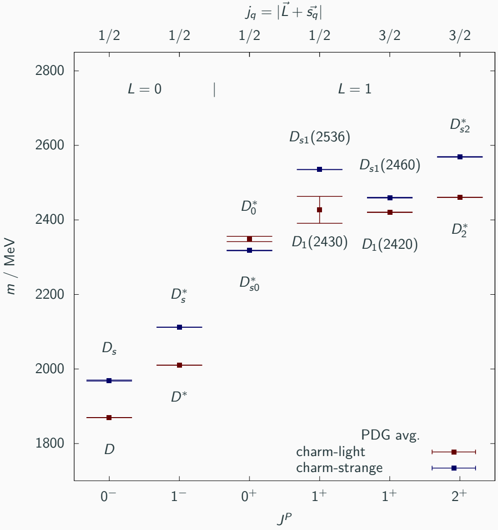
- D_0^* lightest scalar charm-light resonance
- First observed by Belle and FOCUS in 2004: broad enhancement at 2300 - 2400 MeV
- Quark model construction: $q\bar{q}$ in relative P-wave
- Measured mass in agreement with predictions by quark model but has a large width



BELLE Collaboration [arXiv:hep-ex/0307021]

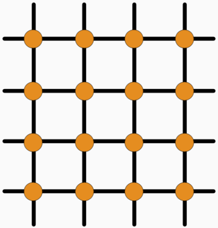
Comparison: D_{s0}^*

- Charm-strange state D_{s0}^* the same from view of quark model
- Mass difference w.r.t. D_0^* due to different light-quark masses
  $\rightarrow$ predicted above D_0^*
- However: observed as narrow peak below DK threshold - well below predictions by quark model; perhaps below D_0^*
  $\rightarrow$ What is going on?
- Proximity in mass of these two states in experiment and differing widths require better theoretical understanding!



Enter: Lattice QCD

- Lattice QCD $\rightarrow$ first principles approach to understand QCD dynamics
- On Lattice: D_0^* as part of $D\pi \rightarrow D\pi$ scattering
- Other HadSpec lattice studies of these systems:
  - D_0^* in $D\pi \rightarrow D\pi$ at $m_\pi = 391$ MeV[1]
  - D_{s0}^* in $DK \rightarrow DK$ at both $m_\pi = 391$ MeV and $m_\pi = 239$ MeV[2]
- Existing lattice study of D_0^* in $D\pi$-scattering found resonance close to PDG average[3]
- Objective here: determine pole position; understand mass ordering and light-quark mass dependence of these two states



[1] G. Moir et al. [arXiv:1607.07093]
[2] G. K. C. Cheung et al. [arXiv:2008.06432]
[3] D. Mohler et al. [arXiv:1208.4059]

Lattice → Finite volume spectrum

- Basis of interpolating operators (quark bilinears and meson-meson) with $C = 1$, $I = 1/2$ projected to irreducible representations (*irreps*) of the lattice
- Contractions make use of distillation framework[1] with 256 vectors
- Principal correlators computed using GEV method: $C_{ij}(t) v_j^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t, t_0) C_{ij}(t_0) v_j^{(\mathfrak{n})}$
- Correlator fits (sum of exponentials) → Finite volume spectrum

a_s	0.11 fm
a_t^{-1}	6.079 GeV
$(L/a_s)^3 \times (T/a_t)$	$32^3 \times 256$
m_π	239 MeV
N_f	$2+1$
N_{cfg}	484

[1] Hadron Spectrum collaboration [arXiv:0905.2160]

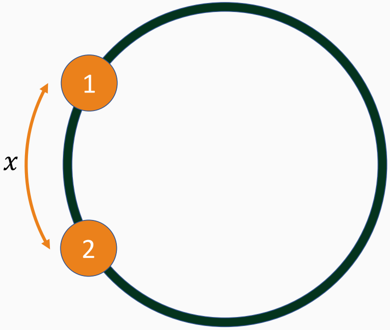
Finite volume spectrum → Infinite volume scattering amplitudes (1D QM)

Analogy: 1-D quantum mechanics:

- 2 bosons in periodic 1-D volume interacting with finite range potential $V(x)$
- Solve Schrödinger equation
  $\rightarrow \Psi = \cos(px + \delta(p))$
  $\rightarrow$ match at $x = \pm L/2$

$$\Rightarrow p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

- $\Rightarrow$ spectrum (determined by p) is discrete and depends on the scattering amplitude (parametrised by $\delta(p)$)



Finite volume spectrum → Infinite volume scattering amplitudes (QFT)

Need a mapping between finite-volume spectrum and infinite volume scattering amplitudes[1]
→ Lüscher quantisation condition

$$\det\left[1 + i\boldsymbol{\rho}(s)\cdot \boldsymbol{t}(s)\cdot (1 + i\boldsymbol{\mathcal{M}}(s,L))\right] = 0$$

- $\rho(s) = 2k(s)/\sqrt{s}$ with $k(s)$ the COM-momentum function and $s = E_{\mathrm{CM}}^2$
- $\boldsymbol{t}(s)$ = infinite volume t-matrix
- $\boldsymbol{\mathcal{M}}(s,L))$ encodes finite-volume effects (dense in partial waves)

Procedure:

- solve determinant equation for a given parametrisation of $\boldsymbol{t}(s)$ to obtain a spectrum
- vary the parameters in $\boldsymbol{t}(s)$ in a χ^2-minimisation to best match the spectrum obtained from the lattice

[1] Arbitrary two-particle QC from QFT: R. A. Briceño [arXiv:1401.3312];
Extension to three particles: M. T. Hansen et al. [arXiv:1901.00483]

t-matrix Parametrisations

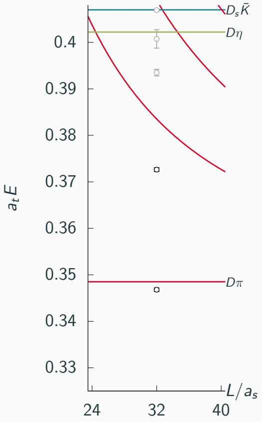
- Parametric form of $\boldsymbol{t}$-matrix undetermined by Lüscher condition
- Unitarity, causality and analyticity provide constraints
- Using a single parametrisation could introduce bias
- We use a range of different parametrisations:
  - K-matrix: $(t^{(\ell)})^{-1}(s) = \frac{1}{(2k)^\ell} K^{-1}(s) \frac{1}{(2k)^\ell} + I(s)$
  - Effective range
  - Breit Wigner
  - Unitarized chiral amplitude[2]

[2] Z.-H. Guo et al. [arXiv:1811.05585]

Spectra at rest

- Irreps are labelled $[\vec{d}]\Lambda^{(P)}$ - with parity P and lattice momentum $\vec{P} = 2\pi\vec{d}/L$
- At rest: neat separation of lowest partial waves
    - A_1^+: S-wave
    - T_1^-: P-wave
    - E^+: D-wave

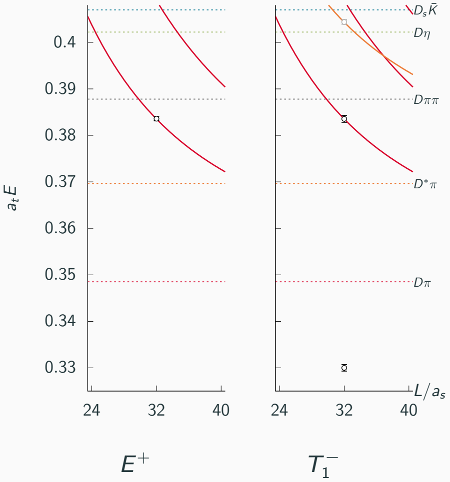
  (higher partial waves subduce but are negligible)
- A_1^+: additional level around $a_t E_{\text{cm}} = 0.37$; levels above and below shifted up and down respectively → suggestive of non-trivial interactions



A_1^+

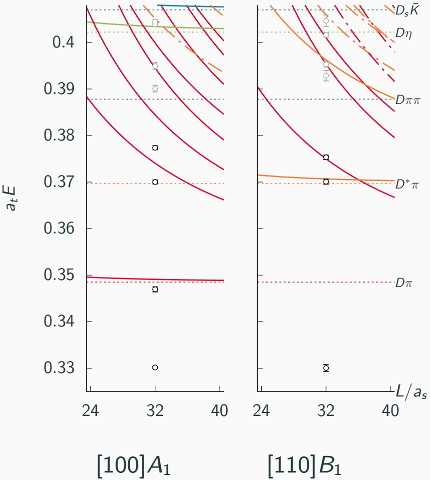
Spectra at rest

- T_1^-: level far below threshold; little interaction above threshold
- E^+: level sits right on non-interacting energy $\rightarrow$ negligible D-wave interaction (we showed that the $D\pi$ D-wave phase shift is consistent with zero)
- Higher partial waves will be ignored (threshold suppression $\propto k^{2l}$)



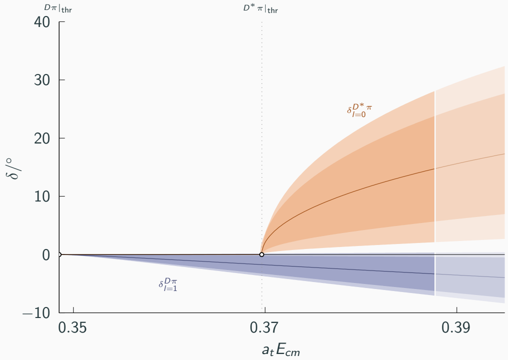
Spectra at non-zero momentum

- Moving-frame $\to$ rotational symmetry further broken $\to$ further mixing of partial waves
- A_1 irreps have contributions from $D\pi$ S- and P-wave
- $[110]B_1/B_2$ and $[100]E_2$ irreps have a contribution from $D^*\pi$ S-wave and $D\pi$ P-wave but no $D\pi$ S-wave



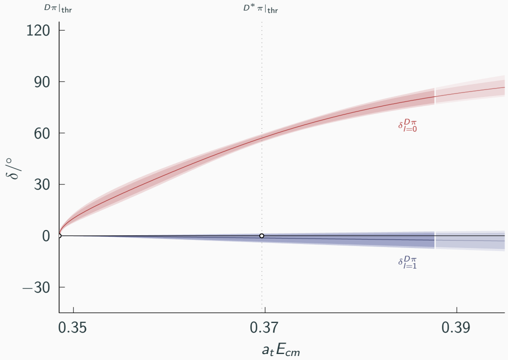
$D\pi$ P-wave and $D^*\pi$ S-wave

- Determined from spectrum fits in $[000]T_1^-$, $[100]E_2$, $[110]B_1$ and $[110]B_2$
- Deeply bound level in all irreps $\rightarrow$ $J^P = 1^-$ D^* bound state
- $D^*\pi$ S-wave $\rightarrow$ contribution in moving frames
- Parametrisation: K-matrix with 2 channels with a pole term in $D\pi$ P-wave
- Phase shift indicates very weak effect of P-wave above threshold



$D\pi$ S- and P-wave

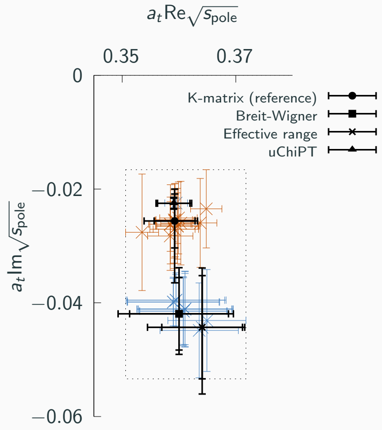
- Fit of energy levels below $D\pi\pi$ threshold in A_1^+, T_1^- & moving-frame A_1 irreps
- Excluding irreps that have $D^*\pi$ contribution
- Deeply bound level in all irreps with P-wave contribution; "extra" level in irreps with S-wave contribution
- Parametrisation: K-matrix for 2 partial waves, both containing a pole term



Poles

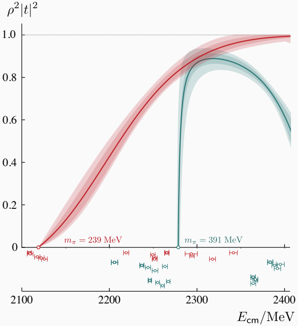
- Cluster of poles from 30 different parametrisations; all above threshold
  → Resonance
- Amplitudes similar at real energies but differ in complex plane; pole common feature
- Scatter of poles: single parametrisation might underestimate uncertainties
- Mass and coupling considering all parametrisations:

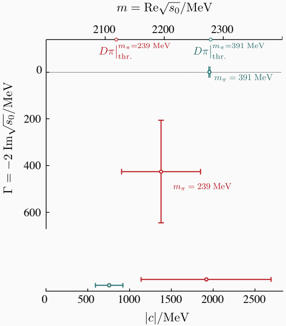
$$\sqrt{s_0}/\mathrm{MeV} = (2196 \pm 64) - \tfrac{i}{2}(425 \pm 224)$$
$$c/\mathrm{MeV} = (1916 \pm 776)\exp i\pi(-0.59 \pm 0.41)$$



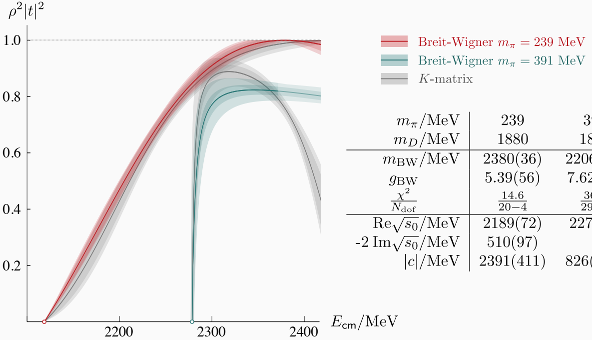
$D\pi$ at different light-quark masses

- Earlier study of $D\pi \to D\pi$ at $m_\pi = 391$ MeV: shallow bound-state ($\approx 2 \pm 1$ MeV below threshold)
- At 239 MeV: pole migrates into complex plane ($\approx 77 \pm 64$ MeV above threshold)
- Mass below reported experimental value (despite heavier-than-physical light quarks)
- Strong coupling of poles to $D\pi$ channel in both cases





Study: parametrisations of the $D\pi$ S-wave amplitude

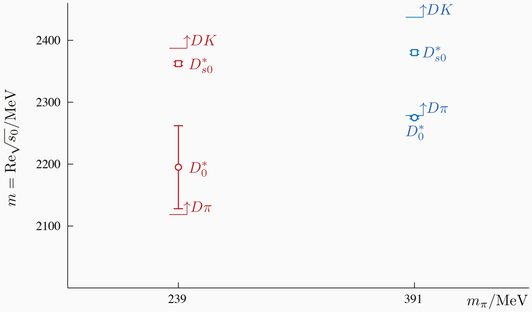


m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\text{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\text{Re}\sqrt{s_0}$/MeV	2189(72)	2275(1)
-2 $\text{Im}\sqrt{s_0}$/MeV	510(97)	-
$\lvert c\rvert$/MeV	2391(411)	826(133)

- Comparison: K-matrix and Breit-Wigner
- Real parts of the poles are compatible between both parametrisations
- Breit-Wigner mass parameter incompatible with pole location

D_0^* and D_{s0}^*: mass hierarchy

- Locations of poles follow expectations from SU(3) symmetry
- D_0^* shallow bound state at $m_\pi = 391$ MeV
  - becomes a resonance at $m_\pi = 239$ MeV
- Pole mass decreases with pion mass $\rightarrow$ extrapolation to physical pion mass would suggest D_0^* well below D_{s0}^*
- D_{s0}^* bound at both masses
- Recent study[1] performing UChPT amplitude fit to LHCb data also suggests a lower mass of the D_0^*

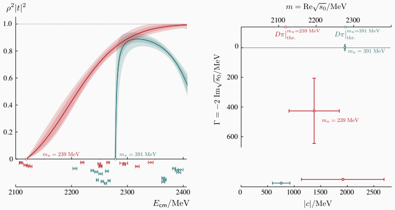


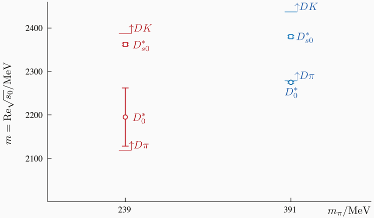
[1]M.-L. Du et al. [arXiv:2012.04599]

Summary

- Found D_0^* resonance pole with
  - $m = (2194 \pm 64)$ MeV
    (77 ± 64) MeV above $D\pi$ threshold
  - $\Gamma = (425 \pm 224)$ MeV

  from first principles
  (no external inputs after fixing quark masses)
- Range of parametrisations
  (major contribution to uncertainty)
- Strongly coupled to $D\pi$ channel;
  compatible with $D_{s0}^* \to DK$
- Slight decrease in pole mass with decreasing pion mass
- Significantly lower than current PDG value
  $\to$ puzzling D_0^* heavier than D_{s0}^* not reproduced by Lattice





Questions?

Correlators on the lattice

- Compute matrix of (euclidean) correlators:

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle \,,$$

- $\mathcal{O}_i(t)$ have quantum numbers of $I = 1/2\ D\pi$
- Find "optimal" interpolators by solving *Generalised Eigenvalue* (GEV) problem

$$C_{ij}(t)v_j^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{(\mathfrak{n})},$$

- Fit Principal correlators (eigenvalues):

$$\lambda_{\mathfrak{n}}(t,t_0) = (1-A_{\mathfrak{n}})e^{-E_{\mathfrak{n}}(t-t_0)} + A_{\mathfrak{n}}e^{-E'_{\mathfrak{n}}(t-t_0)}.$$

Operator Table (S-wave)

$A_1^+[000]$	$A_1[100]$	$A_1[110]$	$A_1[111]$	$A_1[200]$
$D_{[000]}\ \pi_{[000]}$	$D_{[000]}\ \pi_{[100]}$	$D_{[000]}\ \pi_{[110]}$	$D_{[000]}\ \pi_{[111]}$	$D_{[100]}\ \pi_{[100]}$
$D_{[100]}\ \pi_{[100]}$	$D_{[100]}\ \pi_{[000]}$	$D_{[100]}\ \pi_{[100]}$	$D_{[100]}\ \pi_{[110]}$	$D_{[110]}\ \pi_{[110]}$
$D_{[110]}\ \pi_{[110]}$	$D_{[100]}\ \pi_{[110]}$	$D_{[110]}\ \pi_{[000]}$	$D_{[110]}\ \pi_{[100]}$	$D_{[200]}\ \pi_{[000]}$
$D_{[111]}\ \pi_{[111]}$	$D_{[100]}\ \pi_{[200]}$	$D_{[110]}\ \pi_{[110]}$	$D_{[111]}\ \pi_{[000]}$	$D_{[210]}\ \pi_{[100]}$
$D_{[000]}\ \eta_{[000]}$	$D_{[110]}\ \pi_{[100]}$	$D_{[111]}\ \pi_{[100]}$	$D_{[211]}\ \pi_{[100]}$	$D_{[200]}\ \eta_{[000]}$
$D_{[100]}\ \eta_{[100]}$	$D_{[110]}\ \pi_{[111]}$	$D_{[210]}\ \pi_{[100]}$	$D^*{}_{[110]}\ \pi_{[100]}$	
$D_{s\,[000]}\ \bar{K}_{[000]}$	$D_{[111]}\ \pi_{[110]}$	$D^*{}_{[100]}\ \pi_{[100]}$	$D_{[111]}\ \eta_{[000]}$	
	$D_{[200]}\ \pi_{[100]}$	$D^*{}_{[111]}\ \pi_{[100]}$	$D_{s\,[111]}\ \bar{K}_{[000]}$	
	$D_{[210]}\ \pi_{[110]}$	$D_{[110]}\ \eta_{[000]}$		
	$D_{[000]}\ \eta_{[100]}$	$D_{s\,[110]}\ \bar{K}_{[000]}$		
	$D_{[100]}\ \eta_{[000]}$			
	$D_{s\,[000]}\ \bar{K}_{[100]}$			
	$D_{s\,[100]}\ \bar{K}_{[000]}$			
$8 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$18 \times \bar{\psi}\mathbf{\Gamma}\psi$	$9 \times \bar{\psi}\mathbf{\Gamma}\psi$	$16 \times \bar{\psi}\mathbf{\Gamma}\psi$

Operators used in the S-wave fits. Subscripts indicate momentum types. $\mathbf{\Gamma}$ represents some monomial of γ matrices and derivatives.

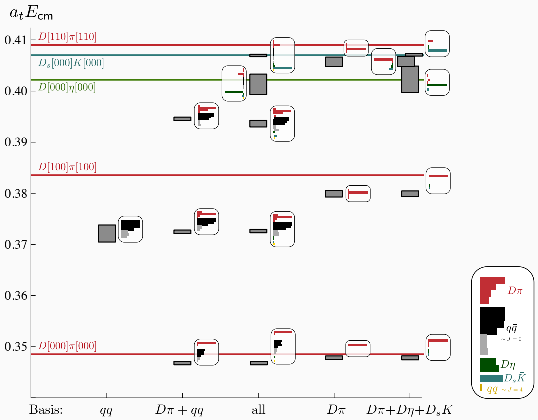
Operator Table (P-wave)

$T_1^-[000]$	$E_2[100]$	$B_1[110]$	$B_2[110]$
$D_{[100]}\ \pi_{[100]}$	$D_{[100]}\ \pi_{[110]}$	$D_{[100]}\ \pi_{[100]}$	$D_{[100]}\ \pi_{[111]}$
$D_{[110]}\ \pi_{[110]}$	$D_{[110]}\ \pi_{[100]}$	$D_{[110]}\ \pi_{[110]}$	$D_{[110]}\ \pi_{[110]}$
$D^*{}_{[100]}\ \pi_{[100]}$	$D^*{}_{[000]}\ \pi_{[100]}$	$D_{[210]}\ \pi_{[100]}$	$D_{[111]}\ \pi_{[100]}$
	$D^*{}_{[100]}\ \pi_{[000]}$	$D^*{}_{[100]}\ \pi_{[100]}$	$D^*{}_{[000]}\ \pi_{[110]}$
		$D^*{}_{[110]}\ \pi_{[000]}$	$D^*{}_{[100]}\ \pi_{[100]}\ \{2\}$
			$D^*{}_{[110]}\ \pi_{[000]}$
			$D^*{}_{[111]}\ \pi_{[100]}$
$6 \times \bar{\psi}\boldsymbol{\Gamma}\psi$	$18 \times \bar{\psi}\boldsymbol{\Gamma}\psi$	$18 \times \bar{\psi}\boldsymbol{\Gamma}\psi$	$20 \times \bar{\psi}\boldsymbol{\Gamma}\psi$

Operators used in the P-wave fits. Subscripts indicate momentum types. $\boldsymbol{\Gamma}$ represents some monomial of γ matrices and derivatives. The number in curly parentheses indicates the number of operators of this momentum combination.

Operator basis variations

- Varying the basis affects the spectrum
- $I = 1/2$ allows both meson-meson and $q\bar{q}$-like operator constructions
- Interpolating the complete spectrum requires both types of operator
- Other meson-meson operators do not play a significant role below coupled-channel threshold



Spectrum in A_1^+

How are operators constructed?

- Two types of interpolating operator:
  - quark bilinears: $\bar{\psi}\Gamma D...\psi$
  - meson-meson like operators: $\sum_{\vec{p_1}+\vec{p_2}=\vec{P}} \mathcal{C}(\vec{p_1},\vec{p_2})\Omega^{\dagger}_{M_1}(\vec{p_1})\Omega^{\dagger}_{M_2}(\vec{p_2})$
- Rotational symmetry broken $\Rightarrow$ eigenstates labelled by irreducible representations of O_h or $LG(\vec{P})$ (*irreps*)
- Continuum spins *subduce* into one or more finite volume irreps; operators are projected into irreps
- Correlators are computed using distillation with 256 vectors

Subduction Table

$\vec{P}$	Irrep Λ	J^P ($\vec{P} = \vec{0}$) $\lvert\lambda\rvert^{(\tilde{\eta})}$ ($\vec{P} \neq \vec{0}$)	$D\pi$ $J^P_{[N]}$	$D^*\pi$ $J^P_{[N]}$
[000]	A_1^+	0^+, 4^+	0^+, ...	...
	T_1^-	1^-, 3^-	1^-, ...	...
	E^+	2^+, 4^+	2^+, ...	...
$[n00]$	A_1	$0^{(+)}$, 4	0^+, 1^-, 2^+, ...	...
	E_2	1, 3	1^-, 2^+, ...	1^+, ...
$[nn0]$	A_1	$0^{(+)}$, 2, 4	0^+, 1^-, $2^+_{[2]}$, ...	...
	B_2, B_2	1, 3	1^-, 2^+, ...	1^+, ...
$[nnn]$	A_1	$0^{(+)}$, 3	0^+, 1^-, 2^+, ...	...

Lowest $D\pi$ and $D^*\pi$ continuum J^P and helicity λ subductions by irrep

Masses and thresholds

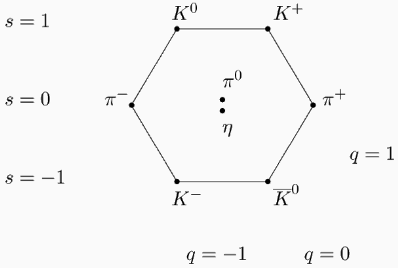
	$a_t m$
π	0.03928(18)
K	0.08344(7)
η	0.09299(56)
D	0.30923(11)
D_s	0.32356(12)
D^*	0.33058(24)

	$a_t E_{\mathrm{threshold}}$
$D\pi$	0.34851(21)
$D\pi\pi$	0.38779(27)
$D\eta$	0.40222(57)
$D_s\bar{K}$	0.40700(14)
$D^*\pi\pi$	0.40914(35)

Left: A summary of the stable hadron masses relevant for this calculation. Right: kinematic thresholds relevant for $I = 1/2$ $D\pi$ scattering.

SU(3) flavour symmetry

- When $m_u = m_d = m_s$ π and K are rows of the same SU(3) octet
  $\rightarrow$ $D\pi$ and DK scattering related by SU(3) flavour symmetry
$$\bar{\mathbf{3}} \otimes \mathbf{8} \rightarrow \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$
- Symmetry is less broken at heavier light-quark masses
- We expect the number of poles to stay the same as function of quark mass



https://en.wikipedia.org/wiki/Eightfold_way_(physics) - Creative Commons

Combined $D\pi$ $S+P$-wave and $D^*\pi$ S-wave

- Sanity check: Fit of all relevant partial waves below three-body threshold
- Fit of energy levels below $D\pi\pi$ threshold in all irreps we computed
- Parametrisation: K-matrix with 2 channels / 3 partial waves
- Pole term in $D\pi$ S- and P-wave
- Constant in $D^*\pi$ S-wave
- Results compatible with fit excluding $D^*\pi$