Υ and η_b mass shifts in nuclear matter and the nucleus bound states

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Motivations

- By studying the interactions of bottomonium states, such as Υ and η_b with nuclei, we can advance in understanding the hadron properties and strongly interacting systems
- Exploring the sources of the biding of bottomonia to atomic nuclei can provide important information on the nature of mesic nuclei
- Estimates made using SU(5) effective Lagrangian densities for Υ and η_b can provide information on the SU(5) symmetry breaking

Introduction

- The in-medium potential for the Υ (η_b) meson comes from the modifications of the B, B^* and B^*B^* meson loop contributions to the Υ (η_b) self-energy relative to those in free space
- Estimates for the self-energies are made using an effective Lagrangian approach at the hadronic level
- Bound state energies are found for various nuclei by solving the Klein-Gordon equation
- B and B^* meson masses and the density distributions in nuclei are calculated within the quark-meson coupling model

The quark meson coupling model

- The QMC model is a quark-based, relativistic mean field model of nuclear matter and nuclei
- The relativistically moving confined light quarks in the nucleon bags (MIT bag) self-consistently interact directly with the scalar-isoscalar σ, vector-isoscalar ω, and vector-isovector ρ mean fields (Hartree approximation) generated by the light quarks in the other nucleons
- The nuclear biding comes from the self-consistent couplings of the confined light quarks

QMC Model

$x = (t, \vec{r})$ ($|\vec{r}| \le$ **bag radius**), $V^q_\sigma \equiv g^q_\sigma \sigma$, $V^q_\omega \equiv g^q_\omega \omega$, $V^q_\rho \equiv g^q_\rho b$

$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega + \frac{1}{2} V^q_\rho\right)\right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0$$

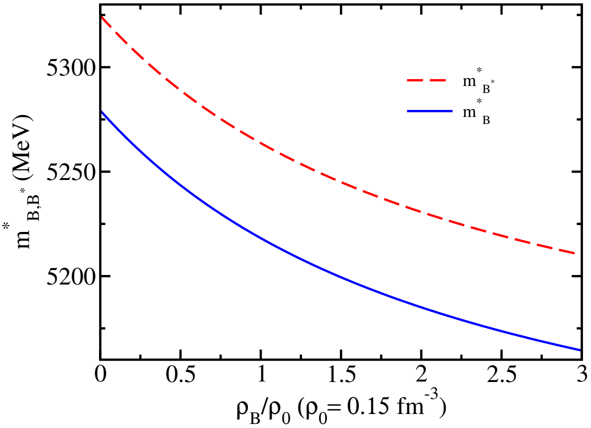
$$\left[i\gamma \cdot \partial_x - (m_q - V^q_\sigma) \mp \gamma^0 \left(V^q_\omega - \frac{1}{2} V^q_\rho\right)\right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0$$

$$[i\gamma \cdot \partial_x - m_b]\, \psi_b(x) \ (\text{or } \psi_{\overline{b}}(x)) = 0$$

$$m^*_h = \sum_{j=q,\overline{q},b,\overline{b}} \frac{n_j \Omega^*_j - z_h}{R^*_h} + \frac{4}{3}\pi R^{*3}_h B_p, \qquad \left.\frac{dm^*_h}{dR_h}\right|_{R_h = R^*_h} = 0$$

$$\Omega^*_q = \Omega^*_{\overline{q}} = [x^2_q + (R^*_h m^*_q)^2]^{1/2}, \text{with } \ m^*_q = m_q - g^q_\sigma \sigma$$

$$\Omega^*_b = \Omega^*_{\overline{b}} = [x^2_b + (R^*_h m_b)^2]^{1/2} \quad (h = B, B^*)$$

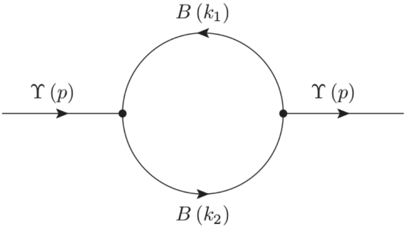


- Quark masses: $m_q = 5$ MeV, $m_b = 4200$ MeV

One loop contribution study for Υ

- SU(5) effective Lagrangian density
- SU(5) universal coupling constant determined by the vector meson dominance model

BB loop cont. to $\Sigma_{\Upsilon}(k^2) \longrightarrow$



- Phenomenological form factors to regularize the self-energy loop integrals
- Cutoff mass values chosen so that the form factors reflect the finite size effect of the mesons

Effective lagrangian

$$\mathcal{L}_0 = Tr\left(\partial_\mu P^\dagger \partial^\mu P\right) - \frac{1}{2} Tr\left(F^\dagger_{\mu\nu} F^{\mu\nu}\right)$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\partial_\mu P \to \partial_\mu P - \tfrac{ig}{2}\left[V_\mu, \mathrm{P}\right],\ F_{\mu\nu} \to \partial_\mu V_\nu - \partial_\nu V_\mu - \tfrac{ig}{2}\left[V_\mu,\ V_\nu\right]$$

$$\begin{aligned}\mathcal{L} \quad &= \mathcal{L}_0 + igTr\left(\partial_\mu P\left[P,\ V_\mu\right]\right) - \tfrac{g^2}{4} Tr\left(\left[\mathrm{P},\ \mathrm{V}_\mu\right]^2\right) \\ &+ igTr\left(\partial^\mu V^\nu\left[V_\mu,\ V_\nu\right]\right) + \tfrac{g^2}{8} Tr\left(\left[V_\mu,\ V_\nu\right]^2\right)\end{aligned}$$

Z. w. Lin and C. M. Ko, Phys. Lett. B 503, 104 (2001)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & \pi^+ & K^+ & \overline{D}^0 & B^+ \\ \pi^- & P_2 & K^0 & D^- & B^0 \\ K^- & \overline{K}^0 & P_3 & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & P_4 & B_c^+ \\ B^- & \overline{B}^0 & \overline{B}_s^0 & B_c^- & P_5 \end{pmatrix}$$

$$P_1 = \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_2 = \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_3 = \frac{-2\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_4 = \frac{-3\eta_c}{\sqrt{12}} + \frac{\eta_b}{\sqrt{20}}$$
$$P_5 = \frac{-2\eta_b}{\sqrt{5}}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} V_1 & \rho^+ & K^{*+} & \overline{D}^{*0} & B^{*+} \\ \rho^- & V_2 & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \overline{K}^{*0} & V_3 & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & V_4 & B_c^{*+} \\ B^{*-} & \overline{B}^{*0} & \overline{B}_s^{*0} & B_c^{*-} & V_5 \end{pmatrix}$$

$$V_1 = \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}}$$
$$V_2 = \frac{-\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}}$$
$$V_3 = \frac{-2\omega}{\sqrt{6}} + \frac{J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}}$$
$$V_4 = \frac{-3J/\Psi}{\sqrt{12}} + \frac{\Upsilon}{\sqrt{20}}$$
$$V_5 = \frac{-2\Upsilon}{\sqrt{5}}$$

$$
\begin{aligned}
\mathcal{L}_{\Upsilon BB} = \quad & i g_{\Upsilon BB} \Upsilon^{\mu} \left(\overline{B} \partial_{\mu} B - \left(\partial_{\mu} \overline{B} \right) B \right), \\
\mathcal{L}_{\Upsilon BB^*} = \quad & \frac{g_{\Upsilon BB^*}}{m_{\Upsilon}} \varepsilon_{\alpha\beta\mu\nu} \left(\partial^{\alpha} \Upsilon^{\beta} \right) \left[\left(\partial^{\mu} \overline{B^*}^{\nu} \right) B + \overline{B} \left(\partial^{\mu} B^{*\nu} \right) \right], \\
\mathcal{L}_{\Upsilon B^* B^*} = \quad & i g_{\Upsilon B^* B^*} \left[\Upsilon^{\mu} \left((\partial_{\mu} \overline{B^*}^{\nu}) B^*_{\nu} - \overline{B^*}^{\nu} \partial_{\mu} B^*_{\nu} \right) \right. \\
& + \left((\partial_{\mu} \Upsilon^{\nu}) \overline{B^*_{\nu}} - \Upsilon^{\nu} \partial_{\mu} \overline{B^*_{\nu}} \right) B^{*\mu} \\
& \left. + \overline{B^*}^{\mu} \left(\Upsilon^{\nu} \partial_{\mu} B^*_{\nu} - (\partial_{\mu} \Upsilon^{\nu}) B^*_{\nu} \right) \right],
\end{aligned}
$$

$$
B = \begin{pmatrix} B^{+} \\ B^{0} \end{pmatrix}, \quad \overline{B} = \begin{pmatrix} B^{-} & \overline{B^{0}} \end{pmatrix}, \quad B^{*} = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}, \quad \overline{B^{*}} = \begin{pmatrix} B^{*-} & \overline{B^{*0}} \end{pmatrix}.
$$

$$
g_{\Upsilon BB} = g_{\Upsilon BB^*} = g_{\Upsilon B^* B^*} = \frac{5g}{4\sqrt{10}} = 13.228 \rightarrow \text{by the VMD Model}
$$

Υ mass shift

$$V = \Delta m_{\Upsilon} = m_{\Upsilon}^{*} - m_{\Upsilon} \qquad m_{\Upsilon}^{2} = \left(m_{\Upsilon}^{0}\right)^{2} + \Sigma\left(k^{2} = m_{\Upsilon}^{2}\right)$$

$$\boxed{\sum_{l}\left(m_{\Upsilon}^{2}\right) = \sum_{l}\left(-\frac{g_{\Upsilon l}^{2}}{3\pi^{2}}\right)\int_{0}^{\infty} dq\,\mathbf{q}^{2} F_{l}\left(\mathbf{q}^{2}\right) K_{l}\left(\mathbf{q}^{2}\right)}$$

$l = BB, BB^{*}, B^{*}B^{*}$

$$F_{BB}\left(\mathbf{q}^{2}\right) = u_{B}^{2}\left(\mathbf{q}^{2}\right),\ F_{BB^{*}}\left(\mathbf{q}^{2}\right) = u_{B}\left(\mathbf{q}^{2}\right) u_{B^{*}}\left(\mathbf{q}^{2}\right),$$
$$F_{B^{*}B^{*}}\left(\mathbf{q}^{2}\right) = u_{B^{*}}^{2}\left(\mathbf{q}^{2}\right)$$

$$u_{B,B^{*}}\left(\mathbf{q}^{2}\right) = \left(\frac{\Lambda_{B,B^{*}}^{2} + m_{\Upsilon}^{2}}{\Lambda_{B,B^{*}}^{2} + 4\omega_{B,B^{*}}^{2}\left(\mathbf{q}^{2}\right)}\right)^{2}$$

$$\boxed{\boxed{\Lambda_{B} = \Lambda_{B^{*}}}\ 2000\ \text{MeV} \leq \Lambda_{B} \leq 6000\ \text{MeV}}$$

Loop contributions

$$K_{BB}\left(\mathbf{q}^2\right) = \frac{1}{\omega_B}\left(\frac{\mathbf{q}^2}{\omega_B^2 - m_\Upsilon^2/4}\right)$$

$$K_{BB^*}\left(\mathbf{q}^2\right) = \frac{\mathbf{q}^2\overline{\omega}_B}{\omega_B\omega_{B^*}}\frac{1}{\overline{\omega}_B^2 - m_\Upsilon^2/4}$$

$$K_{B^*B^*}\left(\mathbf{q}^2\right) = \frac{1}{4m_\Upsilon\omega_{B^*}}\left[\frac{A\left(q^0 = \omega_{B^*}\right)}{\omega_{B^*} - m_\Upsilon/2} - \frac{A\left(q^0 = \omega_{B^*} + m_\Upsilon\right)}{\omega_{B^*} + m_\Upsilon/2}\right]$$

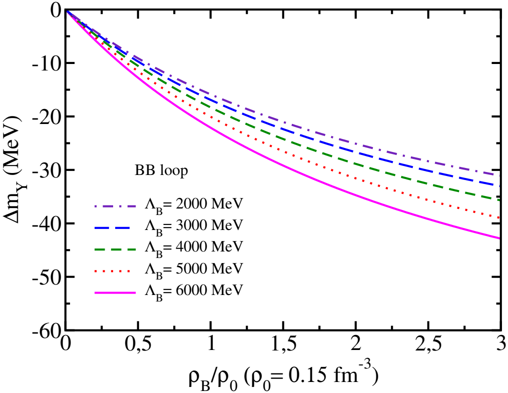
$$\omega_B = \left(\mathbf{q}^2 + m_B^2\right)^{1/2},\ \omega_{B^*} = \left(\mathbf{q}^2 + m_{B^*}^2\right)^{1/2},\ \overline{\omega}_B = \left(\omega_B + \omega_{B^*}\right)$$

$$A(q) = \sum_{i=1}^{4} A_i(q)$$

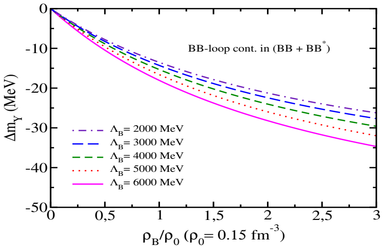
$$\begin{aligned}
A_1(q) &= -4q^2\left[4 - \frac{q^2 + (q-k)^2}{m_{B*}^2} + \frac{[q\cdot(q-k)]^2}{m_{B*}^4}\right] \\
A_2(q) &= 8\left[q^2 - \frac{[q\cdot(q-k)]^2}{m_{B*}^2}\right]\left[2 + \frac{(q^0)^2}{m_{B*}^2}\right] \\
A_3(q) &= 8\left(2q^0 - m_\Upsilon\right)\left[q^0 - \left(2q^0 - m_\Upsilon\right)\frac{q^2 + q\cdot(q-k)}{m_{B*}^2} + q^0\frac{[q\cdot(q-k)]^2}{m_{B*}^4}\right] \\
A_4(q) &= -8\left[q^0 - \left(q^0 - m_\Upsilon\right)\frac{q\cdot(q-k)}{m_{B*}^2}\right]\left[\left(q^0 - m_\Upsilon\right) - q^0\frac{q\cdot(q-k)}{m_{B*}^2}\right]
\end{aligned}$$

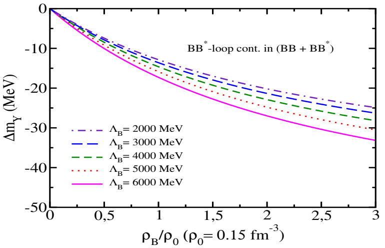
$$q = \left(q^0, \mathbf{q}\right) \qquad k = \left(m_\Upsilon, 0\right)$$

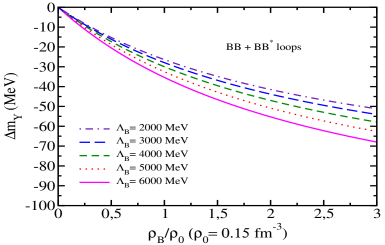
Results for Υ mass shift



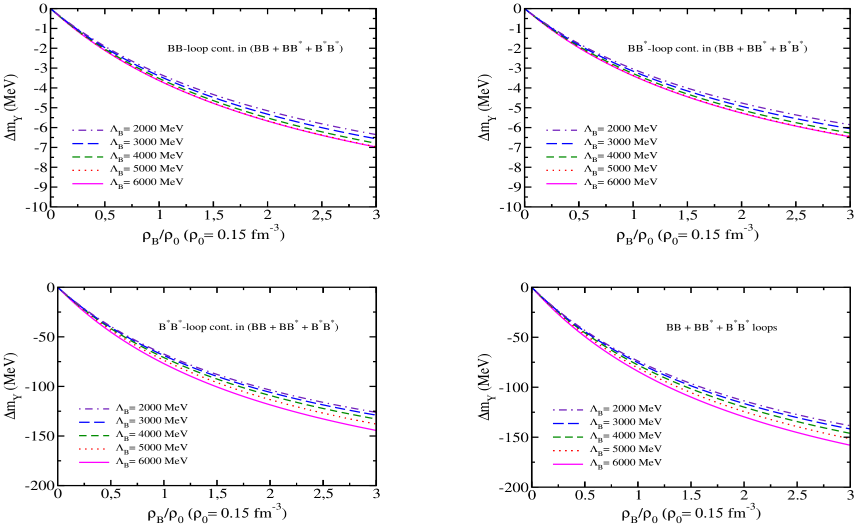
- $\Delta m_{\Upsilon} = -16$ to -22 MeV at ρ_0


BB-loop cont. in (BB + BB*)
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV
Δm_ϒ (MeV)
ρ_B/ρ_0 (ρ_0= 0.15 fm^-3)


BB*-loop cont. in (BB + BB*)
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV
Δm_ϒ (MeV)
ρ_B/ρ_0 (ρ_0= 0.15 fm^-3)


BB + BB* loops
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV
Δm_ϒ (MeV)
ρ_B/ρ_0 (ρ_0= 0.15 fm^-3)

Υ mass shift and bound state energies



- **The negative mass shift increases with** Λ_B

- BB **loop only = -16.22 to -22 MeV** $\rightarrow$ **our prediction!**

- $(BB + BB^*)$ **= -26 to -35 MeV**

- $(BB + BB^* + B^*B^*)$ **= -74 to -84 MeV**

- **The larger contribution from the heavier meson pair indicates the need to consider either different form factors, or adopt an alternative regularization method**

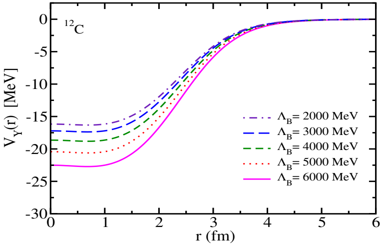
- **Our prediction, therefore, regards only the minimal meson loop contribution**

Υ nuclear potentials

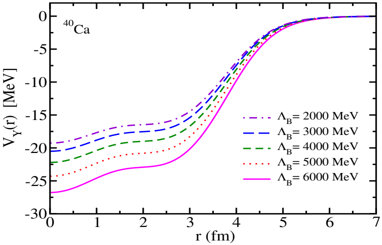
$V_{\Upsilon A}(r) = \Delta m_{\Upsilon}\left(\rho_B^A(r)\right) \rightarrow$ Local density approximation!!!

- **A = ^{4}He, ^{12}C, ^{16}O, ^{40}Ca, ^{48}Ca, ^{90}Zr, ^{208}Pb**

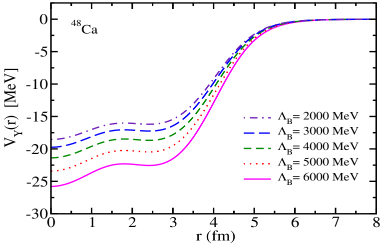
- **Nuclear density distributions calculated within the QMC model**



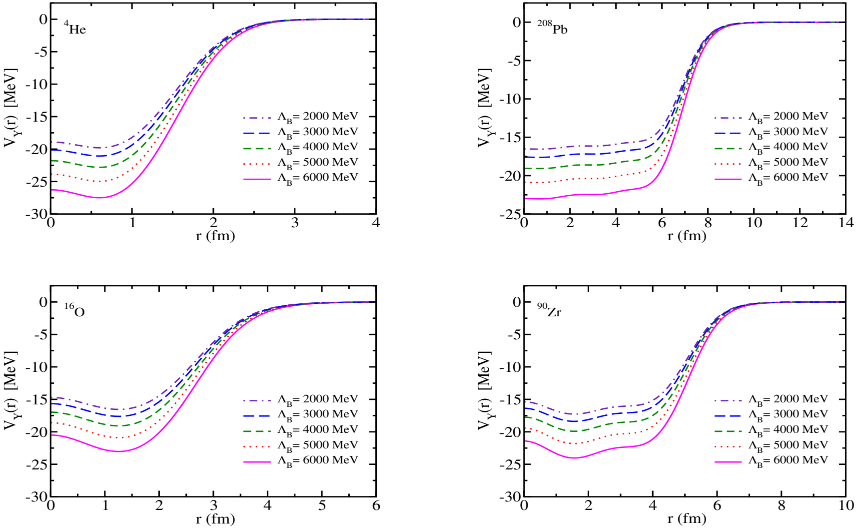
^{12}C
$V_\Upsilon(r)$ [MeV]
r (fm)
0
-5
-10
-15
-20
-25
-30
0 1 2 3 4 5 6
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV



^{40}Ca
$V_\Upsilon(r)$ [MeV]
r (fm)
0
-5
-10
-15
-20
-25
-30
0 1 2 3 4 5 6 7
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV



^{48}Ca
$V_\Upsilon(r)$ [MeV]
r (fm)
0
-5
-10
-15
-20
-25
-30
0 1 2 3 4 5 6 7 8
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV


^{4}He
^{208}Pb
^{16}O
^{90}Zr
$V_\Upsilon(r)$ [MeV]
r (fm)
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV

Bound state energies

Bound states are found (in MeV) for various nuclei by solving the KG equation

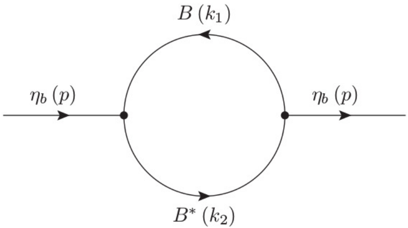
	$n\ell$	Bound state energies $\Lambda_B = 2000$	$\Lambda_B = 3000$	$\Lambda_B = 4000$	$\Lambda_B = 5000$	$\Lambda_B = 6000$
$^{4}_{\Upsilon}$He	1s	-5.6	-6.4	-7.5	-9.0	-10.8

	$n\ell$	Bound state energies $\Lambda_B = 2000$	$\Lambda_B = 3000$	$\Lambda_B = 4000$	$\Lambda_B = 5000$	$\Lambda_B = 6000$
$^{12}_{\Upsilon}$C	1s	-10.6	-11.6	-12.8	-14.4	-16.3
	1p	-6.1	-6.8	-7.9	-9.3	-10.9
	1d	-1.5	-2.1	-2.9	-4.0	-5.4
	2s	-1.6	-2.1	-2.8	-3.8	-5.1
	2p	n	n	n	-0.1	-0.7

Study of η_b

- η_b mass shift studied on the same footing as that for Υ
- SU(5) universal coupling constant

BB^* loop cont. to $\Sigma_{\eta_b}(k^2) \longrightarrow$



- Only the BB^* meson loop contribution is considered (minimal prediction)
- Same range for the cutoff mass values

$$\mathcal{L}_{\eta_b BB^*} = i g_{\eta_b BB^*} \left\{ (\partial^\mu \eta_b) \left(\overline{B^*}_\mu B - \overline{B} B^*_\mu \right) - \eta_b \left[\overline{B^*}_\mu (\partial^\mu B) - (\partial^\mu \overline{B}) B^*_\mu \right] \right\}$$

$$\boxed{g_{\eta_b BB^*} = g_{\Upsilon BB} = g_{\Upsilon B^* B^*} = \frac{5g}{4\sqrt{10}} \to \text{SU(5) scheme}}$$

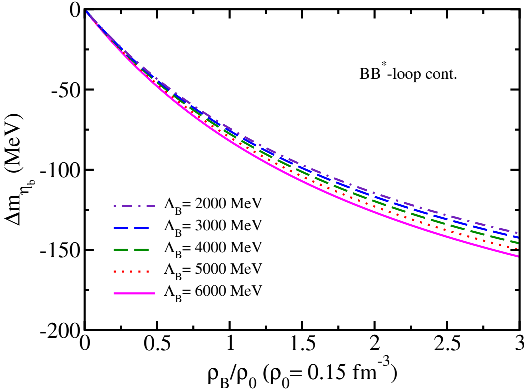
$$\Sigma^{BB^*}_{\eta_b}(\mathbf{q}^2) = \frac{8 g^2_{\eta_b BB^*}}{\pi^2} \int_0^\infty \mathrm{d}q\, \mathbf{q}^2 \tilde{F}_{BB^*}(\mathbf{q}^2) I_{BB^*}(\mathbf{q}^2)$$

$$I_{BB^*}(\mathbf{q}^2) = \left. \frac{m^2_{\eta_b}(-1 + (q^0)^2/m^2_{B^*})}{(q^0 + \omega_{B^*})(q^0 - \omega_{B^*})(q^0 - m_{\eta_b} - \omega_B)} \right|_{q^0 = m_{\eta_b} - \omega_B} + \left. \frac{m^2_{\eta_b}(-1 + (q^0)^2/m^2_{B^*})}{(q^0 - \omega_{B^*})(q^0 - m_{\eta_b} + \omega_B)(q^0 - m_{\eta_b} - \omega_B)} \right|_{q^0 = -\omega_{B^*}}$$

$$\omega_{B,B^*} = (\mathbf{q}^2 + m^2_{B,B^*})^{1/2}, \; \tilde{F}_{BB^*}(\mathbf{q}^2) = \tilde{u}_B(\mathbf{q}^2) \tilde{u}_{B^*}(\mathbf{q}^2)$$

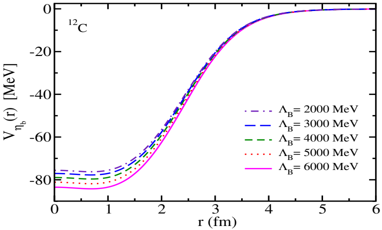
$$\tilde{u}_{B,B^*}(\mathbf{q}^2) = \left(\frac{\Lambda^2_{B,B^*} + m^2_{\eta_b}}{\Lambda^2_{B,B^*} + 4\omega^2_{B,B^*}(q^2)} \right)^2$$

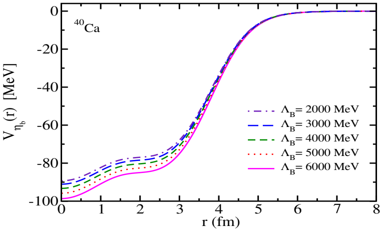
Result for η_b mass shift

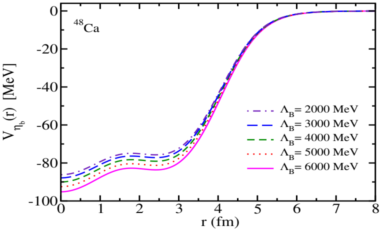


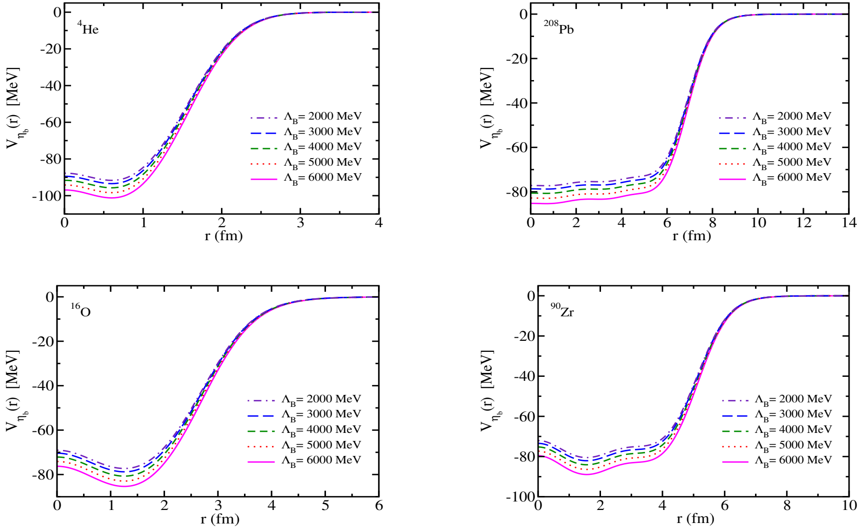
- $\Delta m_{\eta_b} = -75 \text{ to } -82$ MeV at ρ_0

η_b nuclear potentials









⁴He
²⁰⁸Pb
¹⁶O
⁹⁰Zr
$V_{\eta_b}(r)$ [MeV]
r (fm)
Λ_B= 2000 MeV
Λ_B= 3000 MeV
Λ_B= 4000 MeV
Λ_B= 5000 MeV
Λ_B= 6000 MeV

Bound state energies

η_b also form bound states with all nuclei considered

		Bound state energies				
	$n\ell$	$\Lambda_B = 2000$	$\Lambda_B = 3000$	$\Lambda_B = 4000$	$\Lambda_B = 5000$	$\Lambda_B = 6000$
$^{4}_{\eta_b}$He	1s	-63.1	-64.7	-66.7	-69.0	-71.5
	1p	-40.6	-42.0	-43.7	-45.8	-48.0
	1d	-17.2	-18.3	-19.7	-21.4	-23.2
	2s	-15.6	-16.6	-17.9	-19.4	-21.1
	2p	n	n	-0.3	-0.9	-1.7

		Bound state energies				
	$n\ell$	$\Lambda_B = 2000$	$\Lambda_B = 3000$	$\Lambda_B = 4000$	$\Lambda_B = 5000$	$\Lambda_B = 6000$
$^{12}_{\eta_b}C$	1s	-65.8	-67.2	-69.0	-71.1	-73.4
	1p	-57.0	-58.4	-60.1	-62.1	-64.3
	1d	-47.5	-48.8	-50.4	-52.3	-54.4
	2s	-46.3	-47.5	-49.1	-51.0	-53.0
	1f	-37.5	-38.7	-40.2	-42.0	-43.9
	2p	-36.0	-37.1	-38.6	-40.3	-42.2
	1g	-27.1	-28.2	-29.6	-31.3	-33.1
	2d	-25.7	-26.7	-28.1	-29.7	-31.4
	3s	-25.1	-26.2	-27.5	-29.1	-30.8
	1h	-16.6	-17.7	-18.9	-20.4	-22.0
	2f	-15.7	-16.6	-17.8	-19.2	-20.8
	3p	-15.4	-16.3	-17.4	-18.8	-20.3
	1i	-6.1	-6.9	-8.1	-9.4	-10.9
	4s	-6.9	-7.6	-8.5	-9.5	-10.7
	3d	-6.7	-7.4	-8.3	-9.4	-10.6
	2g	-6.3	-7.1	-8.1	-9.3	-10.6
	4p	-1.0	-1.3	-1.8	-2.5	-3.2
	3f	n	-0.3	-0.9	-1.6	-2.4
	2h	n	n	n	-0.2	-1.2

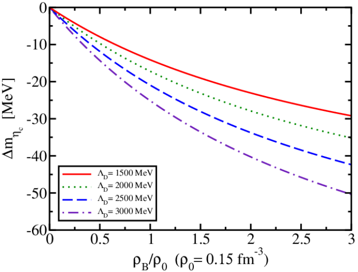
Summary and Conclusion

- We have calculated Υ and η_b mass shifts in nuclear matter by an effective Lagrangian approach
- B and B^* meson masses and the density distributions in nuclei are calculated within the quark-meson coupling model
- The attractions resulted from the negative shifts are strong enough to bind these mesons to various nuclei
- Bound state energies were calculated for all nuclei studied

BONUS: Charm Sector

Results for the Δm_{η_c} and the nucleus bound states

		Bound state energies			
	$n\ell$	$\Lambda_D = 1500$	$\Lambda_D = 2000$	$\Lambda_D = 2500$	$\Lambda_B = 3000$
$^{4}_{\eta_c}$He	1s	-1.49	-3.11	-5.49	-8.55
$^{12}_{\eta_c}$C	1s	-5.91	-8.27	-11.28	-14.79
	1p	-0.28	-1.63	-3.69	-6.33
$^{16}_{\eta_c}$O	1s	-7.35	-9.92	-13.15	-16.87
	1p	-1.94	-3.87	-6.48	-9.63
$^{197}_{\eta_c}$Au	1s	-12.57	-15.59	-19.26	-23.41
	1p	-11.17	-14.14	-17.77	-21.87
	1d	-9.42	-12.31	-15.87	-19.90
	2s	-8.69	-11.53	-15.04	-19.02
	1f	-7.39	-10.19	-13.70	-17.61
$^{208}_{\eta_c}$Pb	1s	-12.99	-16.09	-19.82	-24.12
	1p	-11.60	-14.64	-18.37	-22.59
	1d	-9.86	-12.83	-16.49	-20.63
	2s	-9.16	-12.09	-15.70	-19.80
	1f	-7.85	-10.74	-14.30	-18.37



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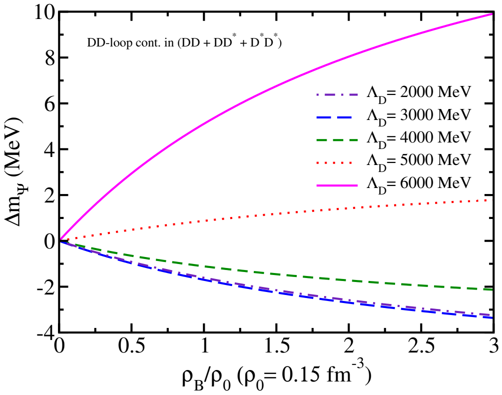
Λ_B range

- It gives more choice and wider information for the readers

- If we omit some and show only the narrower range of the cutoff mass values, readers cannot reproduce the missing results easily

- As long as the form factors reflects the finite sizes of the mesons, one cannot say that the range varies too large

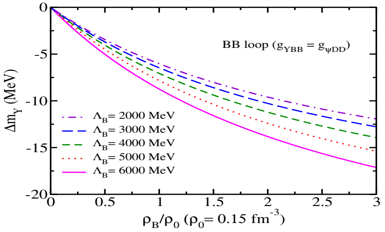


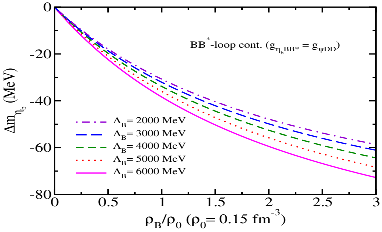
- $\Lambda_D = 5000$ and 6000 MeV may not be justified in this case

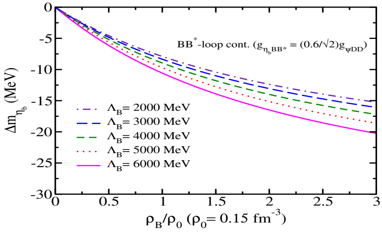
- The closer the cutoff mass values get to the J/Ψ mass, less pronounced the negative mass shift becomes, until it reaches a transition point (when Λ_D is larger than the J/Ψ free space mass), where the potential starts to become positive. At this point, the Compton wavelengths associated with the values of Λ_D become the sizes of the mesons, and the use of the form factors does not make reasonable sense.

- Such behavior is observed for the BB and BB^* meson loop contributions, for the cutoff mass values larger than 10000 MeV.

$g_{\Upsilon BB} = g_{\eta_b BB^*} = g_{J/\psi DD}$ and $g_{\eta_b BB^*} = (0.6/\sqrt{2}) g_{J/\psi DD}$







- Using different couplings for the vertices ΥBB and $\eta_b BB^*$ considering the heavy quark/meson symmetry

- $g_{\Upsilon BB} = g_{J/\Psi DD} \rightarrow BB$ loop cont. for Υ = -6 to -8.74 MeV

- $g_{\eta_b BB^*} = g_{J/\Psi DD} \rightarrow BB^*$ loop cont. for η_b = -30.73 to -38.36 MeV

- $g_{\eta_b BB^*} = (0.6/\sqrt{2}) g_{J/\Psi DD} \rightarrow BB^*$ loop cont. for η_b = -18.8 to -24.32 MeV

Different form factor

$$u_{B,B^*}(\mathbf{q}^2) = \left(\frac{\Lambda^2_{B,B^*}}{\Lambda^2_{B,B^*} + \mathbf{q}^2} \right)^2$$