Theory and Phenomenology of the Three-Gluon Vertex

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The holy grail of QCD

Start from the QCD Lagrangian :

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc}(\partial^\mu \bar{c}^a) A^b_\mu c^c$$

+ Quarks

↓ SDE, BSE, lattice …

and obtain

Dynamical generation of a fundamental mass scale in pure Yang-Mills (mass gap)

Quark constituent masses and chiral symmetry breaking

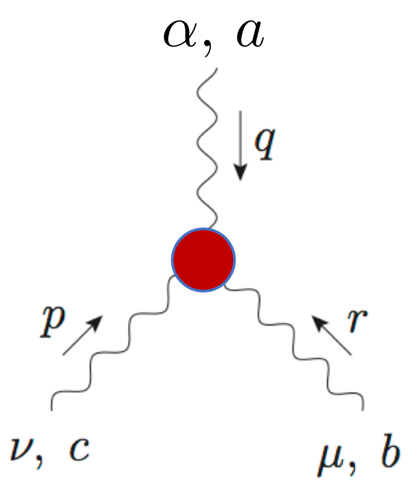
Bound state formation: mesons, hadrons, glueballs, hybrids, exotics …

Signals of Confinement

…

An indispensable ingredient in this quest is the three-gluon vertex

$q + r + p = 0$



$$= g f^{abc} \Gamma_{\alpha\mu\nu}(q, r, p)$$

- *Purely non-Abelian (no QED analogue)*
- *Instrumental for asymptotic freedom*
- *Rich kinematic and tensorial structure (14 form factors)*
- *Displays Bose symmetry*
- $\Gamma^{(0)}_{\alpha\mu\nu}(q, r, p) = (q - r)_\nu g_{\alpha\mu} + (r - p)_\alpha g_{\mu\nu} + (p - q)_\mu g_{\alpha\nu}$

Tensorial decomposition (Ball-Chiu basis)

$$\Gamma^{\alpha\mu\nu}(q,r,p) = \Gamma_L^{\alpha\mu\nu}(q,r,p) + \Gamma_T^{\alpha\mu\nu}(q,r,p)$$

Saturates the STI (for $\Gamma_L^{\alpha\mu\nu}$)

Automatically conserved, when contracted by q^α, r^μ, p^ν (for $\Gamma_T^{\alpha\mu\nu}$)

$$\Gamma_L^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\ell_i^{\alpha\mu\nu}$$

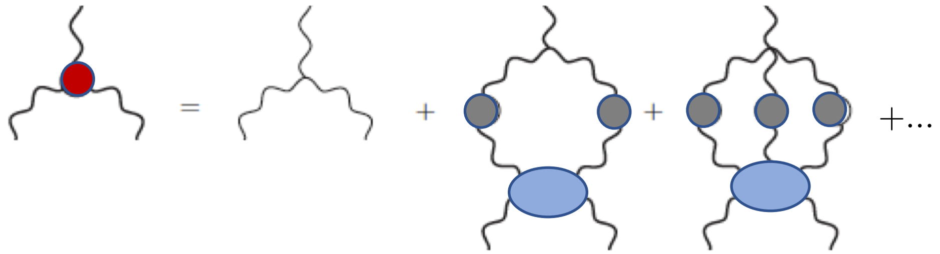
$$\begin{aligned}
\ell_1^{\alpha\mu\nu} &= (q-r)^\nu g^{\alpha\mu} & \ell_2^{\alpha\mu\nu} &= -p^\nu g^{\alpha\mu} & \ell_3^{\alpha\mu\nu} &= (q-r)^\nu[q_\mu r^\alpha - (q\cdot r)g_{\alpha\mu}] \\
\ell_4^{\alpha\mu\nu} &= (r-p)^\alpha g^{\mu\nu} & \ell_5^{\alpha\mu\nu} &= -q^\alpha g^{\mu\nu} & \ell_6^{\alpha\mu\nu} &= (r-p)^\alpha[r^\nu p^\mu - (r\cdot p)g^{\mu\nu}], \\
\ell_7^{\alpha\mu\nu} &= (p-q)^\mu g^{\alpha\nu} & \ell_8^{\alpha\mu\nu} &= -r^\mu g^{\alpha\nu} & \ell_9^{\alpha\mu\nu} &= (p-q)^\mu[p^\alpha q^\nu - (p\cdot q)g^{\alpha\nu}] \\
\ell_{10}^{\alpha\mu\nu} &= q^\nu r^\alpha p^\mu + q^\mu r^\nu p^\alpha
\end{aligned}$$

$$\Gamma_T^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{4} Y_i(q,r,p)t_i^{\alpha\mu\nu}$$

$$\begin{aligned}
t_1^{\alpha\mu\nu} &= [(q\cdot r)g^{\alpha\mu} - q^\mu r^\alpha][(r\cdot p)q^\nu - (q\cdot p)r^\nu] \\
t_2^{\alpha\mu\nu} &= [(r\cdot p)g^{\mu\nu} - r^\nu p^\mu][(p\cdot q)r^\alpha - (r\cdot q)p^\alpha] \\
t_3^{\alpha\mu\nu} &= [(p\cdot q)g^{\nu\alpha} - p^\alpha q^\nu][(q\cdot r)p^\mu - (p\cdot r)q^\mu] \\
t_4^{\alpha\mu\nu} &= g^{\mu\nu}[(p\cdot q)r^\alpha - (r\cdot q)p^\alpha] + g^{\nu\alpha}[(q\cdot r)p^\mu - (p\cdot r)q^\mu] + g^{\alpha\mu}[(r\cdot p)q^\nu - (q\cdot p)r^\nu] \\
&\quad + p^\alpha q^\mu r^\nu - r^\alpha p^\mu q^\nu
\end{aligned}$$

Nonperturbative dynamics of the three-gluon vertex

1 *Schwinger-Dyson equation (Landau gauge)*



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M. Pelaez, M. Tissier, N. Wschebor, Phys. Rev. D88 (2013) 125003.
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic,Phys. Rev. D89,105014 (2014)
A. Blum, M.Q. Huber, M. Mitter, L. von Smekal, Phys. Rev. D 89, 061703(R) (2014)
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W.Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)

2 *Slavnov-Taylor identities*

$$p^\nu \Gamma_{\alpha\mu\nu}(q,r,p) = \underbrace{F(p^2)}_{\text{ghost dressing}} \left[\Delta^{-1}(r^2) P^\sigma_\mu(r) \underbrace{H_{\sigma\alpha}(r,p,q)}_{\text{ghost-gluon kernel}} - \underbrace{\Delta^{-1}(q^2)}_{\text{gluon propagator}} P^\sigma_\alpha(q) H_{\sigma\mu}(q,p,r)\right]$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

Determines fully the longitudinal part $\Gamma_L^{\alpha\mu\nu}(q,r,p)$

A.C.Aguilar, M.N.Ferreira, C.T.Figueiredo, J.P, Phys. Rev. D 99, no. 3, 034026 (2019)
Phys. Rev. D 99, 094010 (2019)

3 *Lattice simulations*

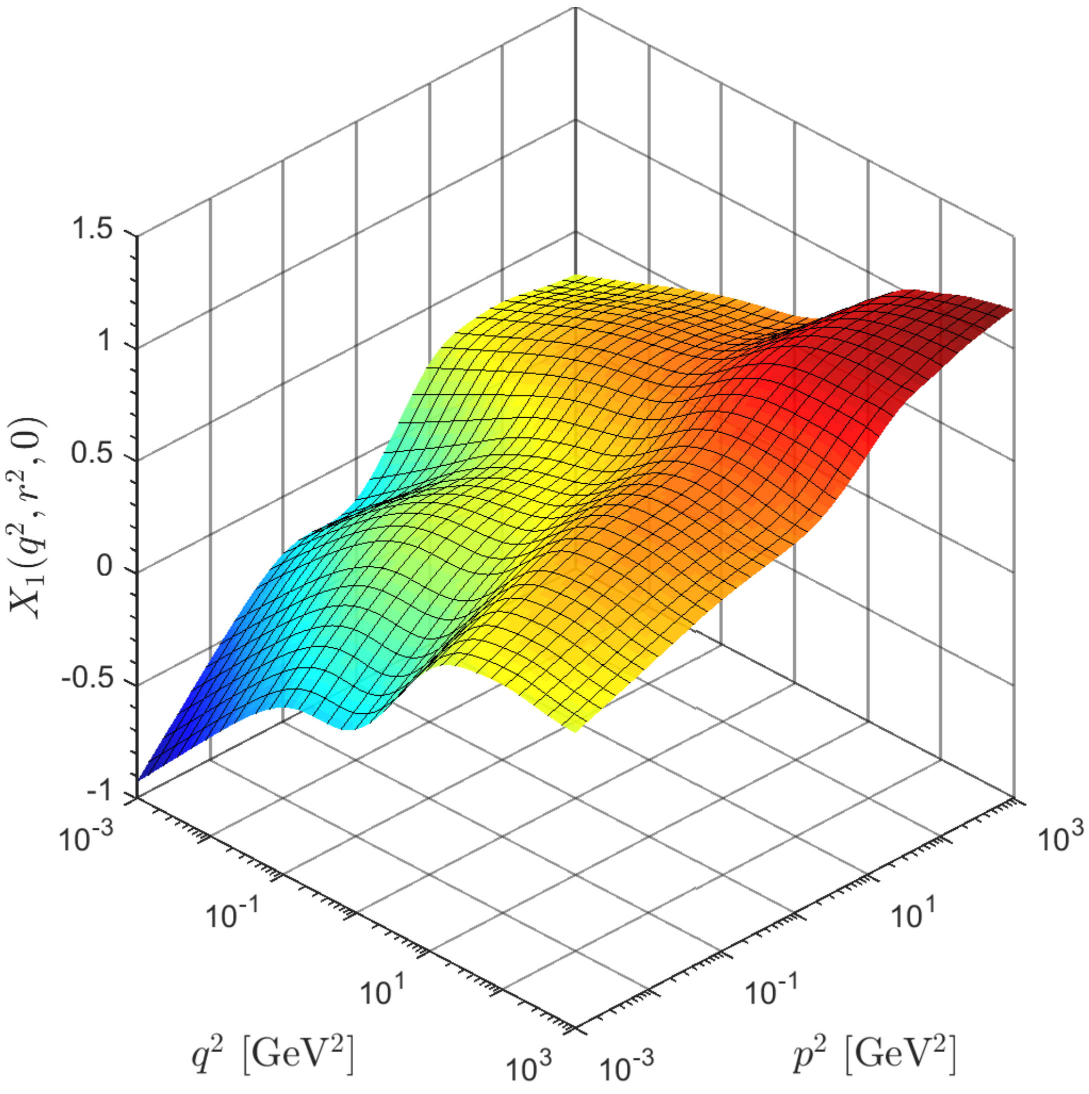
Direct evaluation of $\langle \widetilde{A}^a_\mu(q)\widetilde{A}^b_\nu(r)\widetilde{A}^c_\rho(p)\rangle$

$$L(q,r,p) = \frac{W^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma^{\alpha\mu\nu}(q,r,p)}{W^{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}$$

Simple kinematic configurations (single momentum) are chosen

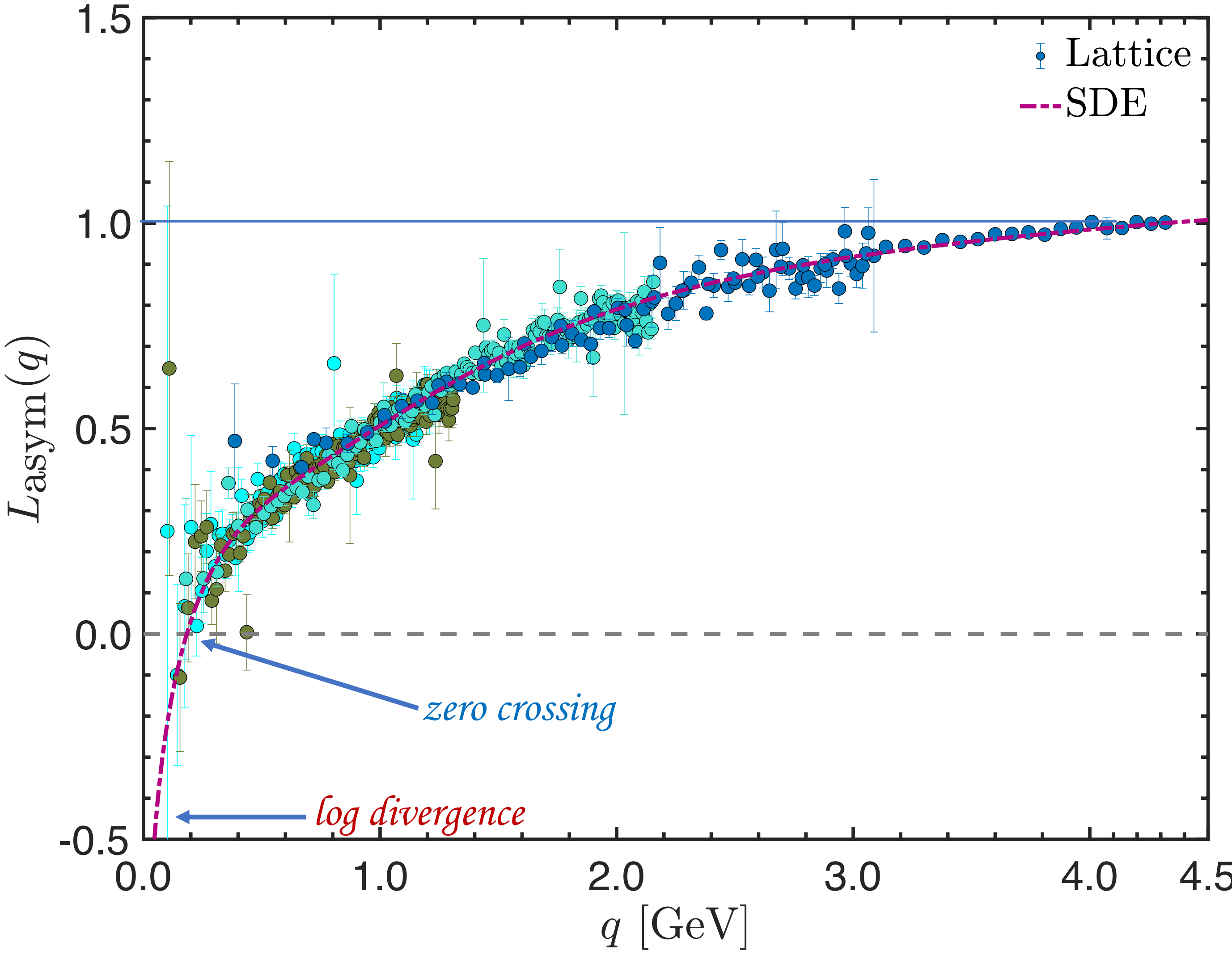
A. Cucchieri, A. Maas, and T. Mendes, Phys. Rev.D74, 014503 (2006)
Phys. Rev.D77, 094510 (2008)
A. G. Duarte, O. Oliveira, P. J. Silva, Phys. Rev.D94, 074502 (2016)
A.Athenodorou et al, Phys. Lett. B761, 444 (2016)
A.~C.~Aguilar et al , Eur.Phys.J.C 80 (2020)
A.C. Aguilar, F. De Soto, M.N.Ferreira, J.P. , J.Rodriguez-Quintero,
Phys. Lett. B818 (2021) 136352

Exceptional nonperturbative features: infrared suppression and zero crossing



Form factor of the tree-level tensor

General Euclidean (space-like) momenta



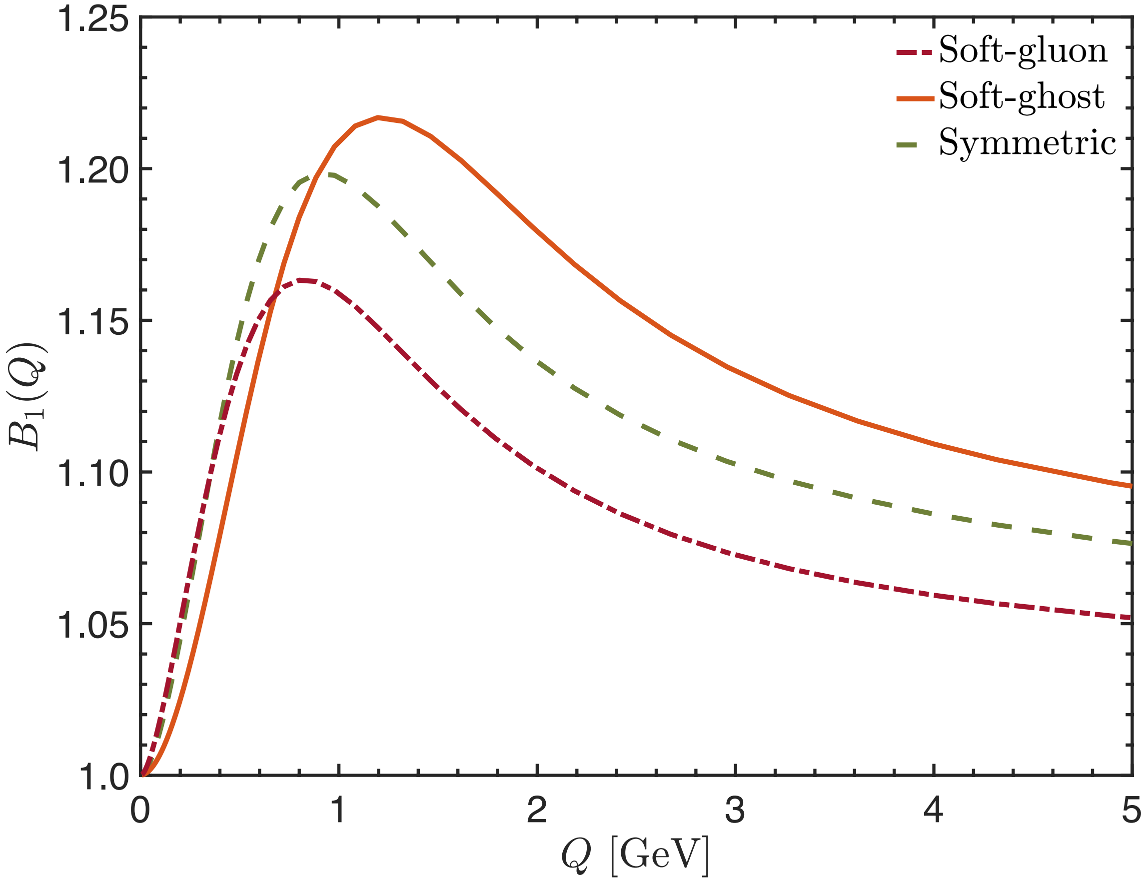
Asymmetric (soft gluon) configuration

$p = 0, \quad r = -q$

To be contrasted to:

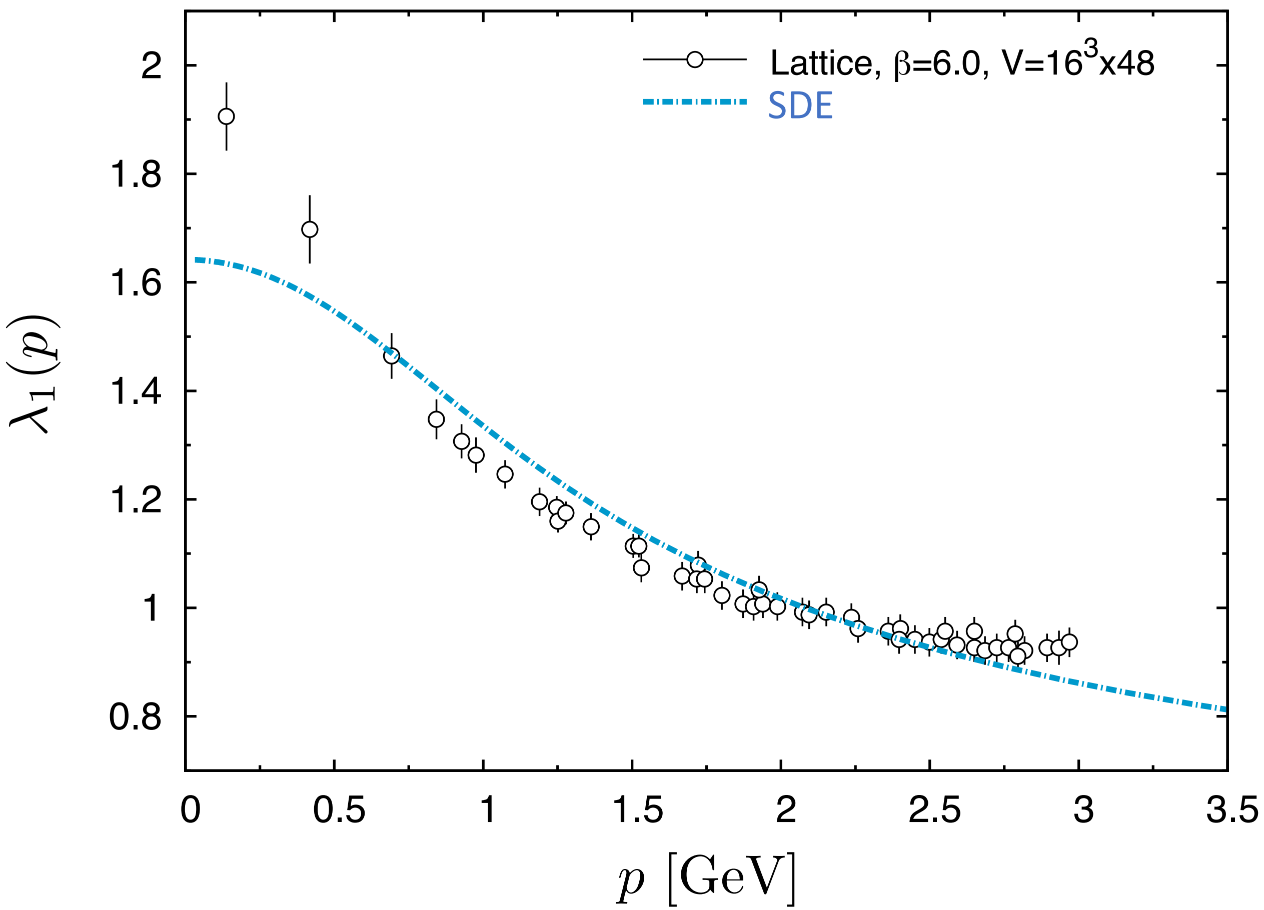
Ghost-gluon vertex

Form factor of tree-level tensor



Quark-gluon vertex

Form factor of tree-level tensor



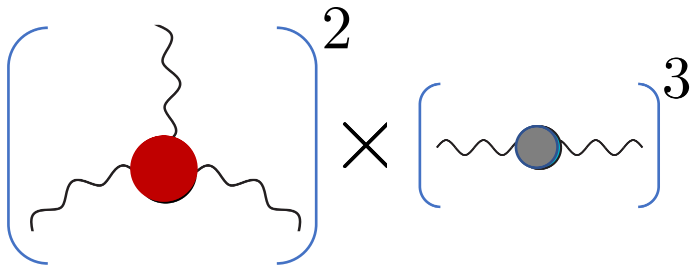
Both cases exhibit ***enhancement*** *with respect to their tree-level value*

Infrared suppression of the effective (running) coupling

Compare suppression by means of renormalization-group invariant quantites

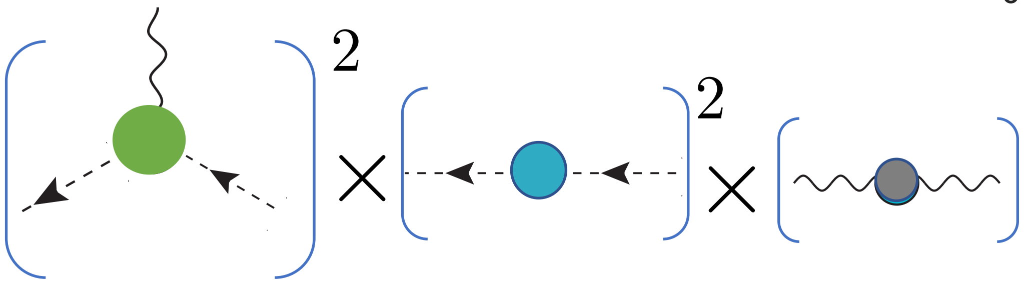
1 *Running coupling from the three-gluon vertex*

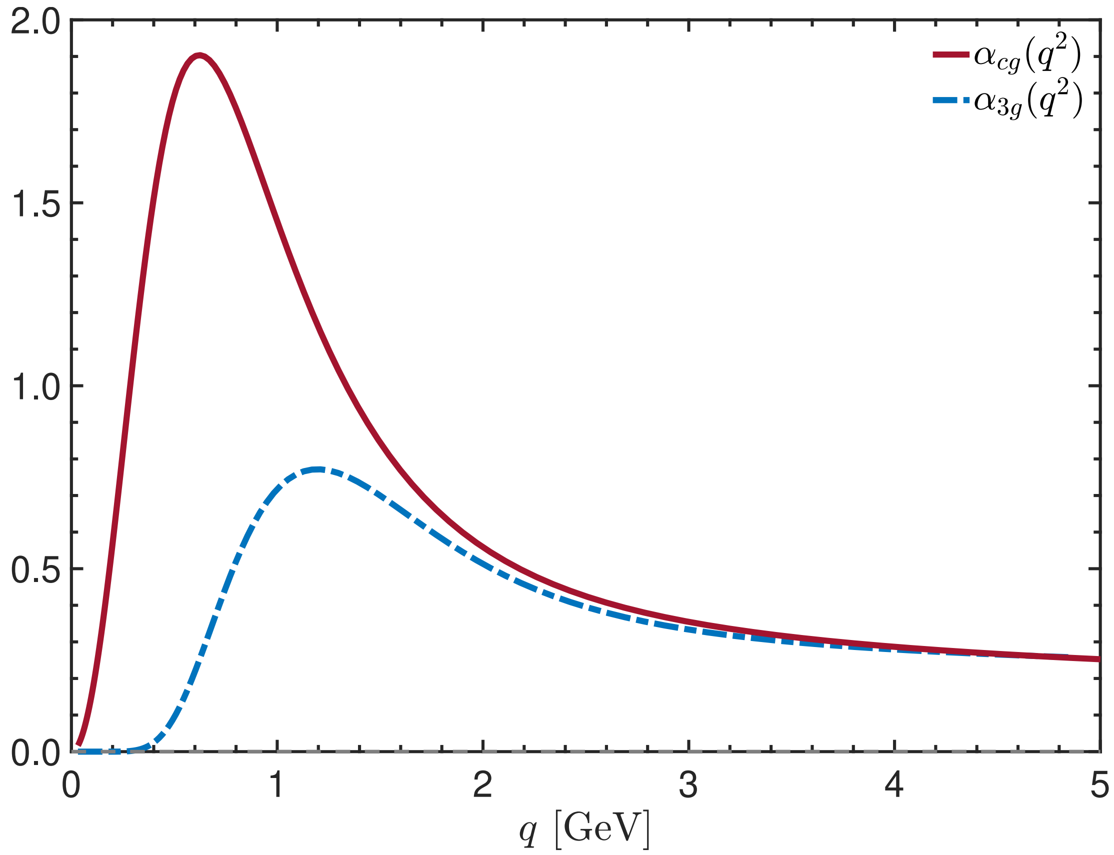
$\alpha_{3g}(q^2) \sim$



2 *Running coupling from the ghost-gluon vertex*

$\alpha_{cg}(q^2) \sim$

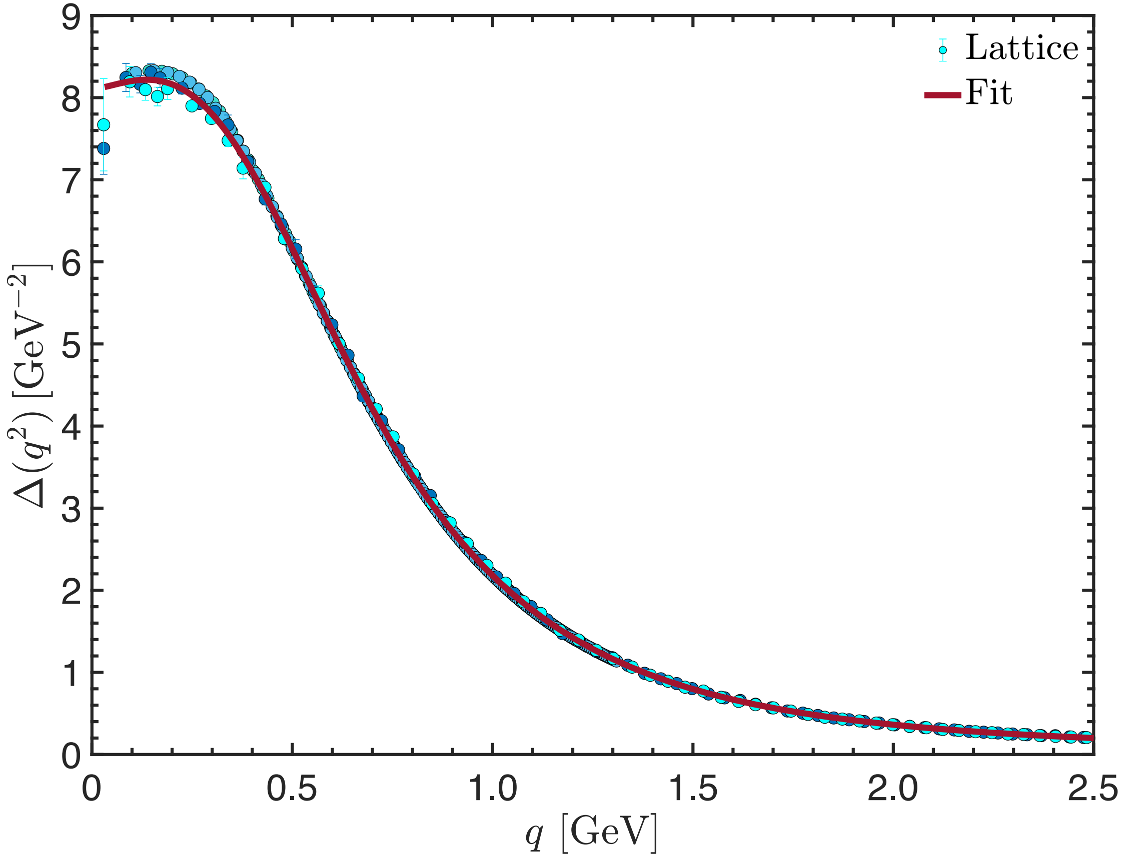


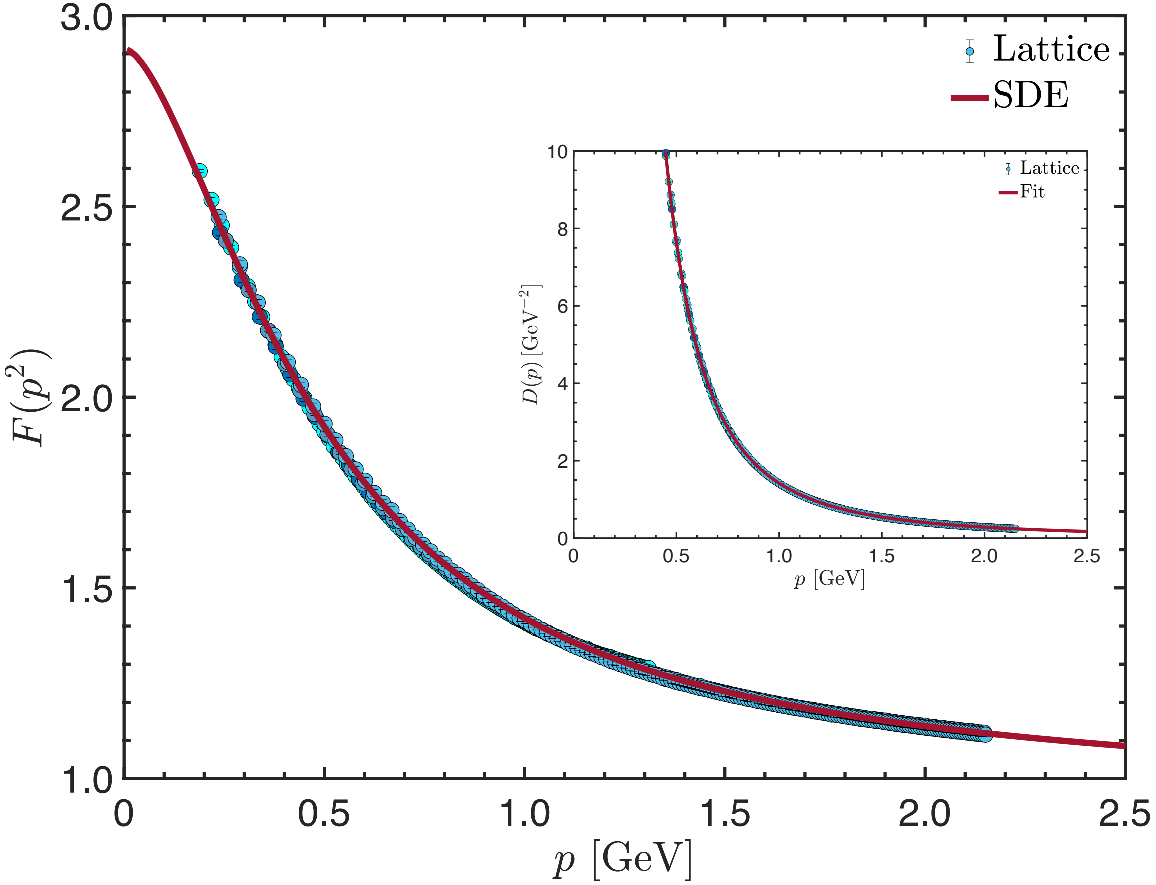


A.C. Aguilar, C.O. Ambrosio, F. De Soto, M.N. Ferreira, B.M. Oliveira, J.P and J. Rodriguez-Quintero, *arXiv:2107.00768*

Origin of the suppression: two crucial ingredients

I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, PoS LATTICE2007, 290 (2007).





The gluon propagator saturates in the deep infrared

Emergence of a gluonic mass scale

The ghost remains masslesss nonperturbatively

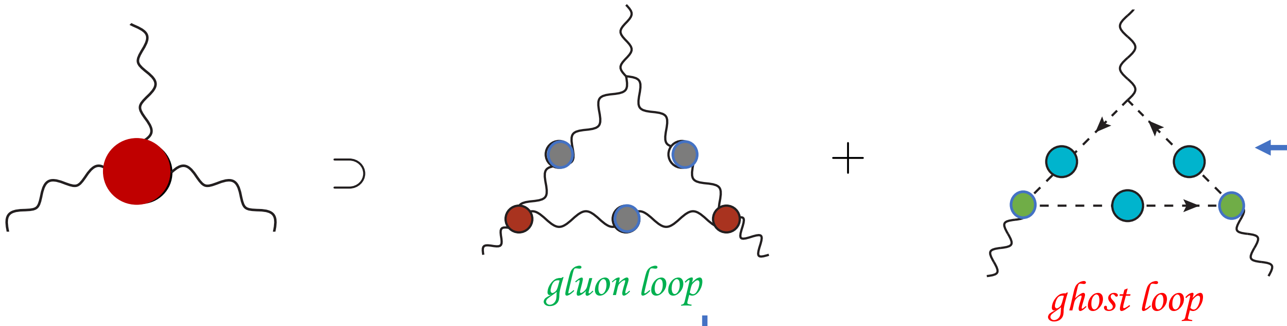
Dressing functions saturates in the deep infrared

$$D(p^2) = \frac{F(p^2)}{p^2}$$

A.C. Aguilar, D.Binosi, J.P, Phys. Rev. D 78, 025010 (2008)

P. Boucaud, J. Leroy, L. Y. A., J. Micheli, O. Pène, J. Rodríguez-Quintero, JHEP 06 (2008) 099.

"Competition" between massive and massless loops



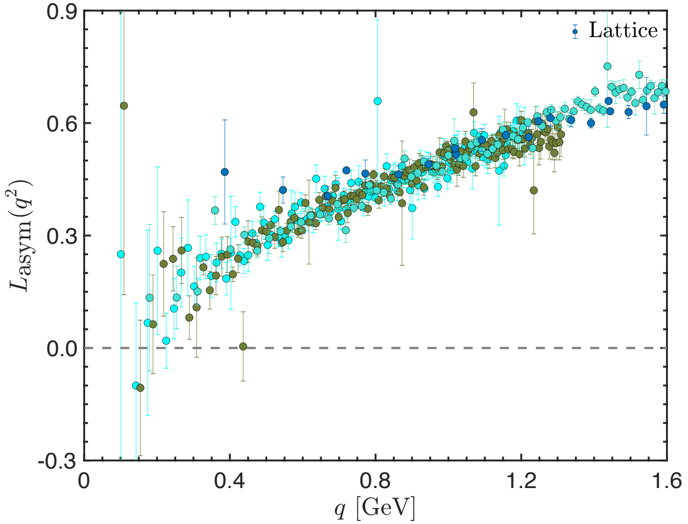
"Primary" ghost loop
Not present in the quark-gluon vertex, nor in the ghost-gluon vertex

$$L(q^2) \approx a + b\ln\left(\frac{q^2+m^2}{\Lambda^2}\right) + c\ln\left(\frac{q^2}{\Lambda^2}\right)$$

"protected"
infrared finite

"unprotected"
infrared divergent

$$L(0) \to -\infty$$



A.C.Aguilar, D.Binosi, D.Ibañez and J.P., Phys. Rev. D 89, no.8, 085008 (2014)

Conclusions

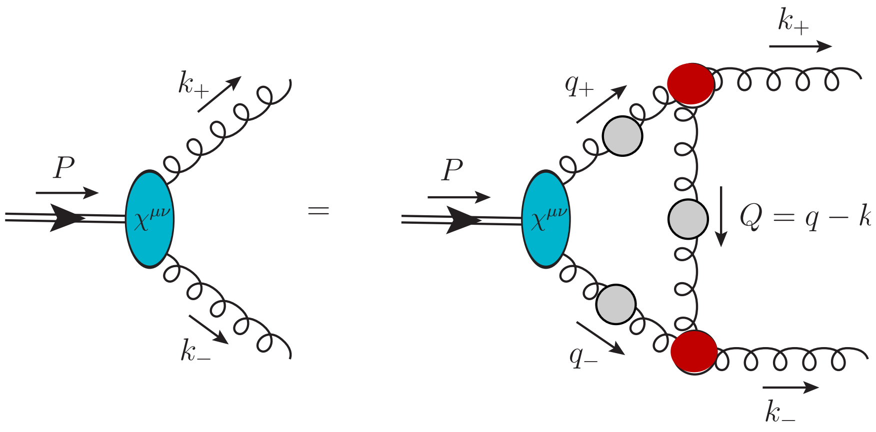
- The three-gluon vertex is host to a multitude of tightly interwoven nonperturbative effects

- Most exceptional feature: Infrared suppression driven by a logarithmic singularity.
  Subtle interplay between massive and massles loops

- Intimately connected to fundamental emergent phenomena, such as the generation of a gluonic mass scale.

- Rich phenomenology

Main example : Glueballs



(see talk by M.Q. Huber)

- New frontier of QCD