From QCD Green's functions to hadron phenomenology

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Introduction

The simple but rich QCD lagrangian:

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=u,d,s,\ldots} \bar{q}_f[\gamma\cdot\partial + ig\tfrac{1}{2}\lambda^a\gamma\cdot A^a + m_f]q_f + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$ + gauge-fixing term

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu$$

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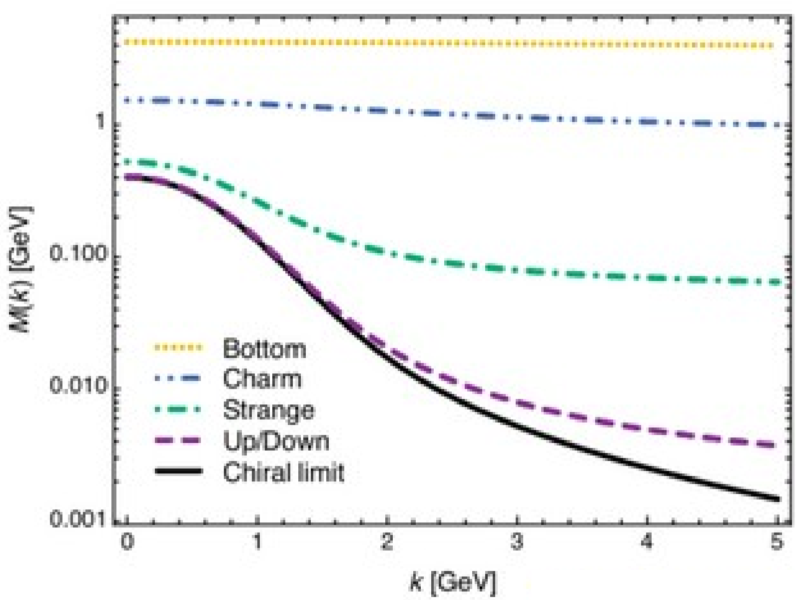
Matter sector:
Chiral symmetry for massless quarks;
Spontaneously broken → Goldstone bosons

Quark mass function from functional methods:



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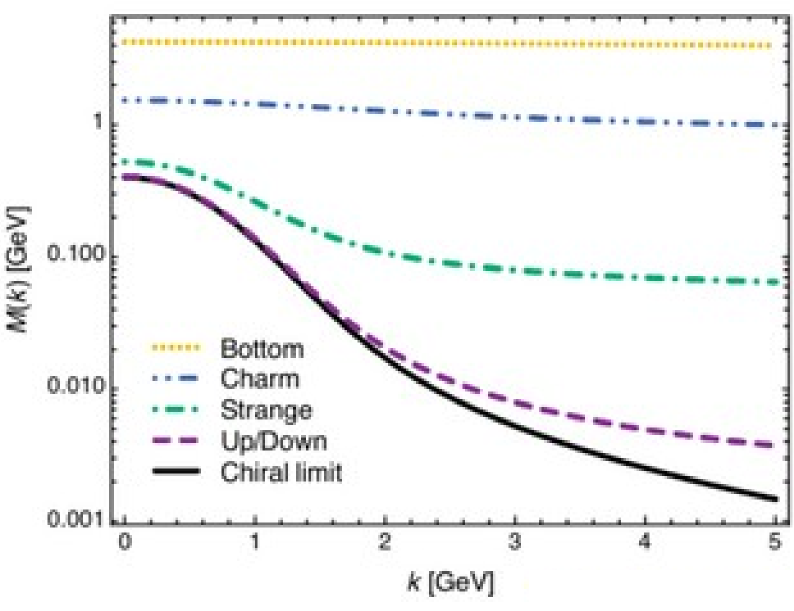

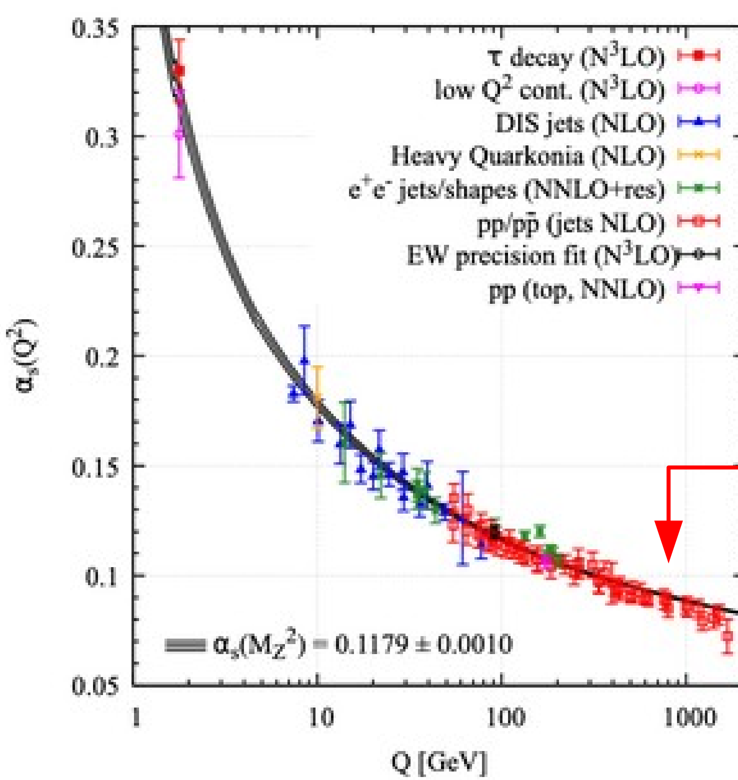

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Key also to explain the low-energy properties of strong interactions: confinement, ...

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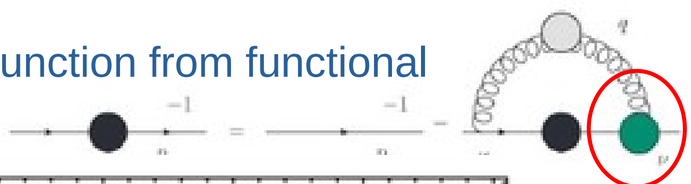

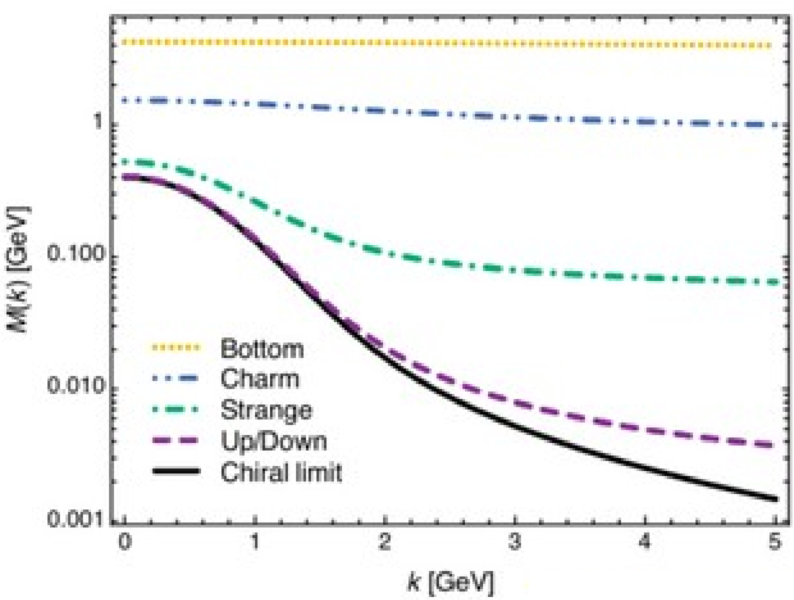

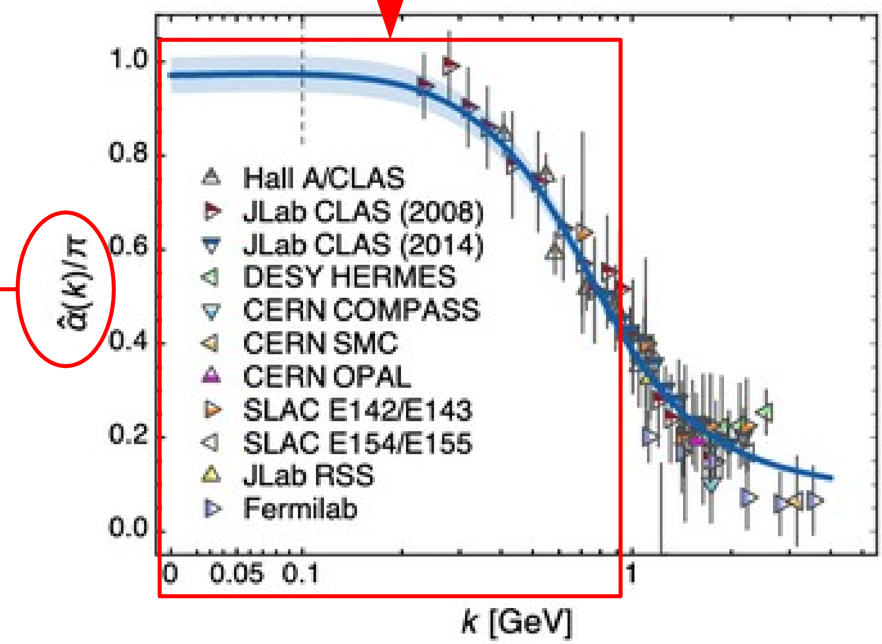

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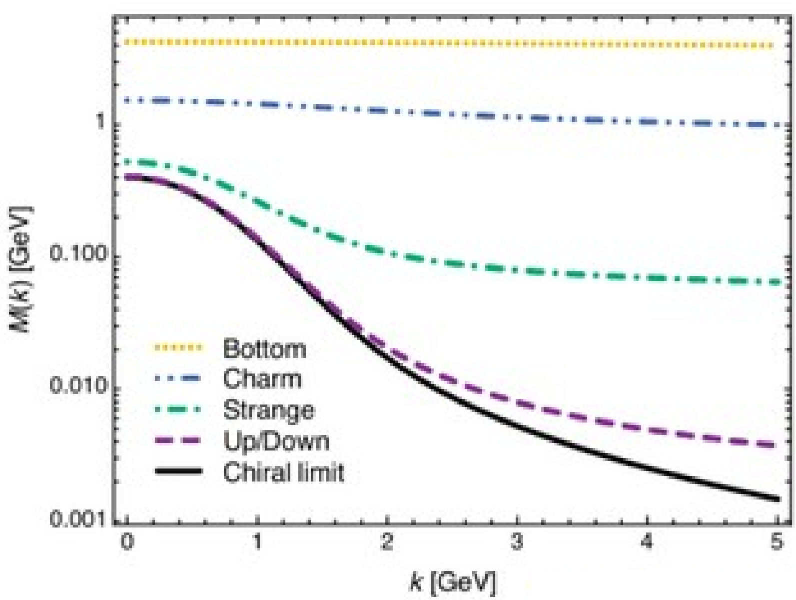
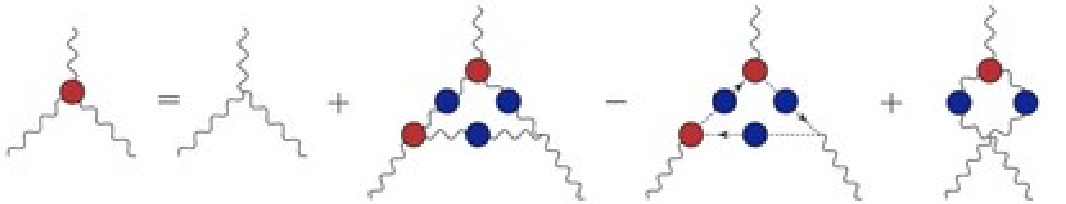


Responsible for the QCD elementary 3- and 4-gluon vertices.



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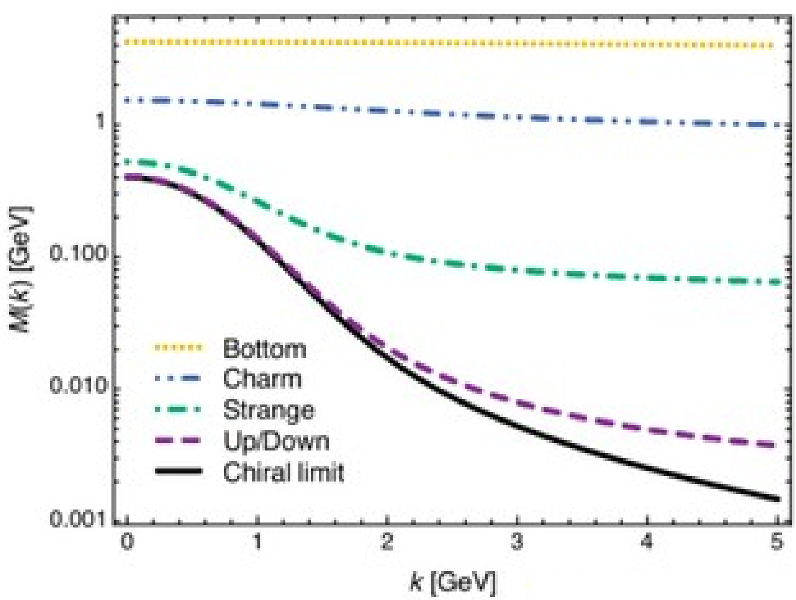

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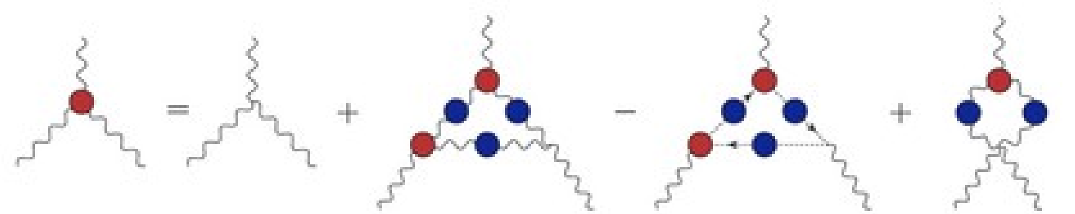


The 3-gluon function can be obtained by solving the corresponding one-loop dressed Dyson-Schwinger equation. Alternatively, combining DSEs and STIs, a deeper insight on it can be gained and results compared to results from lattice QCD.

Green's functions: 2-point sector

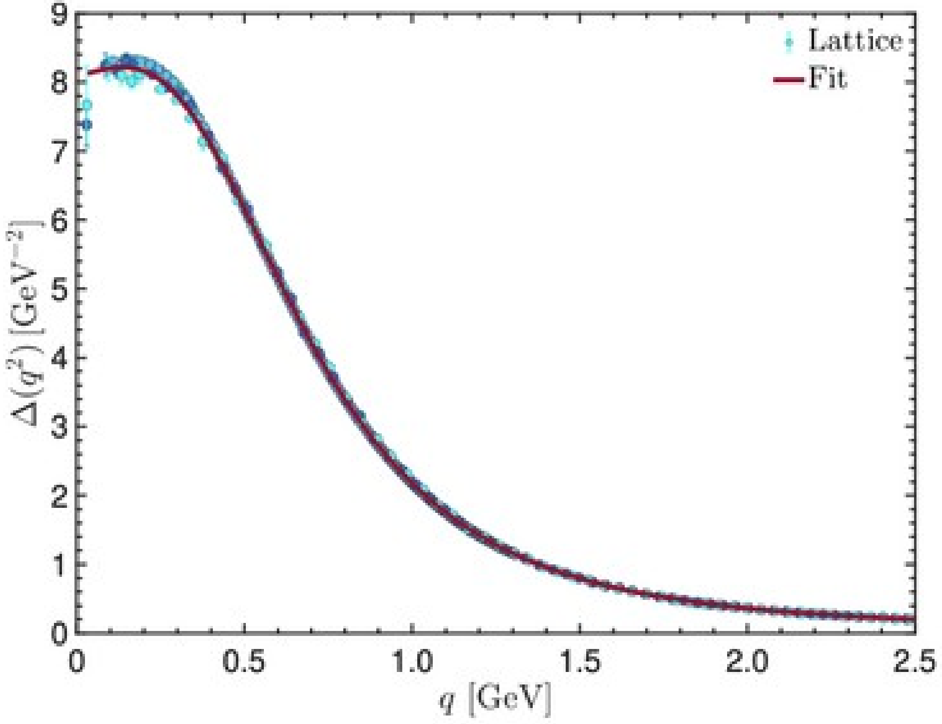
Gluon propagator (in landau gauge):

$$\Delta^{ab}_{\mu\nu}(p) = \langle \widetilde{A}^a_\mu(p)\widetilde{A}^b_\mu(-p)\rangle = \Delta(p^2)\delta^{ab}P_{\mu\nu}(p)$$

$$P_{\mu\nu}(p) = g_{\mu\nu} - p_\mu p_\nu/p^2$$

Lattice QCD gluon propagators are "massive", as suggested by the Schwinger mechanism for mass generation, and in consistence with PT-BFM DSEs. They are our basic piece of information for subsequent continuum QCD computations

$N_f = 0$



Boucaud et al., Phys.Rev.D98(2018)114515 [ArXiv:1809.05776]
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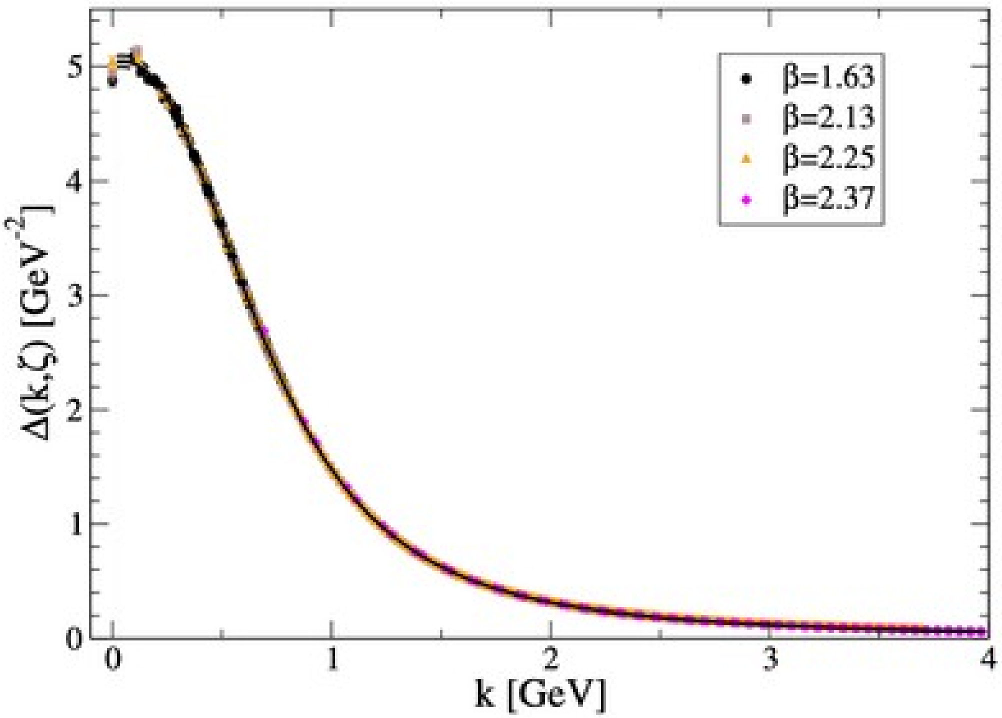

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Cui et al., Chin.Phys.C44 (2020) 083102 [ArXiv:1912.08232]

Green's functions: 2-point sector

Ghost propagator (in landau gauge):

$$D^{ab}(q^2) = i\delta^{ab} D(q^2)$$

$$D(q^2) = F(q^2)/q^2$$

Gap equation:

$$F^{-1}(k^2;\zeta^2) = \tilde{Z}_3(\zeta^2,\Lambda^2) - 3g^2(\zeta^2) \qquad (p = k + q)$$
$$\times \int_{dq}^{\Lambda} \left[1 - \frac{k\cdot q}{k^2 q^2}\right] B_1(q,p)\Delta(q^2;\zeta^2)\frac{F(p^2;\zeta^2)}{p^2}$$

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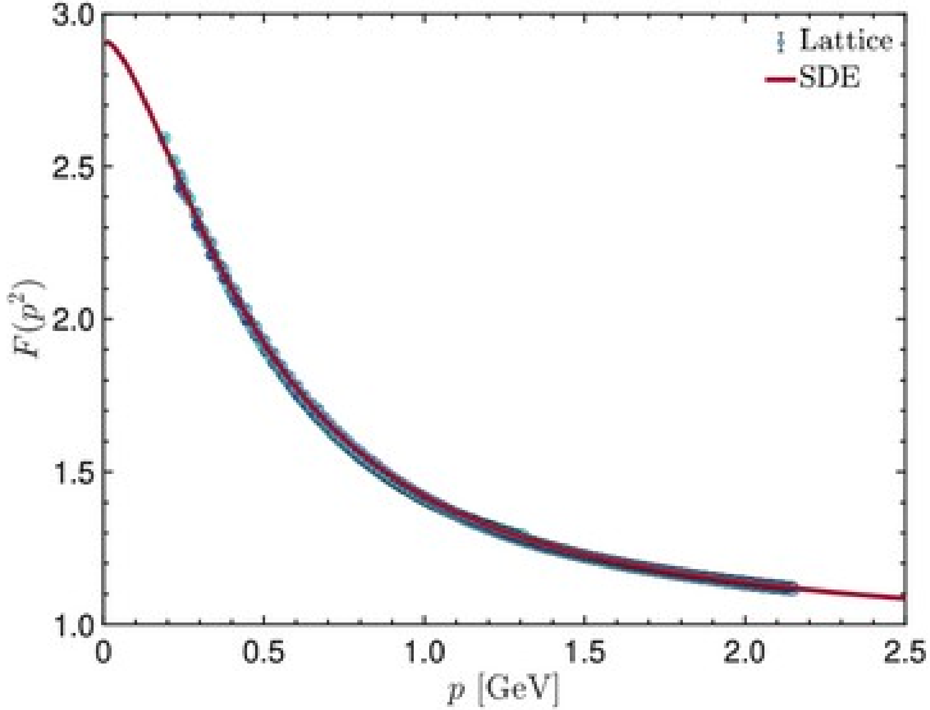

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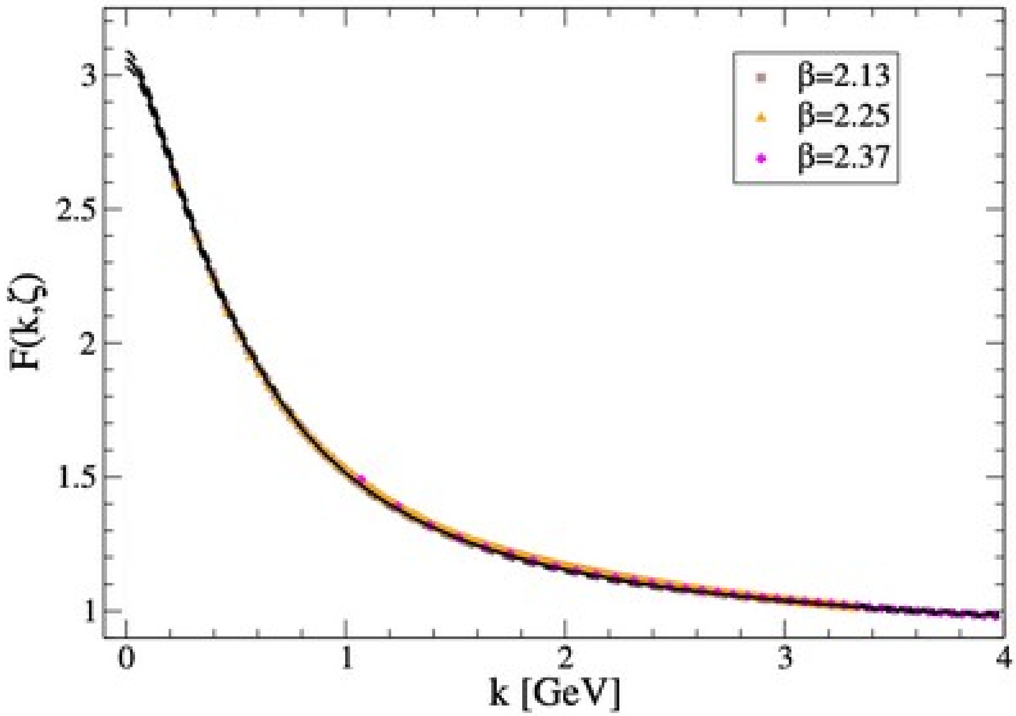

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Green's functions: 3-point sector

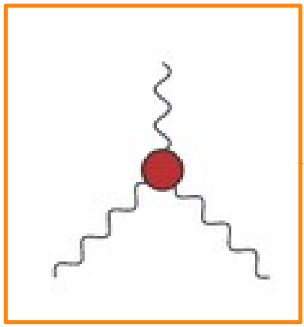
Aguilar et al., Phys.Lett.B818(2021)136352 [ArXiv:2102.04959]

3-gluon vertex (in landau gauge):

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle \widetilde{A}^a_\alpha(q)\widetilde{A}^b_\mu(r)\widetilde{A}^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p)$$

$q + r + p = 0$

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)$$



$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) = \mathrm{I\Gamma}^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)$$

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + V_{\alpha\mu\nu}(q,r,p)$$

Longitudinally coupled massless poles

Symmetric momenta configuration: $q^2 = p^2 = r^2 := s^2$

$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \overline{\Gamma}^{\rm sym}_1(s^2)\lambda_1^{\alpha\mu\nu}(q,r,p) + \overline{\Gamma}^{\rm sym}_2(s^2)\lambda_2^{\alpha\mu\nu}(q,r,p)$$

$$\lambda_1^{\alpha\mu\nu}(q,r,p) = \overline{\Gamma}_0^{\alpha\mu\nu}(q,r,p)\,,$$

$$\lambda_2^{\alpha\mu\nu}(q,r,p) = \frac{(q-r)^\nu(r-p)^\alpha(p-q)^\mu}{s^2}$$

Asymmetric momenta configuration:

$p \to 0,\ r = -q$

$$\overline{\Gamma}^{\alpha\mu\nu}(q,-q,0) = \overline{\Gamma}^{\rm asym}_3(q^2)\lambda_3^{\alpha\mu\nu}(q,-q,0)$$

$$\lambda_3^{\alpha\mu\nu}(q,-q,0) = 2q^\nu P^{\alpha\mu}(q)$$

$$L(q,p,r;\lambda) = \frac{\mathcal{G}_{\alpha\mu\nu}(q,r,p)\lambda^{\alpha\mu\nu}(q,r,p)}{\lambda_{\alpha\mu\nu}(q,r,p)\lambda^{\alpha\mu\nu}(q,r,p)}$$

$$g\overline{\Gamma}^{\rm sym}_i(s^2)\Delta^3(s^2) = L(\bar{\lambda}_i)\Big|_{q^2=r^2=p^2:=s^2}$$

$$g\overline{\Gamma}^{\rm asym}_3(q^2)\Delta^2(q^2)\Delta(0) = L(\lambda_3)\Big|_{r^2=q^2;\,p^2\to 0}$$

Green’s functions: 3-point sector

Slavnov-Taylor identity relating 3-gluon and 2-point gauge Green’s functions (NP gauge technique)

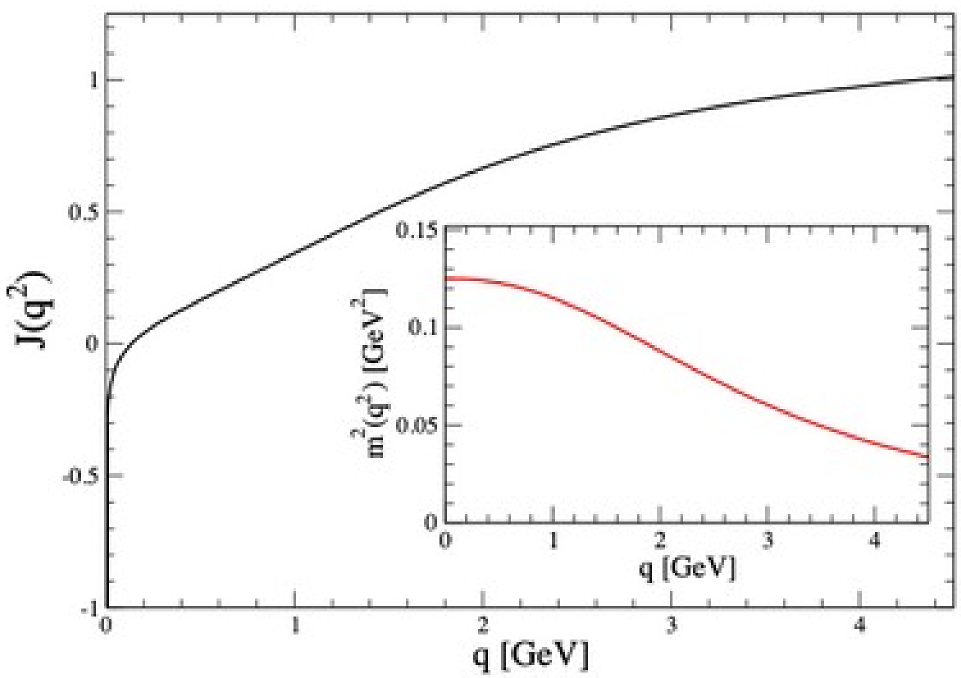
$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[p^2 J(p^2) P^\alpha_\nu(p) H_{\alpha\mu}(p,q,r) - r^2 J(r^2) P^\alpha_\mu(r) H_{\alpha\nu}(r,q,p)]$$

$$\Delta^{-1}(q^2) = q^2 \boxed{J(q^2)} + m^2(q^2)$$

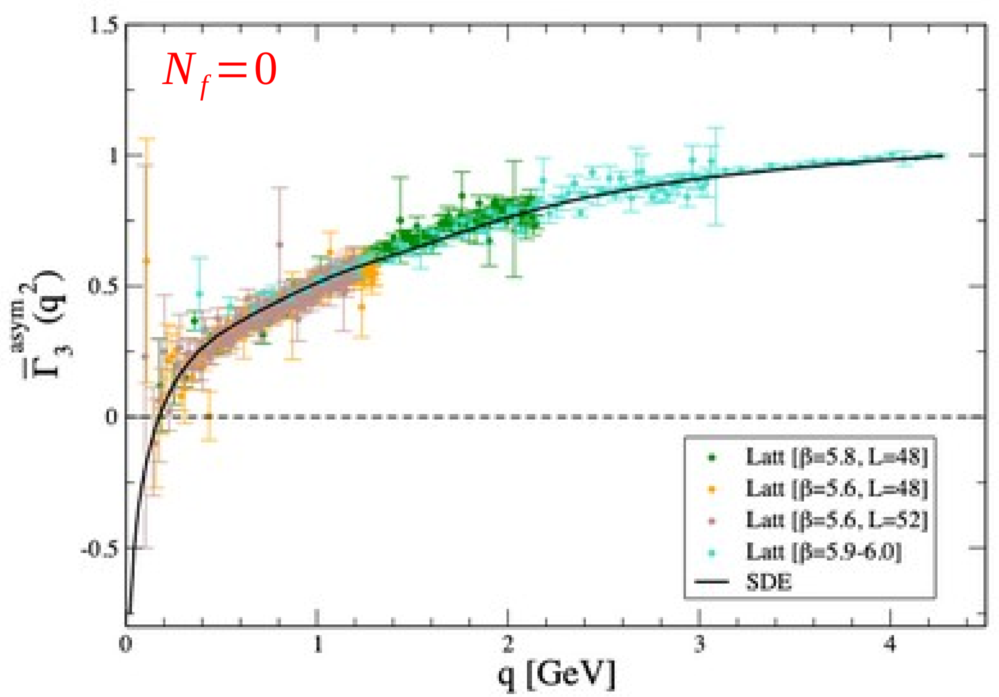
In PT-BFM scheme, gluon propagator can be naturally cast as the sum of a kinetic and a gluon running mass term.

The mass term can be obtained by solving its dynamical equation with gluon propagator lattice data as an ingredient

$$m^2(q^2) = \int \frac{d^4 k}{(2\pi)^4} m^2(k)\Delta(k)\Delta(k+q)\mathcal{K}(k,q,\alpha_s)$$



Asymmetric configuration (soft-gluon limit)



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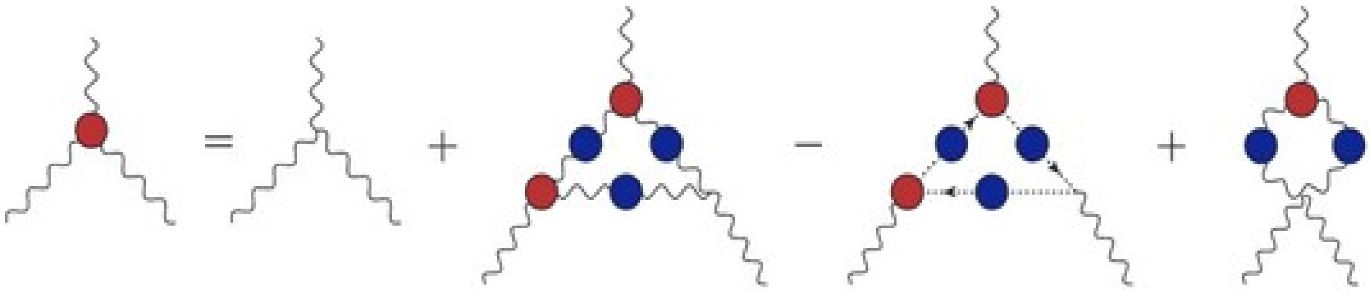
Some information beyond the soft-gluon limit is elusive when using NP gauge technique, but an approximated analysis of the corresponding 3-g DSE,



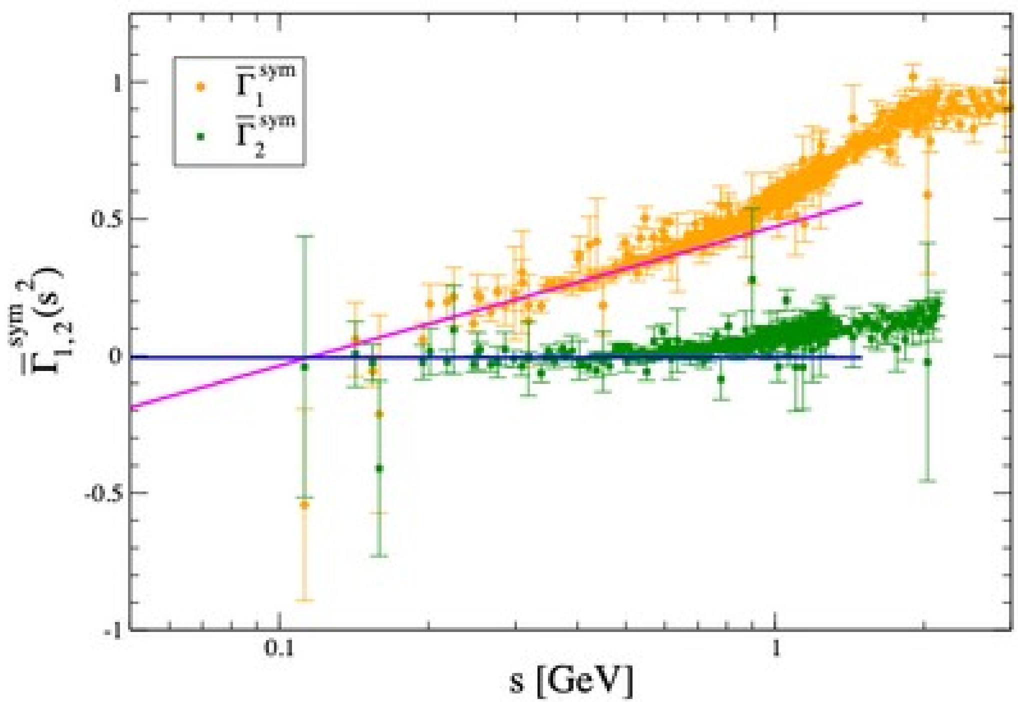
allows for the following IR asymptotic estimates

$$\overline{\Gamma}_1^{\rm sym}(s^2) \underset{s^2\to 0}{\simeq} Z_1^{\rm sym} F(0)\, a \ln(s^2/\mu^2) \approx 0.110(6)\, \ln(s^2/\mu^2)$$

$$\overline{\Gamma}_2^{\rm sym}(s^2) \underset{s^2\to 0}{\simeq} -\frac{3}{4}\left[Z_1^{\rm sym} F(0)\, a + \frac{c}{2} + \frac{d}{3}\right] \approx -0.006(5)$$

which describes strikingly well the lattice data!!!

$N_f=0$ Symmetric configuration



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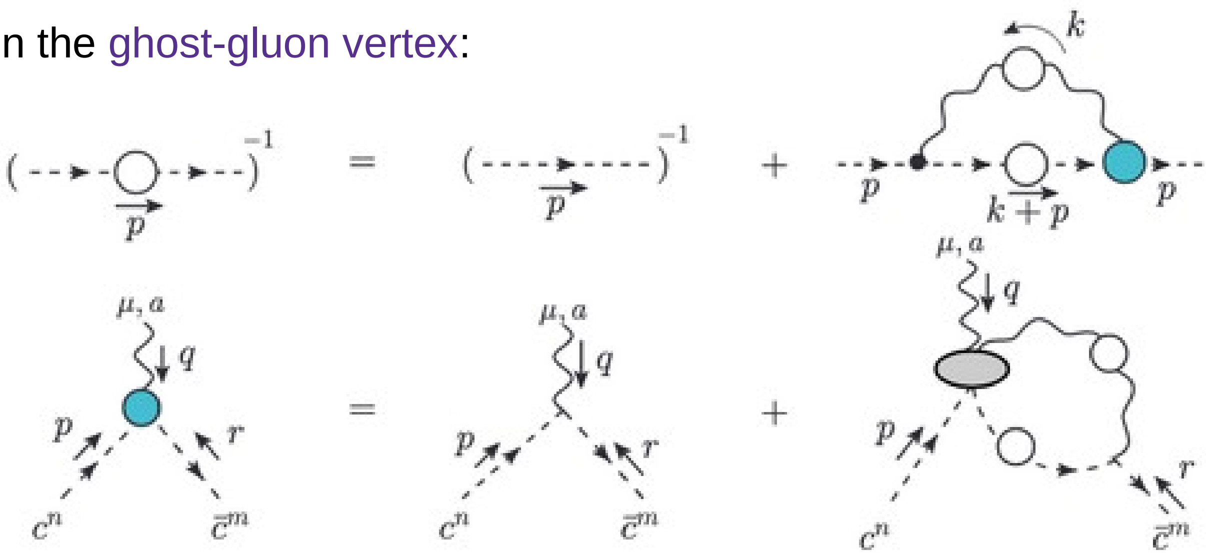
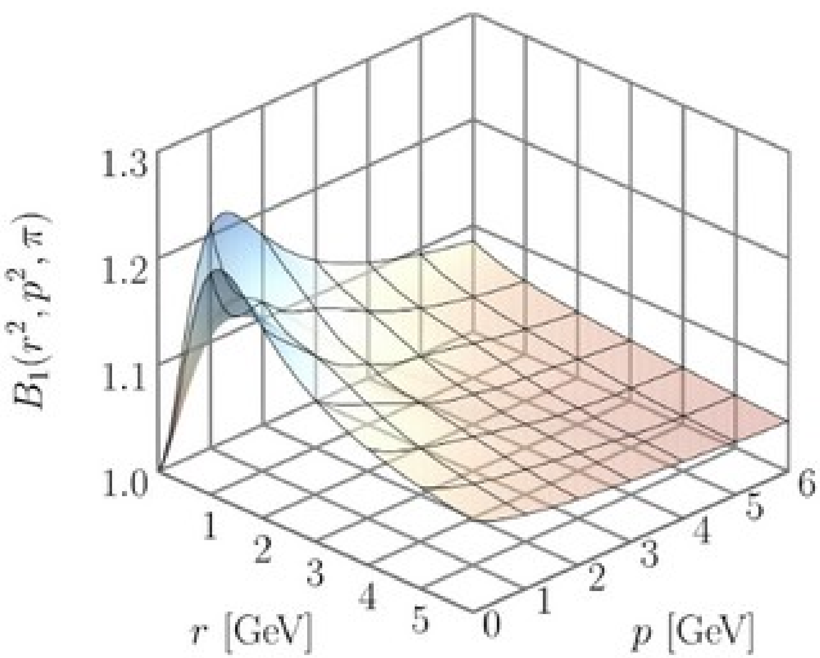
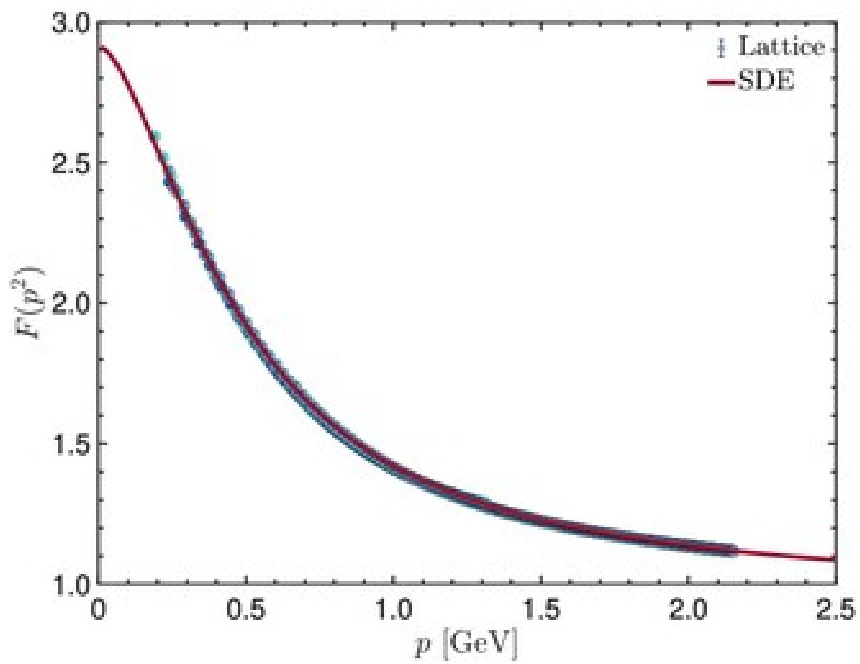


The ghost-gluon vertex DSE solution, plugged into the gap equation produces a very good description of lattice data!





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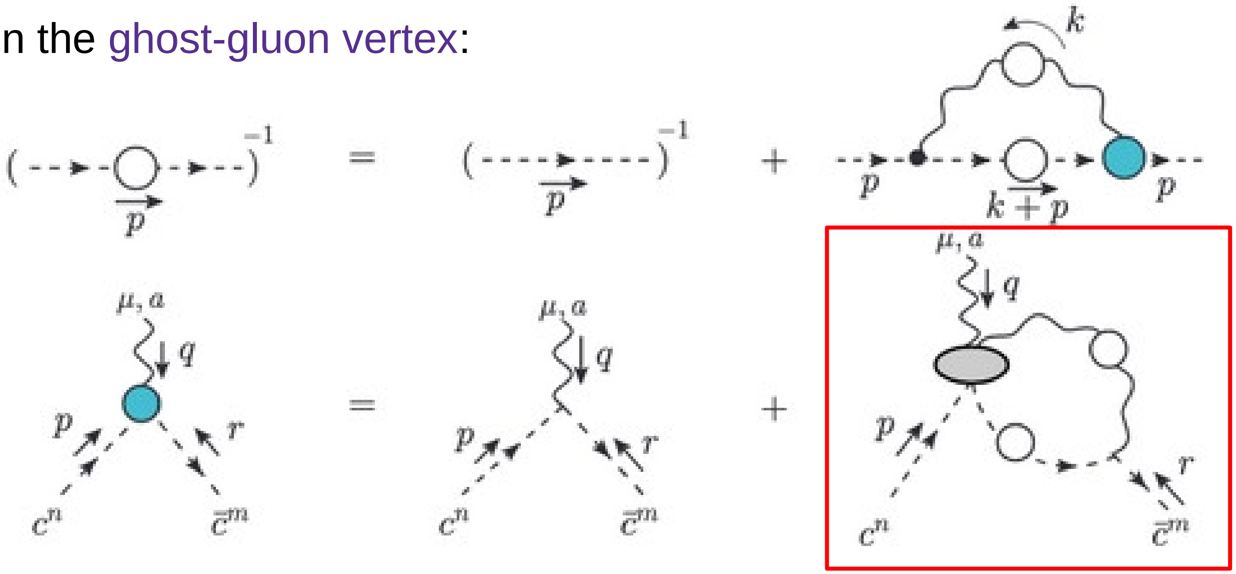

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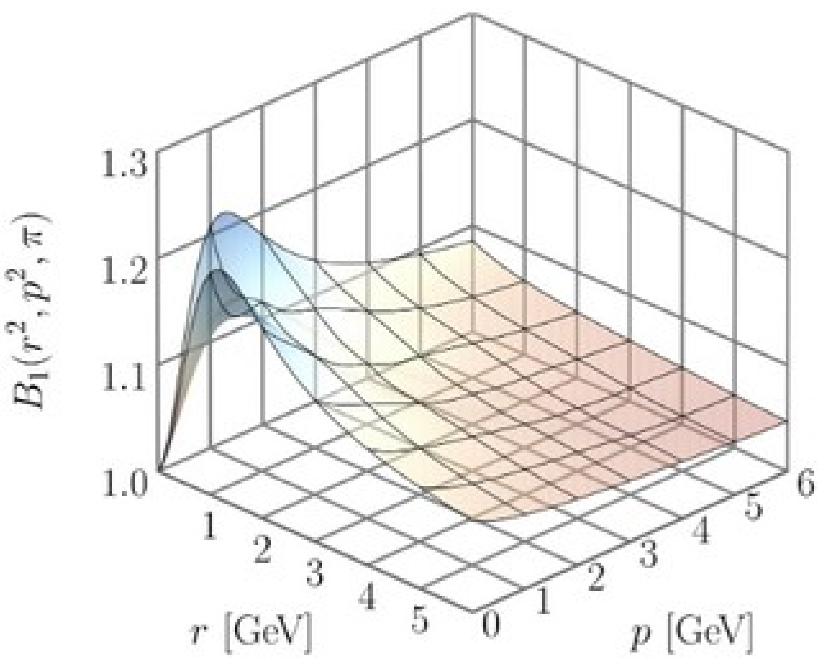
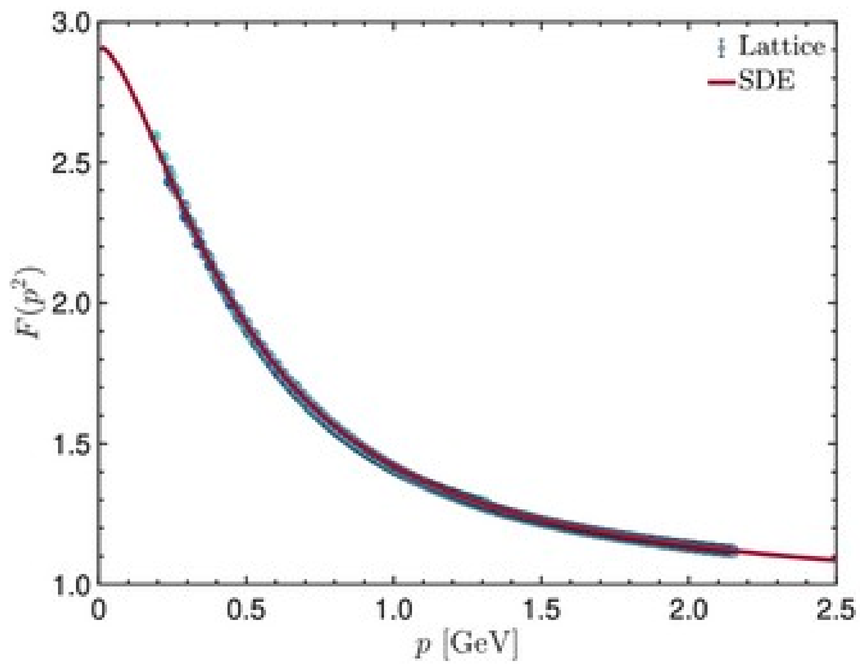
Skeleton expansion for the 1PI four-point scattering kernel and further approximations are needed





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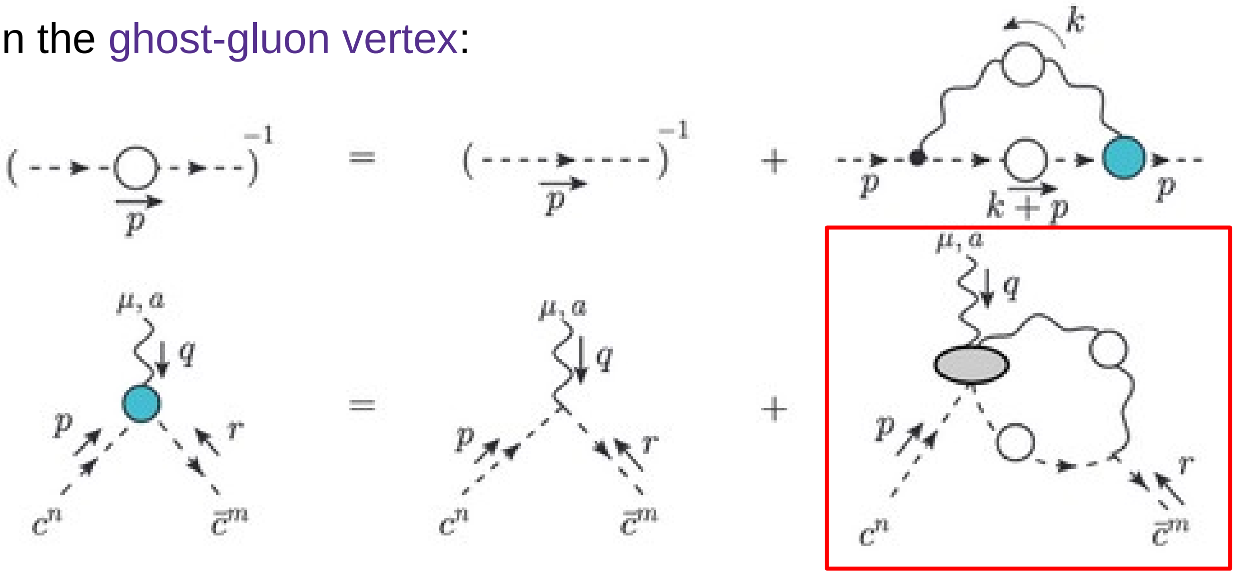

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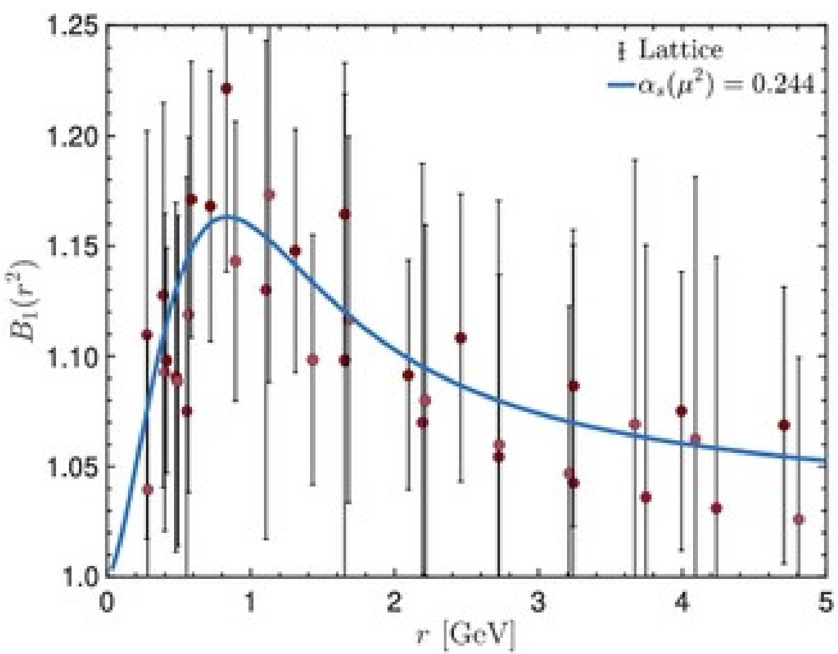
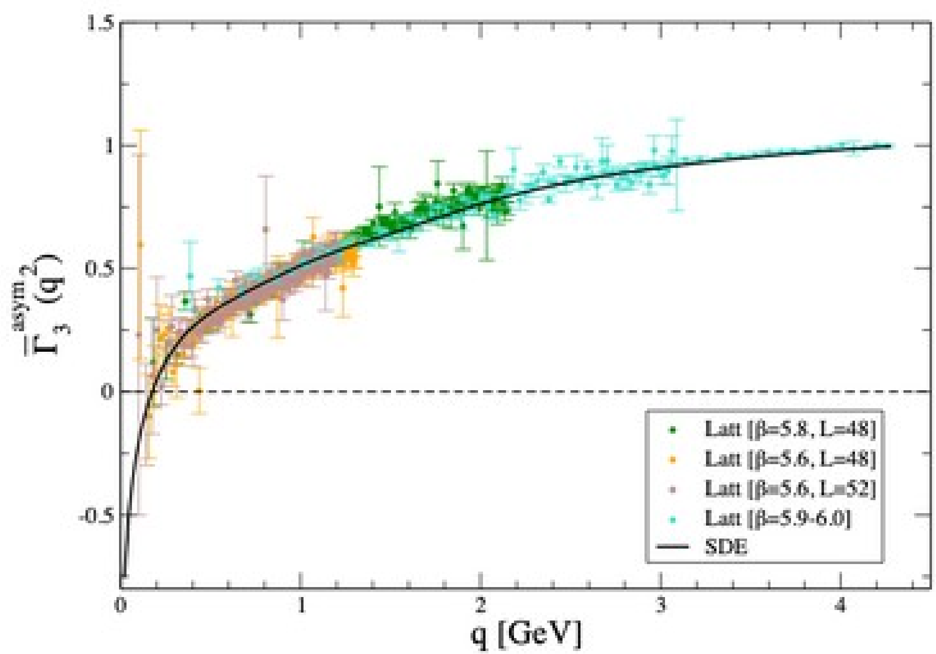
Skeleton expansion for the 1PI four-point scattering kernel and further approximations are needed, which become exact in the soft-gluon limit where the vertex becomes tightly linked to the asymmetric 3-g





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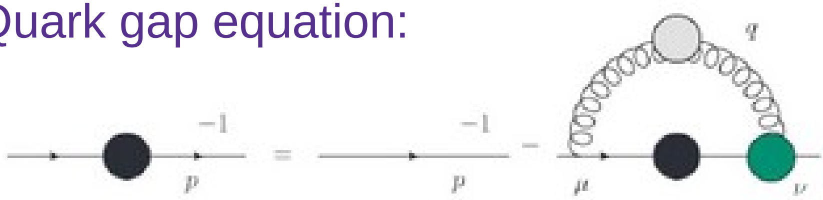
Linking matter to gauge sector: effective charge

Quark gap equation:



PT-BFM scheme rearranges the SDE expansions such that GFs obey linear STIs and, owing to this, gluon vacuum polarization captures the correct RG behavior and a unique QCD effective charge can be defined from two-point GFs which is an analogue of the QED Gell-Mann-Low running charge

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + M_f(p^2)) = Z_2 S_{f(0)}^{-1}(p) + \Sigma_f(p)$$

$$\Sigma_f(p) = \frac{4}{3} Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2)\, P_{\mu\nu}(k)\gamma_\mu S_f(q) \widehat{\Gamma}_\nu^f(p,q)$$

$$k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}$$

$$\widehat{d}(k^2) = \widehat{\alpha}(k^2)\mathcal{D}(k^2)$$

$$\alpha_{\rm T}(k^2) = \alpha(\zeta^2) k^2 \Delta(k^2;\zeta^2) F^2(k^2;\zeta^2)$$

$L(k^2;\zeta^2)$ Longitudinal component of the ghost-gluon scattering kernel, obeying its own DSE

where $\mathcal{D}(k^2)$ is RGI quantity behaving as a free massive boson obtained from lattice gluon propagator

Binosi et al., Phys.Rev.D96(2017)054026 [ArXiv:1612.04836]
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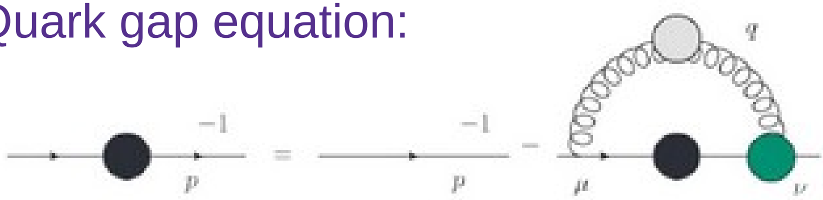

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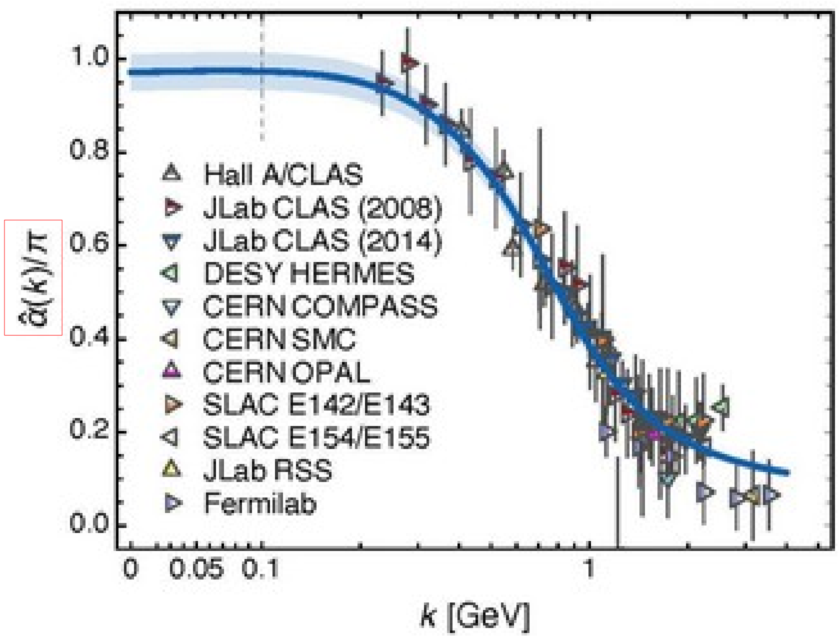

- The running charge is cured from the landau pole and compares very well with world data for the Bjorken-sum-rule charge.
- Below a given mass scale, the interaction become scale-independent and QCD (practically) conformal again
- Then: Modern continuum & lattice QCD analyses in the gauge sector deliver a process-independent, parameter-free prediction for the low-momentum charge saturation.

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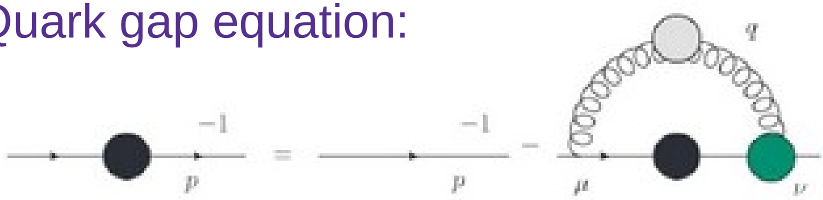

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$$\Sigma_f(p) = \frac{4}{3} Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2)\, P_{\mu\nu}(k)\gamma_\mu S_f(q)\widehat{\Gamma}_\nu^f(p,q)$$

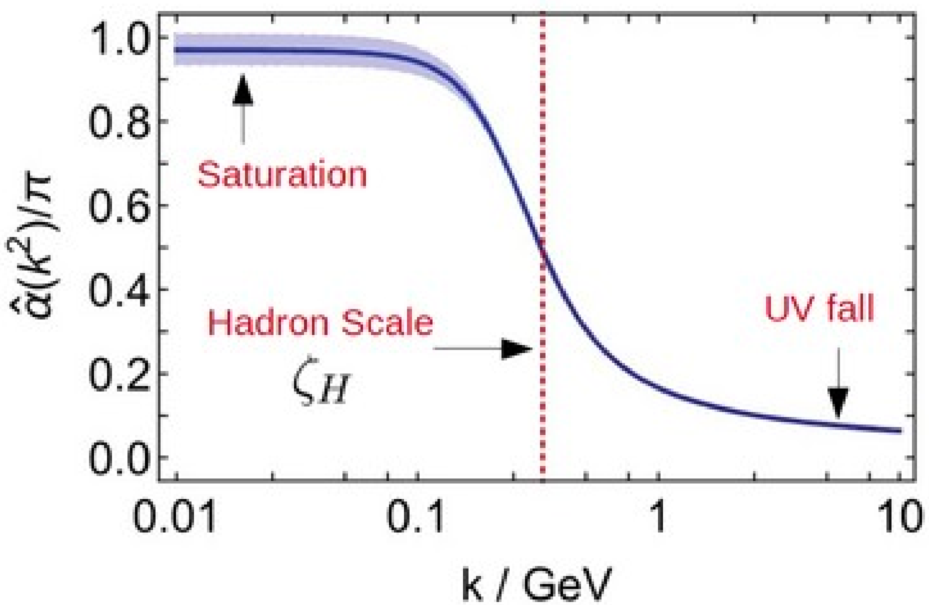
$$k^2\widehat{d}(k^2) = \frac{\alpha_T(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}$$

$$\alpha_T(k^2) = \alpha(\zeta^2)k^2\Delta(k^2;\zeta^2)F^2(k^2;\zeta^2)$$

$L(k^2;\zeta^2)$ Longitudinal component of the ghost-gluon scattering kernel, obeying its own DSE

$$\widehat{d}(k^2) = \widehat{\alpha}(k^2)\mathcal{D}(k^2)$$

where $\mathcal{D}(k^2)$ is RGI quantity behaving as a free massive boson obtained from (Nf=2+1) lattice gluon propagator



One thus defines:

$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{QCD}^2}\right]}\,;\ \alpha(0) = 0.97(4)$$

where

$$\mathcal{M}(k^2 = \Lambda_{QCD}^2) := m_G = 0.331(2)\ \text{GeV}$$

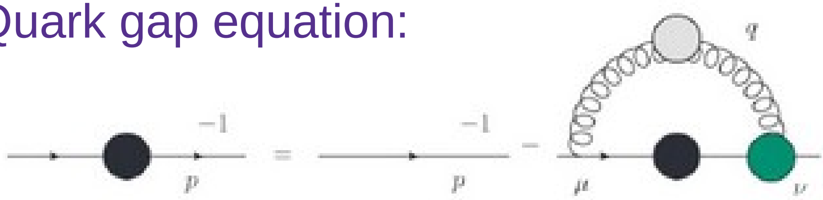
defines the screening mass and an associated wavelength, such that larger gluon modes decouple.

Then, we identify: $\zeta_H := m_G(1 \pm 0.1)$

Cui et al., Eur.Phys.J.C80(2020)1064 [ArXiv:2006.14075]

Linking matter to gauge sector: effective charge

Quark gap equation:



PT-BFM scheme rearranges the SDE expansions such that GFs obey linear STIs and, owing to this, gluon vacuum polarization captures the correct RG behavior and a unique QCD effective charge can be defined from two-point GFs which is an analogue of the QED Gell-Mann-Low running charge

$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + M_f(p^2)) = Z_2 S_{f(0)}^{-1}(p) + \Sigma_f(p)$$

$$\Sigma_f(p) = \frac{4}{3} Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2)\, P_{\mu\nu}(k)\gamma_\mu S_f(q)\widehat{\Gamma}_\nu^f(p,q)$$

$$k^2 \widehat{d}(k^2) = \frac{\alpha_T(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}$$

$$\widehat{d}(k^2) = \widehat{\alpha}(k^2)\mathcal{D}(k^2)$$

$$\alpha_T(k^2) = \alpha(\zeta^2)k^2\Delta(k^2;\zeta^2)F^2(k^2;\zeta^2)$$

$L(k^2;\zeta^2)$ Longitudinal component of the ghost-gluon scattering kernel, obeying its own DSE

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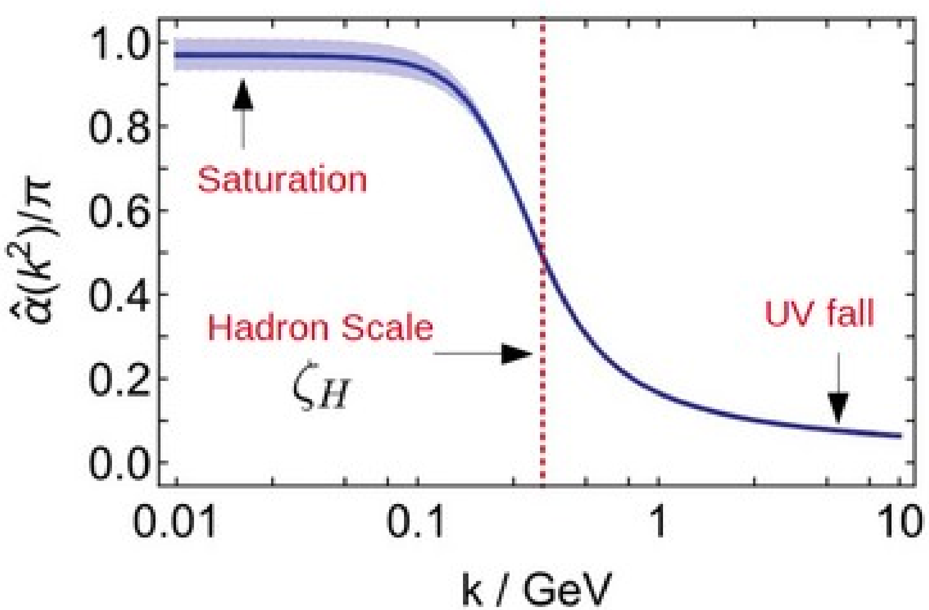
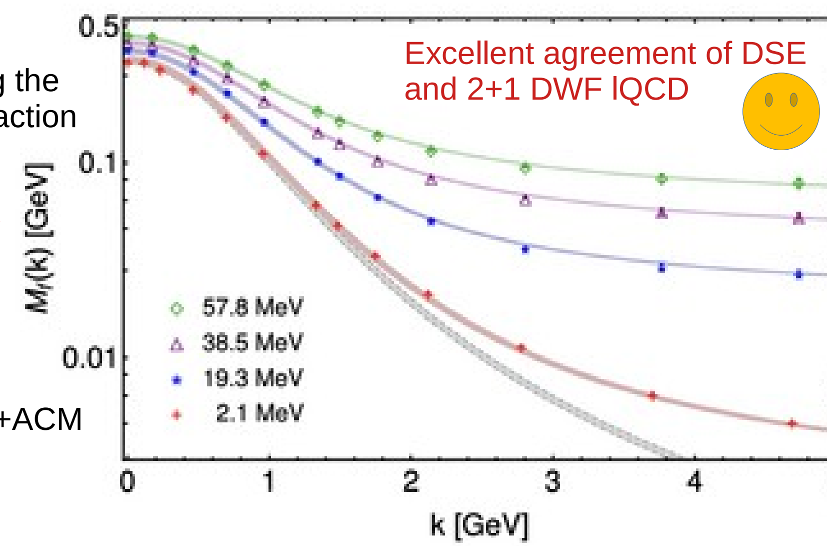


Thus defining the running interaction

And a QGV including BC+ACM



Chang et al., [ArXiv:2105.06596]

Effective charge for QCD evolution of pion PDF

DGLAP leading-order evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi}\underbrace{\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

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Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2\frac{d}{d\zeta^2}\mathbb{1}+\frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix}\gamma_{qq}^{(n)} & 0 & 0\\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)}\\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)}\end{pmatrix}\right\}\begin{pmatrix}\langle x^n\rangle_{\rm NS}(\zeta)\\ \langle x^n\rangle_{\rm S}(\zeta)\\ \langle x^n\rangle_{g}(\zeta)\end{pmatrix}=0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_0,\zeta_f)\right)\langle x^n(\zeta_0)\rangle_q \qquad q=u,\bar{d}$$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\,\alpha(t)$$

$$t = \ln\frac{\zeta^2}{\Lambda_{\rm QCD}^2}$$

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

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Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi}S(\zeta_0,\zeta_f)\right)\langle x^n(\zeta_0)\rangle_q = \langle x^n(\zeta_H)\rangle_q\left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$$q = u, \bar{d}$$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

$$t = \ln\frac{\zeta^2}{\Lambda_{\rm QCD}^2}$$

This ratio encodes the information of the charge

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \mathbb{1} + \boxed{\frac{\alpha(\zeta^2)}{4\pi}} \begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f \gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix} \right\} \begin{pmatrix} \langle x^n \rangle_{\rm NS}(\zeta) \\ \langle x^n \rangle_{\rm S}(\zeta) \\ \langle x^n \rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

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Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_0,\zeta_f)\right) \langle x^n(\zeta_0)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\langle 2x(\zeta_f)\rangle_q\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$$q = u, \bar{d}$$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

$$t = \ln \frac{\zeta^2}{\Lambda_{\rm QCD}^2}$$

This ratio encodes the information of the charge

Use Isospin symmetry:

$$\langle x(\zeta_H)\rangle_u = \langle x(\zeta_H)\rangle_{\bar{d}} = 1/2$$

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi} \begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f \gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix} \right\} \begin{pmatrix} \langle x^n \rangle_{\rm NS}(\zeta) \\ \langle x^n \rangle_{\rm S}(\zeta) \\ \langle x^n \rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f) \rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_0, \zeta_f) \right) \langle x^n(\zeta_0) \rangle_q = \langle x^n(\zeta_H) \rangle_q \left(\langle 2x(\zeta_f) \rangle_q \right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$$q = u, \bar{d}$$

$$S(\zeta_0, \zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{\rm QCD})}^{2\ln(\zeta_f/\Lambda_{\rm QCD})} dt\, \alpha(t)$$

$$t = \ln \frac{\zeta^2}{\Lambda_{\rm QCD}^2}$$

This ratio encodes the information of the charge

Use Isospin symmetry:

$$\langle x(\zeta_H) \rangle_u = \langle x(\zeta_H) \rangle_{\bar{d}} = 1/2$$

Then, after computing "all" the PDF Mellin moments, only one input is needed for their being evolved up and used for a PDF reconstruction at experimental scales.

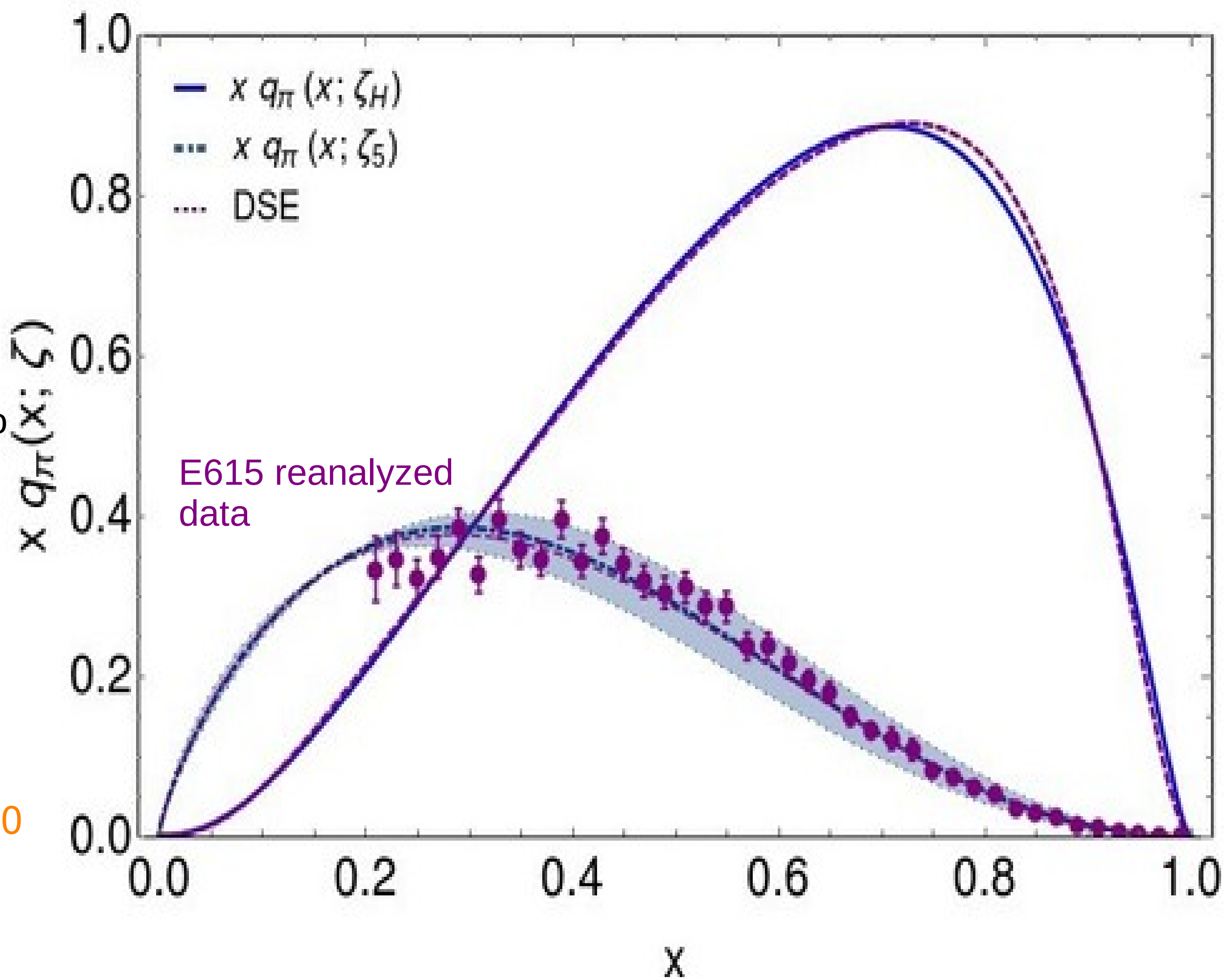
Effective charge for QCD evolution of pion PDF

DSE+BSE-based results for the pion PDF, obtained at the hadronic scale in Ding et al.,Chin.Phys.C44(2020)4, evolved at the E615 scale and agreeing very well with the reanalyzed experimental data (accounting for soft-gluon resummation) delivered in Aicher et al., Phys.Rev.Lett.105(2010)252003.

The only input required for QCD evolution is the light-cone momentum fraction carried by the valence-quark at the experimental scale. So far, this input could have been successfully taken from the lattice or have been just fitted to reproduce the experiment.

ζ_5	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
Ref. [55]	0.18(3)	0.064(10)	0.030(5)
Herein	0.20(2)	0.074(10)	0.035(6)

Lattice : R.S. Sufian et al., arXiv:2001.04960



Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2} \mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi} \begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f \gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix} \right\} \begin{pmatrix} \langle x^n \rangle_{\rm NS}(\zeta) \\ \langle x^n \rangle_{\rm S}(\zeta) \\ \langle x^n \rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f = 4)$

$$\langle 2x(\zeta_f) \rangle_q = \exp\left(-\frac{8}{9\pi} S(\zeta_H, \zeta_f)\right), \qquad q = u, \bar{d};$$

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \boxed{\frac{\alpha(\zeta^2)}{4\pi}}\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

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Implication 2: glue and sea-quark DFs $(n_f = 4)$

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d}; \\
\langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}), \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}$$

Effective charge for QCD evolution of pion PDF

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$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_{g}(\zeta) \end{pmatrix} = 0$$

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Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $\left(n_f = 4\right)$

$$\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d}; \\
\langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - \left(\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}\right), \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}$$

Momentum sum rule:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1$$

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix}\right\}\begin{pmatrix} \langle x^n\rangle_{\rm NS}(\zeta) \\ \langle x^n\rangle_{\rm S}(\zeta) \\ \langle x^n\rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f = 4)$

Momentum sum rule:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1$$

$$\begin{aligned} \langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d}; \\ \langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - \left(\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}\right), \\ &= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\ \langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right); \end{aligned}$$

$\zeta_f/\zeta_H \to \infty$

A textbook result:
G. Altarelli, Phys. Rep. 81, 1 (1982)

Effective charge for QCD evolution of pion PDF

DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{\zeta^2\frac{d}{d\zeta^2}\mathbb{1}+\frac{\alpha(\zeta^2)}{4\pi}\begin{pmatrix}\gamma_{qq}^{(n)} & 0 & 0\\ 0 & \gamma_{qq}^{(n)} & 2n_f\gamma_{qg}^{(n)}\\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)}\end{pmatrix}\right\}\begin{pmatrix}\langle x^n\rangle_{\rm NS}(\zeta)\\ \langle x^n\rangle_{\rm S}(\zeta)\\ \langle x^n\rangle_g(\zeta)\end{pmatrix}=0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution.

Implication 2: glue and sea-quark DFs $(n_f=4)$

$$\begin{aligned}\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar d;\\ \langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar q}-(\langle x(\zeta_f)\rangle_u+\langle x(\zeta_f)\rangle_{\bar d}),\\ &= \frac{3}{7}+\frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4}-\langle 2x(\zeta_f)\rangle_u\\ \langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right);\end{aligned}$$

ζ_5	$\langle 2x\rangle_q^\pi$	$\langle x\rangle_g^\pi$	$\langle x\rangle_{\rm sea}^\pi$
Ref.[55]	0.412(36)	0.449(19)	0.138(17)
Herein	0.40(4)	0.45(2)	0.14(2)

R.S. Sufian et al., arXiv:2001.04960

Effective charge for QCD evolution of pion PDF

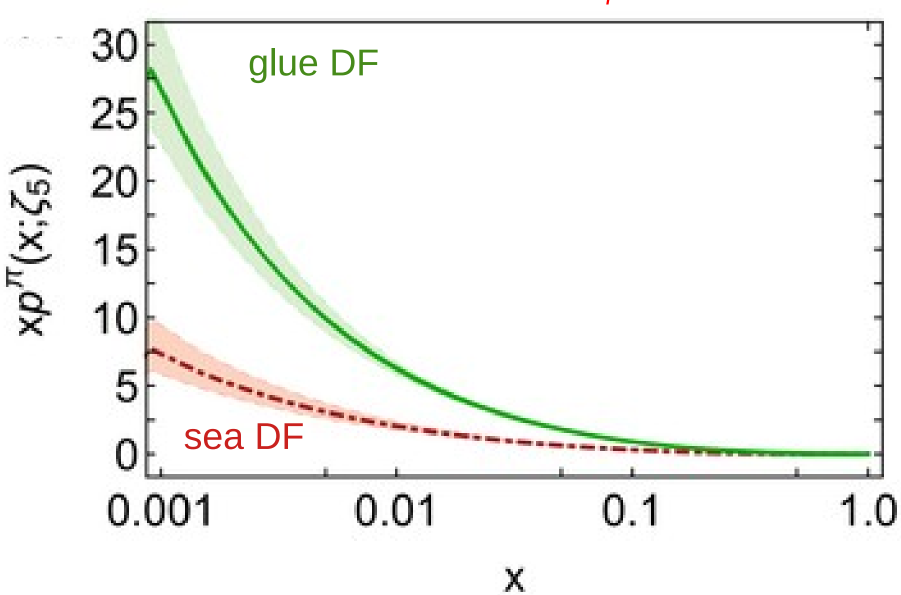
DGLAP ~~leading-order~~ evolution of PDFs:

$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \mathbb{1} + \frac{\alpha(\zeta^2)}{4\pi} \begin{pmatrix} \gamma_{qq}^{(n)} & 0 & 0 \\ 0 & \gamma_{qq}^{(n)} & 2n_f \gamma_{qg}^{(n)} \\ 0 & \gamma_{gq}^{(n)} & \gamma_{gg}^{(n)} \end{pmatrix} \right\} \begin{pmatrix} \langle x^n \rangle_{\rm NS}(\zeta) \\ \langle x^n \rangle_{\rm S}(\zeta) \\ \langle x^n \rangle_g(\zeta) \end{pmatrix} = 0$$

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$

Approach: a charge is defined such that the leading-order evolution kernel gives all-orders evolution. Then, computing all the moments a reconstructing:

Implication 2: glue and sea-quark DFs $(n_f = 4)$.

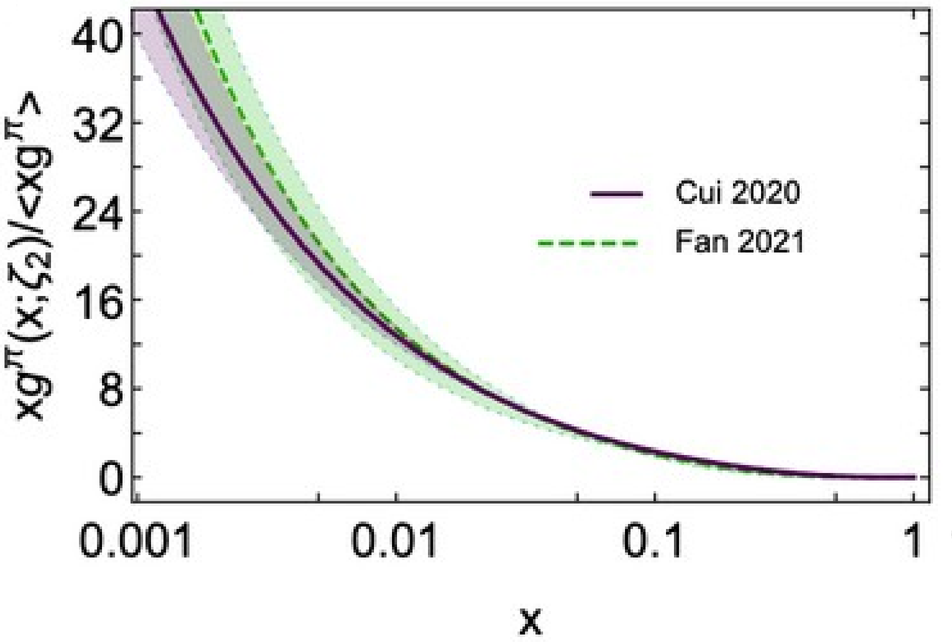


ζ_5	$\langle 2x \rangle_q^\pi$	$\langle x \rangle_g^\pi$	$\langle x \rangle_{\rm sea}^\pi$
Ref.[55]	0.412(36)	0.449(19)	0.138(17)
Herein	0.40(4)	0.45(2)	0.14(2)

R.S. Sufian et al., arXiv:2001.04960

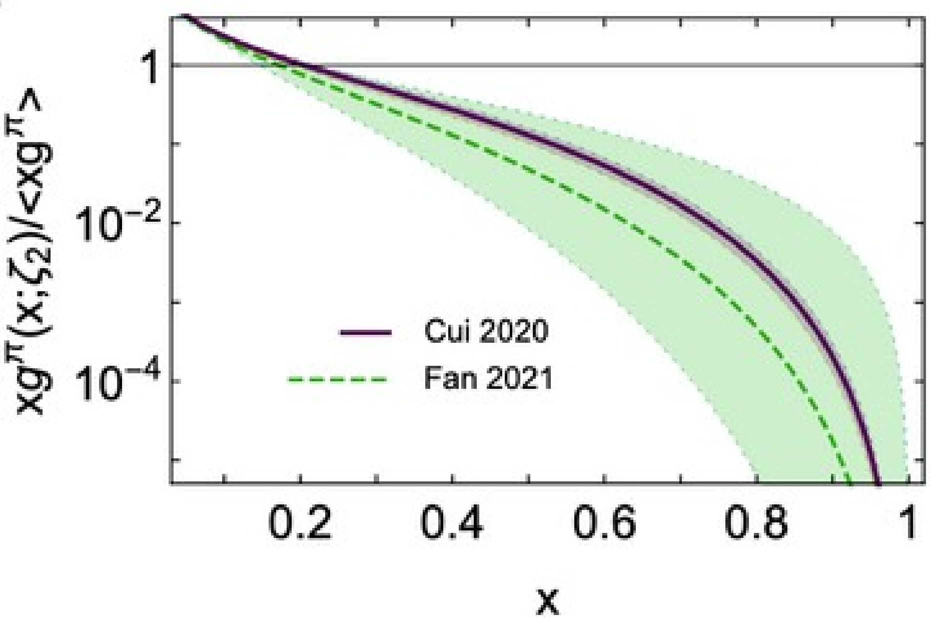
Effective charge for QCD evolution of pion PDF

Lei et al., [ArXiv:2106.08451]



Focus on glue DF and compare on the domain $x \in (0.001, 1)$ with recent lattice results: [Z. Fan and H-W. Lin, arXiv:2104.06372]

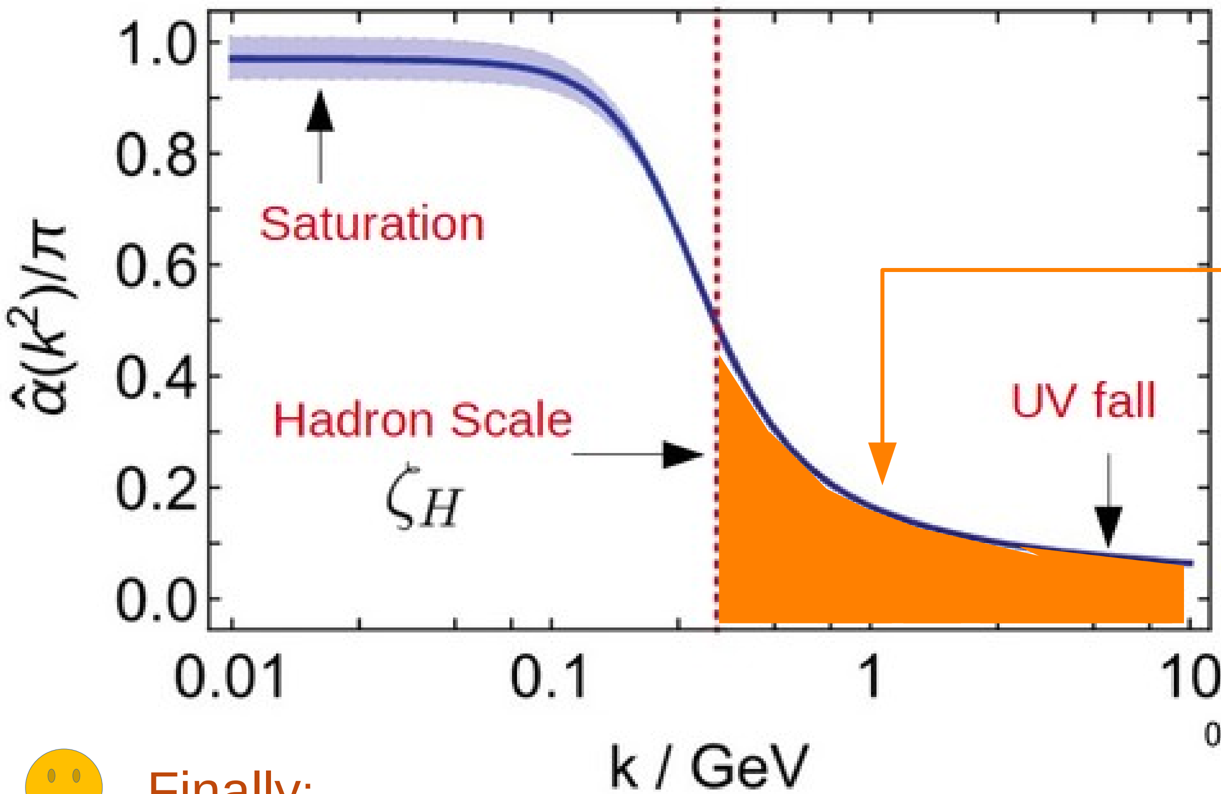
Highlight: pion's glue DF is obtained (via all-orders QCD evolution with an effective charge) from the valence-quark DF computed at the hadronic scale from a direct evaluation of Mellin moments in DSE/BSE [M. Ding et al, Chin.Phys.C44(2020)3,031002]).



Excellent agreement!

So far, QCD evolution is just encoded by the valence-quark momentum fraction at the final scale: $\langle 2x(\zeta_f)\rangle_q$

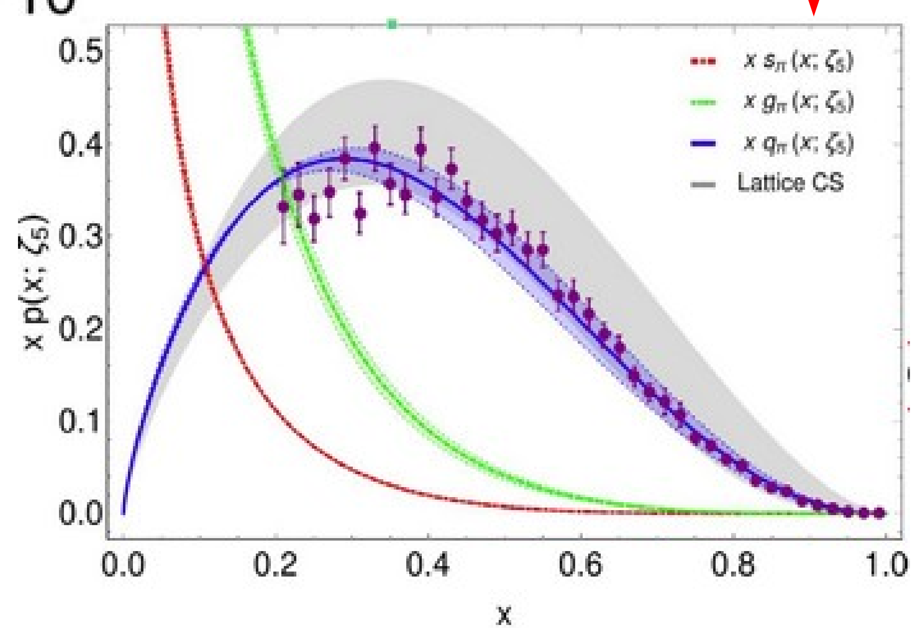
Effective charge for QCD evolution of pion PDF



The strength of the charge defines the input for the evolution

$$S(\zeta_H,\zeta_f) = \int_{2\ln(\zeta_H/\Lambda_{QCD})}^{2\ln(\zeta_f/\Lambda_{QCD})} dt\, \widehat{\alpha}(t)$$

$$\langle x(\zeta_5)\rangle_q^\pi = \frac{1}{2}\exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_5)\right) = 0.20(2)$$



Finally:

Then, the glue, valence- and sea-quark DFs can be predicted, with no tuned parameter, on the ground of the effective charge definition, from the symmetry-preserving DSE/BSE computation of the valence-quarks Mellin moments (and further reconstruction)

[M. Ding et al, CPC44(2020)3,031002]
[Z-F. Cui et al, EPJC80(2020)11,1064]
[Z-F. Cui et al, EPJA57(2021)1,5]

Conclusions

- 2-points and 3-points QCD Green's function obtained from a combination of lattice and continuum methods have been introduced and they feature a consistent emerging picture.

- A process-independent effective charge obtained from gauge-field two-point Green's function is shown, and shown to saturate at vanishing momentum, displaying no landau pole and opening therewith an IR "conformality window".

- This effective charge have been then applied to derive an "all-orders" QCD evolution scheme for PDFs, which succeeds in explaining results from lQCD and from the experiment, respectively, in the glue and valence-quark sectors.