Parton distribution function of nucleon

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Outline

QCD and dynamical mass generation

The key feature of QCD is the running mass behaviour:

Asymptotic free behavior at large energy scale

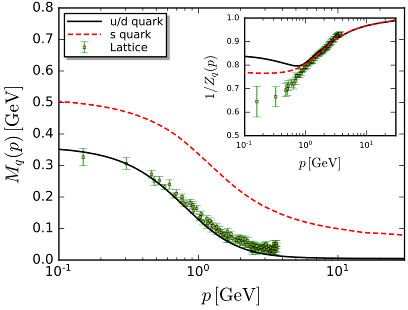
Dynamical mass generation at lower energy scale

For pion:

- massless at chiral limit as Goldstone Boson;

For nucleon (simply through the mass budget):

- roughly three constituent quark at hadron scale;
- with valence, sea quark, gluon for larger scale.



lattice:
P. O. Bowman et al, PRD71, 054507 (2005)
DSE:

FG, J. Papavassiliou, J. M. Pawlowski, PRD 102, 034027 (2020)

Parton distribution functions

Parton distribution functions (PDF) of nucleon:

- Experimentally:
  PDFs could be extracted through the Deep inelastic scattering process and Drell-Yan process.

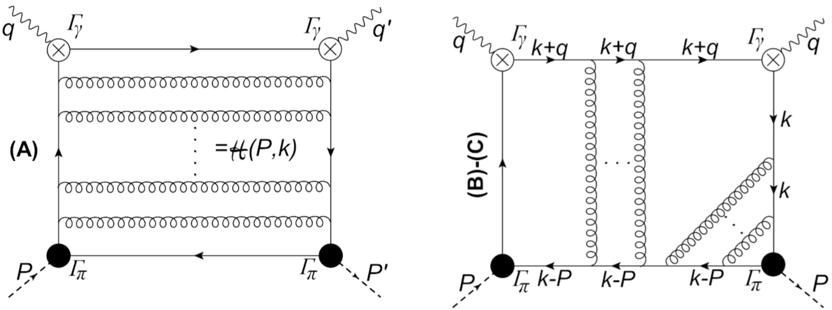
- Theoretically:
  PDF reveals the inside structure of nucleon directly as a function of the momentum fraction of partons;
  Especially, it is with great interest to understand the sea quark and glue distribution since the nucleon is no longer just three valence quarks at large scale.

at hadron scale, only valence quark PDF;
glue and sea distribution generates through DGLAP evolution to larger scale.

Diagrams for PDF

Considering the quark-diquark model for nucleon.

A corrected leading-order expression of parton distribution function includes two diagrams:



The first diagram can be described in terms of the derivative of propagator based on ward identity:

$$\Gamma_n = n_\mu \frac{\partial S(k \pm P)}{\partial k_\mu} \tag{1}$$

The Second diagram can be similarly described by the derivative of the vertex:

$$\tilde{\Gamma}_n = n_\mu \frac{\partial \Gamma(k; P)}{\partial k_\mu} \tag{2}$$

Only by considering both diagrams, people can obtain the correct momentum sum rule:

- without meson-cloud corrections and dressed-gluon distribution , $< x >_q^\pi = \frac{1}{2}$

Quark-diquark model for nucleon

Considering the quark-diquark model for nucleon. Giving the algebraic model for quark propagator $S(k)$, diquark propagator $D^{s,av}(k)$ and the quark-diquark amplitude $\Gamma^{s,av}(k;P)$as followings:

$$
\begin{aligned}
S^{-1}(k) &= i\not{k} + M, \\
D^{s}(k) &= \frac{1}{k^2 + M_s^2} \\
D^{av}(k) &= (\delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}) \frac{1}{k^2 + M_{av}^2}, \\
\Gamma^{s}(k;P) &= \int_{-1}^{1} dz \rho(z) \big(F(k_z)^3 + i(\not{k} - \frac{\not{P} k \cdot P}{P^2}) F(k_z)^{5/2} \big), \\
\Gamma^{av}_{\mu}(k;P) &= \lambda \frac{1}{\sqrt{3}} (\gamma_\mu - \frac{P_\mu k \cdot P}{P^2}) \int_{-1}^{1} dz \rho(z) F(k_z)^{5/2},
\end{aligned}
\tag{3}
$$

There will exist $[ud]$-scalar diquark, $[ud]$-axial vector and $[uu]$-axial vector diquark.

quark distribution in diquark

The quark distribution in diquark can be taken into account by:

$$\tilde{u}_{s,av} = \int_x^1 \frac{1}{y} f^{s,av}(y) f_{q/s,av}(x/y).$$

where $f_{q/s,av}$ is chosen as:$f_{q/s,av}(z) = 30z^2(1-z)^2$. The final parton distribution functions of u and d quark are:

$$\begin{aligned} u(x) &= f^u(x) + \tilde{u}^s(x) + 5\tilde{u}^{av}(x) \\ d(x) &= f^d(x) + \tilde{d}^s(x) + \tilde{d}^{av}(x) \end{aligned}$$

$x \to 1$

Behaviour at $x \to 1$:

The denominator in the integrand of the pdf is:

$$\frac{1}{(k_- P_+ (x + (z-1)/2) + k_\perp^2 + M^2)^a (k_- P_+ (x + (z'-1)/2) + k_\perp^2 + M^2)^a} \times \frac{1}{(k_- P_+ x + k_\perp^2 + M^2)^2 (k_- P_+ (x-1) + k_\perp^2 + M^2)}.$$

If employing $\rho(z) = (1 - z^2)$ which leads to the asymptotic behaviour of QCD, the denominator always goes to $(1-x)^5$. The behaviour of the numerator,

- The quark pdf includes $L = 0$ of scalar diquark contributes $(1-x)^0$ and thus, totally, it will be $(1-x)^5$.
- The quark pdf from $L = 1$ and also the axial vector diquark is $(1-x)^5 \times \frac{k_\perp^2}{(1-x)^2} \sim (1-x)^3$.

$x \to 1$

For the diquark pdf, the direct computation gives:

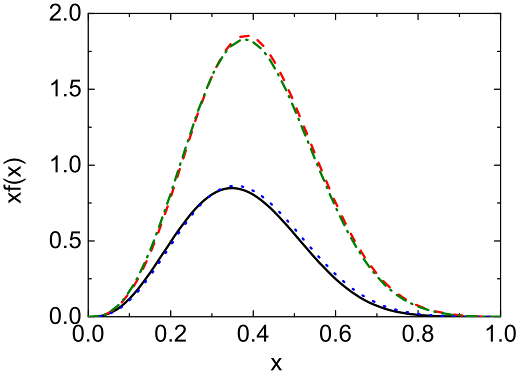
From $L = 0$ of scalar diquark is $(1-x)^4$ and from the others are $(1-x)^2$, which is harder than the quark distribution. This is because we employ a symmetric distribution for quark-diquark system, which is not the case. Here we modify it as $\rho(z) = (1-z^2)(1-z)$, then the diquark distribution has the same large-x behaviour.

The quark distribution inside diquark:

From $L = 0$ of scalar diquark is $(1-x)^8$ and from the others are $(1-x)^6$.

- *The behaviour of nucleon pdf at $x \to 1$ is $(1-x)^3$*
- *The leading contribution comes from the quark distribution with $L = 1$ scalar diquark and also the axial vector diquark.*

The valence-quark pdf $u(x)$ and $d(x)$ at $Q^2 = \zeta_H^2$ with different proportion of axial vector diquark component:



The PDF satisfy the relation:

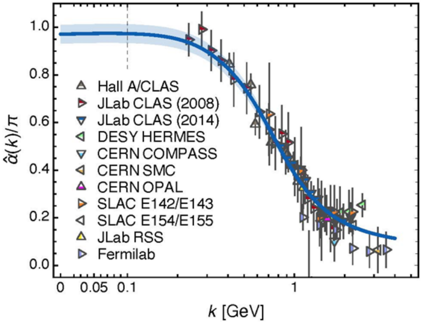
$$\int dxu(x) = 2 \int dxd(x) = 2$$

$$\int dxx(u(x) + d(x)) = 1$$

DGLAP evolution and effective charge

It naturally implies a hadron scale $\zeta_H \sim 0.33$ GeV:

- below ζ_H, the evolution is frozen.
- above ζ_H, sea quarks and glue distributions are generated through evolution.



DGLAP with all orders starting from ζ_H:

Details seen in Prof. Jose RODRIGUEZ-QUINTERO's talk.

$$\frac{d}{dt}q(x;t) = -\frac{\alpha_s(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y;t)P(\frac{x}{y})$$

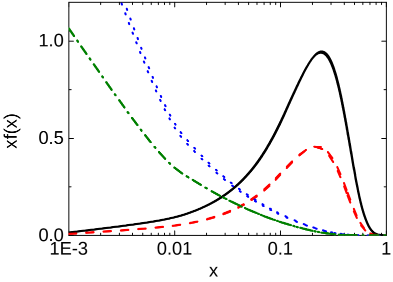
A. Deur, S. J. Brodsky and G. F. de Teramond, PPNP 90 (2016) 1-74

Daniele Binosi et al., PRD 96 (2017) 054026/1-7

Zhu-Fang Cui et al., CPC 44 (2020) 083102/1-10

pdf at 2 GeV

After evolution, Distribution of u, d, s, $\bar{u}$, $\bar{d}$, $\bar{s}$, *glue* at 2 GeV:

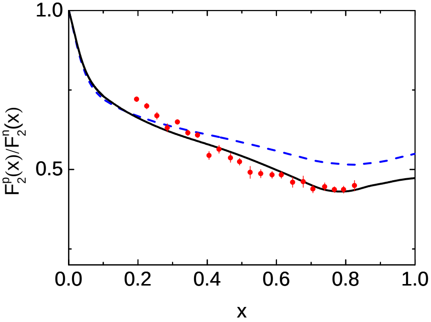


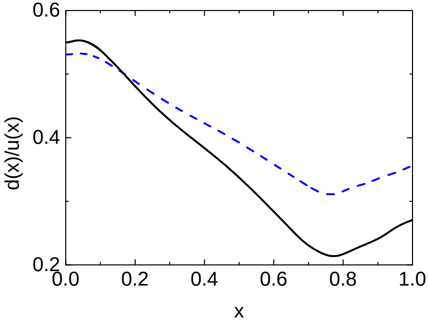
$\lambda = 0.355$	u_v	d_v	g	$\bar{u}_s$	$\bar{d}_s$	$\bar{s}_s$
ζ_H	0.68	0.32	0	0	0	0
2 GeV	0.36	0.17	0.40	0.024	0.024	0.024
$\lambda = 0.6$	u_v	d_v	g	$\bar{u}_s$	$\bar{d}_s$	$\bar{s}_s$
ζ_H	0.68	0.32	0	0	0	0
2 GeV	0.36	0.17	0.40	0.024	0.024	0.024

scale evolution

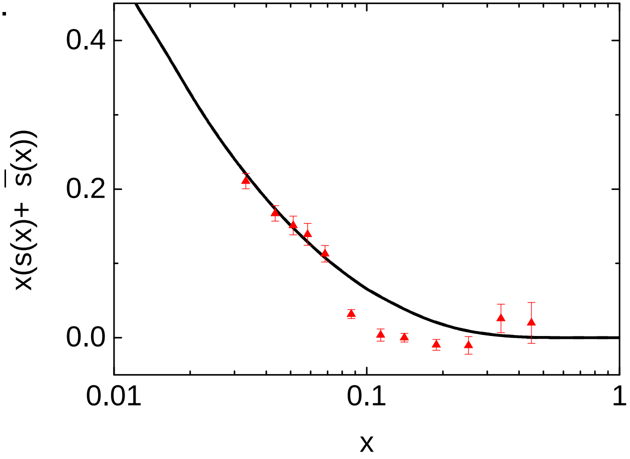
The MARATHON data of $F_2^p(x)/F_2^n(x)$ from the data of $^3H/^3He$. The ratio of $F_2^p(x)/F_2^n(x)$ can be expressed with quark distribution as:

$$\frac{F_2^p(x)}{F_2^n(x)} = \frac{(u(x)+\bar{u}(x)) + 4(d(x)+\bar{d}(x)) + (s(x)+\bar{s}(x))}{4(u(x)+\bar{u}(x)) + (d(x)+\bar{d}(x)) + (s(x)+\bar{s}(x))}$$





The strange quark distribution is also obtained which coincide with the HERMES data very well.



analysis of flavour asymmetry

Analysis of flavour asymmetry from DGLAP evolution:

- with flavour changing process, for example, $u \to d$ via pion, the evolution $\delta f = f_{\bar{d}}(x, Q) - f_{\bar{u}}(x, Q)$ is

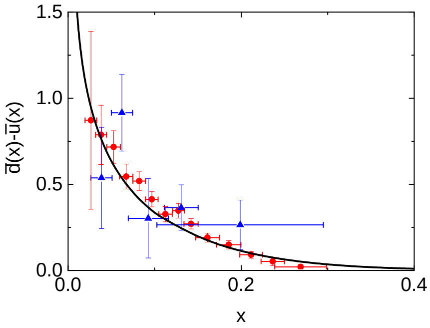
$$\frac{d}{dlogQ}\delta f = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} (P_q - P_{ud})\delta f$$

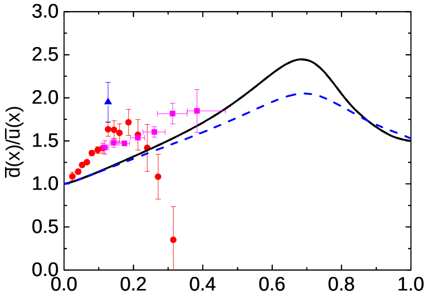
  and thus do not contribute to flavour asymmetry.
- only with $u/d \to \pi$ process, and with asymmetry of pi^+ and pi^- distribution $\delta f_\pi = f_{\pi^+} - f_{\pi^-}$:

$$\frac{d}{dlogQ}\delta f_{\bar{q}} = \frac{\alpha_{s/\pi}}{\pi} \int_x^1 \frac{dz}{z} \left[P_{\bar{q}\leftarrow\bar{q}}\delta f_{\bar{q}} + P_{\bar{q}\leftarrow\pi}\delta f_\pi\right]$$

$$\frac{d}{dlogQ}\delta f_\pi = \frac{\alpha_\pi}{\pi} \int_x^1 \frac{dz}{z} \left[P_{\pi\leftarrow q,\bar{q}}(f_{u_v} - f_{d_v}) + C\delta(1-z)\delta f_\pi\right]$$

The difference could be directly obtained from above evolution equation, which is very well consistent with the NuSea/E866-E906 and HERMAS data





Comparison of the zeroth moment of $\bar{d} - \bar{u}$:

x_{min}	x_{max}	$\int_{x_{min}}^{x_{max}}(\bar{d}-\bar{u})$	Q^2	sources	Ref.
0.0	1	0.147±0.39	4	NMC	M. Arneodo et al. PRD 50, 1(1994)
0.0	1	0.118±0.012	54	NUSEA	R. Towell et al., PRD 64, 052002 (2001)
0.001	1	0.114	54	CT10nlo	S. Dulat et al., PRD 93, no.3, 033006 (2016)
0.001	1	0.116	4	CT10nlo	S. Dulat et al., PRD 93, no.3, 033006 (2016)
0.0	1	0.13(7)	4	Lattice	H. W. Lin, PoS LATTICE2016, 005 (2016)
0.0	1	0.116	4	this work	

In summary

Results for nucleon PDF:

A corrected leading-order expression of parton distribution function is employed here to compute the nucleon pdf in the quark-diquark picture

- The $x \to 1$ behaviour of nucleon pdf is $(1-x)^3$, which is contributed from the quark distribution with the $L = 1$ component of scalar diquark and also axial quark;
- d and u valence-quark distribution at hadron scale have been obtained
- After considering the $q \to \bar{q}\pi$ process in the DGLAP evolution, the sea asymmetry is induced by the valence quark asymmetry.