Recent Advances in Global Analyses of Pion PDFs

Patrick Barry (Jefferson Lab)

HADRON 2021

Jefferson Lab
Exploring the Nature of Matter

Jefferson Lab Angular Momentum Collaboration

What do we want?

To study the makeup of nuclear matter

Building blocks of nature are quarks and gluons

What’s the problem?

Quarks and gluons are not directly measurable!

Motivation

- QCD allows us to study the structure of hadrons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

Game plan

What to do:

- Define a structure of hadrons in terms of quantum field theories
- Identify theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
- Perform global QCD analysis as structures are universal and are the same in all processes

Complicated Inverse Problem

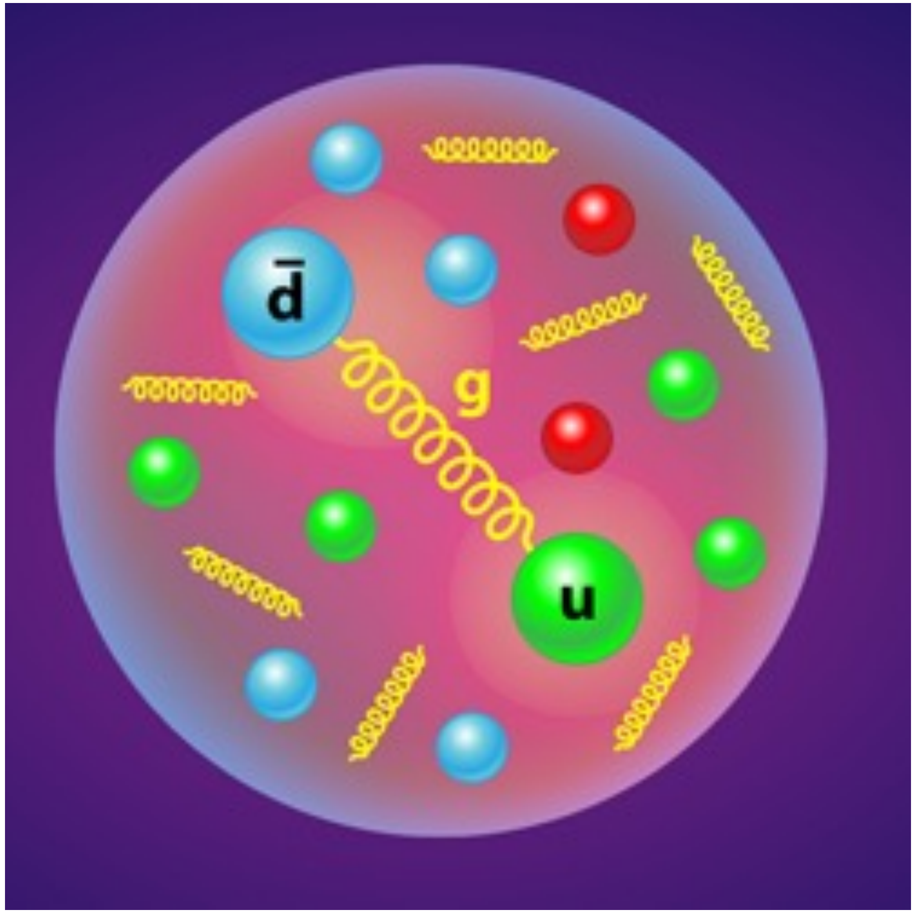
- Factorization theorems involve convolutions of hard perturbatively calculable physics and non-perturbative objects

$$\frac{d\sigma}{d\Omega} \propto \mathcal{H} \otimes f = \int_x^1 \frac{d\xi}{\xi} \mathcal{H}(\xi) f\left(\frac{x}{\xi}\right)$$

- Parametrize the non-perturbative objects and perform global fit

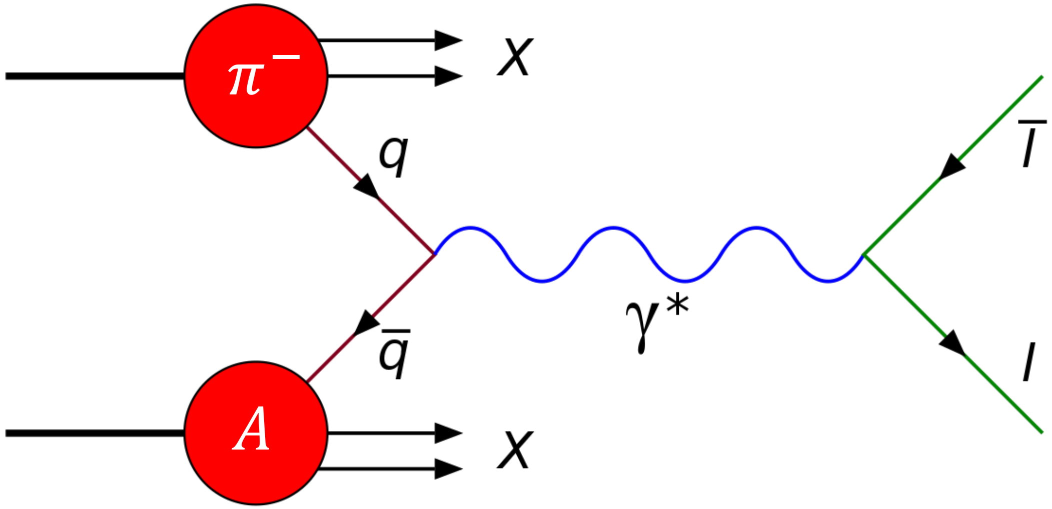
Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a pseudoscalar meson made up of q and $\bar{q}$ constituents



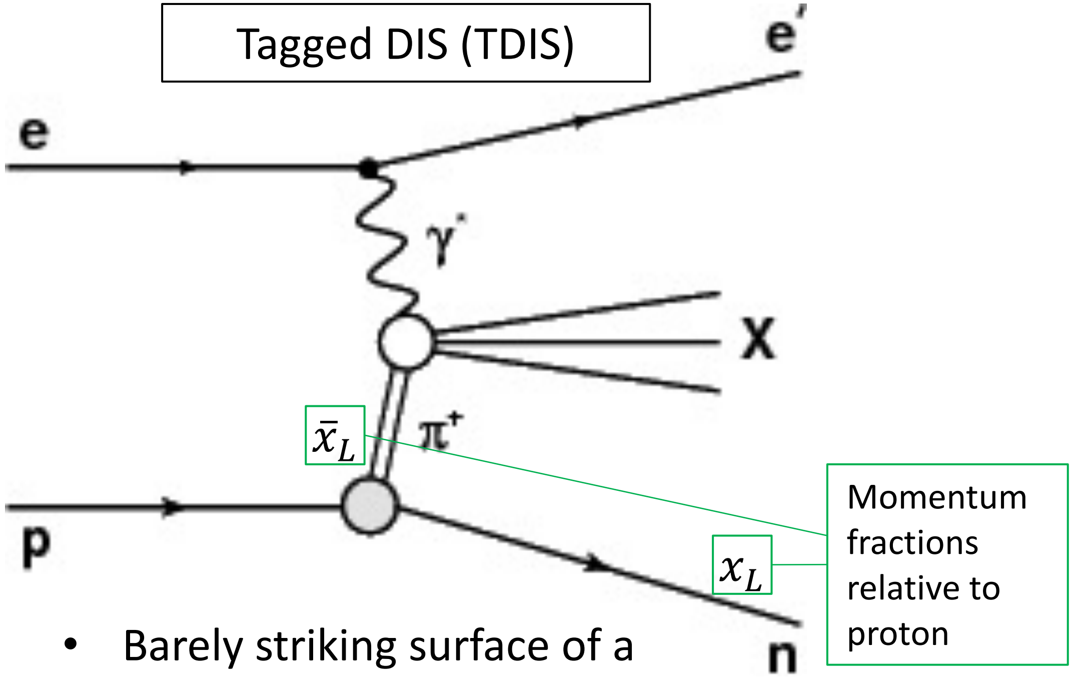
Experiments to Probe Pion Structure

- Drell-Yan (DY)



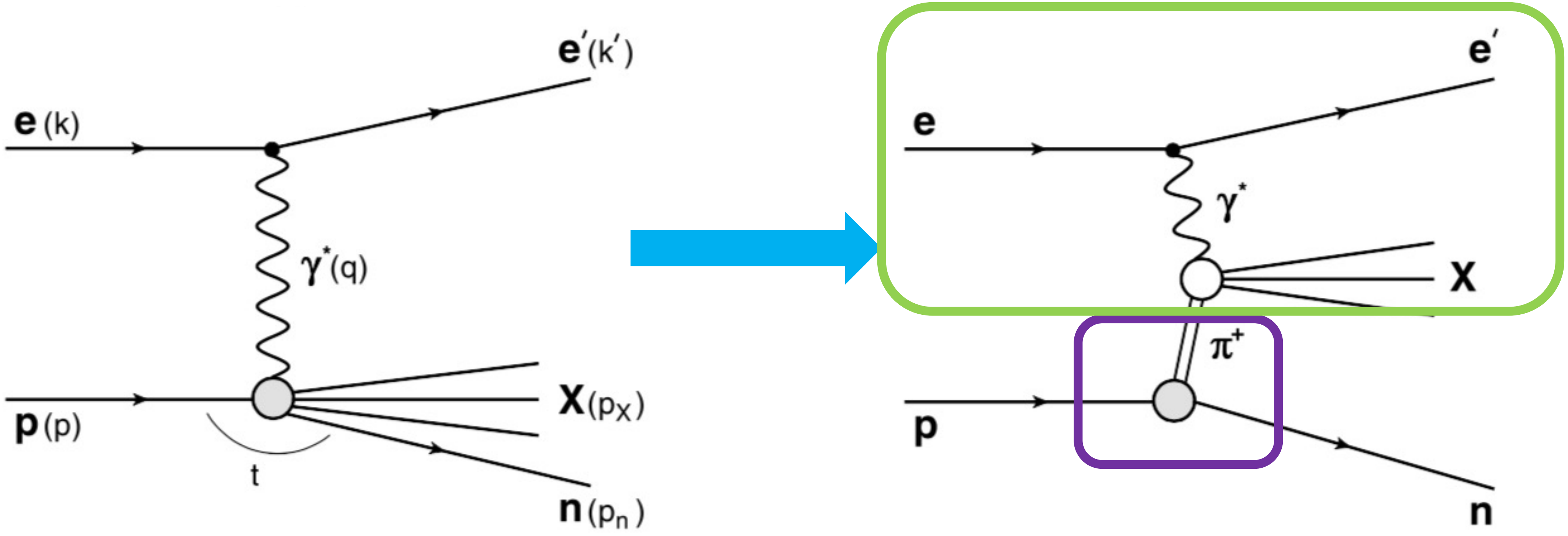
- Accelerating pion allows for time dilation and longer lifetime

- Leading Neutron (LN)



- Barely striking surface of a target proton knocks out an almost on-shell pion to probe

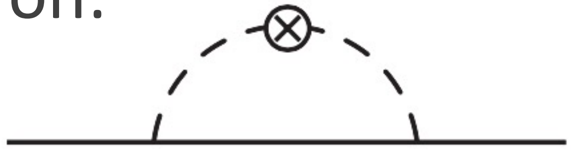
Leading Neutron (LN)



$$\frac{d\sigma}{dx\,dQ^2\,d\bar{x}_L} \propto f_{\pi N}(\bar{x}_L) \times \sum_i \int_{x/\bar{x}_L}^{1} \frac{d\xi}{\xi} C(\xi) f_i(\frac{x/\bar{x}_L}{\xi}, \mu^2)$$

Splitting Function and Regulators

Amplitude for proton to dissociate into a π^+ and neutron:



$$f_{\pi N}(\bar{x}_L) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{\bar{x}_L \left[k_\perp^2 + \bar{x}_L^2 M^2\right]}{x_L^2\, D_{\pi N}^2} |\mathcal{F}|^2,$$

$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1-y}[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]$$

Best fit

$$\mathcal{F} = \begin{cases} \text{(i)} & \exp\left((M^2 - s)/\Lambda^2\right) & s\text{-dep. exponential} \\ \text{(ii)} & \exp\left(D_{\pi N}/\Lambda^2\right) & t\text{-dep. exponential} \\ \text{(iii)} & (\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & t\text{-dep. monopole} \\ \text{(iv)} & \bar{x}_L^{-\alpha_\pi(t)} \exp\left(D_{\pi N}/\Lambda^2\right) & \text{Regge} \\ \text{(v)} & \left[1 - D_{\pi N}^2/(\Lambda^2 - t)^2\right]^{1/2} & \text{Pauli-Villars} \end{cases}$$

- We examine five regulators, and we fit Λ
- $\mathcal{F}$ is a UV regulator, which the data chooses

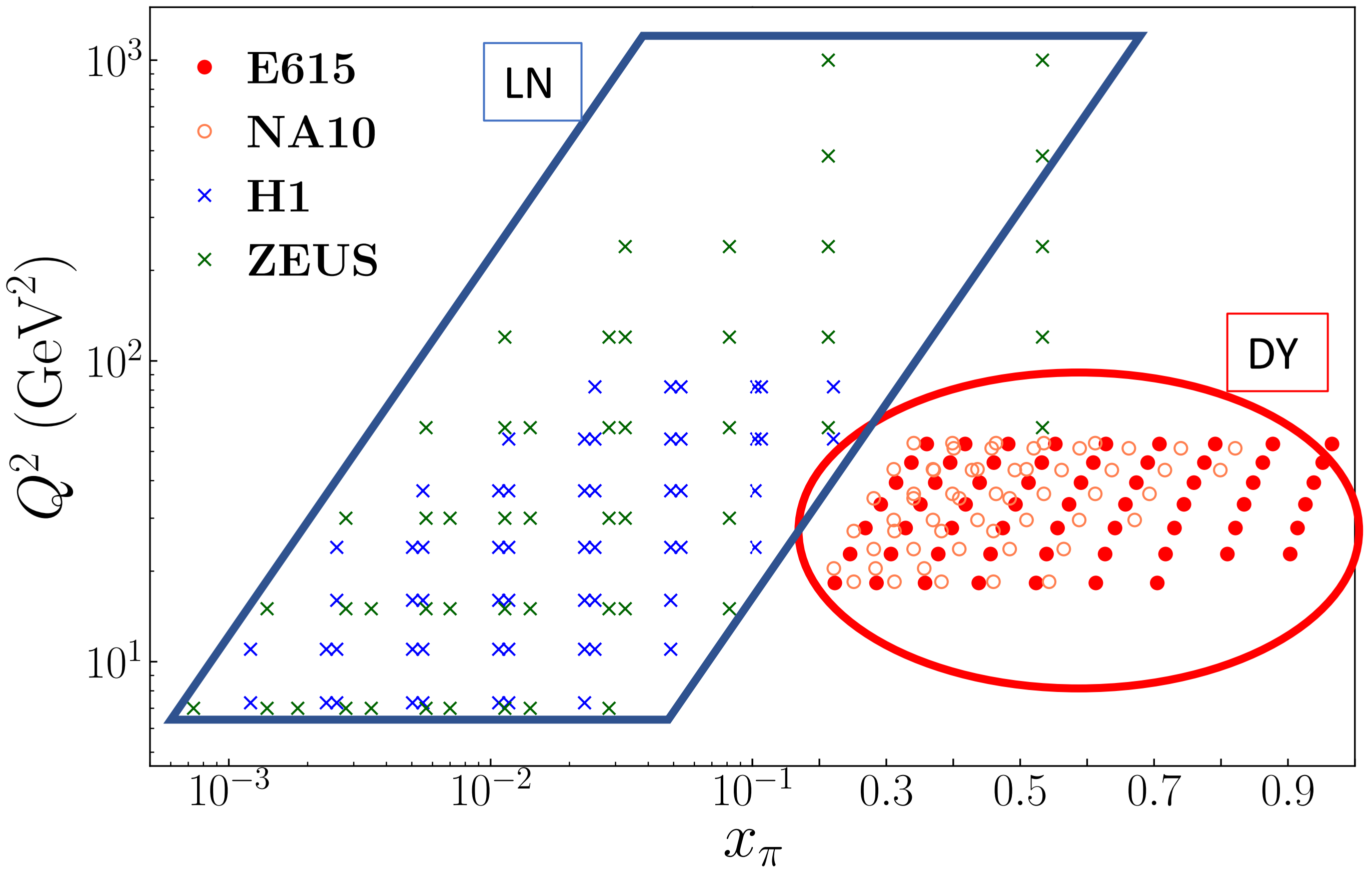
Datasets -- Kinematics

- Large x_π -- Drell-Yan (DY)
- Small x_π -- Leading Neutron (LN)
- Not much data overlap
- In DY:

$$x_\pi = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right)$$

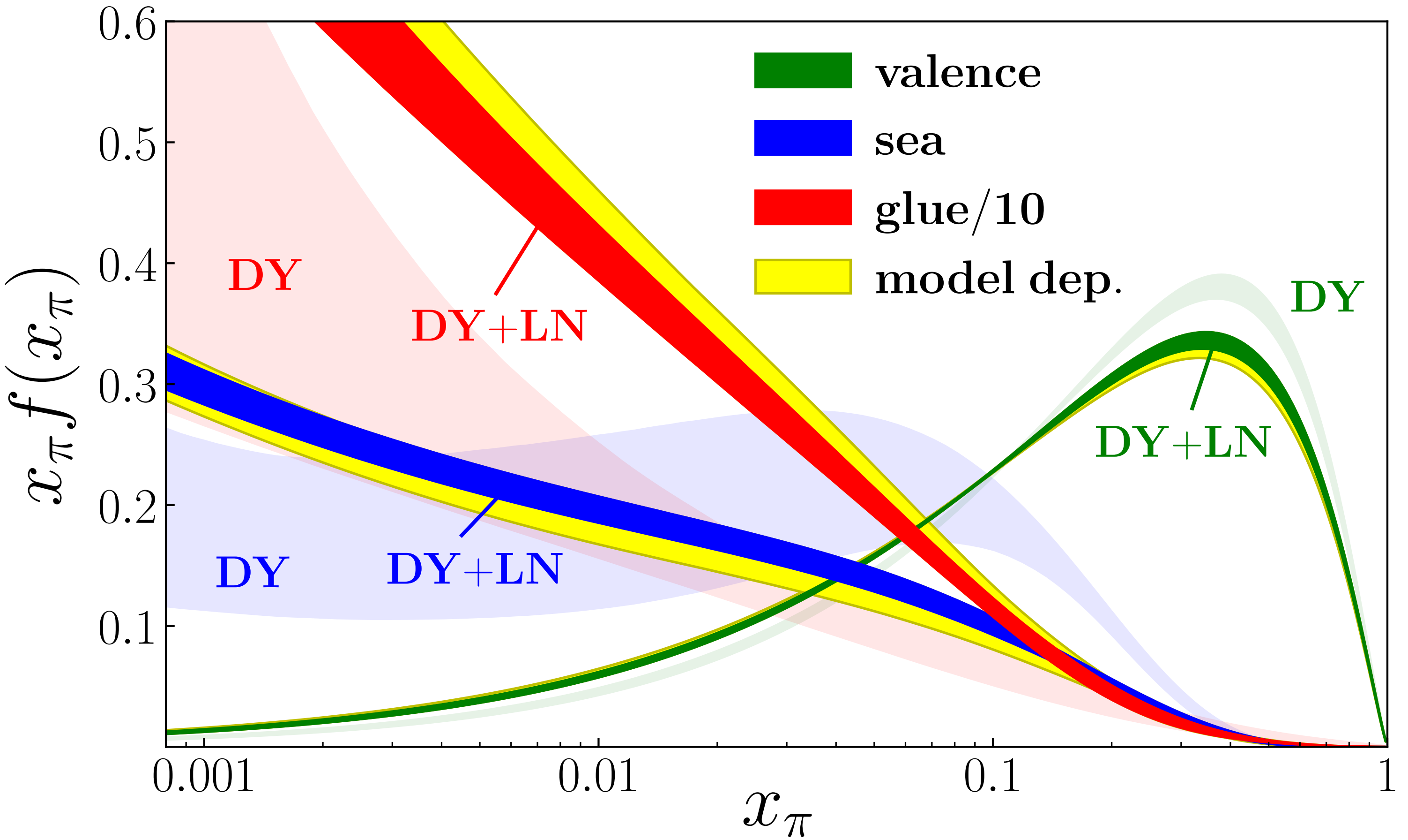
- In LN:

$$x_\pi = x_B / \bar{x}_L$$



JAM18 Pion PDFs

- Lightly shaded bands – only Drell-Yan data
- Solid bands – fit to both Drell-Yan and LN data



PCB, N. Sato, W. Melnitchouk and Chueng-Ryong Ji, Phys. Rev. Lett. **121**, 152001 (2018).

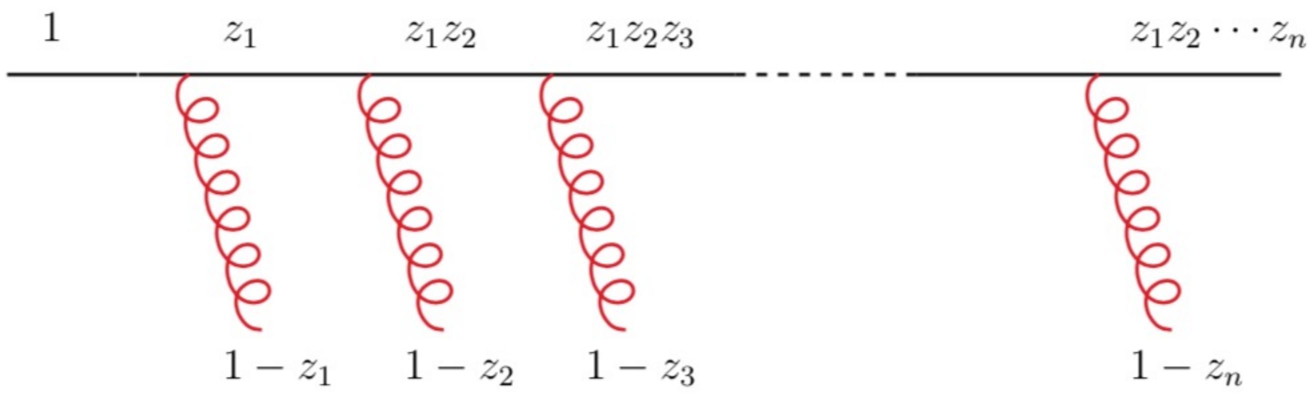
Large-x_π behavior

- Generally, the parametrization lends a behavior as $x_\pi \to 1$ of the valence quark PDF of $q_v(x) \propto (1-x)^\beta$
- For a fixed order analysis, we find $\beta \approx 1$
- Debate whether $\beta = 1$ or $\beta = 2$
- Aicher, *et al.* (2010) found $\beta = 2$ with threshold resummation

Threshold Resummation in Pion Drell-Yan

PCB, Chueng-Ryong Ji (NCSU), N. Sato (Jefferson Lab), W. Melnitchouk (Jefferson Lab)

Soft Gluon Resummation



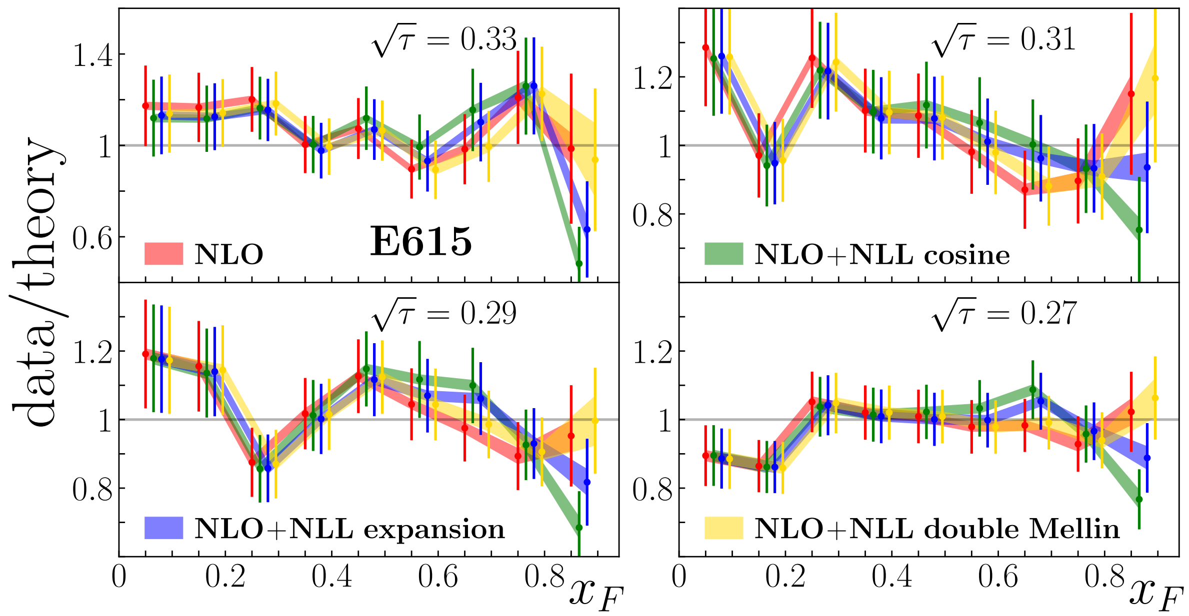
- Fixed-target Drell-Yan notoriously has large-x_F contamination of higher orders
- Large logarithms may spoil perturbation
- Focus on corrections to the most important $q\bar{q}$ channel
- Resum contributions to all orders of α_s

Methods of Resummation

- Resummation is performed in conjugate space
- Drell-Yan data needs two transformations
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects

Data and Theory Comparison – Drell-Yan

- Cosine method tends to overpredict the data at very large x_F
- Double Mellin method is qualitatively very similar to NLO
- Resummation is largely a high-x_F effect

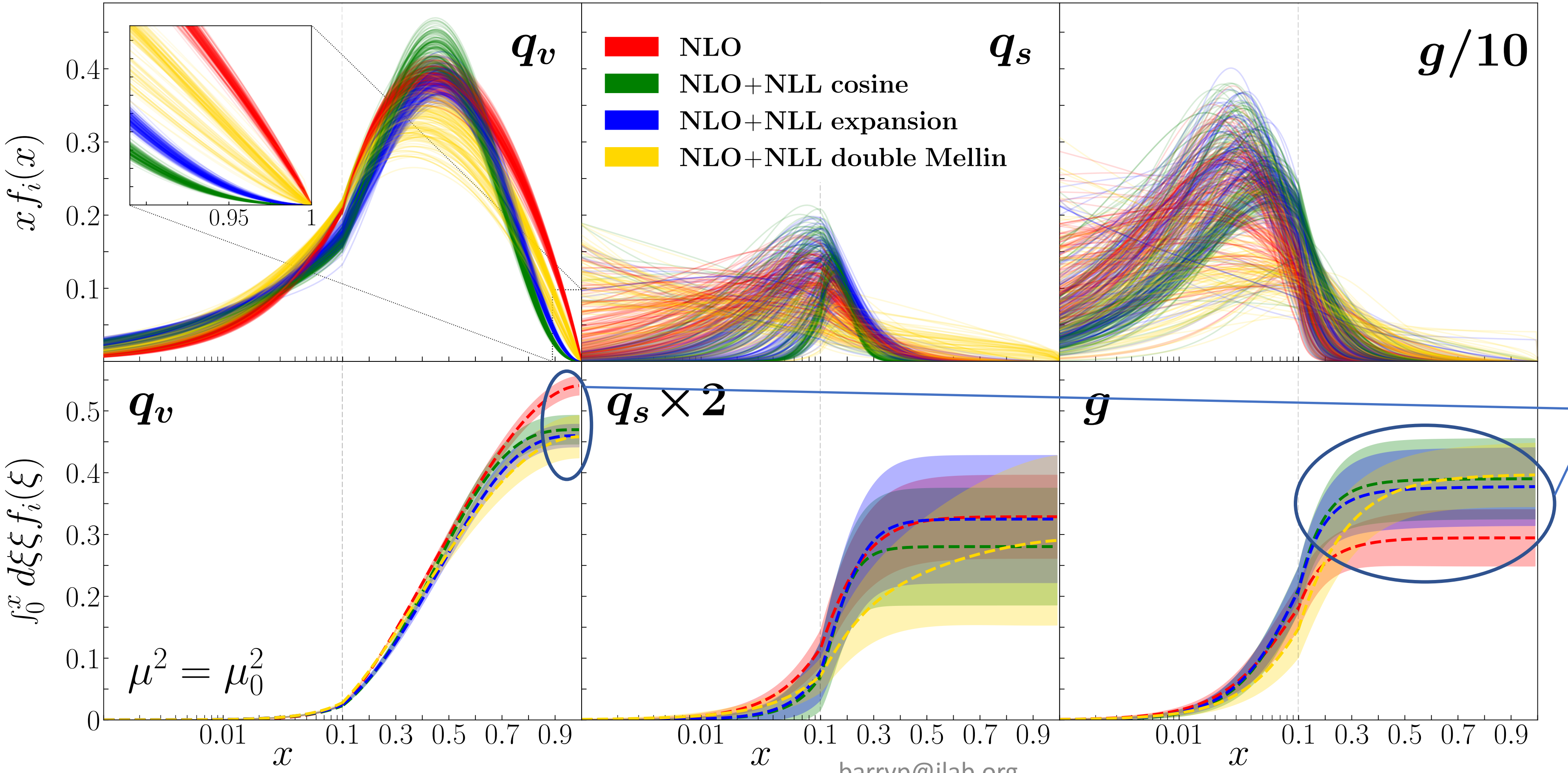


Method	χ^2/npts
NLO	0.85
NLO+NLL cosine	1.29
NLO+NLL expansion	0.95
NLO+NLL double Mellin	0.80

Slightly disfavored

PDF Results

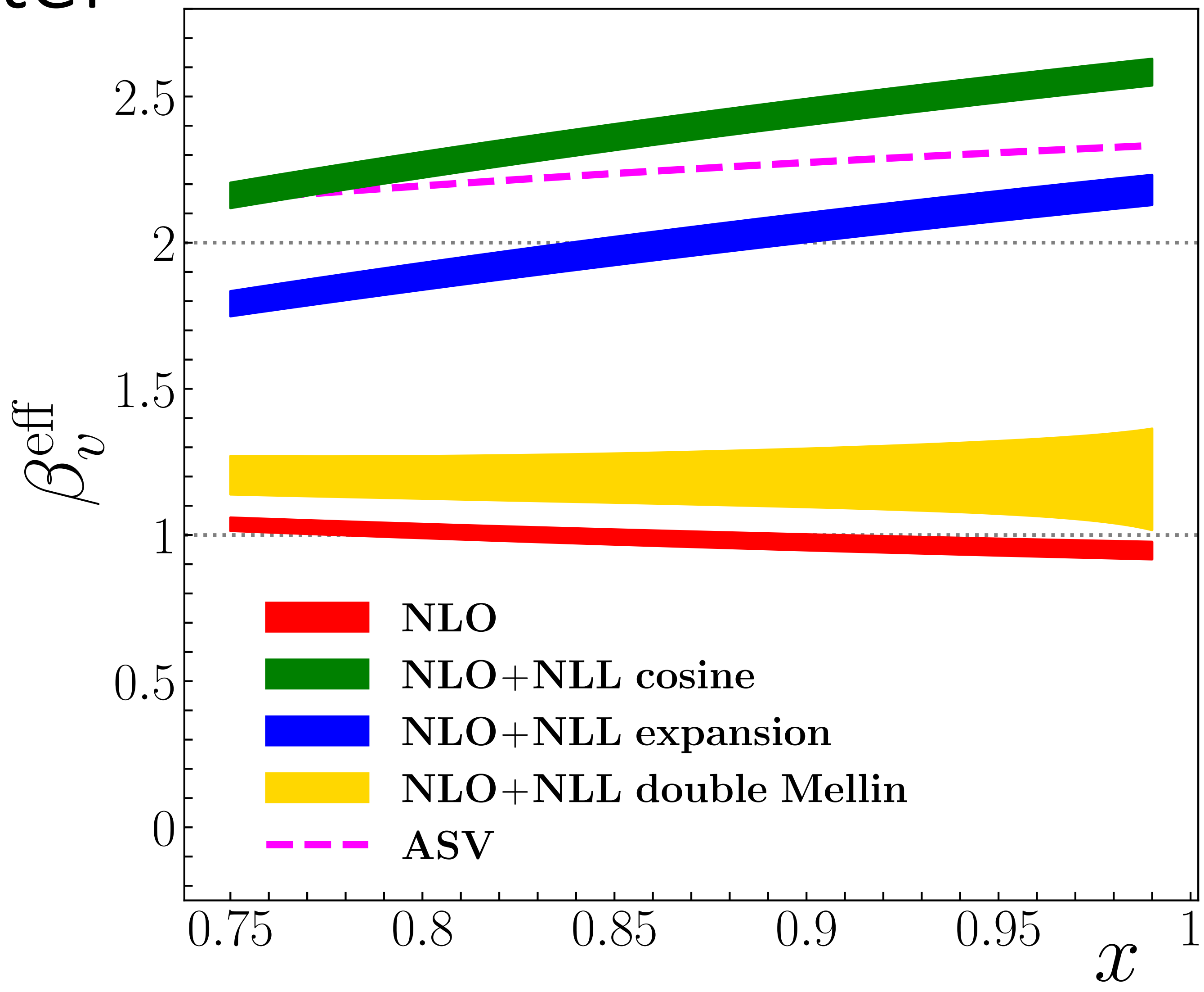
- Large x behavior in valence depends on prescription



Large x momentum fraction in the valence shifts to low x gluon momentum fraction

Effective β_v parameter

- $q_v(x) \sim (1-x)^{\beta_v}$ as $x \to 1$
- Threshold resummation does not give universal behavior of β_v
- NLO and double Mellin give $\beta_v \approx 1$
- Cosine and Expansion give $\beta_v > 2$



Transverse Momentum Dependent Drell-Yan

PHYSICAL REVIEW D **103,** 114014 (2021)

Towards the three-dimensional parton structure of the pion: Integrating transverse momentum data into global QCD analysis

N. Y. Cao,[1] P. C. Barry,[2,3] N. Sato,[3] and W. Melnitchouk[3]

Jefferson Lab Angular Momentum (JAM) Collaboration

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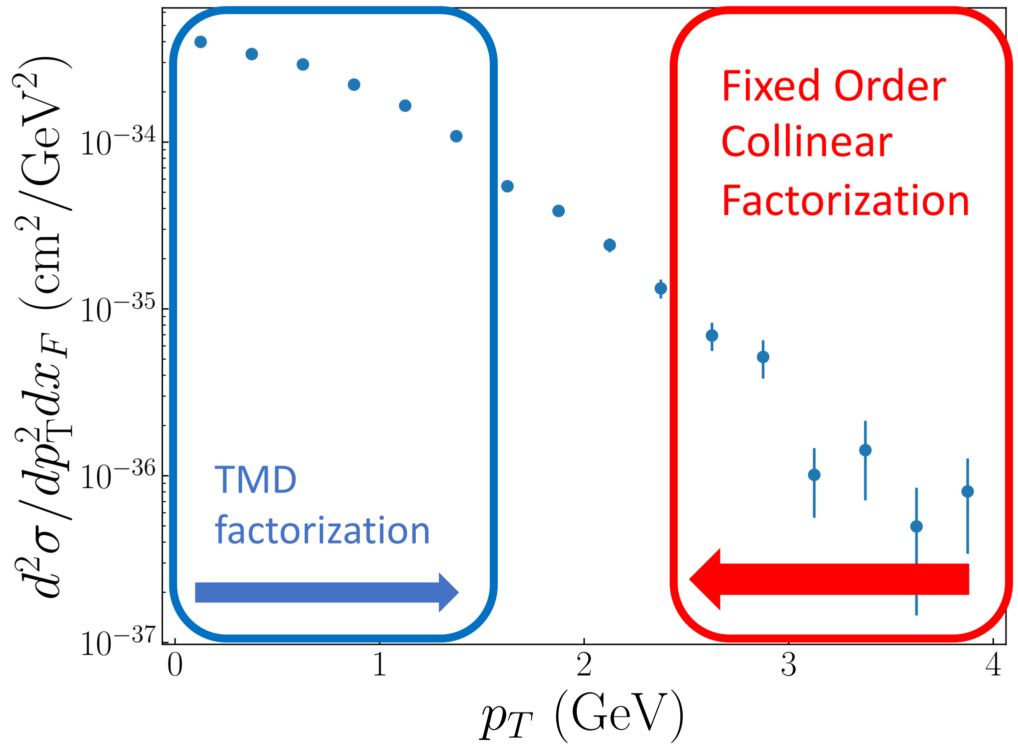
[2]*North Carolina State University, Raleigh, North Carolina 27607, USA*
[3]*Jefferson Lab, Newport News, Virginia 23606, USA*

p_T-dependent spectrum for pion data

- Small-p_T data – TMD factorization – partonic transverse momentum
- Large-p_T data – collinear factorization – recoil transverse momentum

See L. Gamberg on Wed. @ 10:25am



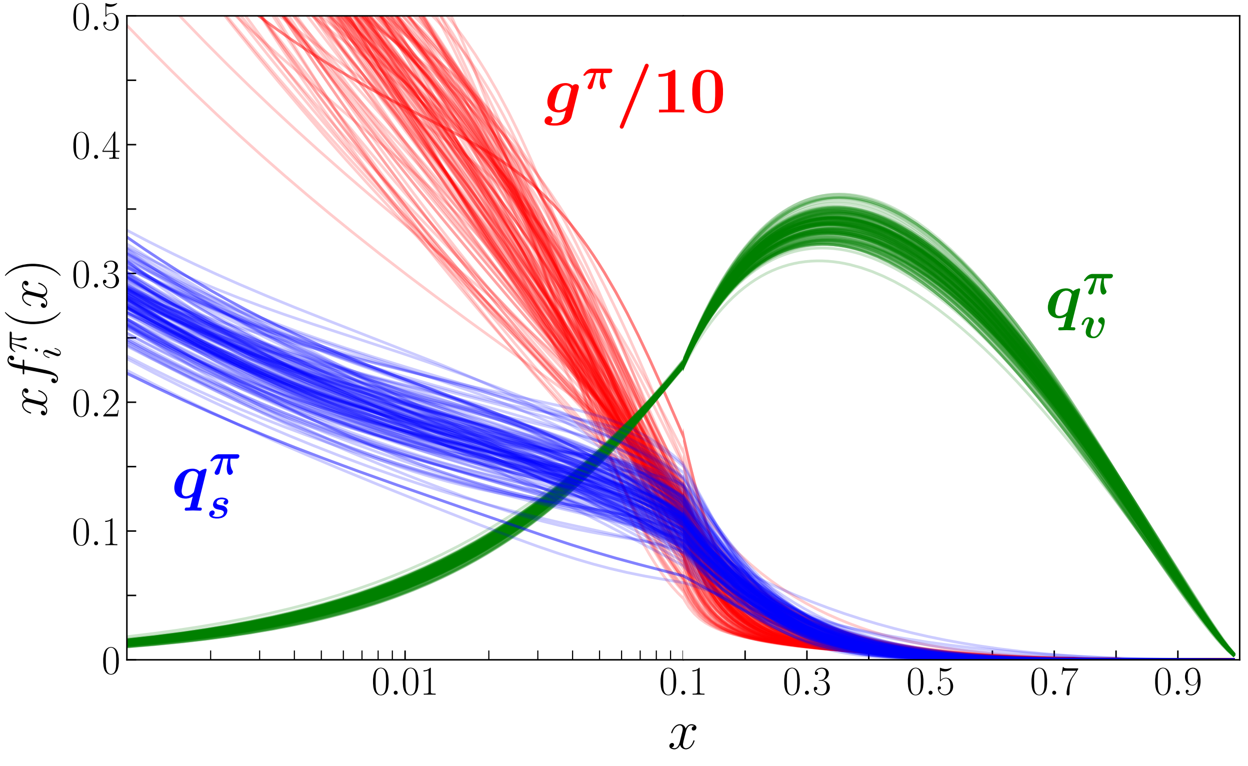
E615 πW Drell-Yan

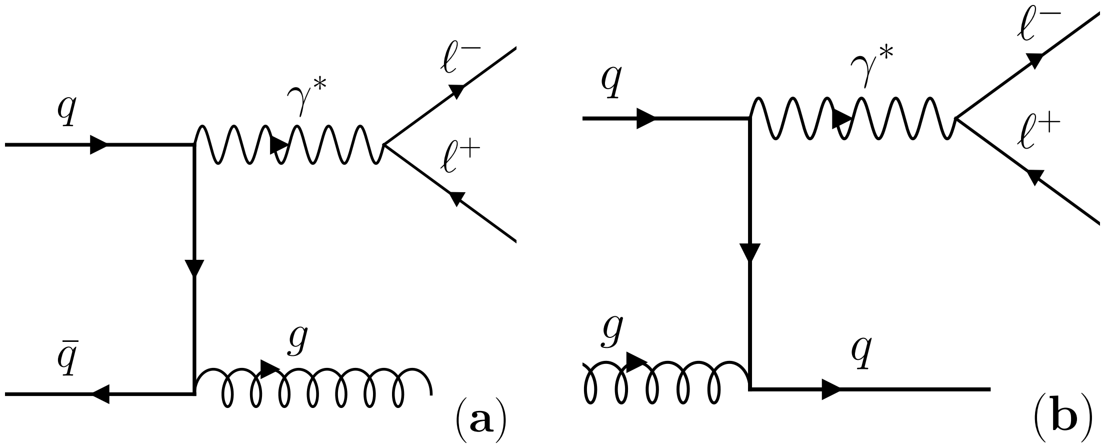
Phys. Rev. D **39**, 92 (1989).

This talk

JAM20 Pion PDFs

Fixed Order Analysis





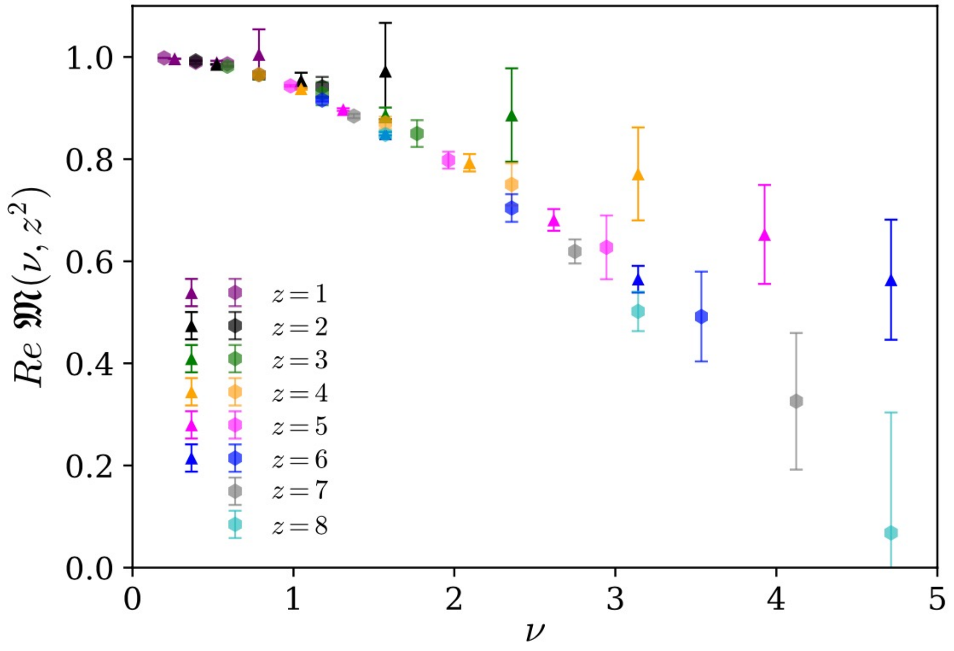
- For the first time, we included large p_{T}-dependent Drell-Yan data, which follows collinear factorization
- Large p_{T} does not dramatically affect the PDF
- Successfully describe data with a scale $\mu = p_{\mathrm{T}}/2$

Inclusion of Lattice Data

PCB, J. Karpie (Columbia), W. Melnitchouk (Jefferson Lab), C. Monahan (William & Mary, Jefferson Lab), K. Orginos (William & Mary, Jefferson Lab), Jian-Wei Qiu (Jefferson Lab), D. Richards (Jefferson Lab), N. Sato (Jefferson Lab), R. S. Sufian (William & Mary, Jefferson Lab), S. Zafeiropoulos (Aix Marseille Univ.)

Lattice data to examine from JLab Hadstruct

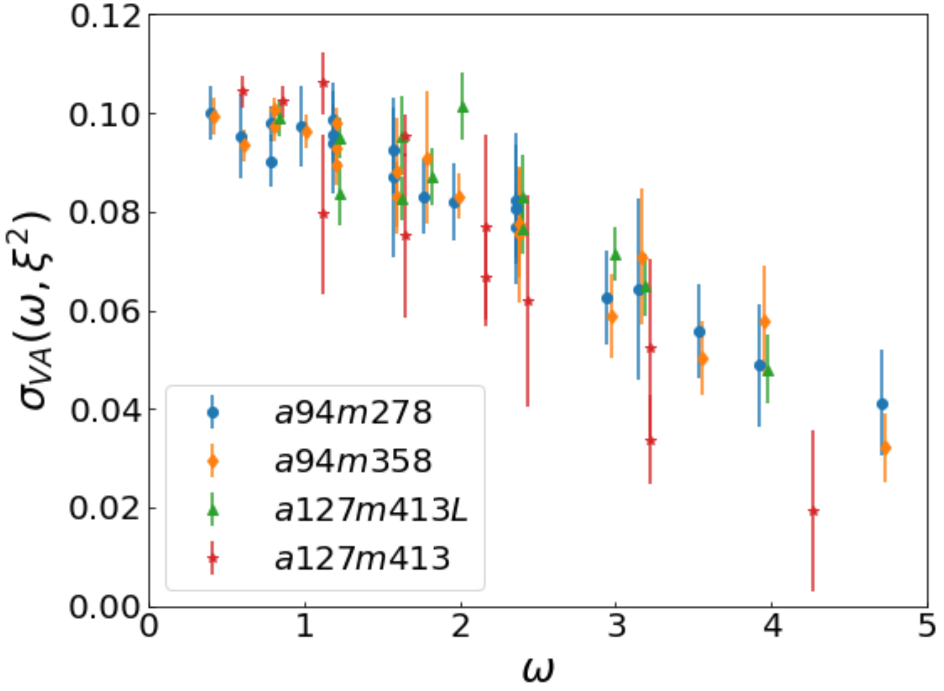
- Reduced pseudo Ioffe time distributions



More precise

B. Joó, J. Karpie, K. Orginos, A. V. Radyushkin, D. G. Richards, R. S. Sufian and S. Zafeiropoulos, Phys. Rev. D **100**, 114512 (2019).

- Current-Current Correlators



Noisier with large uncertainties

R. S. Sufian, C. Egerer, J. Karpie, R. G. Edwards, B. Joó, Y. Q. Ma, K. Orginos, J. W. Qiu and D. G. Richards, Phys. Rev. D **102**, 054508 (2020).

Connection of PDFs with Lattice Data

- Calculate the theoretical observable in a similar fashion as dealing with experimental data
- "Good lattice cross section" with matching is shown by

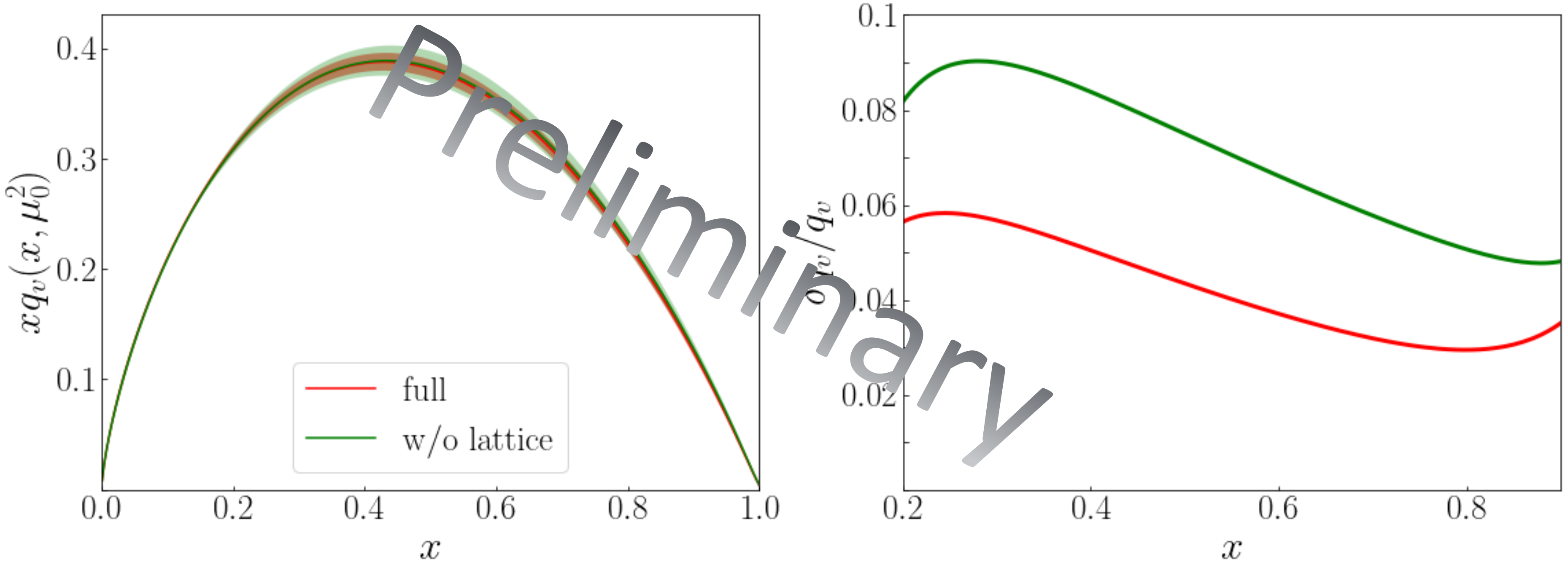
$$\sigma_{n/h}(\omega, \xi^2) \equiv \langle h(p) | T\{\mathcal{O}_n(\xi)\} | h(p) \rangle$$
$$= \sum_i f_{i/h}(x, \mu^2) \otimes K_{n/i}(x\omega, \xi^2, \mu^2)$$
$$+ O(\xi^2 \Lambda^2_{\text{QCD}}) ,$$

Lattice observable such as **reduced pseudo Ioffe time distribution** or **current-current correlators**

PDF

Matching coefficients – observable dependent quantities

Impact of reduced pseudo Ioffe time dependence



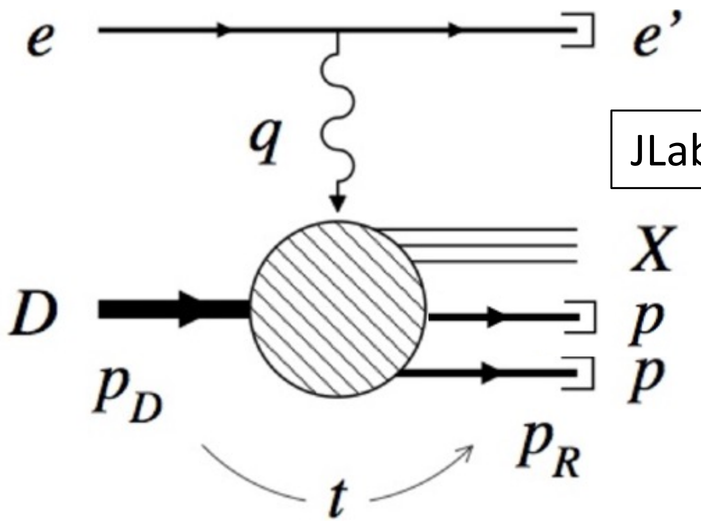
- Central values do not change much
- Uncertainties on valence PDF reduce by 35-45%

Future Experiments

PCB, Chueng-Ryong Ji (NCSU), W. Melnitchouk (Jefferson Lab), N. Sato (Jefferson Lab)

Future Experiments

- TDIS experiment at 12 GeV upgrade from JLab, which will tag a proton in coincidence with a spectator proton
    - Gives leading proton observable, complementary to LN, but with a fixed target experiment instead of collider (HERA)

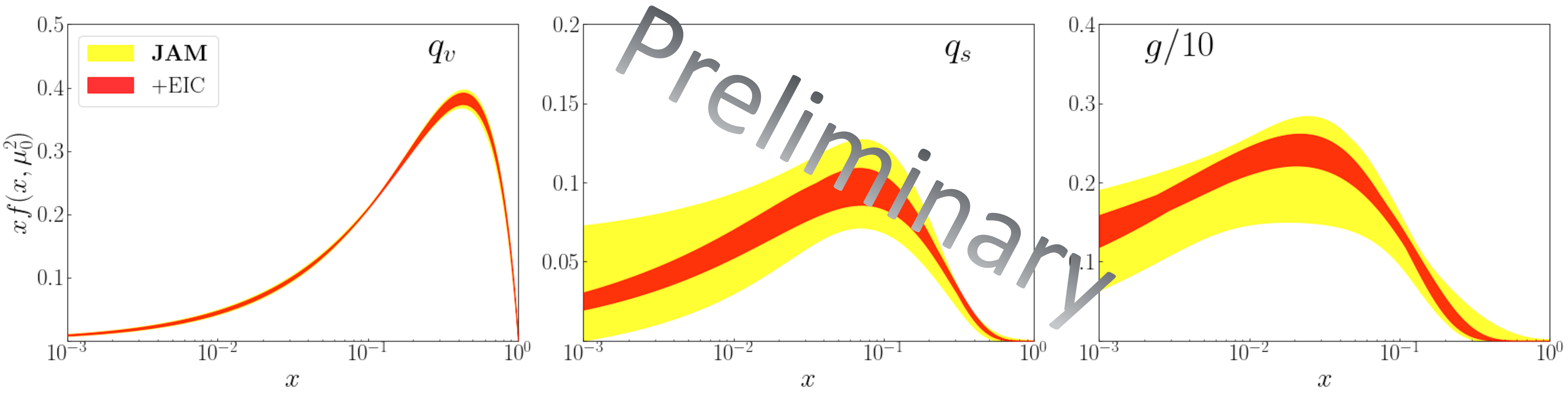


JLab TDIS

- Proposed EIC can measure a LN observable
    - Integrated luminosity is so large that systematics dominate uncertainties
- Proposed COMPASS++/AMBER also give π-induced DY data
    - Both π^+ and π^- beams on carbon and tungsten targets

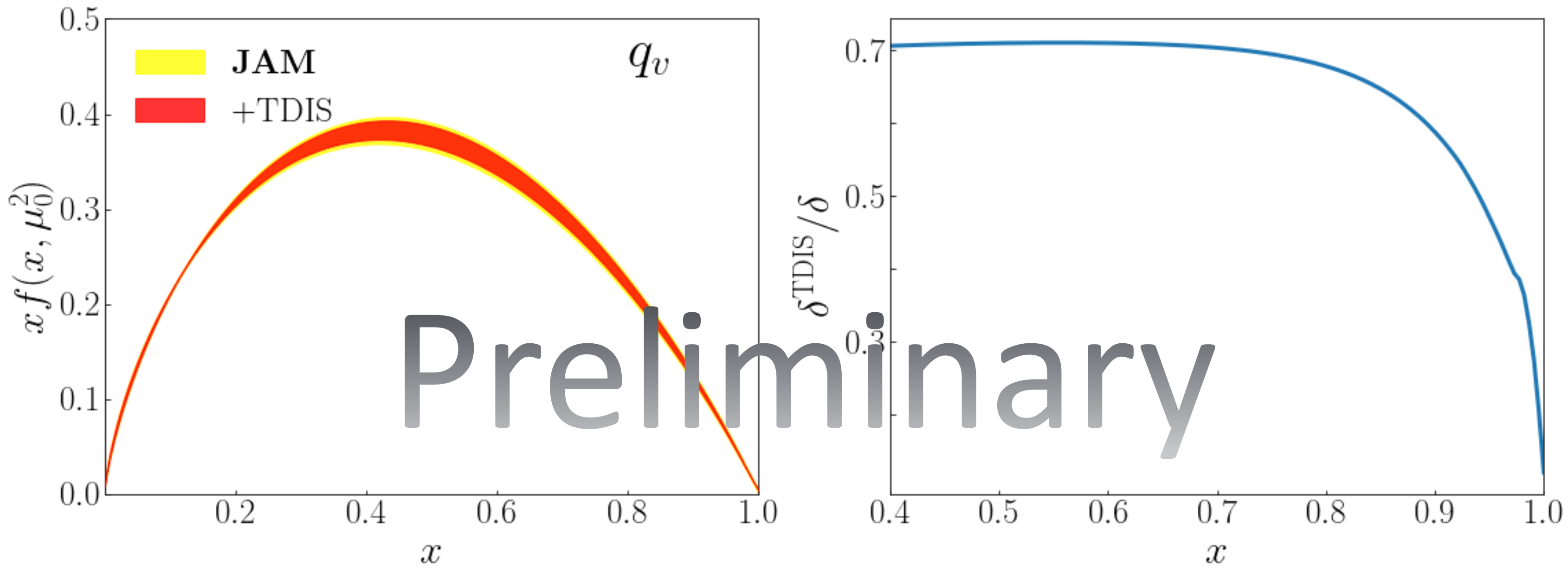
EIC Impact

- Take into account the theoretical systematic errors of changing the UV regulator of the splitting function
- Assume a 1.2% systematic uncertainty



JLab TDIS Impact

- Fixed-target nature of JLab TDIS constrains large-x valence quark PDF
- Assume a 6.5% systematic uncertainty on data



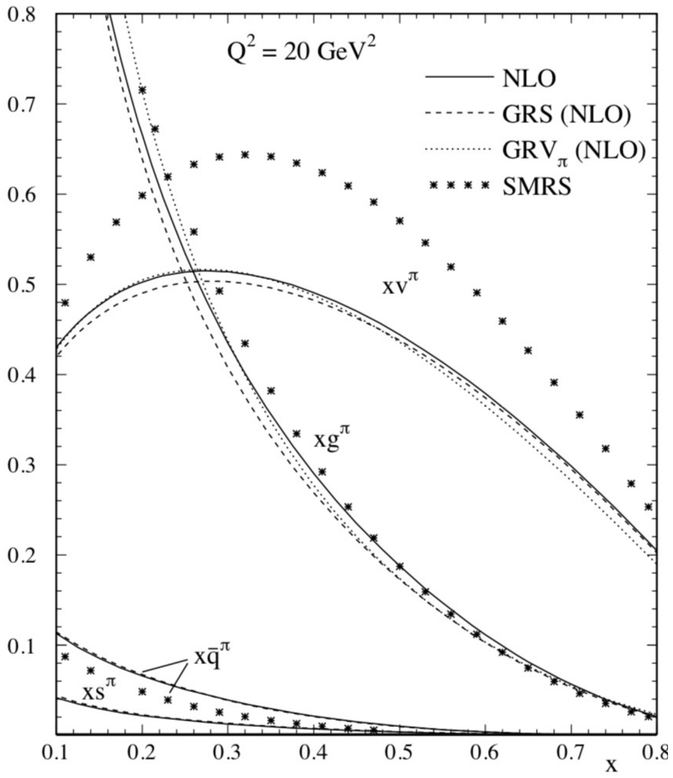
Conclusions

- JAM performs simultaneous fits of non-perturbative objects to world data
- Pion PDF extraction is influenced greatly by the method of threshold resummation used
- Successful description of large p_{T} Drell-Yan data from the pion
- Lattice data constrains the valence quark PDF in the pion
- We look forward to future experiments for further constraints on pion PDFs

Backup

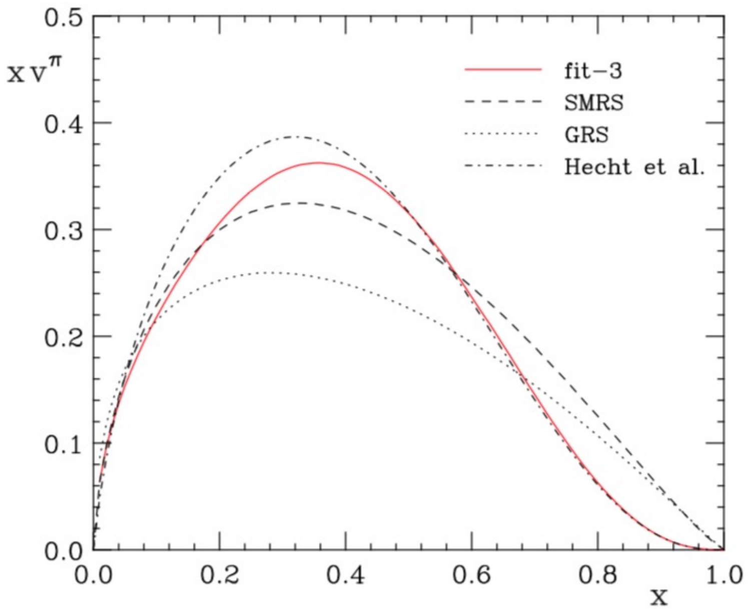
Previous Pion PDFs

- Fits to Drell-Yan, prompt photon, or both



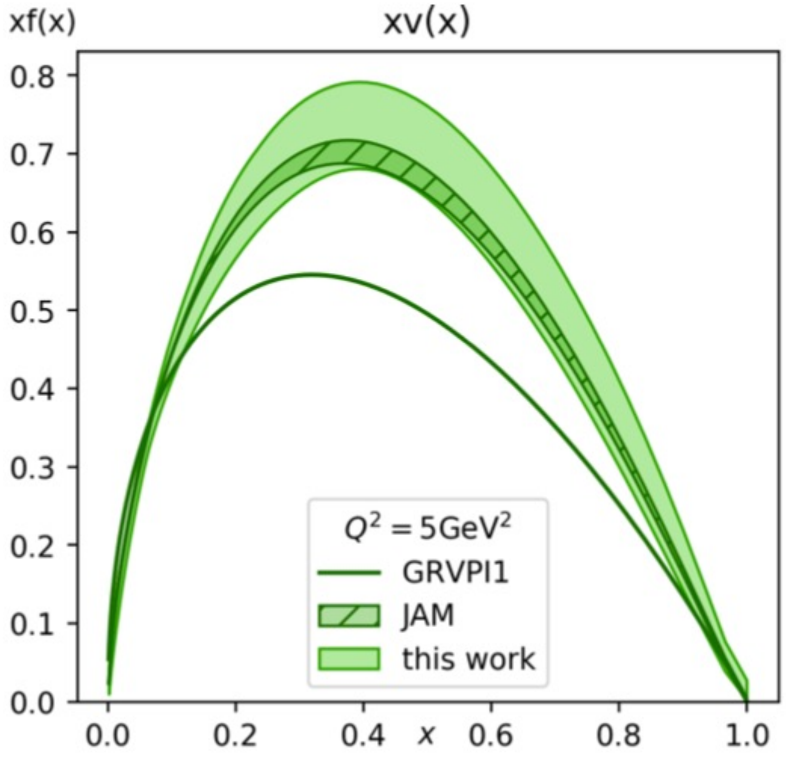
GRS, GRV, and SMRS

Z. Phys. C **67**, 433 (1995).
Eur. Phys. J. C **10** 313 (1997).
Phys. Rev. D **45** 2349 (1992).



ASV valence PDF

Phys. Rev. Lett. **105**, 114023 (2011).



xFitter

Phys. Rev. D **102**, 014040 (2020).

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+ \longrightarrow \hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

$$\tau = \frac{Q^2}{S}$$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

An NLO calculation gathers the $\mathcal{O}(\alpha_S)$ terms

	LL	NLL	…	N^{p}LL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^{k}LO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Add the columns to the rows

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2 , \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1} , \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

barryp@jlab.org

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2(\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k\left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

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