Large-x nucleon PDFs:
Counting rules meet functional mimicry

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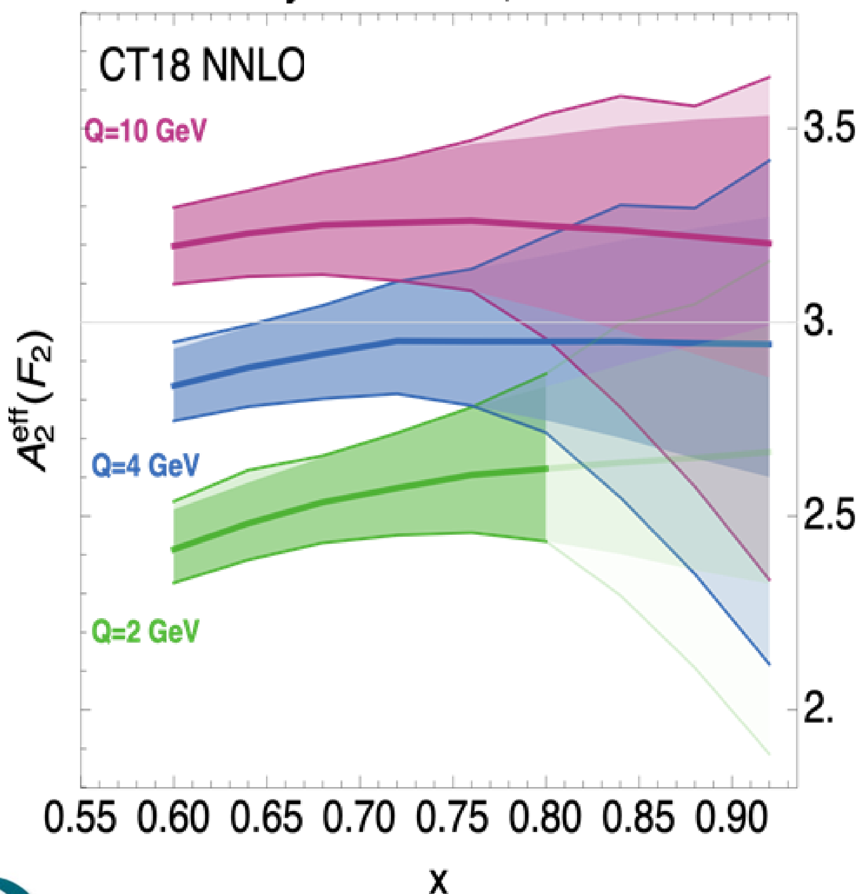
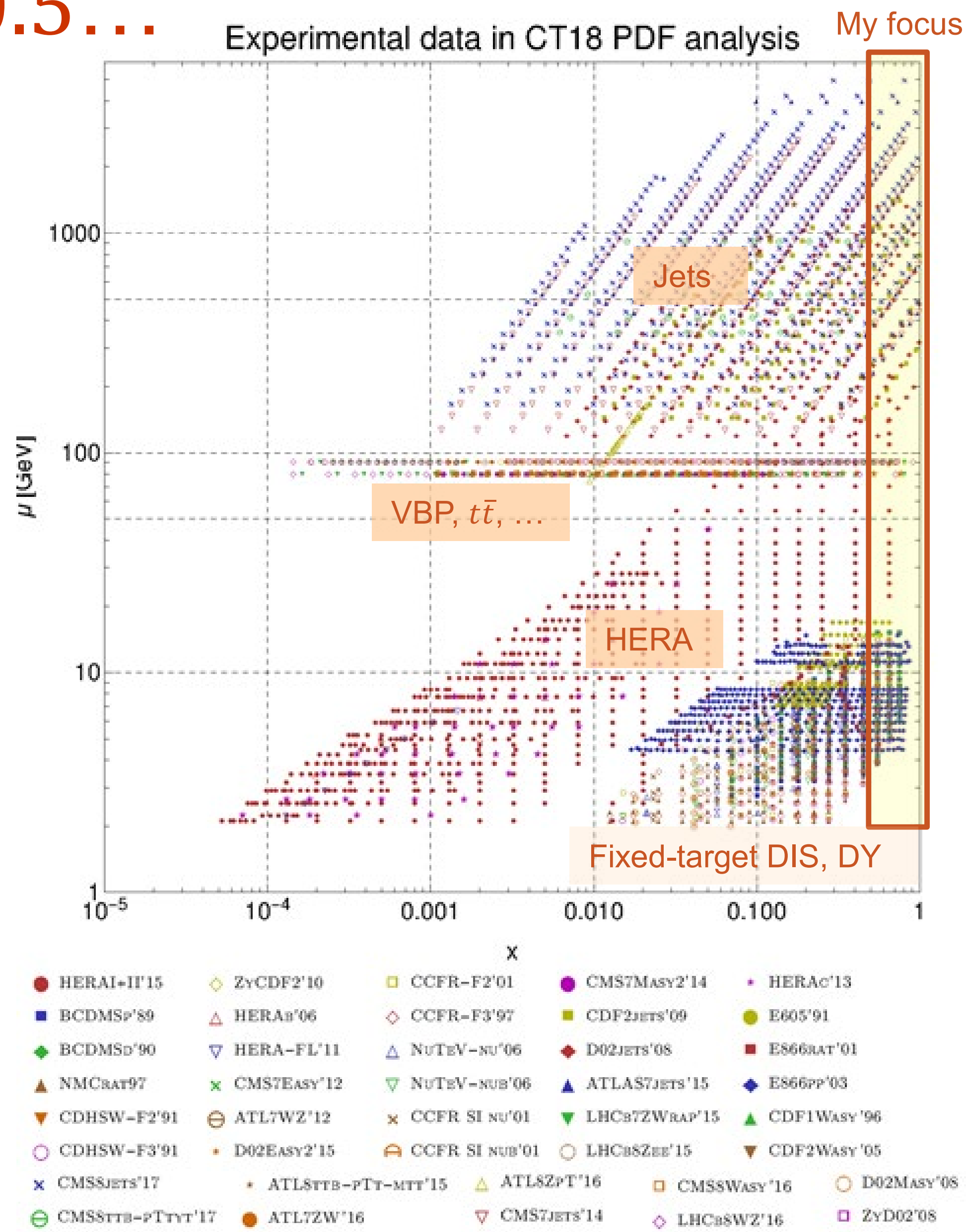
and CTEQ-TEA (Tung Et. Al.) working group

Phys. Rev. D 103, 054029



"FORDECYT-PRONACES"

Proton PDFs at $x > 0.5$…



… are of strong interest for theoretical models of nucleon structure and lattice QCD

… are important for LHC new physics searches

…are **still** constrained predominantly by fixed-target DIS and Drell-Yan experiments
[Hou et al., arXiv:1912.10053]

…are likely to be sensitive to nuclear effects
(⇒ T. Hobbs's talk)

… can be accessed at large Q at colliders

Interconnected questions

1. What can the PDFs at $x > 0.5$ tell us about nonperturbative QCD dynamics?
2. What is the best strategy to confront large-x data with predictions of nonperturbative QCD?
3. Are experimental constraints on large-x PDFs mutually consistent? ⇒ backup slides
4. How can we access the $x > 0.5$ region at colliders (HERA, LHC, EIC)? ⇒ backup slides

Strategy issues are relevant to studies of large-x pion PDFs at JLAB, AMBER@CERN, EIC
[Aguilar et al., 1907.08218; Roberts et al. 2102.01765]

We will use tests of quark counting rules (QCRs) as an example
[See also Ball, Nocera, Rojo, 1604.00024; Barry et al., 1804.01965; Novikov et al., 2002.02902]

What can the PDFs at $x > 0.5$ tell us about nonperturbative QCD dynamics?

Confronting predictions from nonperturbative QCD approaches and lattice QCD with pheno PDFs

Proton and pion PDFs

PDFs in nonperturbative QCD

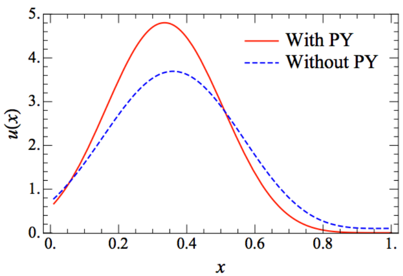
$@\ \mu_0^2 < 1\ GeV^2$

Phenomenological PDFs

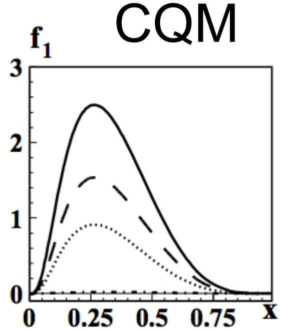
$@\ \mu^2 > 1\ GeV^2$

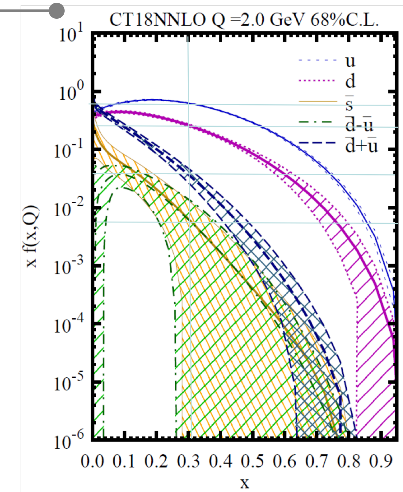
Proton

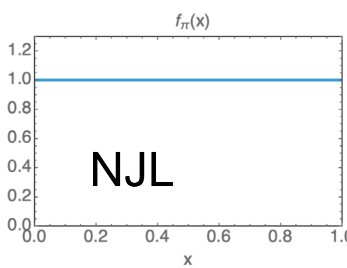
MIT Bag

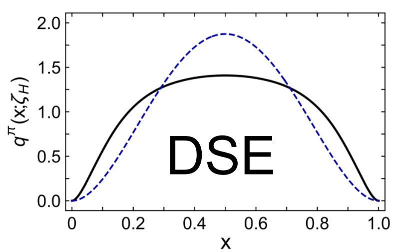


Light-Cone CQM

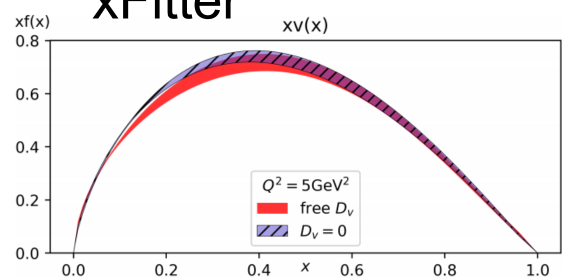








xFitter



Pion

Proton and pion PDFs

PDFs in nonperturbative QCD

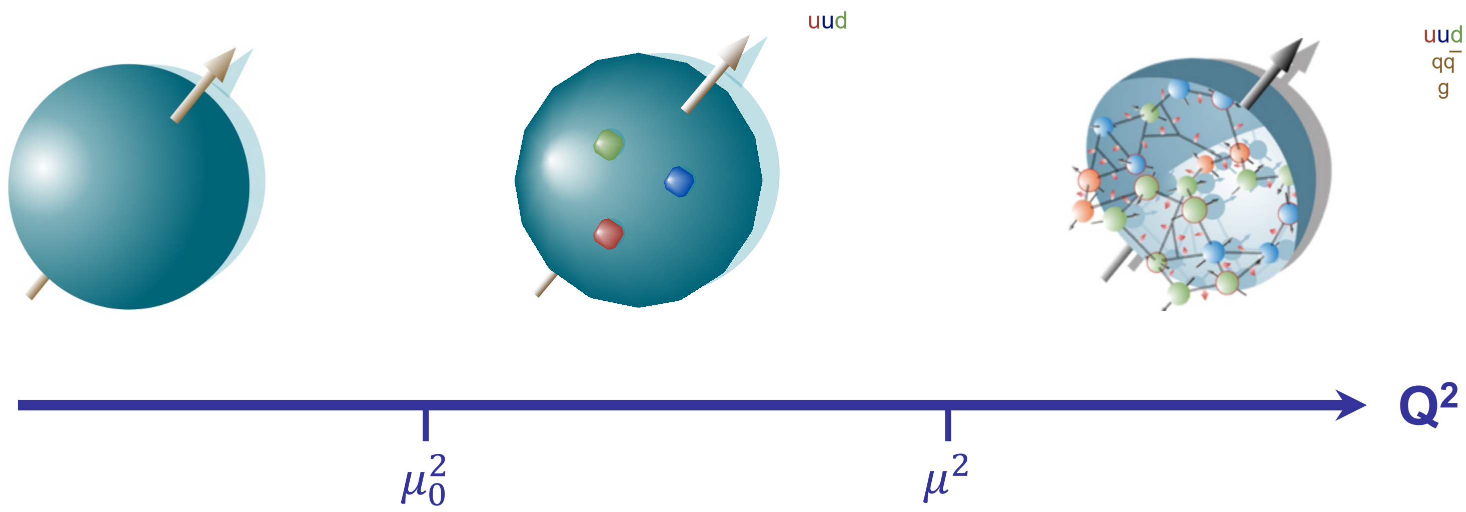
at hadronic scale μ_0^2< 1GeV²

- nonperturbative dynamics
- model's degrees of freedom
- Not factorized

Phenomenological PDFs

at factorization scale μ^2> 1GeV²

- quasi-free partonic degrees of freedom
- defined in the $\overline{MS}$ scheme
- leading-power approximation to full dynamics

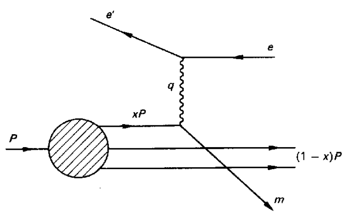


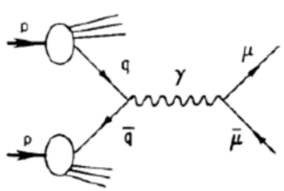
Proton and pion PDFs

PDFs in nonperturbative QCD

Relevant for processes at $Q^2 \approx 1\ GeV^2$?

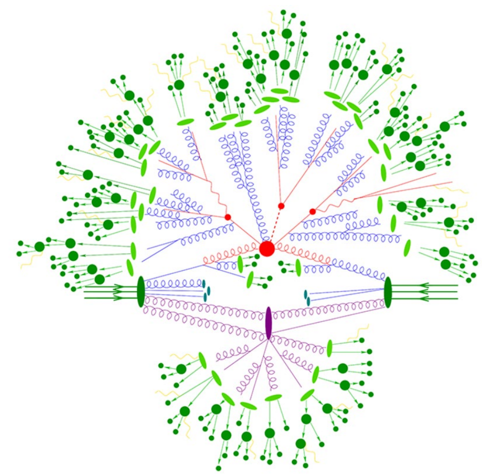
⇨ we can learn about nonperturbative dynamics by comparing predictions to data for the simplest scattering processes (DIS and DY)





Phenomenological PDFs

Determined from processes at $Q^2 \gg 1\ GeV^2$



⇨ pheno PDFs are determined from analyzing many processes with complex scattering dynamics

How to relate the x dependence of the perturbative and nonperturbative pictures?

Does the evidence from primordial dynamics survive PQCD radiation?

For example, can we test quark counting rules?

- No, if we try to deduce the exact analytical form of $f_a(x, Q)$ from data.
- Yes, if we measure a finite-difference derivative $A_2^{eff}(x_B, Q)$ of $F(x_B, Q)$ or PDFs $f_a(x, Q)$.

Quark counting rules in DIS in the threshold limit $x_B \to 1$

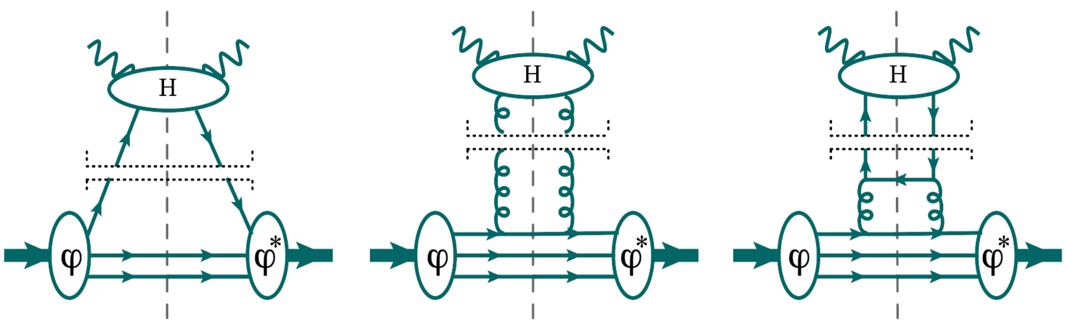
1. For a structure function $\boldsymbol{F}$, assuming $p^+ \gg 1\ GeV$:

$$\lim_{x_B \to 1} F(x_B, Q \approx 1\ GeV, \lambda_q) \propto (1 - x_B)^{2\, n_s - 1 + 2|\lambda_q - \lambda_A|}$$

n_s is the number of spectator fermions, λ_q and λ_A are helicities of struck quark and target

Brodsky and Farrar, PRL31 and PRD11
Ezawa, Nuovo Cim. A23
Berger and Brodsky, PRL42
Soper, PRD15
many others

2. For unpolarized PDFs: $\lim_{x \to 1} f_a(x, \mu_0 \approx 1\ GeV) \propto (1 - x)^{A_2}$



$f(x, \mu_0) \xrightarrow[x \to 1]{} (1 - x)^{A_2 = 3}$ valence quarks

$(1 - x)^{A_2 > 4}$ gluon

$(1 - x)^{A_2 > 5}$ sea quarks

Assuming the simplest PQCD diagrams dominate at $x_B \to 1$

Predictions:

- A_2 is the same for u and d quarks
- At $\mu > \mu_0$, anomalous dimensions increase the *true* A_2.
- The *apparent* A_2 may increase or decrease.

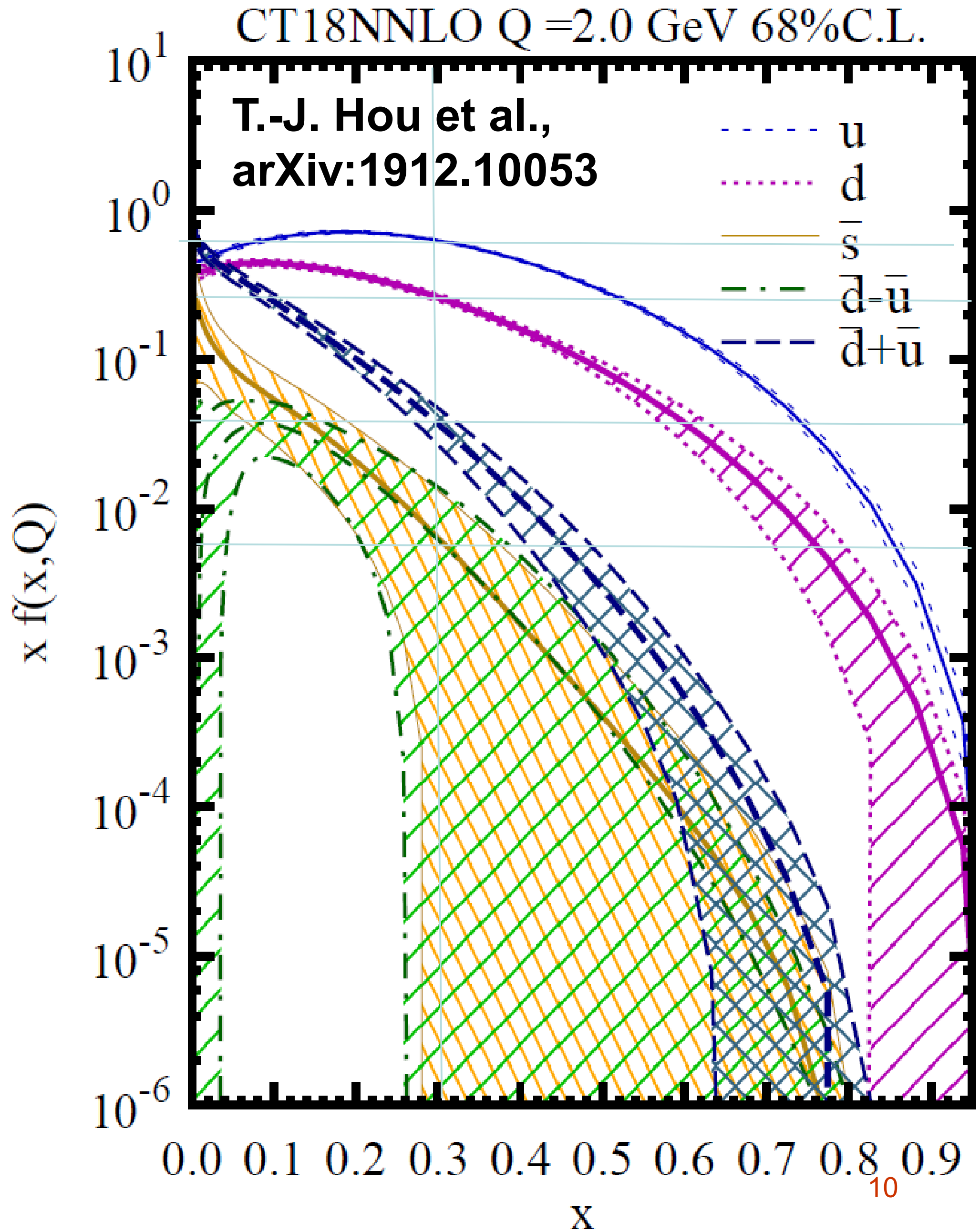
Proton PDFs at $x \to 1$

At $x \to 1$, it is easy to radiate a gluon or a sea quark off a valence quark with a much larger PDF

Mimicry – a fundamental feature of multivariate optimization:

Diverse functional forms of PDFs at the initial scale Q_0 provide equally good description of QCD data from DIS, Drell-Yan pair, jet, and $t\bar{t}$ production

Mathematically, it is not possible to determine the primordial A_2^{true} from discrete, fluctuating data at $x < 1$.

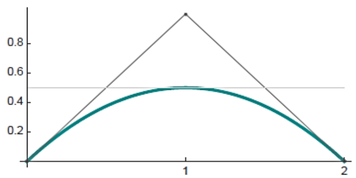


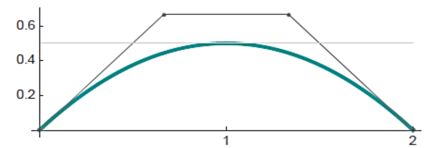
Bézier curves

[AC, Nadolsky, arXiv:2011.10078]

2. <u>Bézier curves</u> give an example of mathematical equivalence of polynomials of different orders

- defined using Bernstein basis polynomials:





$$f(x) = \alpha(1-x)^2 + 2\beta(1-x)x + \gamma x^2$$

$$f(x) = \alpha(1-x)^3 + 3\beta'(1-x)^2 x + 3\beta''(1-x)x^2 + \gamma x^3 \qquad \beta', \beta'' \equiv F[\alpha, \beta, \gamma]$$

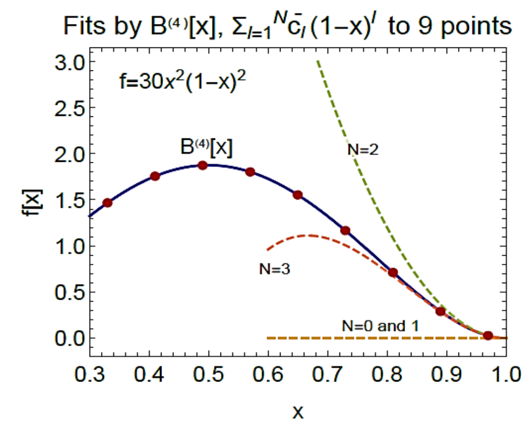
- can be used to interpolate discrete data points

Interpolation by a Bézier curve is unique if the polynomial degree= (# points-1):
there's a closed-form solution to the problem,

$$\mathcal{B}^{(n)}(x) = \sum_{l=0}^{n} c_l \; B_{n,l}(x) \quad \text{with the Bernstein pol.} \quad B_{n,l}(x) \equiv \binom{l}{n} x^l (1-x)^{n-l}.$$

This interpolation can be expanded in monomials of (1-x) about x=1:

$$u_\pi(x \to 1) = \sum_{i=0}^{n} \bar{c}_i \, (1-x)^i$$



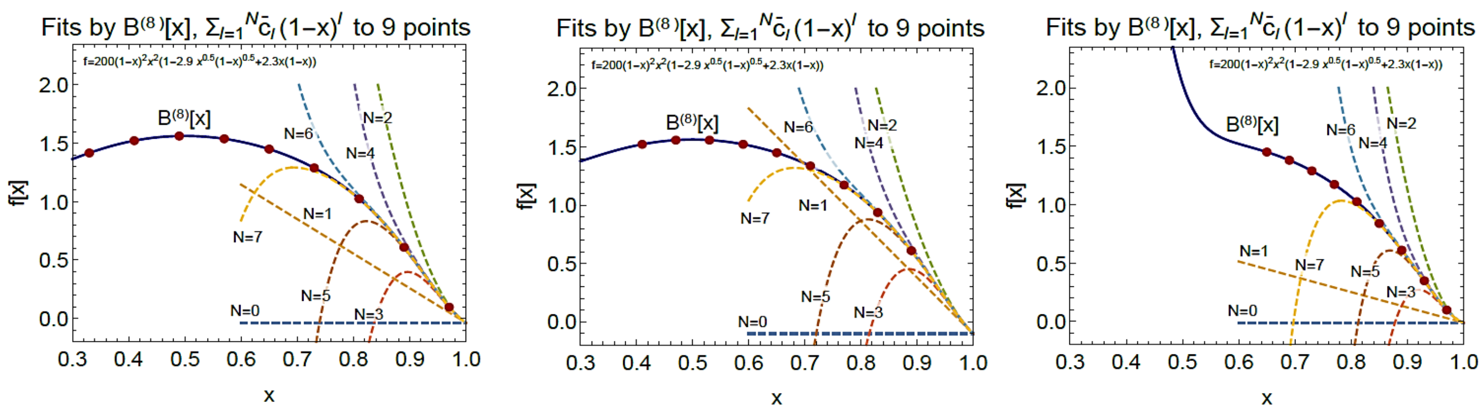
Pinning down the large-x behaviour?

Take a realistic functional form, here $f=200(1-x)^2x^2(1-2.9\,x^{0.5}(1-x)^{0.5}+2.3x(1-x))$

- Sample 9 points with $x < 0.95$ from this function and interpolate by a Bezier curve of order 8. [This interpolation is exact.]
- The lowest coefficients of the monomial expansion (quantified by the truncated solutions with N=1, 2, 3) are spurious and depend on the range and spacing of the sampled data.

⇨ The lowest powers of the monomial expansion $(1-x)^p$ cannot be meaningfully reconstructed.

For each data set sampled from the same f(x) we obtain a different Bézier curve. They differ in derivatives of order > 8.



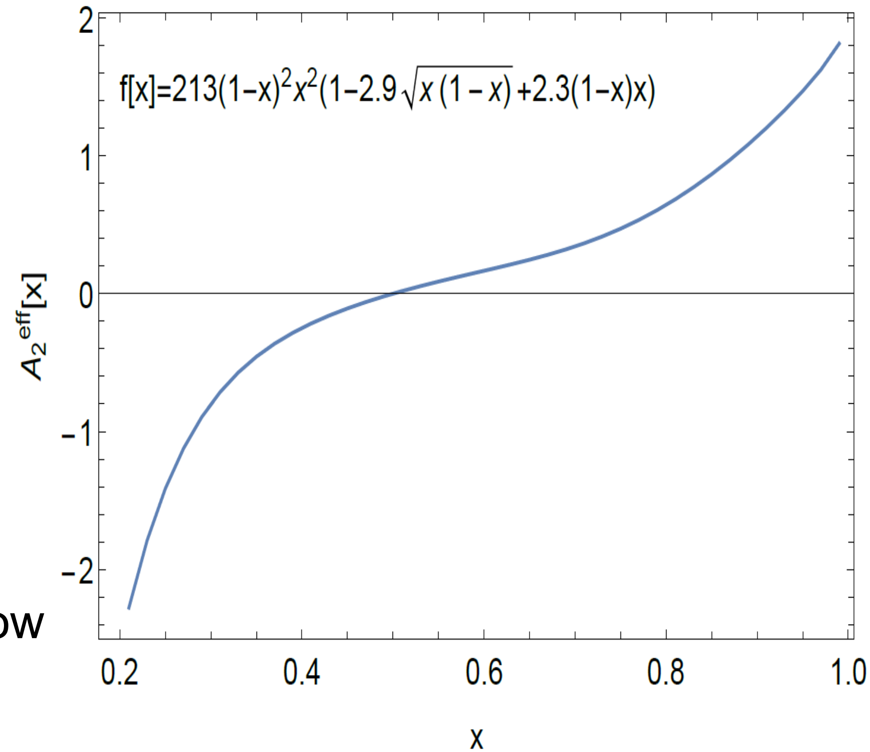
A_2^{eff}: a finite-difference approximation to A_2^{true}

If at $x \to 1$:

$$f(x,Q) = \underbrace{(1-x)^{A_2^{true}}}_{fast\ function} \times \underbrace{\Phi(x)}_{slow\ function},$$

then

$$A_2^{eff}(x,Q) \equiv \frac{\partial \ln\big(f(x,Q)\big)}{\partial \ln(1-x)}$$
$$\approx A_2^{true} + small\ term.$$



Dependence of $A_{2,eff}$ on x and Q indicates how important the higher-order PQCD corrections are.

A_2^{eff} for the DIS structure function $F_2(x,Q)$

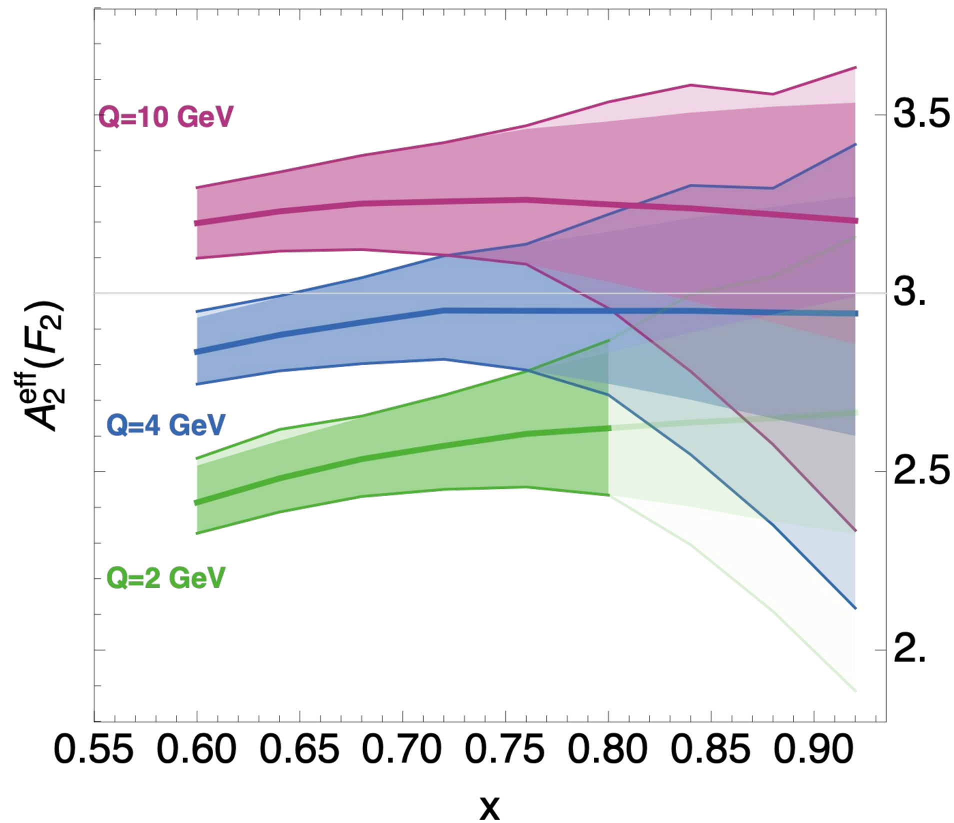
$F_2(x,Q)$ are computed using CT18 NNLO

Dark bands: 68% CL Hessian errors
Light bands: envelopes of PDF parametrization dependence with 363 trial parametrization forms

- A_2^{eff} agrees with the predicted $A_2 = 3$ within a large PDF uncertainty
- $\delta_{PDF} A_2^{eff} \sim 1 \; at \; x \to 1$
- Non-negligible running with Q^2
- The leading-twist picture is meaningful for $W^2 > m_p^2 + (1-x)/x \, Q^2$,

corresponding to $x < 0.8$ at $Q = 2$ GeV

CT18 NNLO, parametrization dependence



This prediction can be directly compared to the large-x, large-Q ZEUS DIS data
[I. Abt et al., PRD 101 (2020)112009]

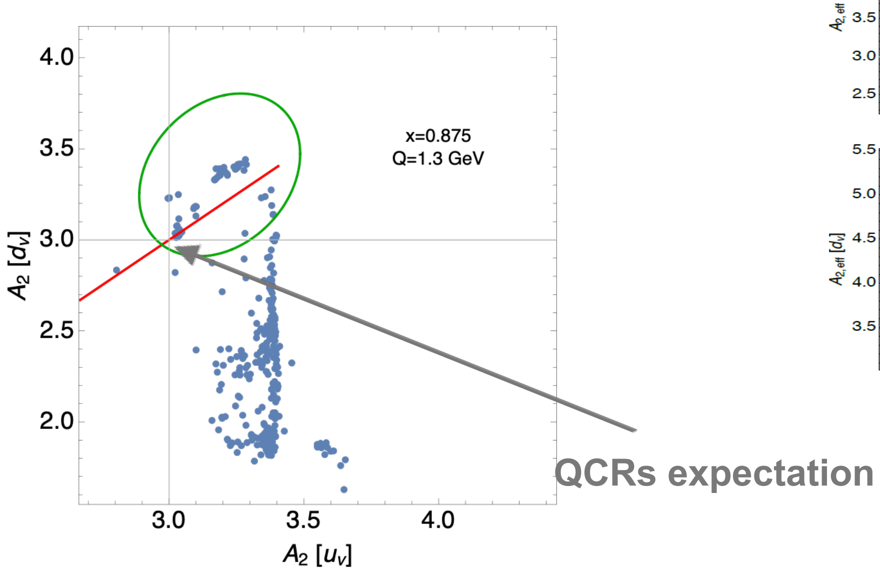
A_2^{eff} for u, d PDFs

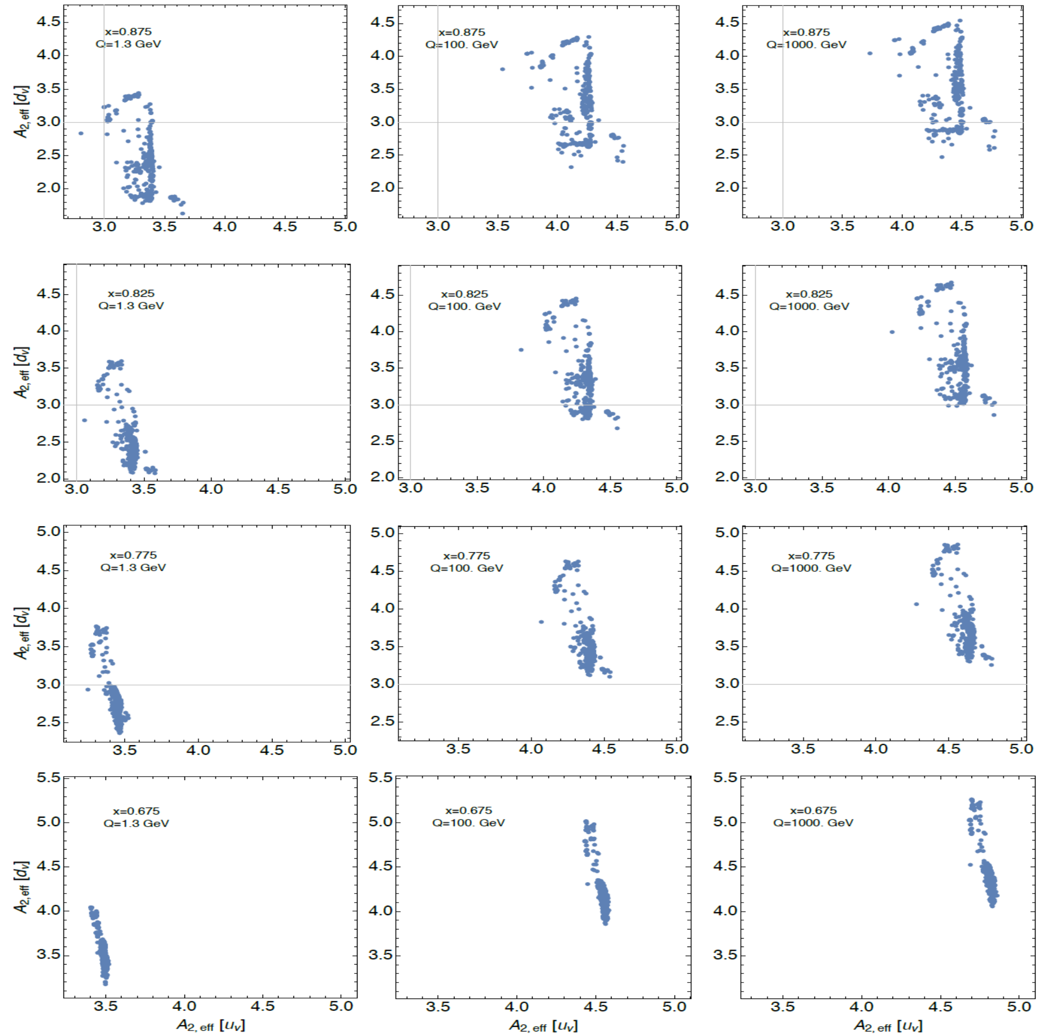
Using CT18 NNLO

Red: Hessian uncertainty on the nominal PDF parameter $A_{2,u} = A_{2,d}$

Green: Hessian error ellipse for $A_{2,u}^{eff}$, $A_{2,d}^{eff}$

Blue: $A_{2,u}^{eff}$, $A_{2,d}^{eff}$ for 363 functional forms with $A_{2,u} \neq A_{2,d}$



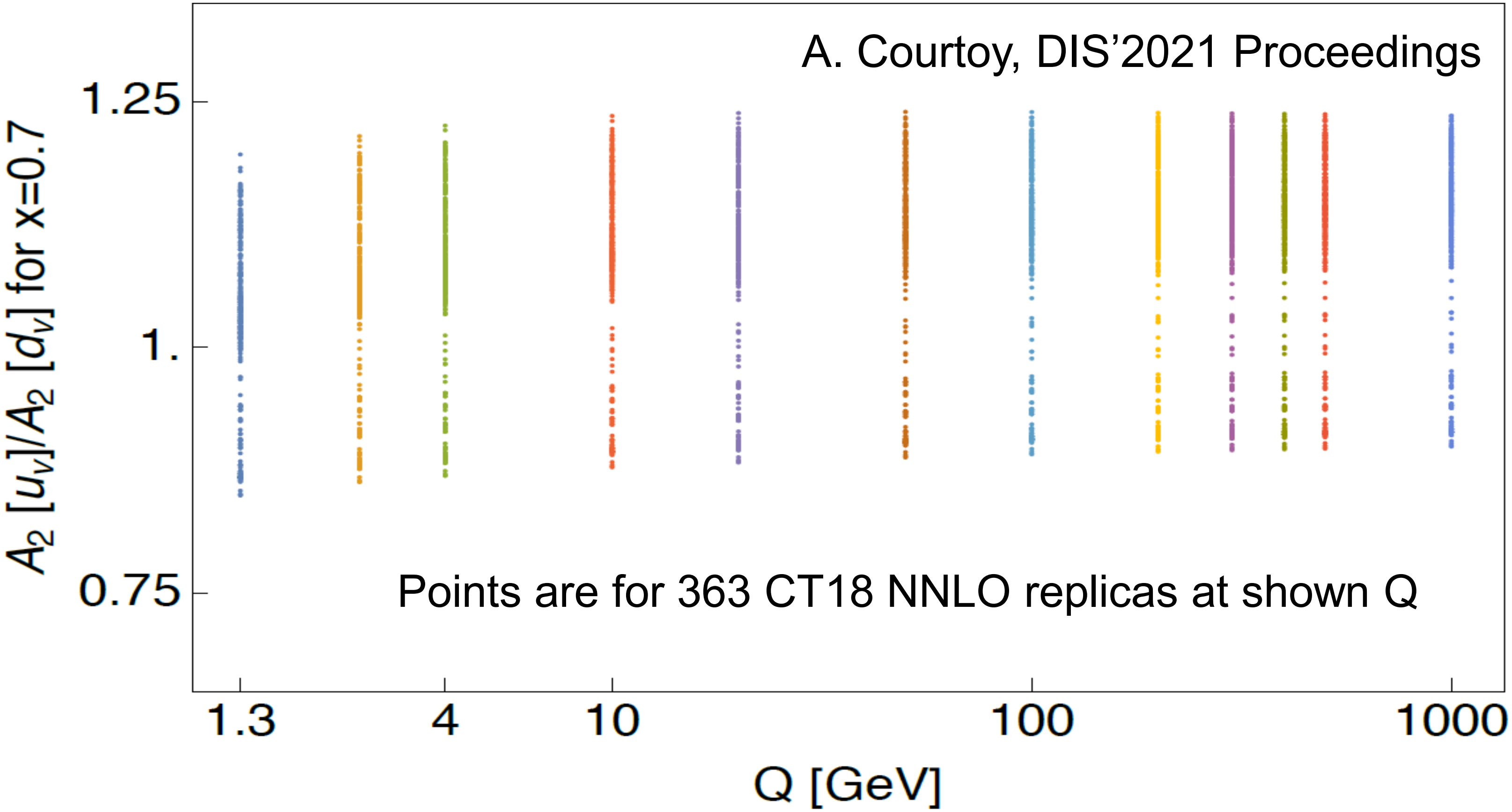


Predictions based on 363 functional forms

Substantial dependence on Q and x: higher-order PQCD is important, but param. dependence preserved to high Q

Examine $A_{2,u}^{eff}/A_{2,d}^{eff}$

NEW

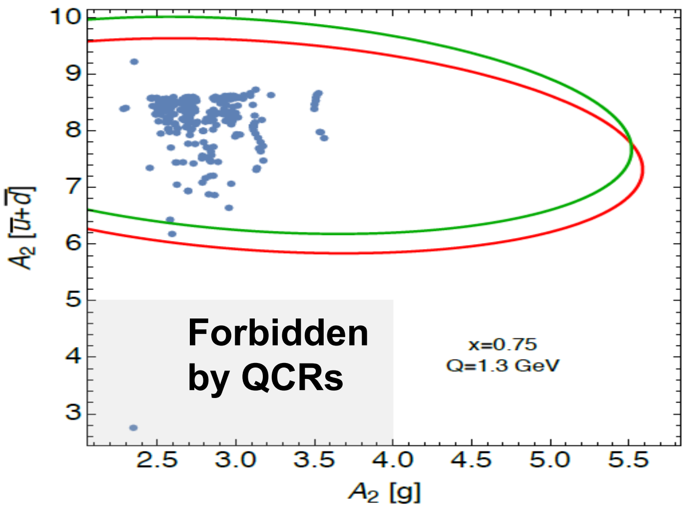


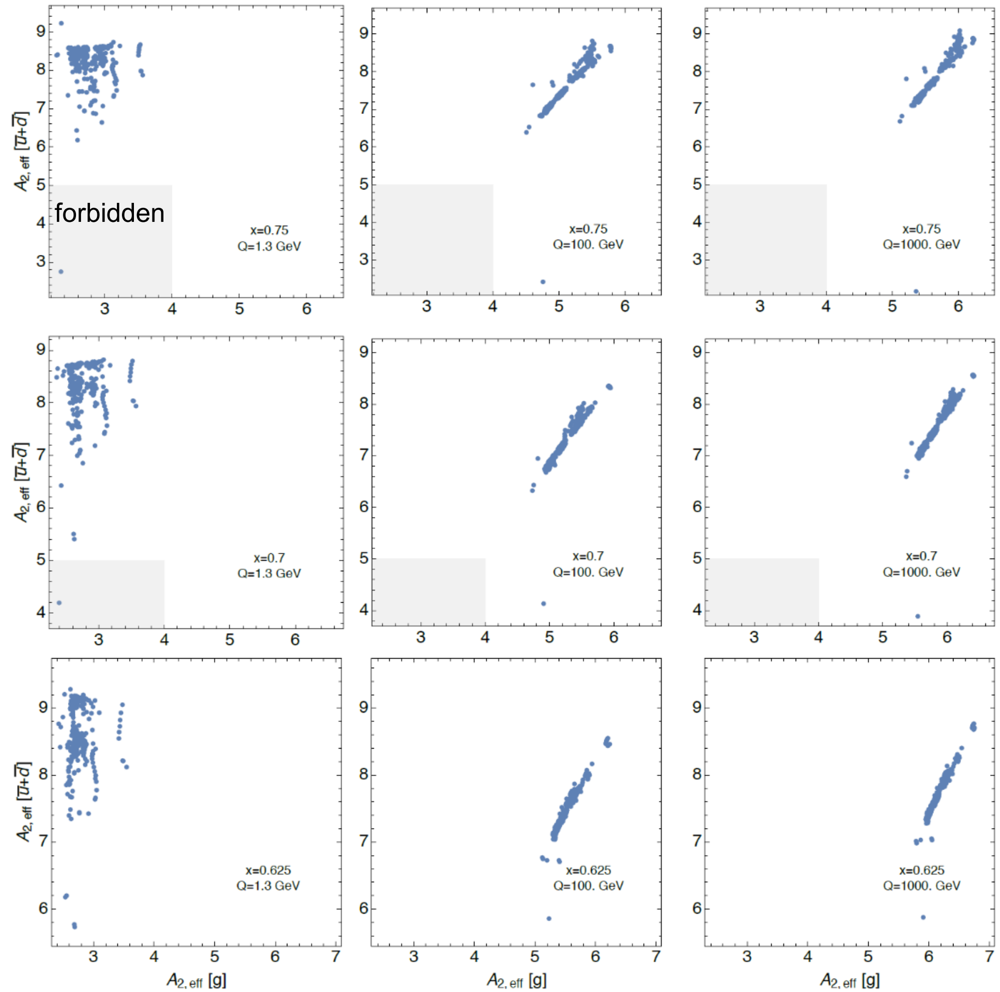
The Q dependences of $A_{2,u}^{eff}$ and $A_{2,d}^{eff}$ cancel out because both obey the non-singlet DGLAP equation

Flavor separation of nonperturbative $A_{2,u}^{eff}$ and $A_{2,d}^{eff}$ can be deduced from high-Q data

A_2^{eff} for g, $\bar{u}+\bar{d}$ PDFs

Using CT18 NNLO
Red: Hessian error ellipse for the nominal PDF parameters $A_{2,g}$, $A_{2,\bar{u}+\bar{d}}$
Green: Hessian error ellipse for $A_{2,g}^{eff}$, $A_{2,\bar{u}+\bar{d}}^{eff}$
Blue: $A_{2,g}^{eff}$, $A_{2,\bar{u}+\bar{d}}^{eff}$ for 363 functional forms

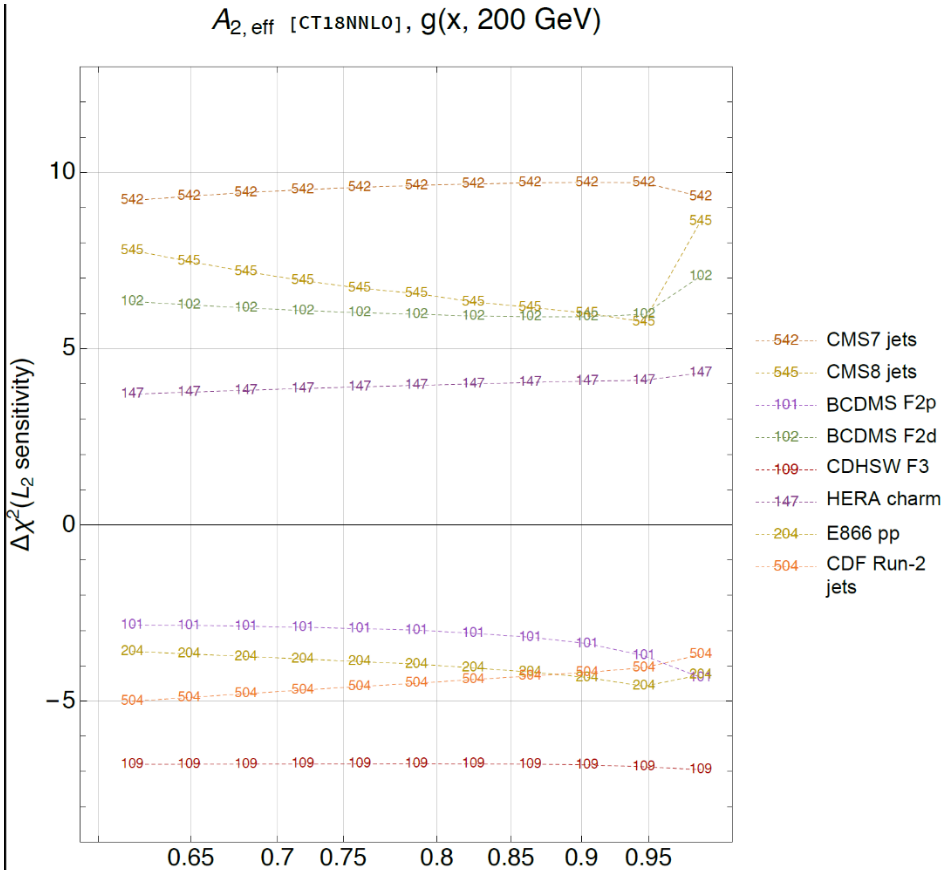


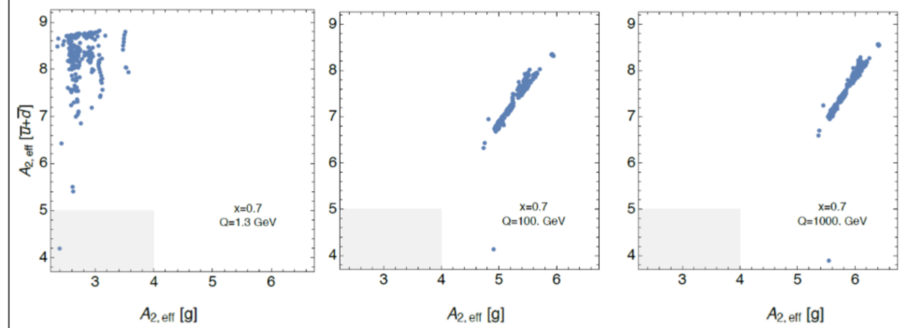


Predictions based on 363 functional forms
Substantial dependence on Q and x for g, less for $\bar{u}+\bar{d}$ (in the downward direction)

NEW

Pulls on A_2^{eff} for gluon





The rate of Q evolution weakly depends on the parametrization form, can be used to reconstruct $A_{2,g}^{eff}$ at $Q\sim 1$ GeV from high-Q measurements.

At $x > 0.65$, $A_{2,g}^{eff}$ is determined to a large degree by the tradeoff between jet production experiments [CMS, CDF, …], νA DIS [CDHSW], BCDMS and E866.

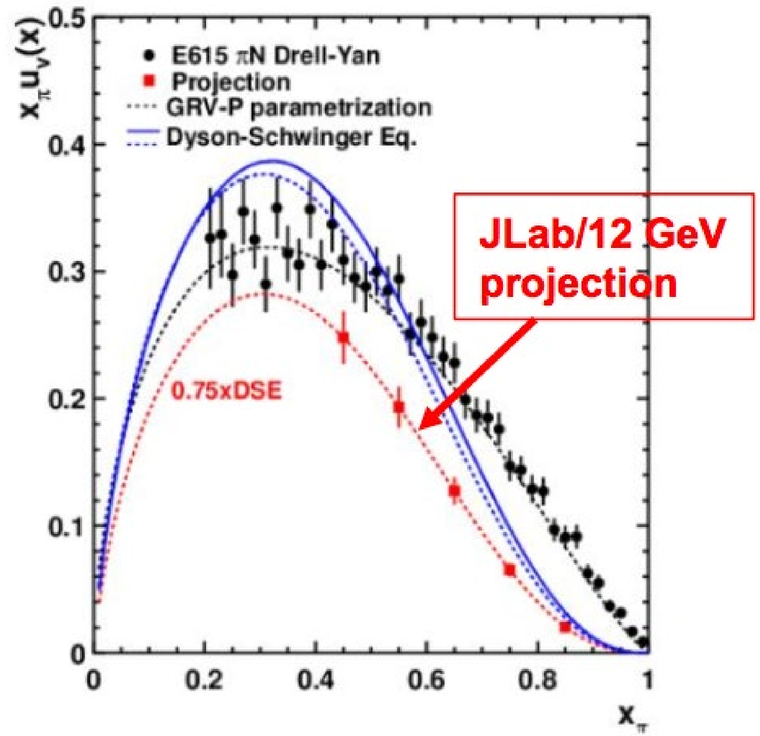
A. Courtoy, DIS'2021 Proceedings

Implications for pion PDFs

x dependence of pion PDFs at $x \to 1$ can offer unique insights about the emergence of hadronic mass

New experiments on $e\pi$ DIS and $p\pi \to \ell\,\overline{\ell\, X}$ envisioned at JLab, AMBER/COMPASS++, EIC

Mimicry, process dependence, QCD corrections, PDF uncertainties, threshold resummation scheme dependence are important in pion global PDF analyses.



As in the nucleon case, we can extract A_2^{eff} rather than A_2^{true} from discrete data, with typical uncertainty of about one unit or more

⇒ talk by P. Barry

Conclusions

We have analyzed the **quark counting rules** for the CT18NNLO global fit of proton PDFs.

We examined their **universality** w.r.t. scattering processes and flavors, as well as for structure functions vs. PDFs.

Global analyses rely on complex processes. Dependence on x generally reflects power-suppressed hadronic activity, not only scaling violations or resummation.

Mimicry reconciles many parametrizations of PDFs with measurements.

How do we cast nonperturbative manifestations into measurable observables? We advocate for using **an effective (1-x)-exponent $\boldsymbol{A_2^{eff}}$**.

The Q^2 dependence of A_2^{eff} is not negligible — supported by other global fits and by PQCD. We can solve for Q^2 dependence to obtain A_2^{eff} at $Q \approx 1$ GeV from collider measurements at HERA, LHC, and EIC.

Thank you
for your attention

Multivariate parametric forms

A typical PDF set may depend on tens to several hundreds of free parameters

PDF functional forms must be flexible to accommodate a variety of behaviors
CT18 parametrizations at initial scale Q_0 are given by

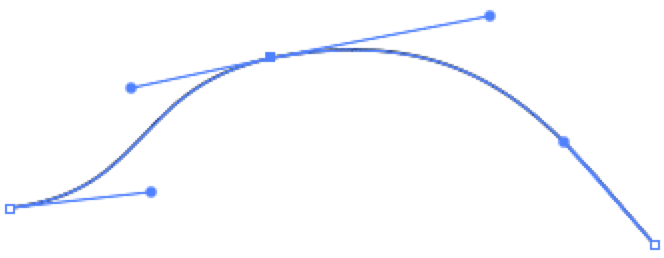
$$f_a\,(x, Q_0) = A x^{a_1} (1-x)^{a_2} B_a^{(n)}(x; a_3, a_4, \dots)$$

$$B_a^{(n)}(x) = \sum_{k=0}^{n} a_{k+2} \binom{n}{k} x^k (1-x)^{n-k}$$

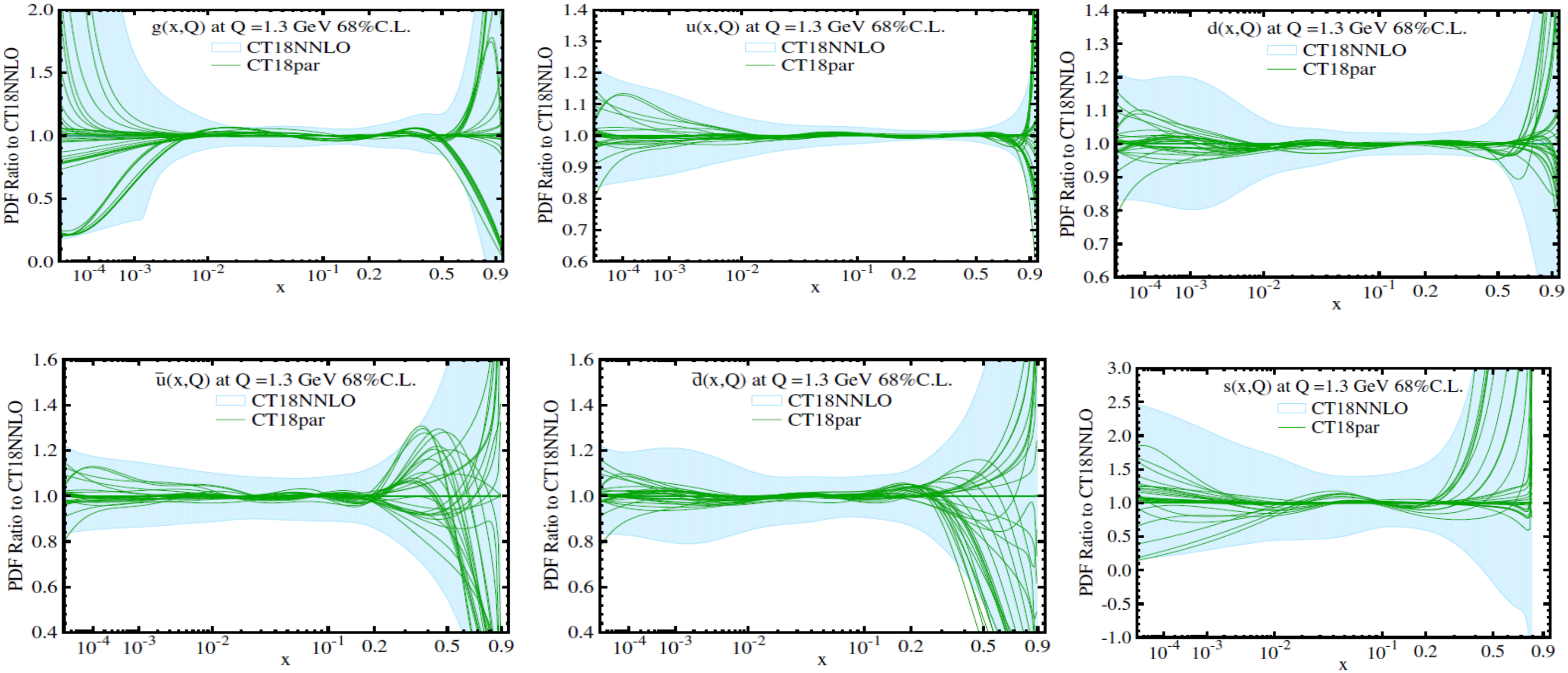
are **Bézier curves** – flexible polynomials familiar from vector graphics programs

Bézier curves can mimic a variety of behaviors of PDFs and their uncertainties. A powerful alternative to neural networks!

[A. Courtoy, P. N., arXiv: 2011.10078, accepted to PRD*]*

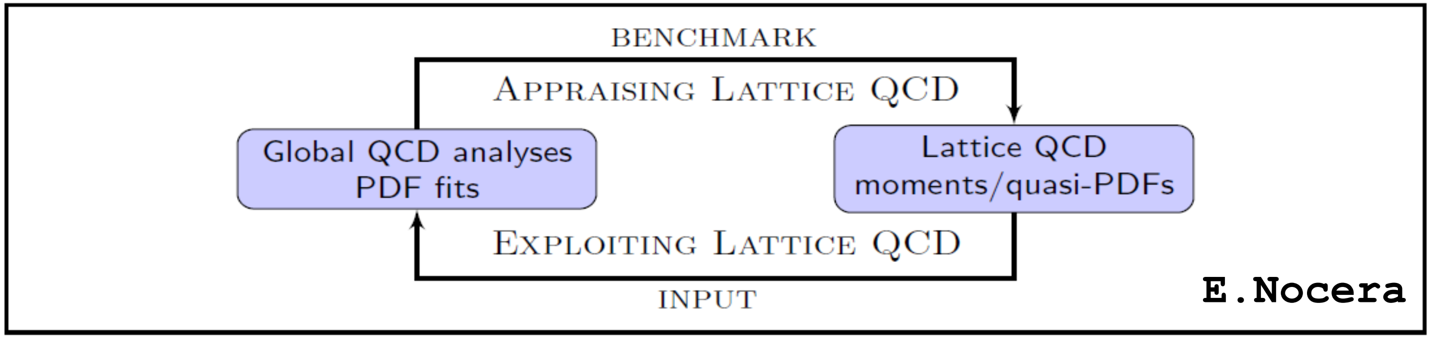


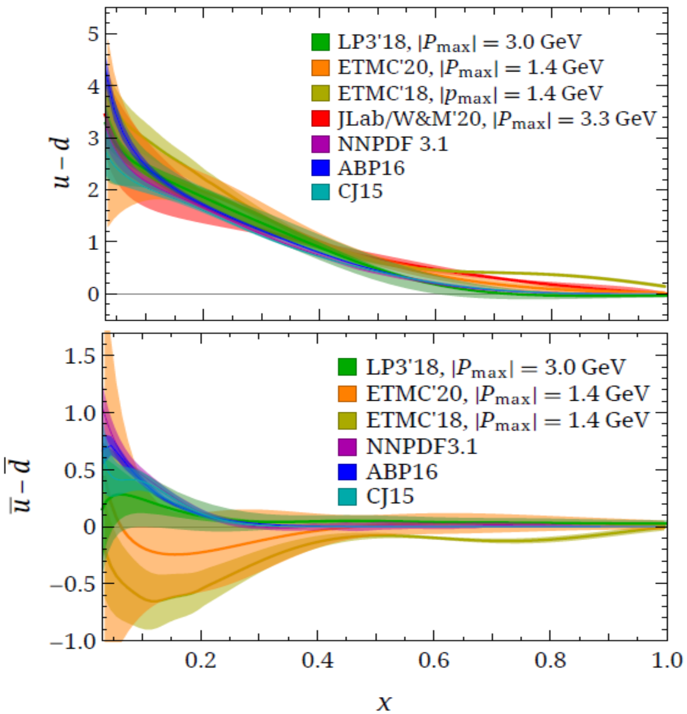
250+ candidate nonperturbative parametrization forms of CT18 PDFs



- CT18par – a sample of **some** non-perturbative parametrization forms tried in CT18
- No data constrain very large x or very small x regions.

Lattice QCD: ab initio computations of PDFs





Lattice QCD computes nonperturbative functions for the hadron structure (Mellin moments, quasi-PDFs, pseudo-PDFs) by discretizing the QCD Lagrangian density

This is a rapidly progressing field: computations of PDFs in several lQCD approaches have been compared against phenomenological PDF models at two workshops:

- PDFLattice2017, Oxford, March 2017
- PDFLattice2019, Michigan State University, Sept. 2019

[Prog.Part.Nucl.Phys. 100 (2018) 107; arXiv:2006.08636]

Pheno PDFs provide empirical benchmarks for lattice QCD computations. Lattice QCD has the potential to predict PDF combinations not accessible in the experiment.

Are experimental constraints on large-x PDFs mutually consistent?

Effective powers may be non-universal among ep and pp processes

QCRs may work better in processes with low hadron multiplicities

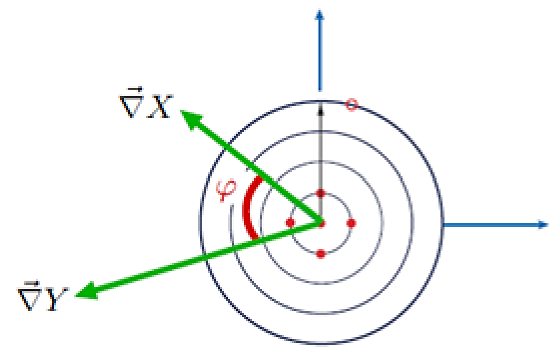
We examine this question in the CT18 NNLO analysis using the method of L_2 sensitivities [**T.Hobbs, B.T. Wang, PN, Olness, 1904.00022**]

L_2 sensitivity, definition

$S_{f,L_2}(E)$ for experiment E is the estimated $\Delta\chi_E^2$ for this experiment when a PDF $f_a(x_i, Q_i)$ increases by the +68% c.l. Hessian PDF uncertainty

Take $X \equiv f_a(x_i, Q_i)$ or $\sigma(f)$; $Y \equiv \chi_E^2$ for experiment E.

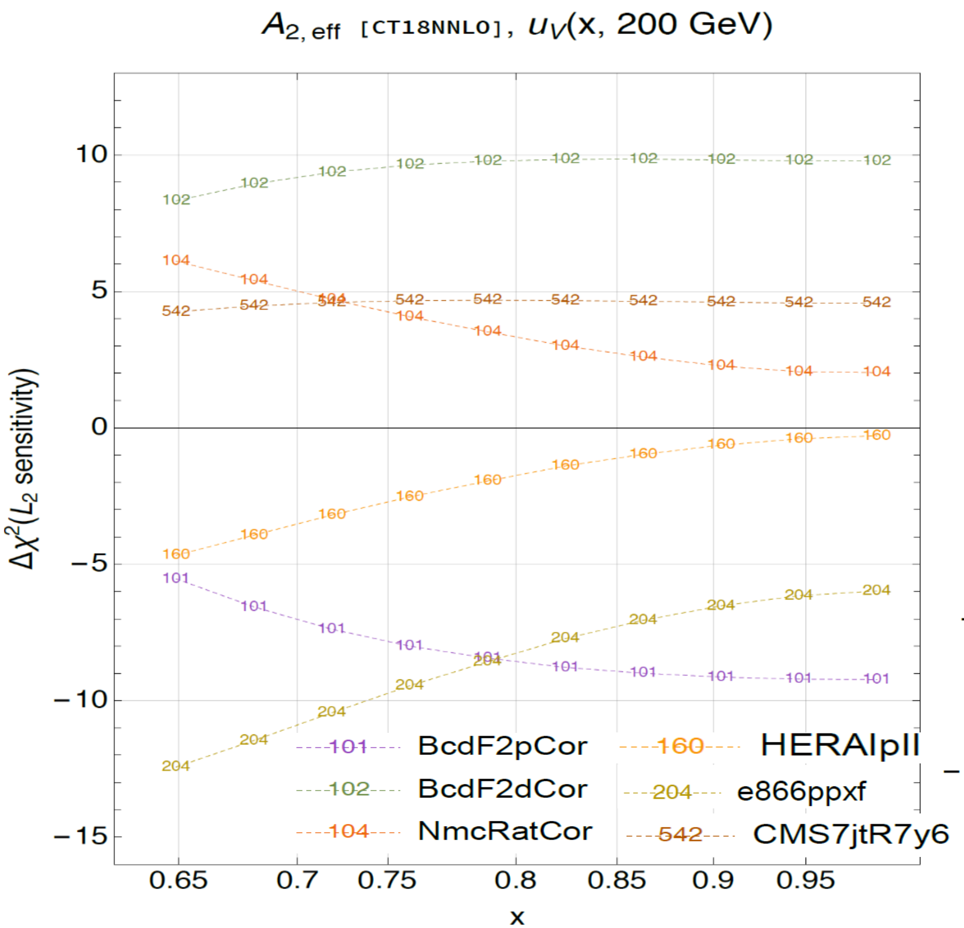
$\hat{z}_X \equiv \nabla X / |\nabla X|$ is the unit vector in direction of the PDF uncertainty of X.

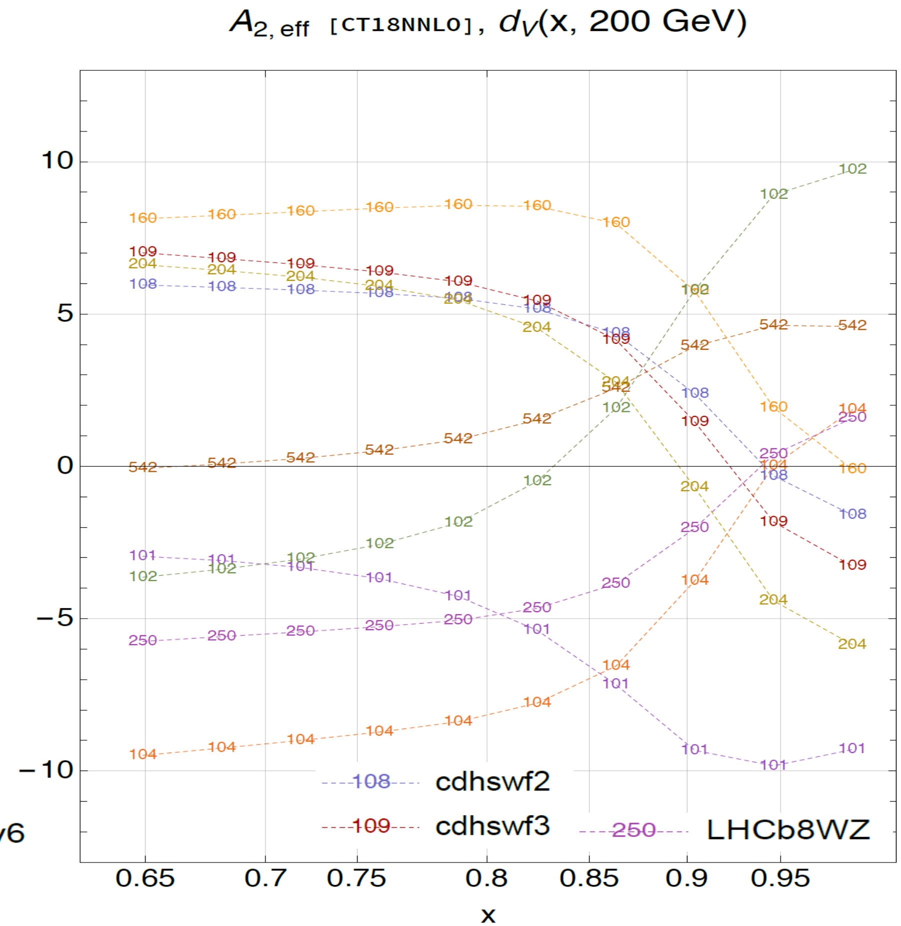


$$S_{X,L_2} \equiv \Delta Y(\hat{z}_X) = \nabla Y \cdot \hat{z}_X = \nabla Y \cdot \frac{\nabla X}{|\nabla X|} = \Delta Y \cos\varphi \,.$$

A fast version of the Lagrange Multiplier scan of χ_E^2 along the direction of $f_a(x_i, Q_i)$!

Pulls on A_2^{eff} for u_v and d_v





* Proton BCDMS and DY E866 favor a larger value of $A_{2,eff}[uv]$
* Deuteron BCDMS favors a smaller value of $A_{2,eff}[uv]$
* $A_2^{eff}[u_v]$ largely follows $A_2^{eff}[F_2]$

* Tradeoff between opposite pulls of HERA and NMC at $x < 0.85$
* $A_{2,d}^{eff} = A_{2,u}^{eff}$ at $x \to 1$ by construction

How can we access the $x > 0.5$ region at colliders (HERA, LHC, EIC)?

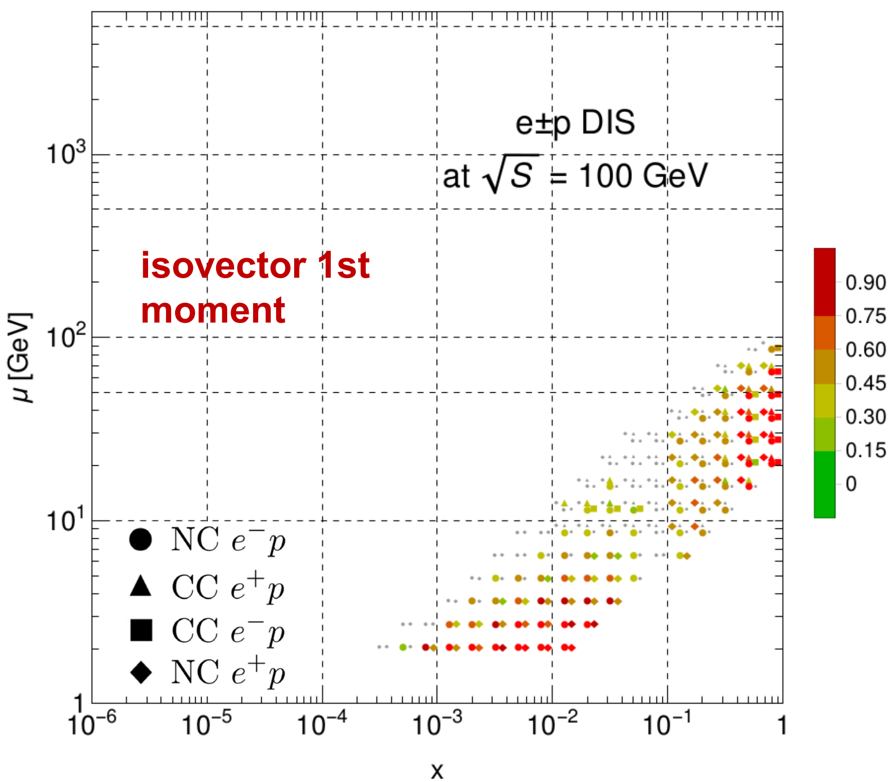
For large-x opportunities at HERA, see A. Caldwell's talk

An EIC would drive lattice phenomenology

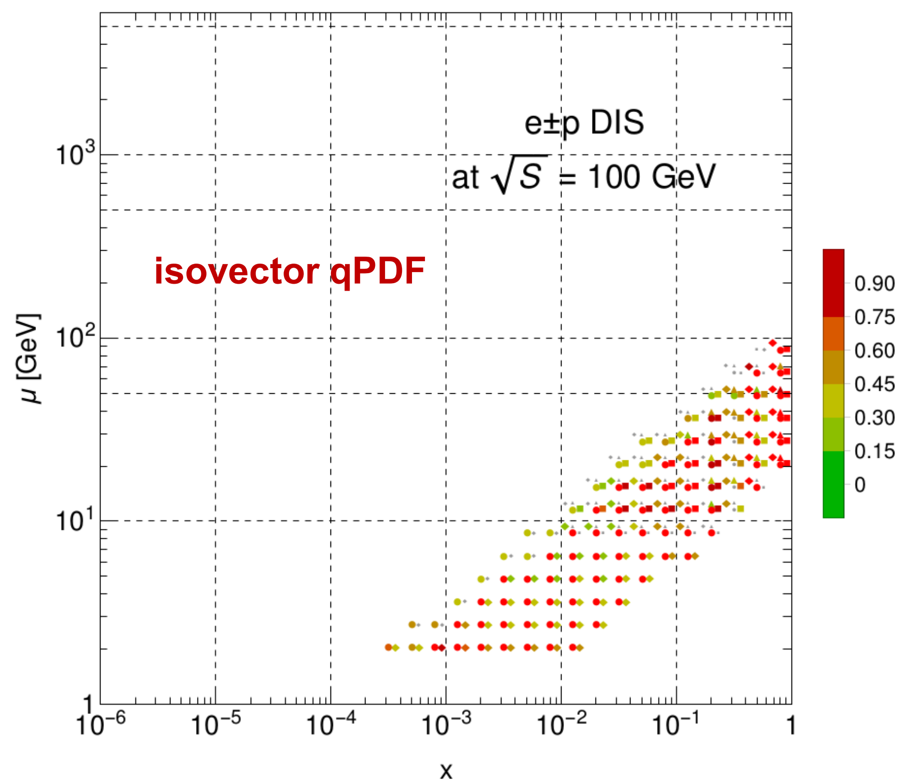
- A high-luminosity lepton-hadron collider will impose very tight constraints on many lattice observables; below, the isovector first moment and qPDF

- Many of the experiments most sensitive to PDF Mellin moments and qPDFs involve nuclear targets $\longrightarrow$ **eA data from EIC would sharpen knowledge of nuclear corrections**

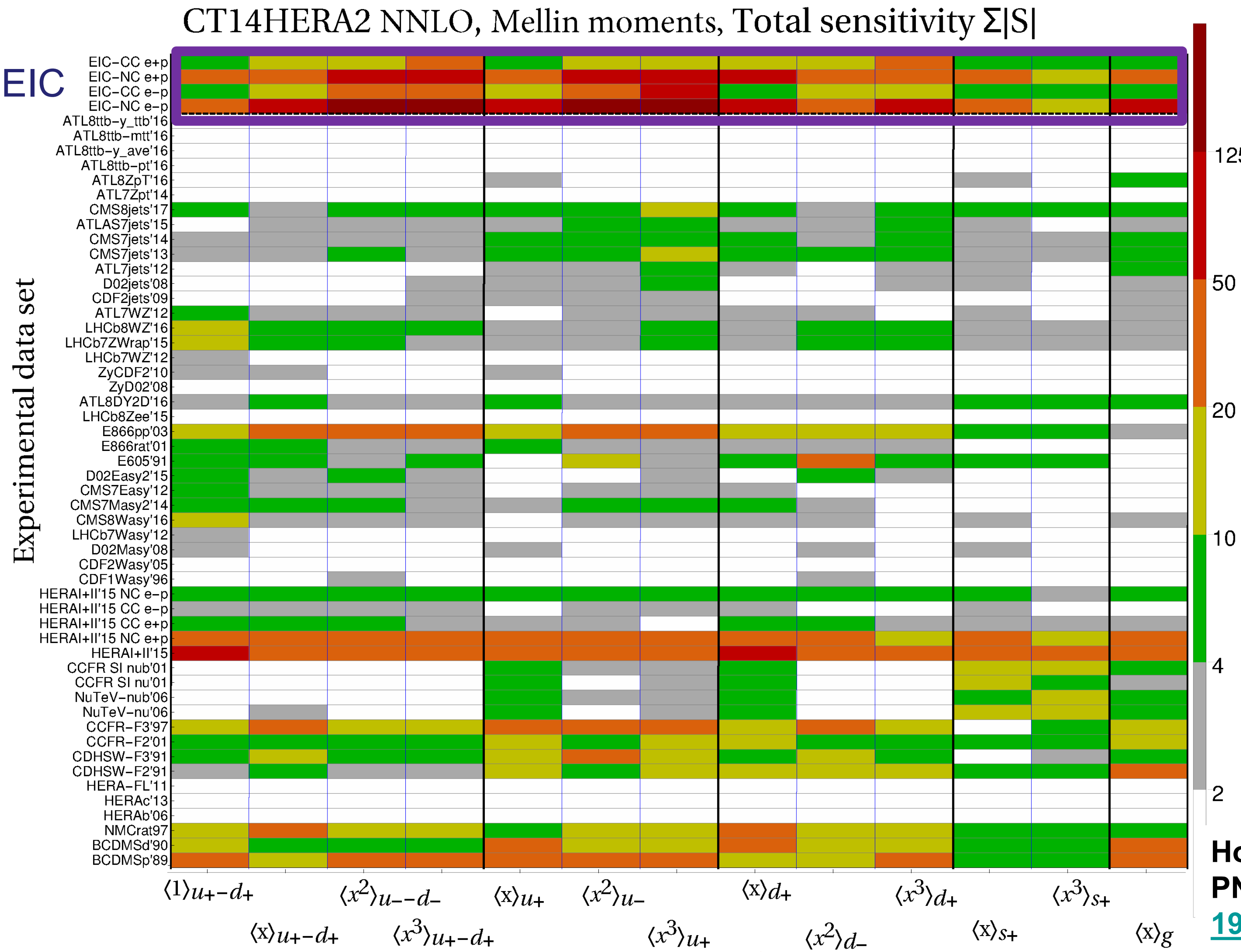
| S_f | for $\langle x^1 \rangle_{u^+-d^+}$, CT14HERA2



| S_f | for $[\tilde{u}-\tilde{d}]$(x=0.85, P_z=1.5GeV), CT14HERA2



Total sensitivity to Mellin moments

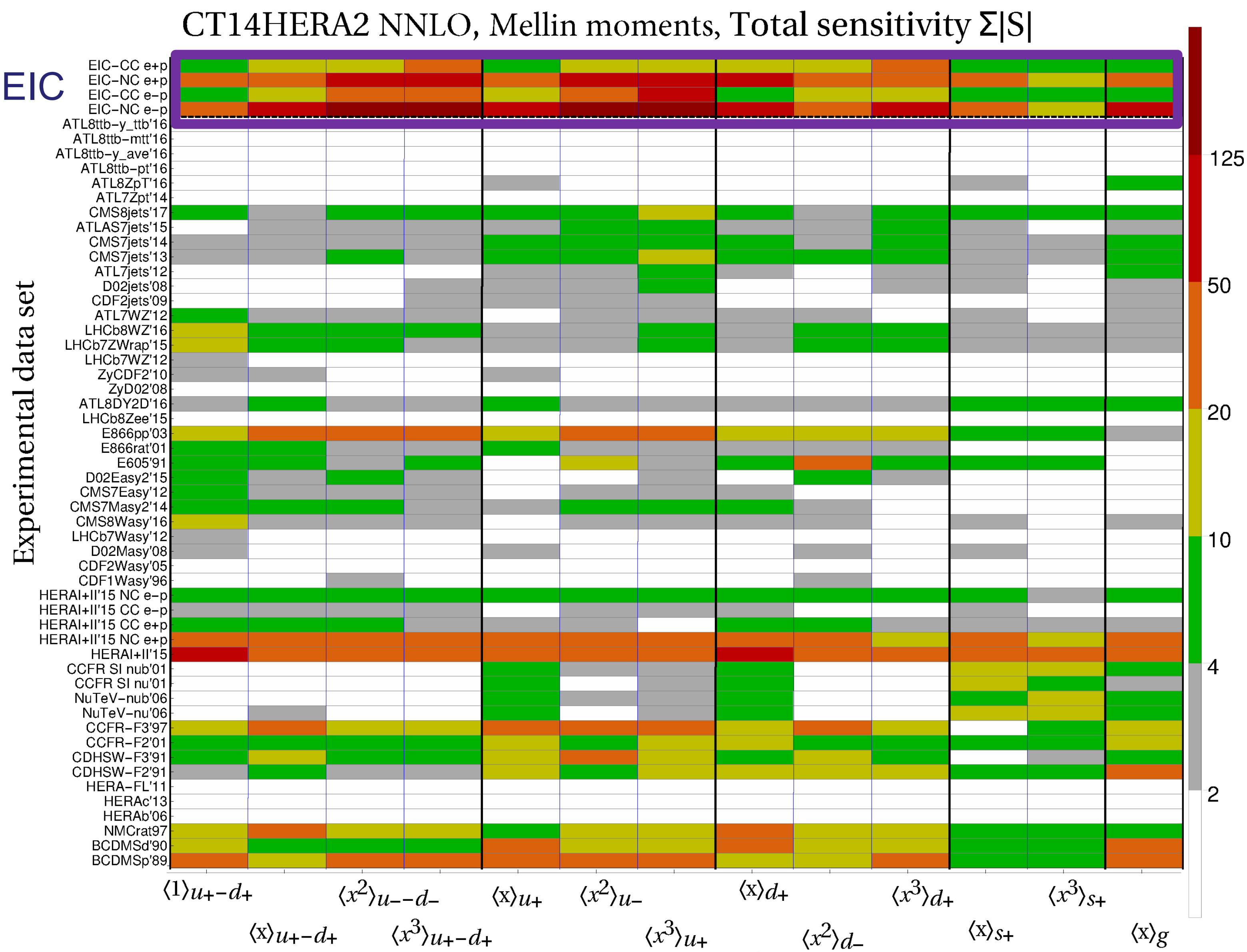


For Mellin moments computable on the lattice at scale 2 GeV

Dark red (white) indicates experiments with highest (lowest) sensitivity of the shown experiment

Hobbs, B. T. Wang, PN, Olness, [1904.00022]

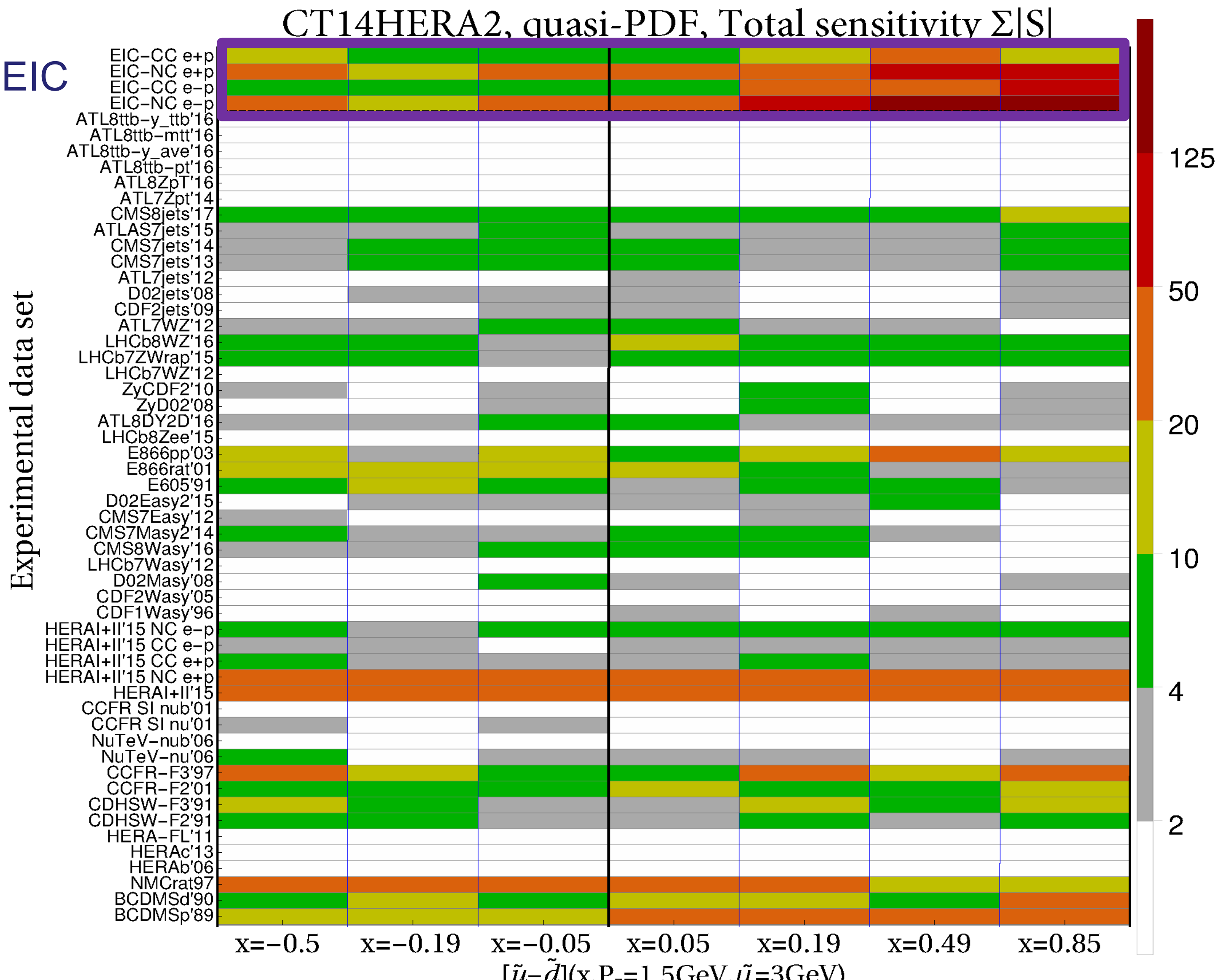
Total sensitivity to Mellin moments



Pre-2019 data: HERA, BCDMS, NMC, E866 DY pair production are most sensitive to the moments

The EIC will dramatically improve the constraints

Total sensitivity to lattice quasi-PDFs



$$\tilde{q}(x) \equiv \begin{cases} q(x), \\ x > 0; \\ -\bar{q}(|x|), \\ x < 0 \end{cases}$$

$Q = 3$ GeV,
$P_z = 1.5$ GeV