ON THE SCALAR πK FORM FACTOR BEYOND THE ELASTIC REGION

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based on arXiv:2103.01966 in collaboration with F. Noël, C. Hanhart, M. Hoferichter and B. Kubis

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MOTIVATION

- Consistent description of πK needed for heavy meson decays, eg. $\mathrm{B} \to J/\Psi \mathrm{K}\pi$

- High accuracy dispersive analysis of low energy phase shifts already exist
  - Based on experimental data and constraints from chiral symmetry

- Problematic when extending to higher energies
  - Inelastic channels become relevant
  - Higher energetic resonances need to be included

- Formalism consistent with unitarity and analyticity which maps to low energy amplitudes and incorporates resonances determining high energy dynamics
  - Firstly introduced as parametrization of the pion vector form-factor [Hanhart, 2012] and applied to $\pi\pi$-scattering [Ropertz et al., 2018]

OMNÉS PROBLEM

- Dispersion theory: express function $f(s)$ obeying unitarity and causality by its imaginary part

$$\text{disc}\, f(s) = 2i\,\text{Im}\, f(s) \qquad f(s) = \frac{1}{\pi}\int_{s_{\text{th}}}^{\infty} \text{d}z \frac{\text{Im}\, f(z)}{z-s}$$

- Consider f(s) to be a two particle amplitude with definite isospin and angular momentum
  - f(s) can be expressed by Watson's theorem in elastic regime with on-shell T-matrix $T(s)$ and scattering phase shift $\delta(s)$

$$\text{Im}\, f = T^{*}\sigma f \quad \to \quad f(s) = |f(s)|e^{i\delta(s)}$$

  - Solution $f(s) = P(s)\Omega(s)$ with polynomial $P(s) \in \mathbb{R}$ and Omnes-function $\Omega(s)$

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \text{d}z \frac{\delta(z)}{z(z-s)}\right)$$

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FORMALISM

- Implement unitarity and analyticity via Bethe-Salpeter equation

$$T_{if} = V_{if} + V_{im} G_{mm} T_{mf}$$

  - V_{if} : scattering potential between initial channel i and final channel f
  - G_{mm} : loop function describing free propagation of particles in channel m
    $\rightarrow V_{if} \in \mathbb{R} \wedge \text{disc}G_{mm} = 2i\sigma_m$

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- Separate scattering potential into two parts $V = V_0 + V_\mathrm{R}$ and thus $T = T_0 + T_\mathrm{R}$
  - T_0 calculated from elastic input phase δ_0
  - V_0 not explicitly needed $\rightarrow$ absorbed into T_0

$$T_0 = V_0 + V_0 G T_0 \qquad T_0(s) = \begin{pmatrix} [\sigma(s)(\cot\delta_0(s) - i)]^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

  - 2 channel setup including πK and $\eta' K$ as ηK decouples

FORMALISM

- Define vertex function $\Omega = \mathbb{1} + T_0 G$
  - disc $\Omega = 2i T_0{}^* \sigma \Omega$
  - Coincides with discontinuity of Omnes function calculated from T_0

$$\Omega_{\pi K} = \exp\left[\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \mathrm{d}z \frac{\delta_0(z)}{z(z-s)}\right] \qquad \Omega(s) = \begin{pmatrix} \Omega_{\pi K}(s) & 0 \\ 0 & 1 \end{pmatrix}$$

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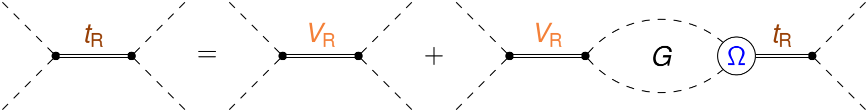
- Use $T_R = \Omega t_R \Omega^T$ to obtain Bethe-Salpeter like equation for t_R

$$t_R = V_R + V_R (G\Omega) t_R$$



  - Selfenergy $\Sigma = G\Omega$ incorporates effects of T_0 in order to conserve unitarity

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FORMALISM

- Define self energy $\Sigma = G\Omega$
  - $\text{disc}\,\Sigma = 2i\Omega^\dagger \sigma \Omega$
  - Express Σ as a once-subtracted dispersion integral

$$\Sigma(s) = \frac{s}{2\pi i}\int_{s_{\text{th}}}^{\infty} \mathrm{d}z \frac{\text{disc}\,\Sigma(z)}{z(z-s)} \qquad \Sigma(s) = \begin{pmatrix} \Sigma_{\pi K}(s) & 0 \\ 0 & \Sigma_{\eta' K}(s) \end{pmatrix}$$

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- General potential with coupling constants g_i, g_j given by

$$\bar{V}_{\text{R}}(s)_{ij} = \sum_r g_i^{(r)} \frac{1}{s - m_r^2} g_j^{(r)}$$

  - Subtract potential at some point s_0 to reduce its impact at low energies

$$V_{\text{R}}(s)_{if} = \bar{V}_{\text{R}}(s)_{ij} - \bar{V}_{\text{R}}(s_0)_{ij} = g_i \frac{s - s_0}{(s - m_r^2)(s_0 - m_r^2)} g_j$$

FORMALISM

- Expression for T-matrix

$$T = T_0 + \Omega \left[\mathbb{1} - V_R \Sigma\right]^{-1} V_R \Omega^{\mathsf{T}}$$

  - T_0 covers effects of $K_0^*(700)$
  - T_{R} explicitly incorporates $K_0^*(1430), K_0^*(1950)$
  - 6 free parameters: 4 couplings constants and 2 resonance masses
- $L = 0$ partial wave measured in combination of $I = ½$ and $I = ³⁄₂$

$$T = T^{1/2} + T^{3/2}/2$$

- Results fixing low energy behaviour taken from [Pelaez and Rodas, 2016]
  - Phase must converge to multiple of π to ensure proper high energy behaviour of Omnes function
  - $I = 3/2$: purly elastic up to $1.8\,\mathrm{GeV}$, no resonances present
  - $I = 1/2$: reduced formalism without $K_0^*(1430/1950)$

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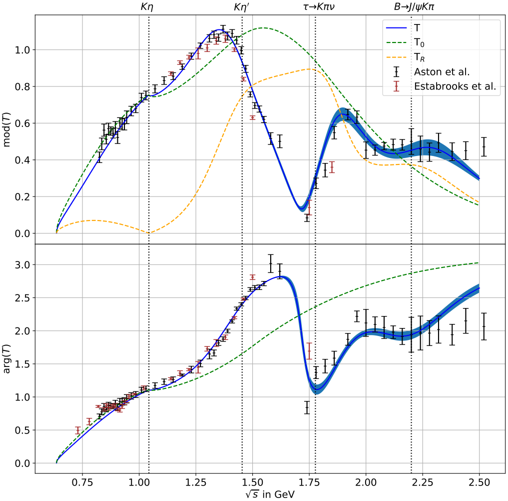
FIT TO PHASE AND MODULUS



- Model reproduces data up to 2.3 GeV
- Fixed $s_0 = (m_K + m_\eta)^2$
  - minimizes T_R at low energies
- Fit to Aston et al. $\chi^2/\text{d.o.f.} = 370/112 \approx 3.5$
- Large incompatibilities between the two data sets
  - Underestimated systematic uncertainties
- Cover πK phase space in
  - $\tau \to K\pi\nu_\tau$
  - $B \to J/\Psi K\pi$

data: [Aston et al., 1988],[Estabrooks et al., 1978]

PARAMETRIZATION OF THE FORM FACTOR

- Calculate form factor Γ with correct analytic structure, containing information of both phase and modulus of the underlying T-matrix

$$\Gamma_i = \Omega_{im}\left[\mathbb{1} - V_R \Sigma\right]^{-1}_{mn} M_n, \qquad M_i = c_i - \sum_r g_i^r \frac{s - s_0}{(s - m_r^2)(s_0 - m_r^2)} \alpha_r$$

- ▸ Couplings g_i^r and resonance masses m_r fixed by previous fit
- ▸ Normalization c_i and resonance coupling to source term α_r are free parameters
- ▸ for πK: Ward identity implicitly fixes $c^{(0)}_{\pi K} = 1$
  U(3) ChPT at leding order fixes $c^{(0)}_{\eta' K} = \sqrt{3}$

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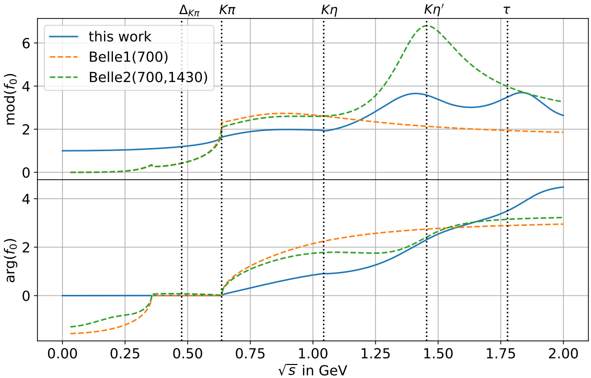
SCALAR FORM FACTOR

scalar form factor resulting from fit to πK invariant mass spectrum obtained from $\tau \to K_S \pi^- \nu_\tau$ decays by Belle [Epifanov et al., 2007]



- correct phase motion in elastic regime by construction
- Callan-Treiman low energy theorem fulfilled up to at least 0.5%

POLE EXTRACTION

- Breit-Wigner parameterization model- and reaction-dependent
  - violate unitarity for overlapping resonances
  - problematic for broad or near-threshold resonances

- Information of a resonance encoded in its pole
  - mass M_{R} and width Γ_{R} from pole position $\sqrt{s_{\mathrm{R}}} = M_{\mathrm{R}} - i\frac{\Gamma_{\mathrm{R}}}{2}$
  - coupling constant $\tilde{g}_j^{\mathrm{R}}$ from residue $\mathcal{R}_{ij} = -\lim_{s \to s_{\mathrm{R}}} T_{ij} \to \tilde{g}_j^{\mathrm{R}} = \mathcal{R}_{ij}/\sqrt{\mathcal{R}_{ii}}$
  - $\mathrm{BR}_{\mathrm{R}\to i} = \Gamma_{\mathrm{R}\to i}/\Gamma_{\mathrm{tot.}}$ from partial width $\Gamma_{\mathrm{R}\to i} = |\tilde{g}_i^{\mathrm{R}}|^2 \rho({M_{\mathrm{R}}}^2)/M_{\mathrm{R}}$
  - coupling C_{us}^{R} to $\bar{s}\gamma^{\mu}u$ current from residue $\lim_{s \to s_{\mathrm{R}}} f_0 \propto \tilde{g}_i^{\mathrm{R}} C_{us}^{\mathrm{R}}$

Convention from PDG resonance review [P.A. Zyla et al., 2020]

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  Convention from PDG resonance review [P.A. Zyla et al., 2020]

- T_0 has complicated analytic structure due to left-hand cuts
  - Use Padé approximants to extract resonance properties

PADÉ APPROXIMANTS

$$P_1^N(s, s_0) = \frac{\sum_{n=0}^{N} a_n^{(N)}(s - s_0^{(N)})^n}{1 + b^{(N)}(s - s_0^{(N)})}, \qquad \Delta_{\text{sys}}^{(N)} = \left|\sqrt{s_R^{(N)}} - \sqrt{s_R^{(N-1)}}\right|$$

- fit $P_1^N(s, s0)$ to T-matrix and form factor
- choose $s_0^{(N)}$ that minimizes $\Delta_{\text{sys}}^{(N)}$ for each N

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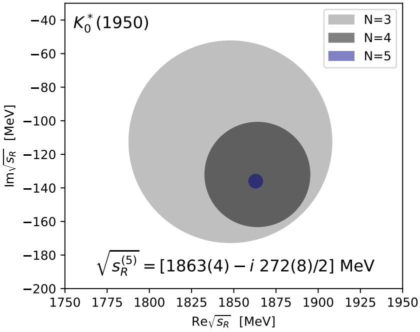
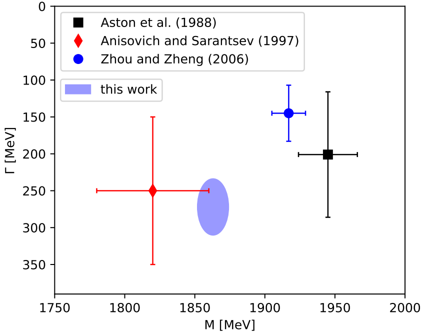
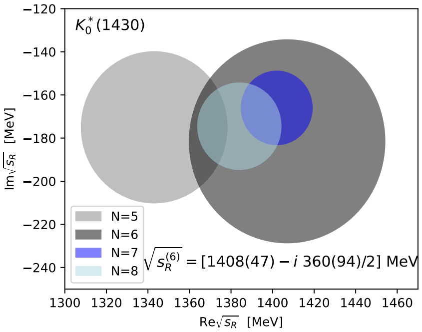


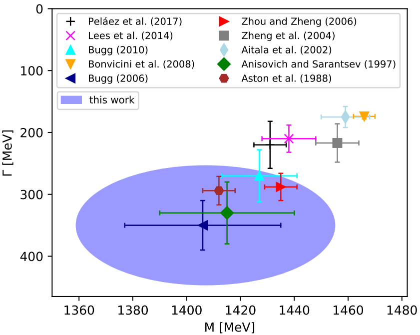


PADÉ APPROXIMANTS

- $K_0^*(1430)$ located in close proximity to the $K\eta'$ threshold
  - use conformal variable $\omega(s)$ to improve convergence of $P_1^N(s, s_0)$

$$\omega(s) = \frac{\sqrt{s - s_1^{\text{th}}} - \sqrt{s_2^{\text{th}} - s}}{\sqrt{s - s_1^{\text{th}}} + \sqrt{s_2^{\text{th}} - s}}$$





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CONCLUSION

- Applied new formalism consistent with unitarity and analyticity for πK scattering and production
  - Low energy regime fixed by input phase
  - High energy dynamics determined by resonances

- Model able to reproduce phase and modulus data up to 2.3 GeV including the $K_0^*(700), K_0^*(1430)$ and $K_0^*(1950)$ resonances
  - extracted pole using Padé approximants:

$$\sqrt{s_{K_0^*(1430)}} = [1408(48) - i\,180(48)]\mathrm{MeV}, \quad \mathrm{BR}_{K_0^*(1430)\to\pi K} = 0.87(12)$$
$$\sqrt{s_{K_0^*(1950)}} = [1863(12) - i\,136(20)]\mathrm{MeV}, \quad \mathrm{BR}_{K_0^*(1950)\to\pi K} = 0.70(8)$$

- Reproduce scalar part of πK invariant mass spectrum in $\tau \to K\pi\nu_\tau$
  - improved estimate for $\mathrm{BR}_{\tau\to K_0^*(1430)\nu_\tau} < 1.6 \times 10^{-4}$ (at 95% confidence level)
  - *CP* asymmetry generated by tensor operator $A_{CP}^{\tau,\mathrm{BSM}} = -0.034(14)\ \mathrm{Im}\, c_T$

LITERATUR I

[1] C. Hanhart. “A New Parameterization for the Pion Vector Form Factor”. In: Phys. Lett. B 715 (2012), pp. 170–177. DOI: 10.1016/j.physletb.2012.07.038. arXiv: 1203.6839 [hep-ph].

[2] S. Ropertz, C. Hanhart, and B. Kubis. “A new parametrization for the scalar pion form factors”. In: Eur. Phys. J. C 78.12 (2018), p. 1000. DOI: 10.1140/epjc/s10052-018-6416-6. arXiv: 1809.06867 [hep-ph].

[3] D. Aston et al. “A Study of K- pi+ Scattering in the Reaction K- p —¿ K- pi+ n at 11-GeV/c”. In: Nucl. Phys. B 296 (1988), pp. 493–526. DOI: 10.1016/0550-3213(88)90028-4.

[4] P. Estabrooks et al. “Study of K pi Scattering Using the Reactions K+- p —¿ K+- pi+ n and K+- p —¿ K+- pi- Delta++ at 13-GeV/c”. In: Nucl. Phys. B 133 (1978), pp. 490–524. DOI: 10.1016/0550-3213(78)90238-9.

LITERATUR II

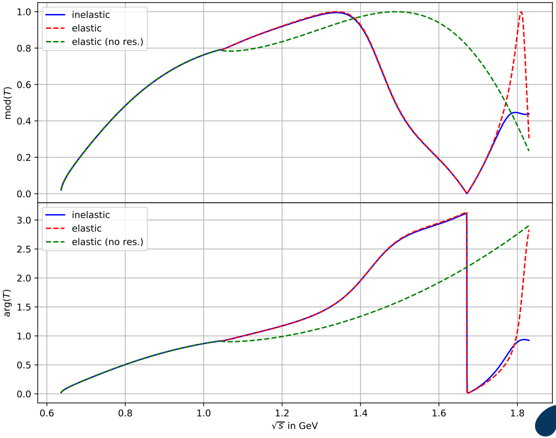
[5] D. Epifanov et al. “Study of tau- —¿ K(S) pi- nu(tau) decay at Belle”. In: Phys. Lett. B 654 (2007), pp. 65–73. DOI: 10.1016/j.physletb.2007.08.045. arXiv: 0706.2231 [hep-ex].

[6] J. Pelaez and A. Rodas. “Pion-kaon scattering amplitude constrained with forward dispersion relations up to 1.6 GeV”. In: Phys. Rev. D 93.7 (2016), p. 074025. DOI: 10.1103/PhysRevD.93.074025. arXiv: 1602.08404 [hep-ph].

PELAEZ AND RODAS PARAMETERIZATION

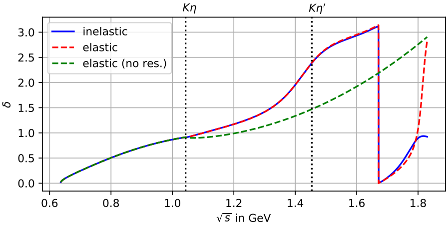
$$t_0^{1/2} = \frac{S_0^b S_1^r S_2^r}{2i\sigma}$$

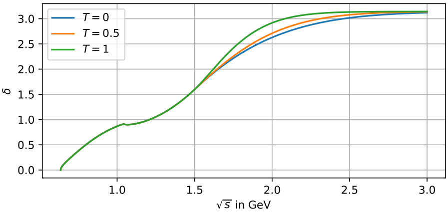
$$S_0^b = \exp\left(2ip_{\eta K}\left(\phi_0 + \phi_1 p_{\eta K}^2\right)\right)$$



INPUT PHASE

- Results fixing low energy behaviour taken from [Pelaez et al., 2016]
  - $I = ½$: reduced formalism with pure elastic phase without resonance contributions





  - Phase must converge to multiple of π to ensure proper high energy behaviour of Omnes function
  - Continuation matched at $\sqrt{s_{\mathrm{m}}} = 1.5\,\mathrm{GeV}$ with additional tuning parameter T

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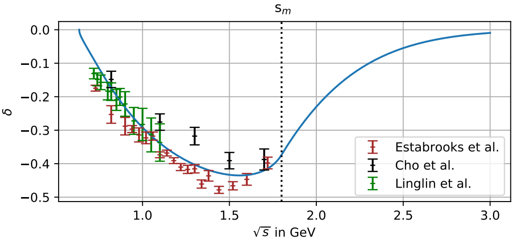
INPUT PHASE

- $I = 3/2$: purly elastic up to $1.8\,\mathrm{GeV}$
  - $T_0^{3/2}$ directly calculated from phase
  - no resonances present in this channel (exotic quantum numbers) $\rightarrow$ phase continued to 0 and $T_R = 0$

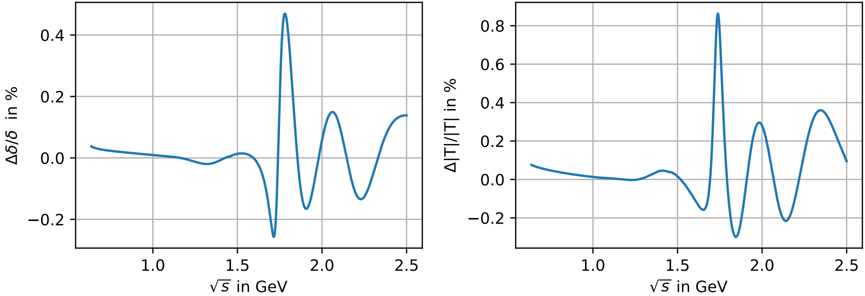


  - original phase taken up to $\sqrt{s_\mathrm{m}} = 1.8\,\mathrm{GeV}$ to conserve its prominent structure

INCLUSION OF ηK-CHANNEL

- Explicit inclusion of ηK-channel statistically insignificant
- Check relative difference between 2-channel formalism $(\pi K, \eta' K)$ and 3-channel formalism $(\pi K, \eta K, \eta' K)$

$$\frac{\Delta\delta}{\delta} = \frac{\arg(T_{2c}) - \arg(T_{3c})}{\arg(T_{2c})} \qquad \frac{\Delta|T|}{|T|} = \frac{\mathrm{mod}(T_{2c}) - \mathrm{mod}(T_{3c})}{\mathrm{mod}(T_{2c})}$$

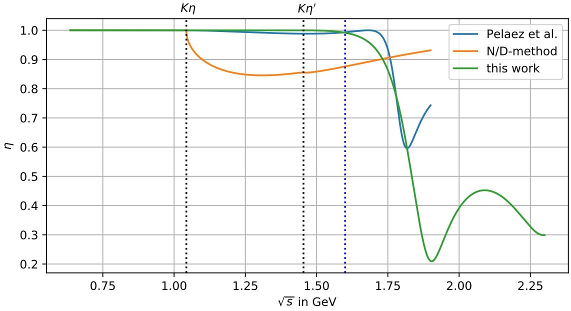


- Fit indicates vanishing couplings of resonances to the ηK-channel πK-channel mostly elastic up to 1.6 $\mathrm{GeV} \rightarrow \eta K$-channel decouples

ELASTICITY

$$\eta_{\pi K} = \mathrm{mod}(1 + 2i\sigma_{\pi K} T_{\pi K})$$

- Elasticity of new model compatible with results from [Pelaez et al., 2016]



FIT TO τ-DECAY SPECTRUM

- $\tau \to K_S \pi^- \nu_\tau$ differential decay rate

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{c_\Gamma}{s}\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right) q_{\pi K} \left(q_{\pi K}^2 |\bar{f}_+|^2 + \frac{3\Delta_{\pi K}^2}{4s(1 + 2\frac{s}{m_\tau^2})} |\bar{f}_0|^2 \right)$$

- defining matrix elements

$$\langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = (p_K - p_\pi)^\mu f_+(s) + (p_K + p_\pi)^\mu f_-(s) \qquad (1)$$

$$\langle \bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}u|0\rangle = \frac{\Delta_{\pi K}}{m_s - m_u} f_0(s)\,, \qquad (2)$$

τ-DECAY SPECTRUM

- πK invariant mass spectrum extracted from $\tau \to K_s \pi^- \nu_\tau$ decays by Belle [Epifanov et al., 2007]
  - Parameterized in terms of scalar F_S and vector F_V form factor

$$\frac{d\Gamma}{d\sqrt{s}} \propto \frac{1}{s}\left(1-\frac{s}{m_\tau^2}\right)\left(1+2\frac{s}{m_\tau^2}\right) P \left[P^2|F_V|^2 + \frac{3(m_K^2-m_\pi^2)^2}{4s(1+2\frac{s}{m_\tau^2})}|F_S|^2\right]$$

  - Form factors of Belle based on addition of Breit-Wigners, thus violating unitarity

$$F_S{}^{\text{Belle}} = \kappa \frac{s}{M^2_{K_0^*(700)}} \text{BW}_{K_0^*(700)} + \gamma \frac{s}{M^2_{K_0^*(1430)}} \text{BW}_{K_0^*(1430)}$$

$$F_V{}^{\text{Belle}} = \frac{1}{1+\beta+\chi}\left[\text{BW}_{K^*(892)} + \beta \text{BW}_{K^*(1410)} + \chi \text{BW}_{K^*(1680)}\right]$$

  - $\kappa \in \mathbb{R} \quad \gamma, \beta, \chi \in \mathbb{C}$ free parameters

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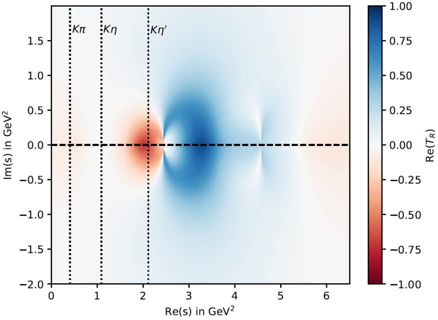
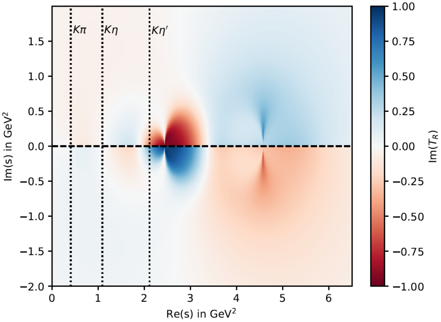
FIRST RIEMANN-SHEET

- T_0 only contains information of the input phase on the real axis
  $\rightarrow$ no analytic continuation to the complex plane
  $\rightarrow$ no unphysical poles by construction
- Can check T_R for anomalous structures





- Extract pole positions via Padé-approximations $\rightarrow$ to be done

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τ-DECAY SPECTRUM

- $\tau \to K_S \pi^- \nu_\tau$ decay spectrum meassured by Belle [Epifanov et al., 2007]

$$\frac{d\Gamma}{d\sqrt{s}} = \frac{c_\Gamma}{s}\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right) q_{\pi K}\left(q_{\pi K}^2 |f_+|^2 + \frac{3\Delta_{\pi K}^2}{4s(1+2\frac{s}{m_\tau^2})}|f_0|^2\right)$$

- use new formalism to calculate scalar form factor f_0 and RChPT to model vector form factor f_+