Dispersive analysis of $\pi\pi$ and πK scattering

Oleksandra Deineka

In collaboration with Igor Danilkin and Marc Vanderhaeghen

Phys. Rev. D **103**, no.11, 114023 (2021)

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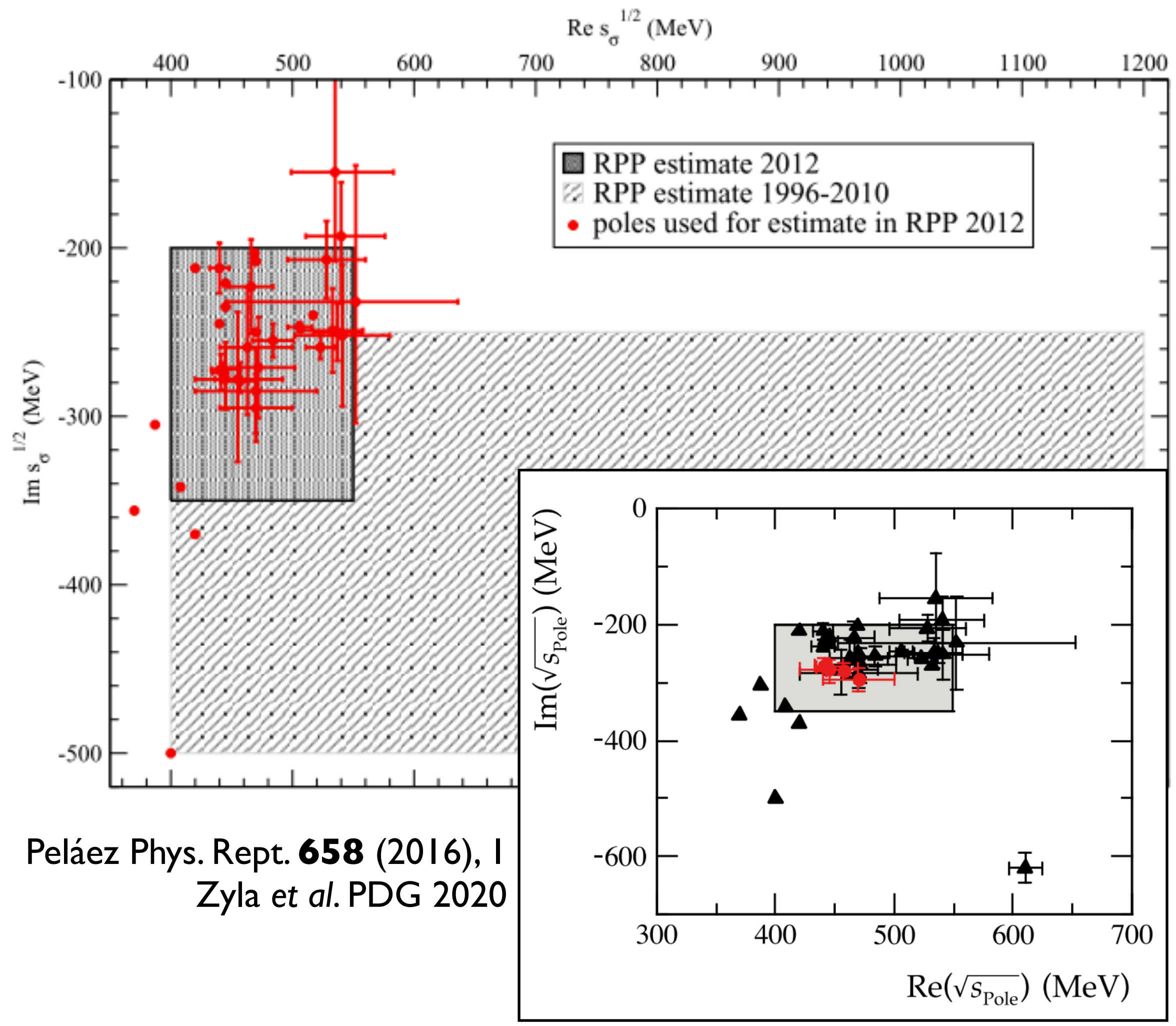
19th INTERNATIONAL CONFERENCE ON HADRON SPECTROSCOPY AND STRUCTURE

THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

- Scalar resonances: $f_0(500)$, $f_0(980)$, $K^*(700)$
- Precise dispersive determination for $f_0(500)$ and $K^*(700)$ from Roy and Roy-Steiner analyses based on all available **experimental** data



Peláez Phys. Rept. **658** (2016), 1
Zyla *et al.* PDG 2020

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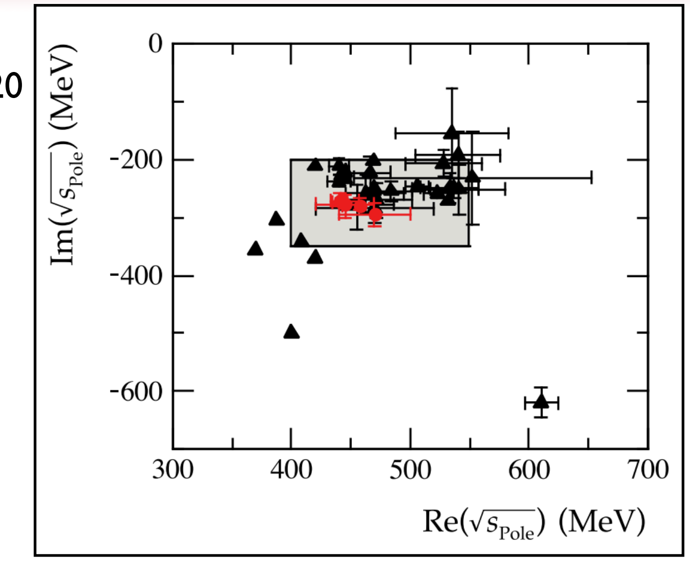
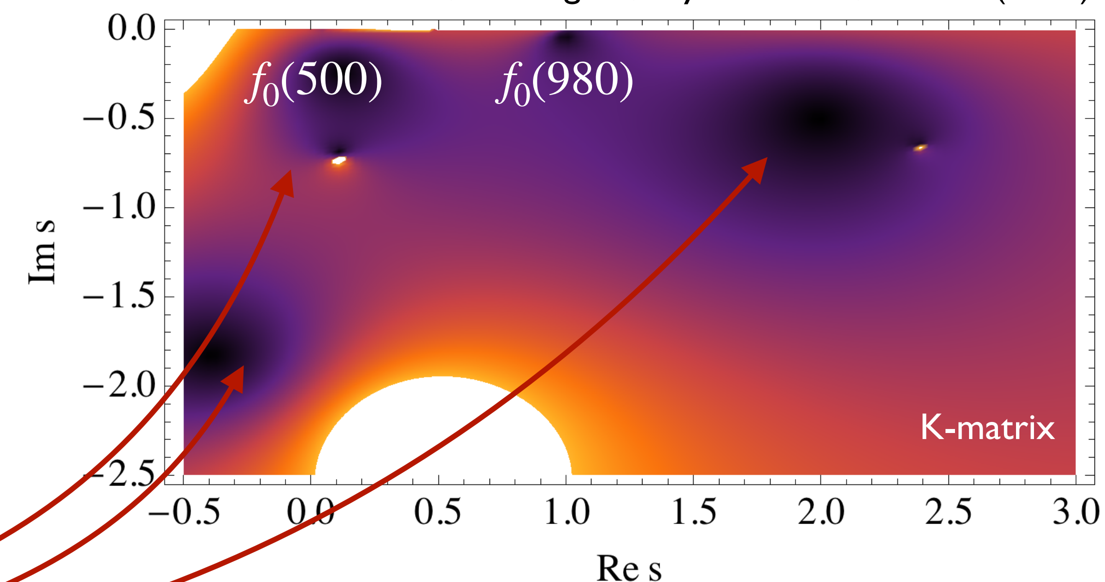
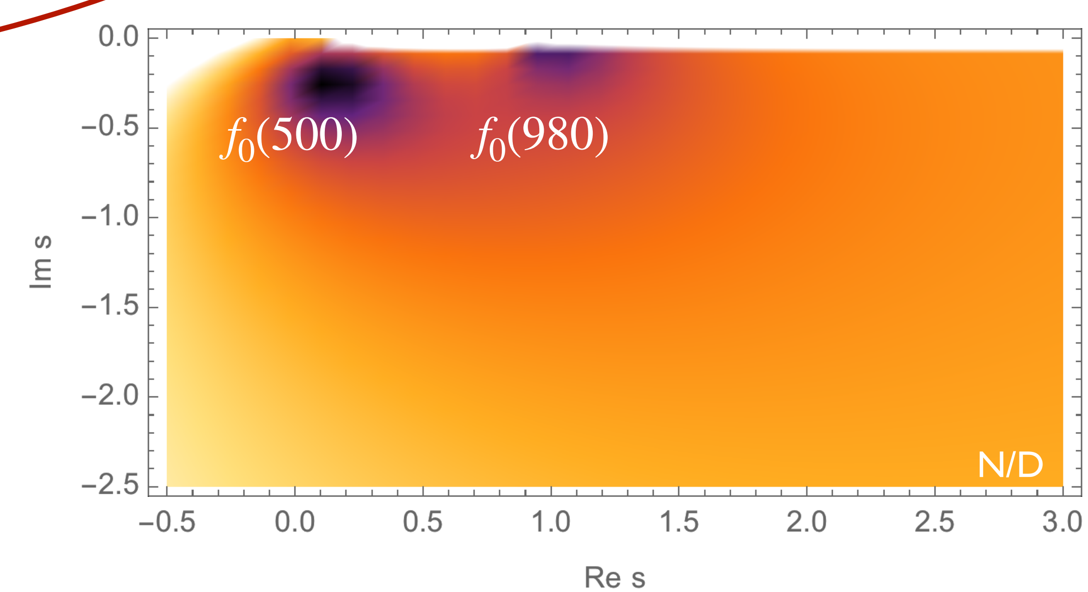
Zyla *et al.* PDG 2020



- Recent **lattice** studies of $\pi\pi \to \pi\pi$ $(\pi K \to \pi K)$ for pion masses as low as 236 (239) MeV
- Lattice data is analysed using K-matrix approach $\Longrightarrow$ not always possible to find a stable pole due to the presence of spurious poles
- We use N/D approach which incorporates both unitarity and analyticity

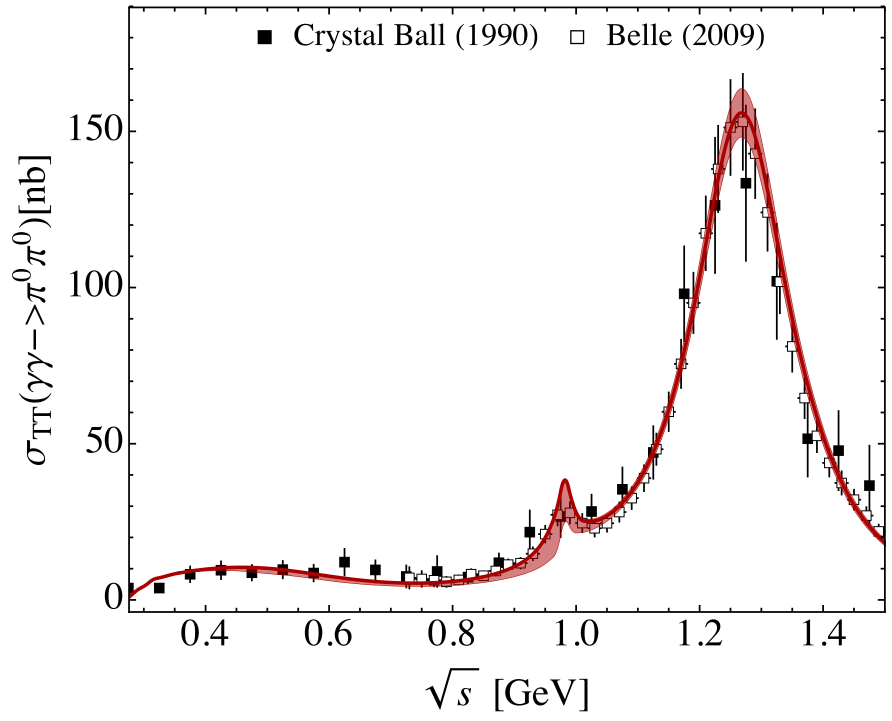
Dai, Pennington, Phys. Rev.D90, 036004 (2014)





Motivation: $\gamma\gamma \to \pi\pi$ and other reactions input

- The **Omnès function** is crucial and universal: it accounts for the hadronic FSI. It arises naturally from the N/D approach as an inverse of the D-function
- Input to muon $(g-2)$ from single and double virtual $\gamma^*\gamma^* \to \pi\pi, K\bar{K}, \pi\eta$ processes
- Recent calculation of $f_0(980)$ contribution $a_\mu^{\text{HLbL}}[f_0(980)] = -0.2(2) \times 10^{-11}$ arXiv: 2105.01666 [hep-ph]



Acta Phys.Polon.B 50 (2019) 1901-1910

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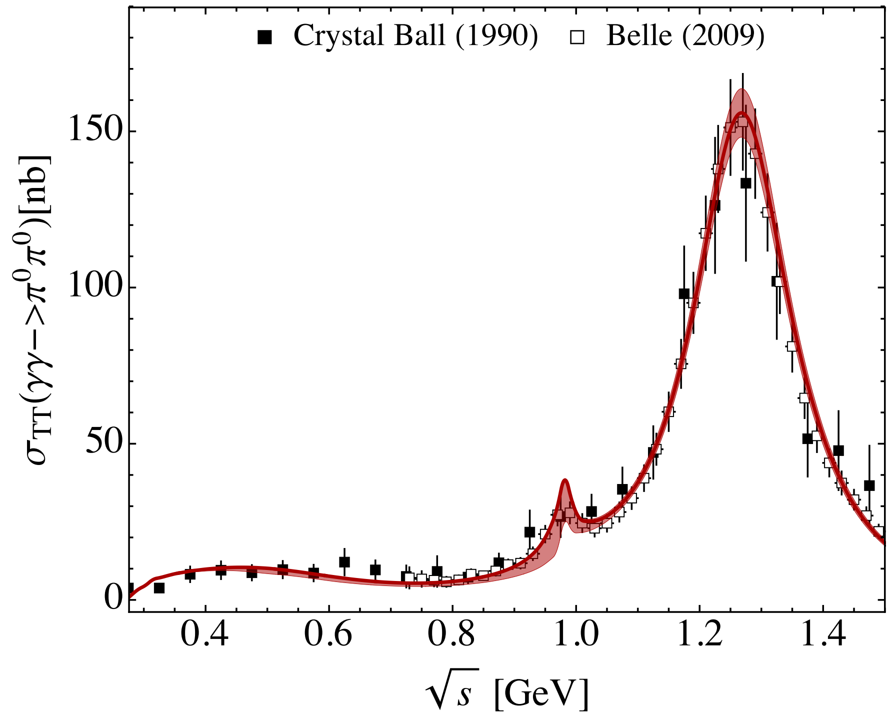
- Further application to exotics: $Z_c(3900)$ from $e^+e^- \to J/\psi\pi\pi(K\bar{K})$



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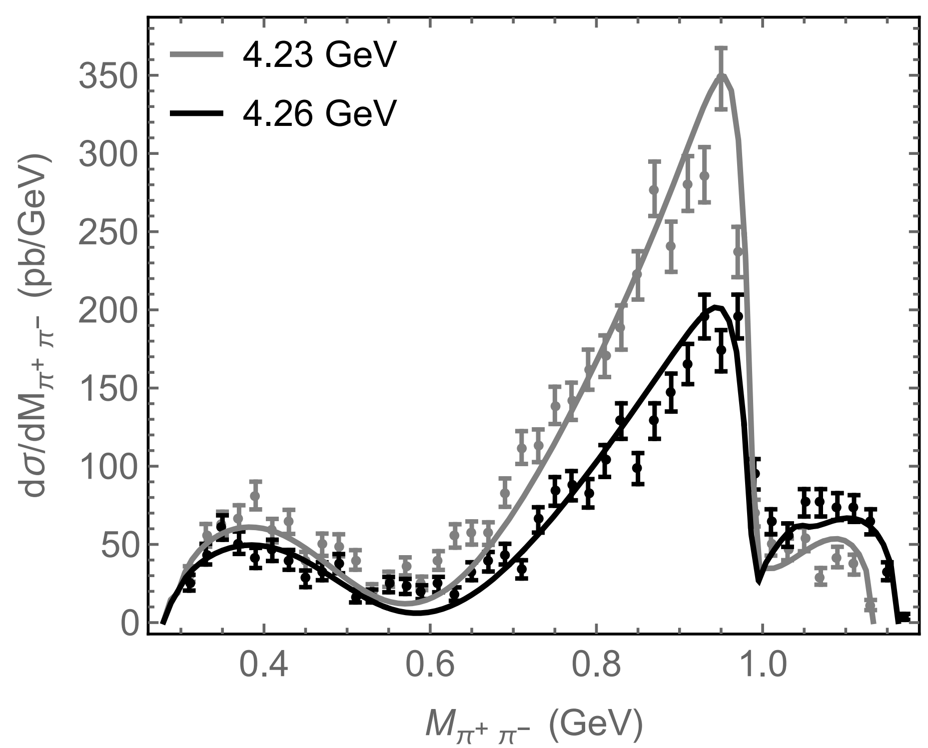


Phys. Rev. D 102 1, 016019, (2020)

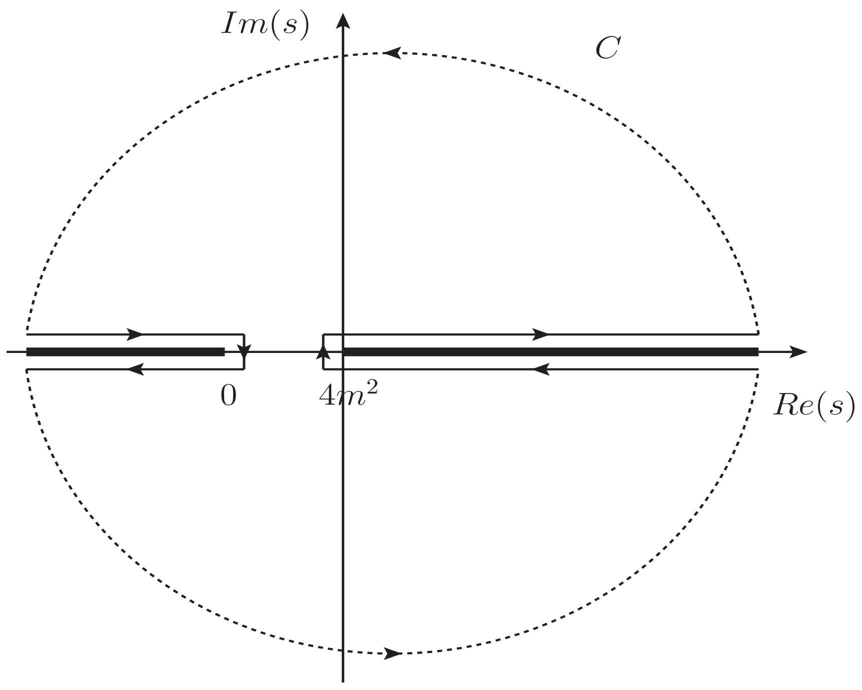
Partial wave dispersion relation

Unitarity relation for the partial wave amplitudes $\Longrightarrow$ guarantees that p.w. amplitudes behave asymptotically **no worse than a constant**

$$\text{Disc}\, t_{ab}(s) = \sum_c t_{ac}(s)\, \rho_c(s)\, t^*_{cb}(s)$$

From the **maximal analyticity** principle one can write **dispersion relation**

$$t_{ab}(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\text{Disc}\; t_{ab}(s')}{s'-s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc}\; t_{ab}(s')}{s'-s}$$



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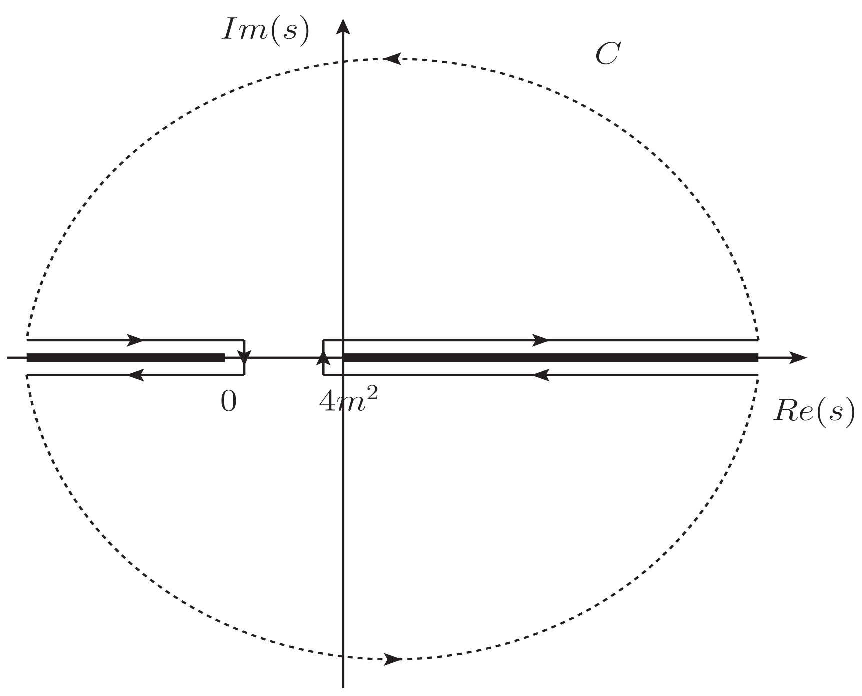

Which we subtract once in accordance with **unitarity bound**

$$t_{ab}(s) = U_{ab}(s) + \frac{s-s_M}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'-s_M} \frac{\text{Disc}\, t_{ab}(s')}{s'-s}$$

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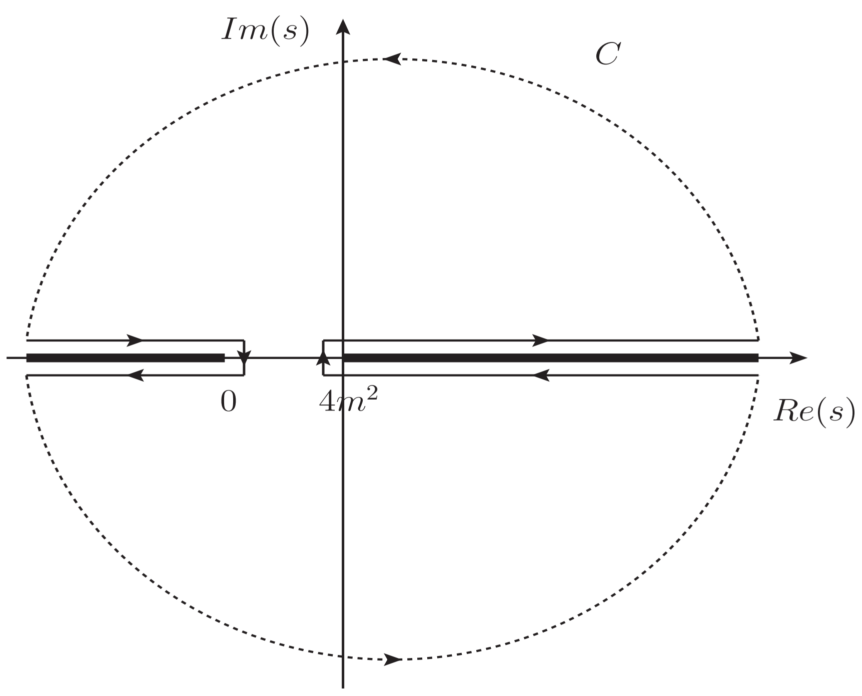

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included subtraction constant and left-hand cuts, asymptotically bounded **unknown function**

N/D method

Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s' - s}$$

can be solved using N/D method with input from $U_{ab}(s)$ **above threshold**

$$t_{ab}(s) = \sum_c D^{-1}_{ac} N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ac}(s')\rho_c(s')(U_{cb}(s') - U_{cb}(s))}{s' - s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ab}(s')\rho_b(s')}{s' - s} = \Omega^{-1}_{ab}(s)$$

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need to solve integral equation

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Omnès function

Conformal mapping expansion

We approximate left-hand cuts as an expansion in a **conformal mapping variable** $\xi(s)$

Gasparyan, Luts, Nucl. Phys. A848, 126 (2010)

$$U_{ab}(s) = t_{ab}(0) + \frac{s}{\pi}\int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\text{Im}\; t_{ab}(s')}{s' - s} \simeq \sum_{n=0}^{\infty} C_{ab,n}\, \xi_{ab}^{n}(s)$$

to be determined from the fits

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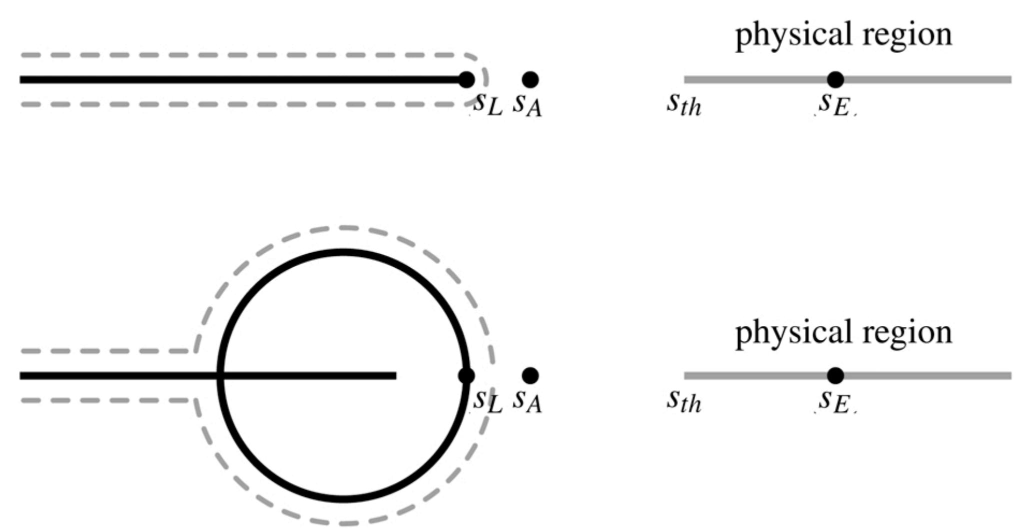

The exact form of the conformal map

$\{\pi\pi, K\bar{K}\}$
$$\xi(s) = \frac{\sqrt{s-s_L} - \sqrt{s_E - s_L}}{\sqrt{s-s_L} + \sqrt{s_E - s_L}}$$

πK
$$\xi(s) = -\frac{(\sqrt{s} - \sqrt{s_E})(\sqrt{s}\sqrt{s_E} + s_L)}{(\sqrt{s} + \sqrt{s_E})(\sqrt{s}\sqrt{s_E} - s_L)}$$

source of the systematic uncertainties

$$\xi(s_E) = 0$$

$$\sqrt{s_E} = \frac{1}{2}\left(\sqrt{s_{th}} + \sqrt{s_{max}}\right)$$

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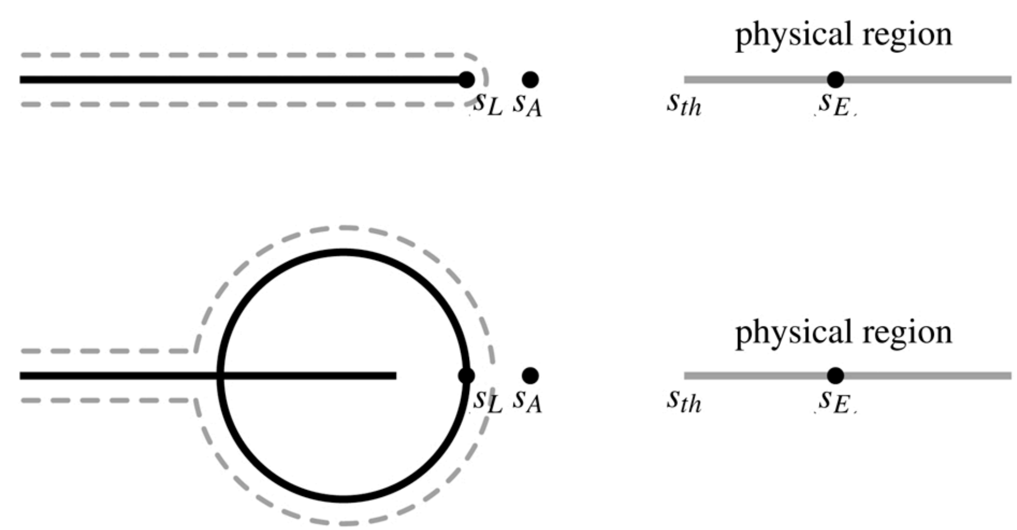

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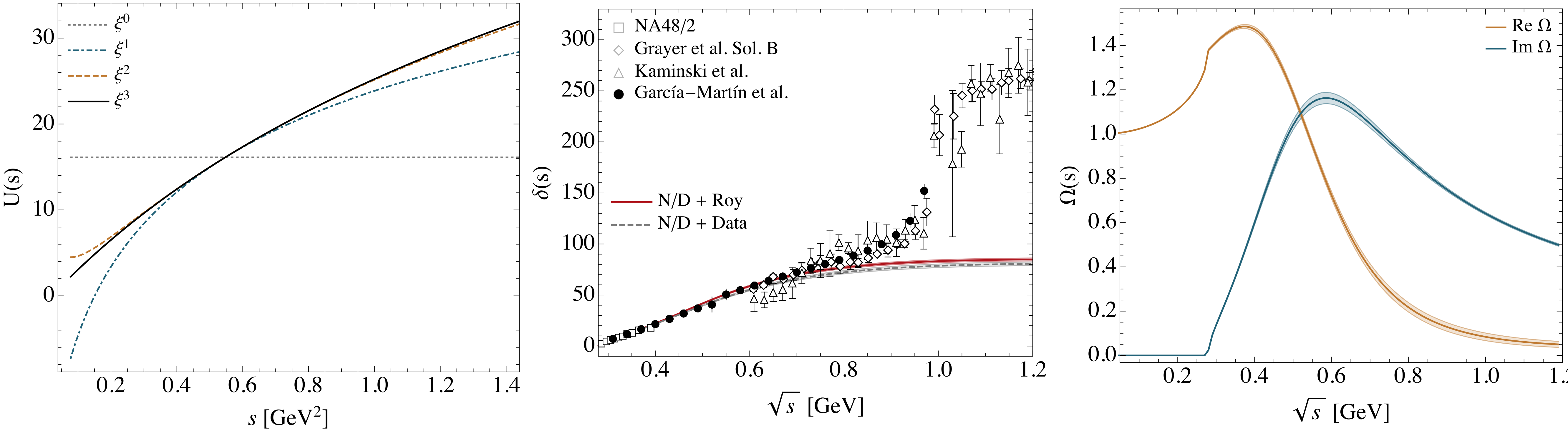
source of the systematic uncertainties

N/D approach requires solving an integral equation $\Longrightarrow$ we use **parametric bootstrap** technique for fitting and error analysis

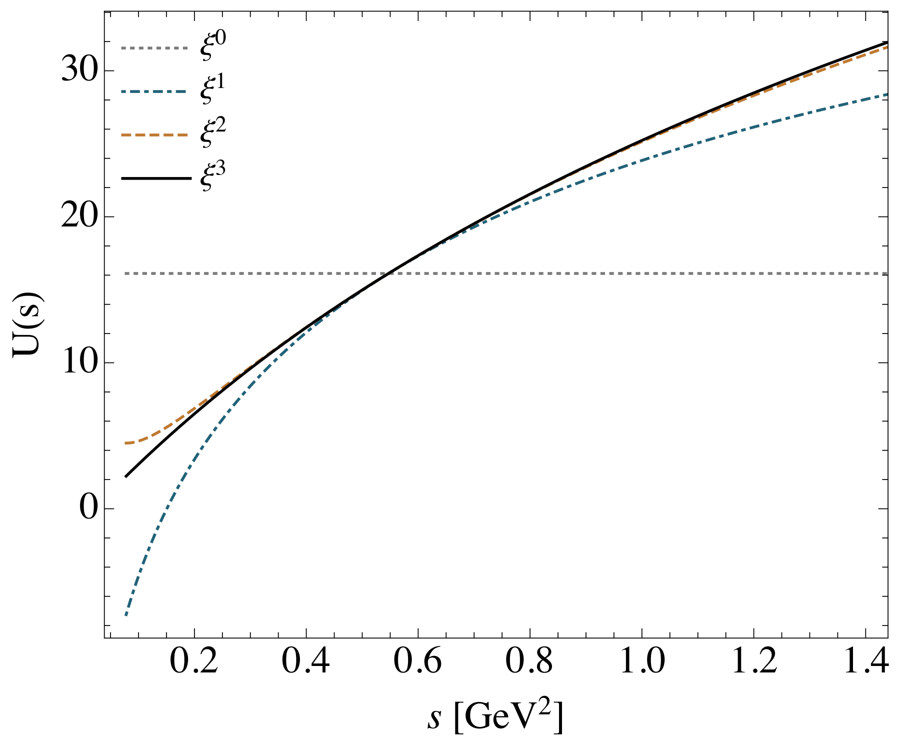
Single channel $\pi\pi$ scattering

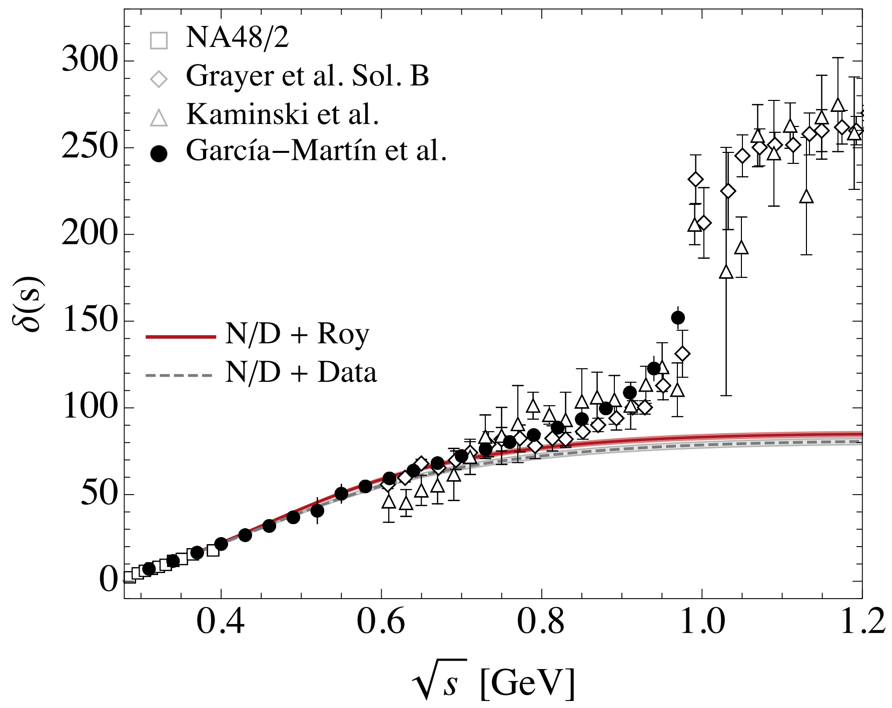


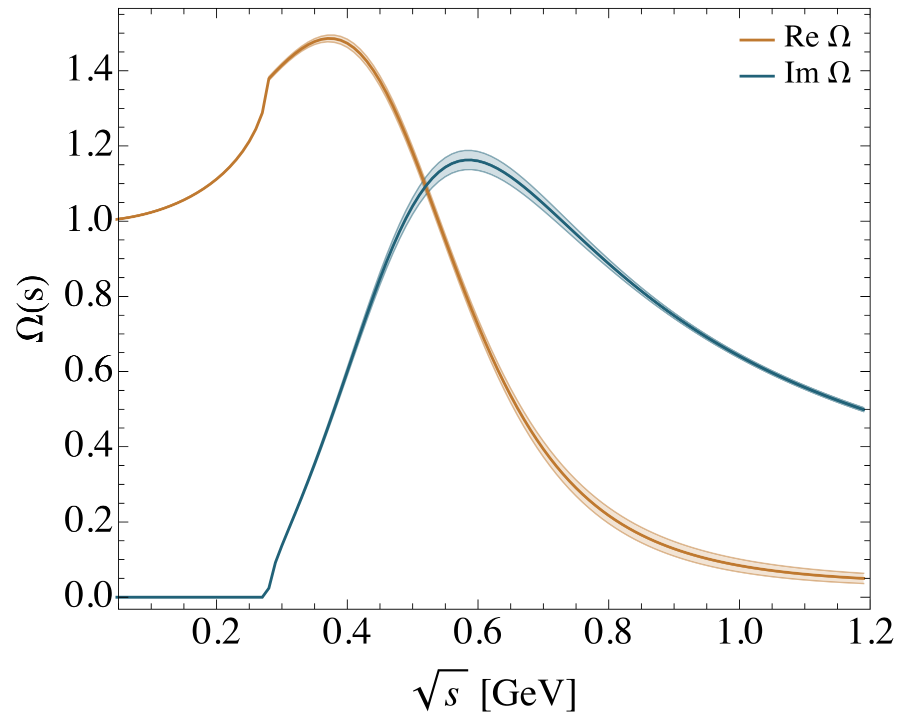
Input: experimental data/Roy analysis + NNLO χPT a, b + NLO χPT Adler zero

4 parameters fit

Single channel $\pi\pi$ scattering







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4 parameters fit

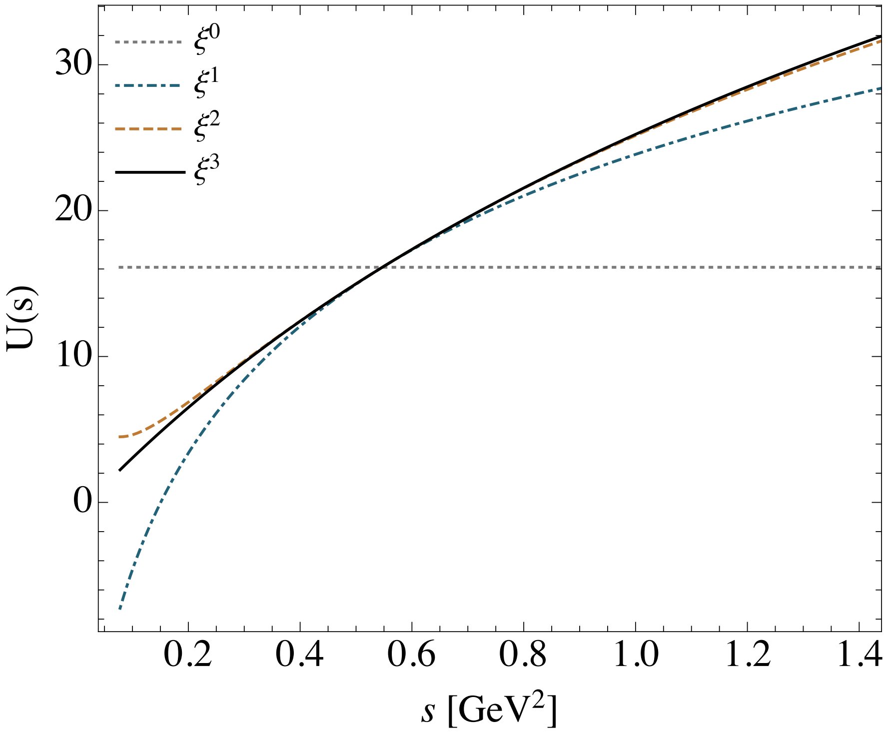
$$\sqrt{s_{f_0(500),ND+exp}} = 435(7)^{+6}_{-8} - i\,250(5)^{+6}_{-8}\ \text{MeV}$$

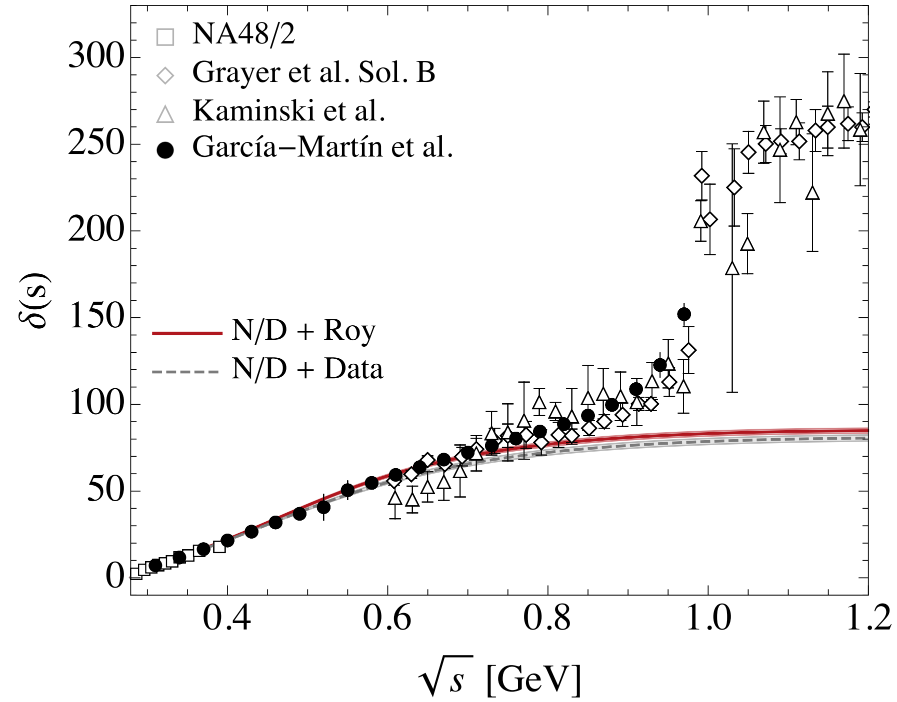
$$\sqrt{s_{f_0(500),ND+Roy}} = 458(7)^{+4}_{-10} - i\,245(6)^{+7}_{-10}\ \text{MeV}$$

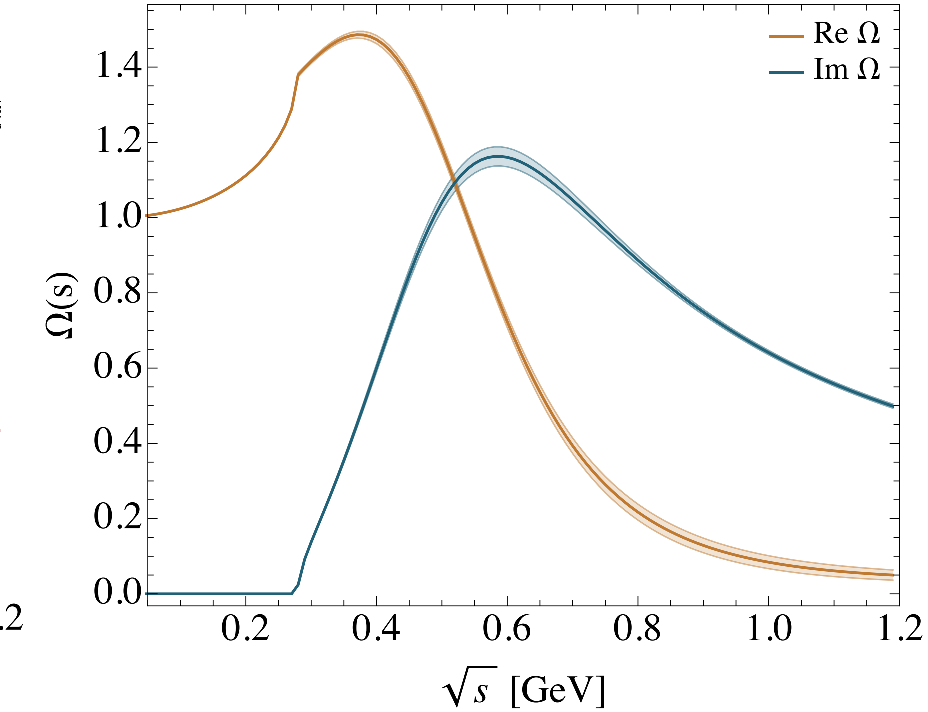
$$\sqrt{s_{f_0(500),Roy}} = 449^{+22}_{-16} - i\,275(15)\ \text{MeV}$$

García-Martín *et al.* Phys.Rev.D 83 (2011) 074004
Caprini *et al.* Phys. Rev. Lett. 96, 132001 (2006)

Single channel $\pi\pi$ scattering







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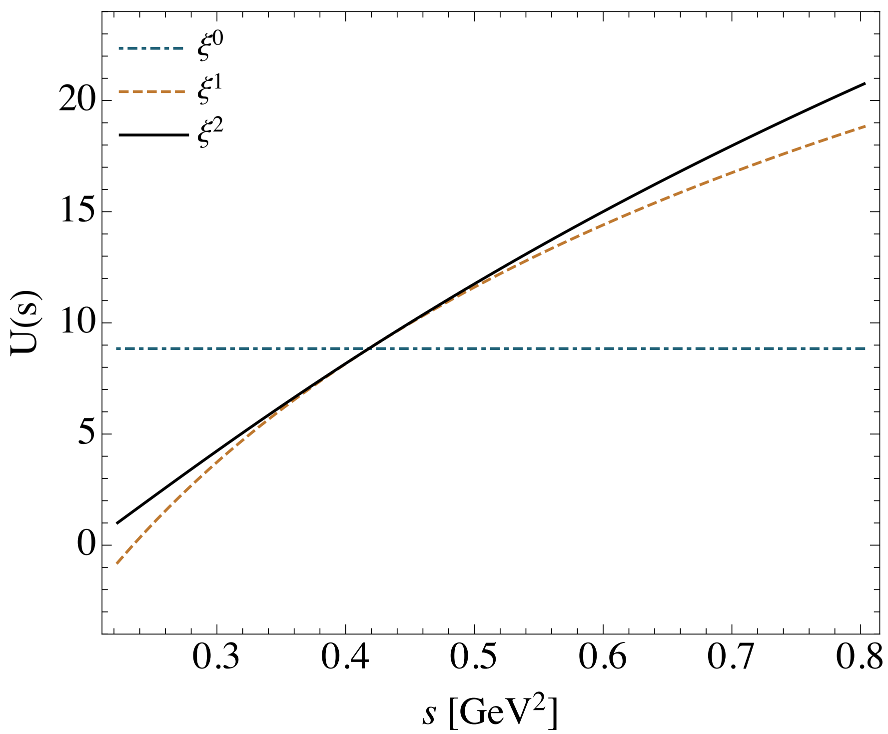
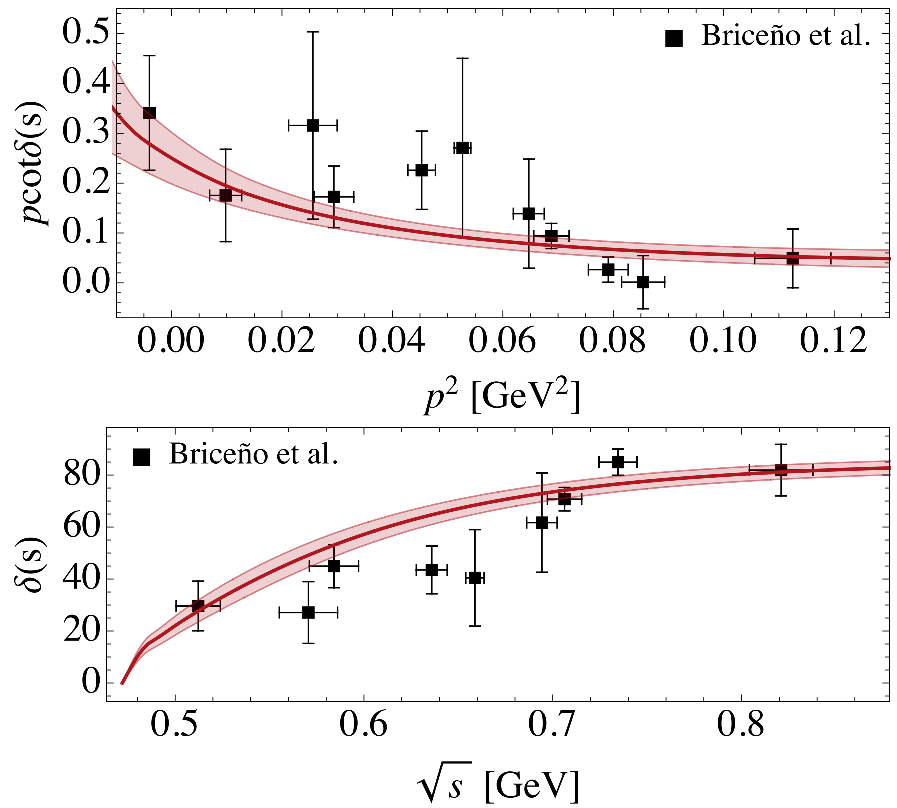
$$\Omega(s) = D^{-1}(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right)$$

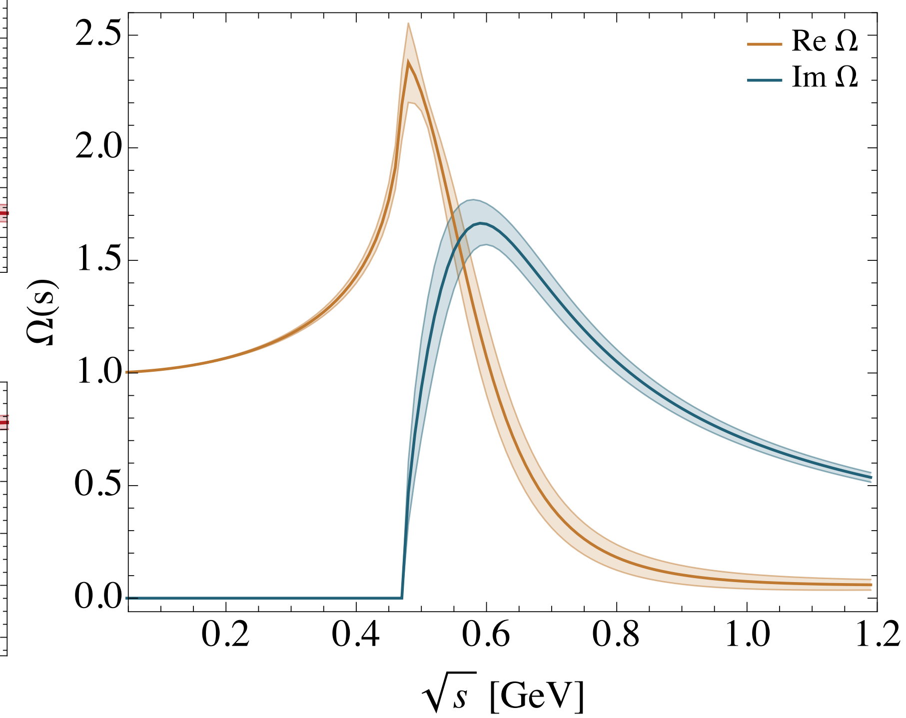
Similar results for single channel $\pi\pi$ phase-shift and Omnès can be obtained by using mIAM with the χPT input

Gomez Nicola *et al.* Phys. Rev. D 77, 056006 (2008)

Single channel $\pi\pi$ scattering for $m_\pi = 236$ MeV







Input: Lattice data + NLO χPT Adler zero

3 parameters fit

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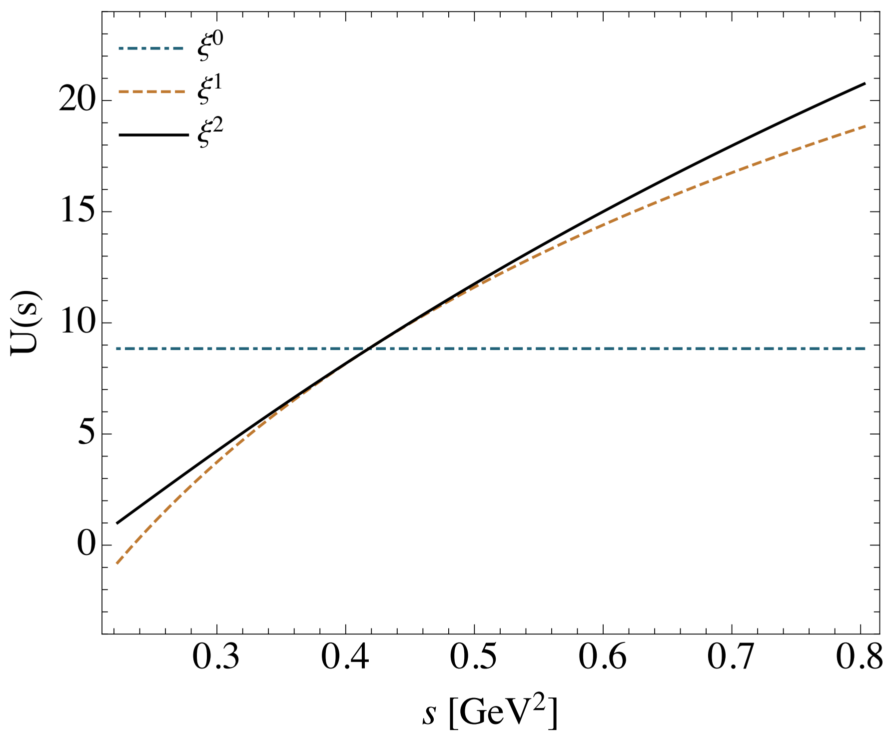
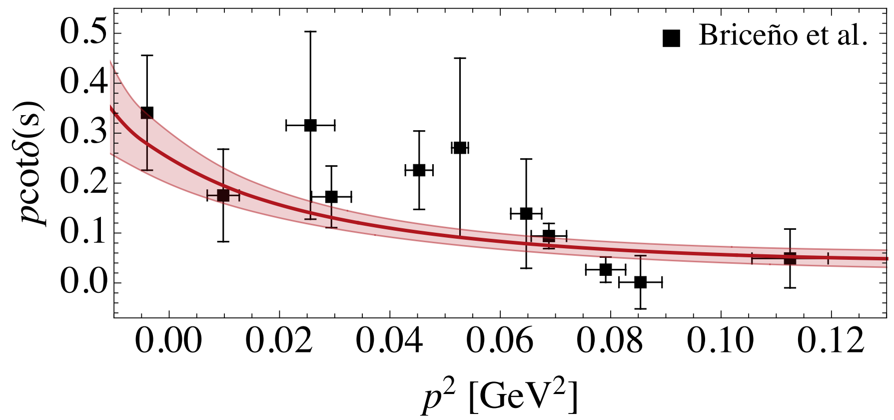
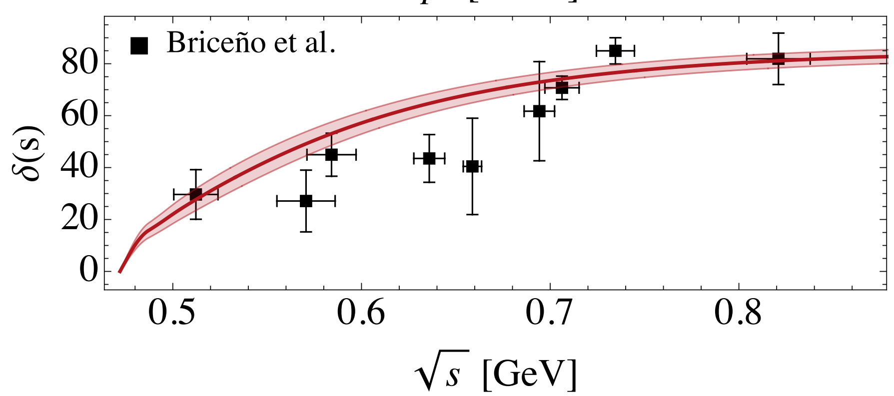
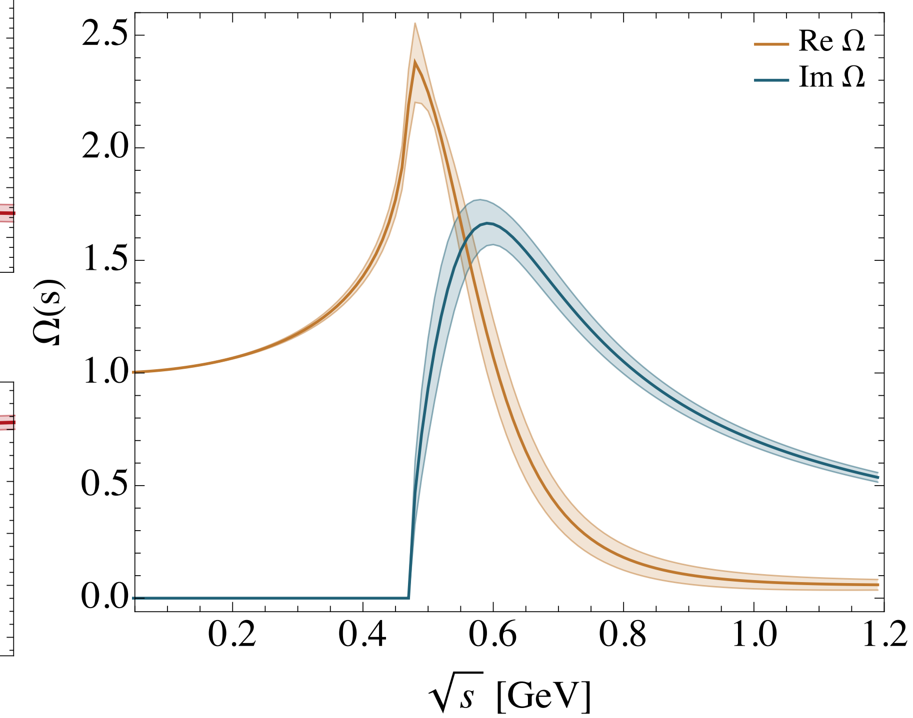








Input: Lattice data + NLO χPT Adler zero

3 parameters fit

N/D analysis of the lattice data:

$$\sqrt{s_{f_0(500),ND+Latt}} = 498(21)^{+12}_{-19} - i\,138(13)^{+5}_{-10} \text{ MeV}$$

Briceño *et al.* Phys.Rev.Lett. 118 (2017) 2, 022002

Predictions:

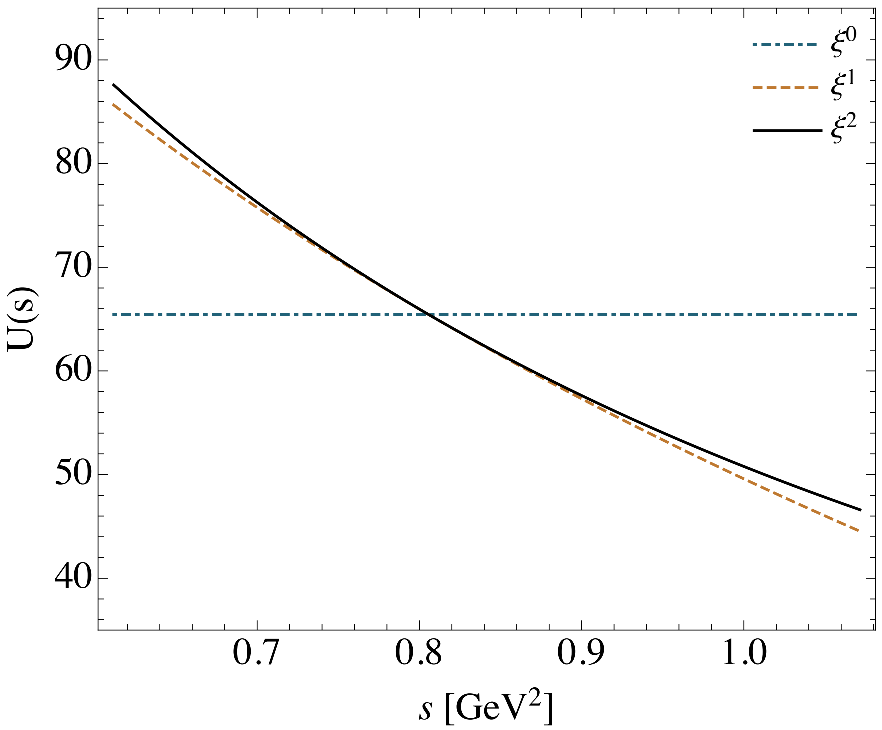
$$\sqrt{s_{f_0(500),mIAM_{NNLO}}} = 510 - i\,175 \text{ MeV}$$

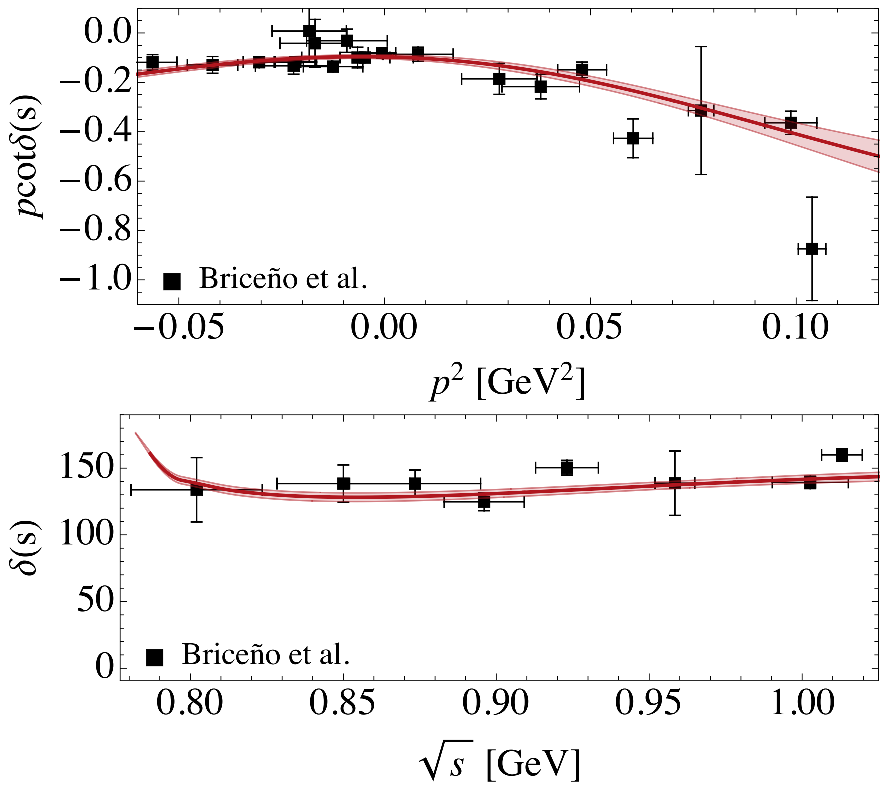
Peláez, Rios Phys.Rev.D 82 (2010) 114002

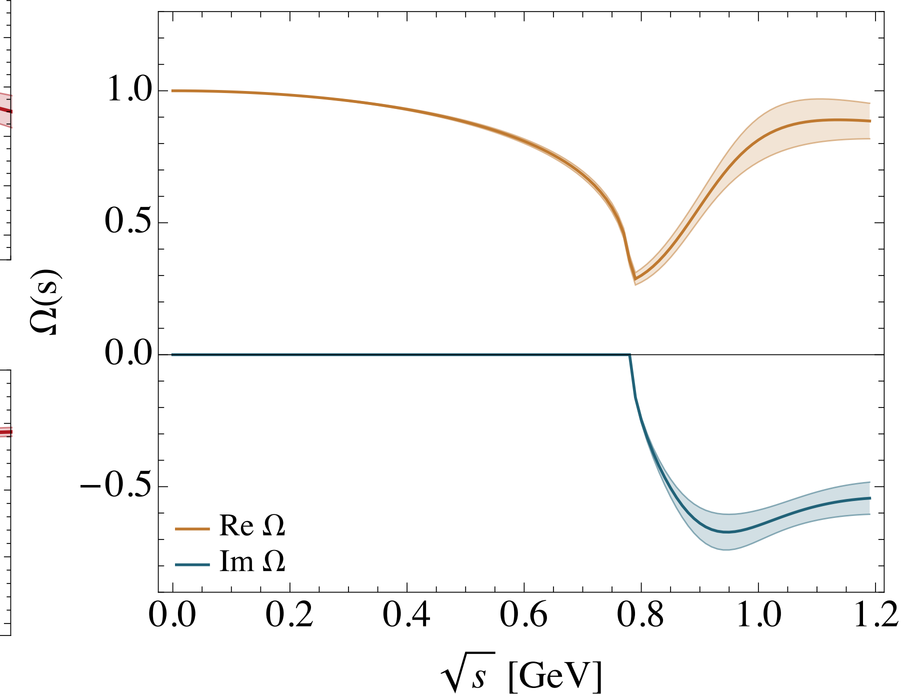
$$\sqrt{s_{f_0(500),BSE_{NLO}}} = 490(15) - i\,180(10) \text{ MeV}$$

Albaladejo, Oller Phys. Rev. D86, 034003 (2012)

Single channel $\pi\pi$ scattering for $m_\pi = 391$ MeV







Input: Lattice data 3 parameters fit

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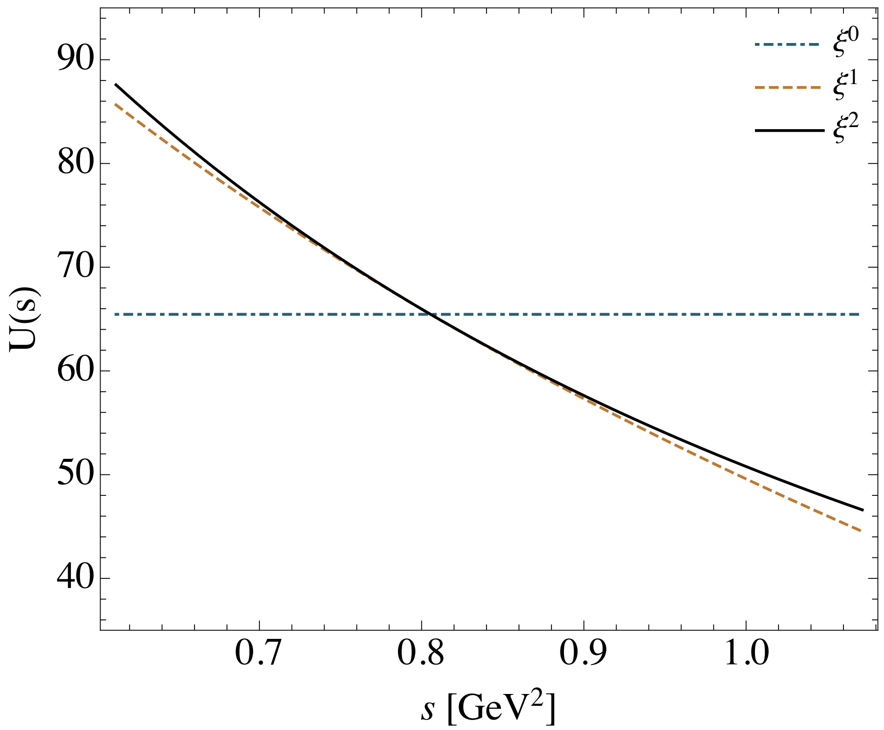
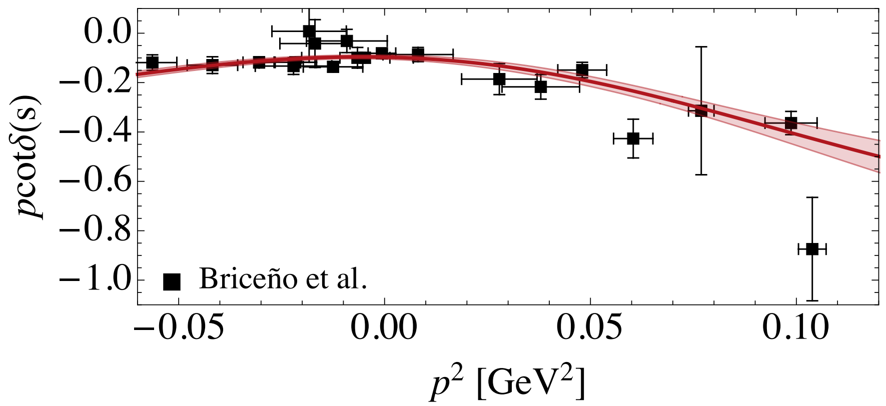
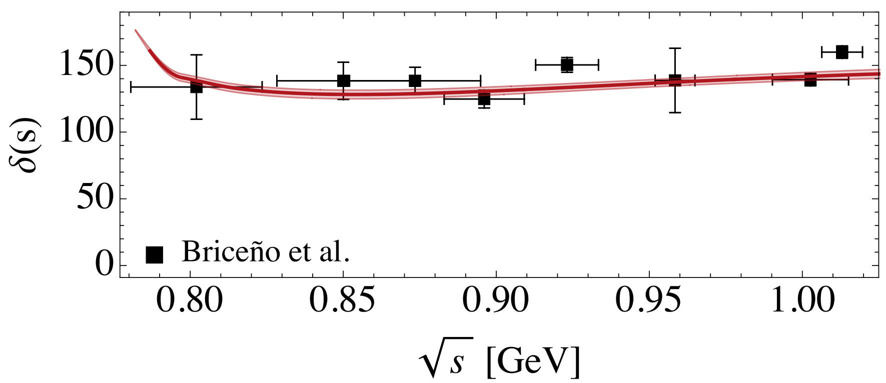
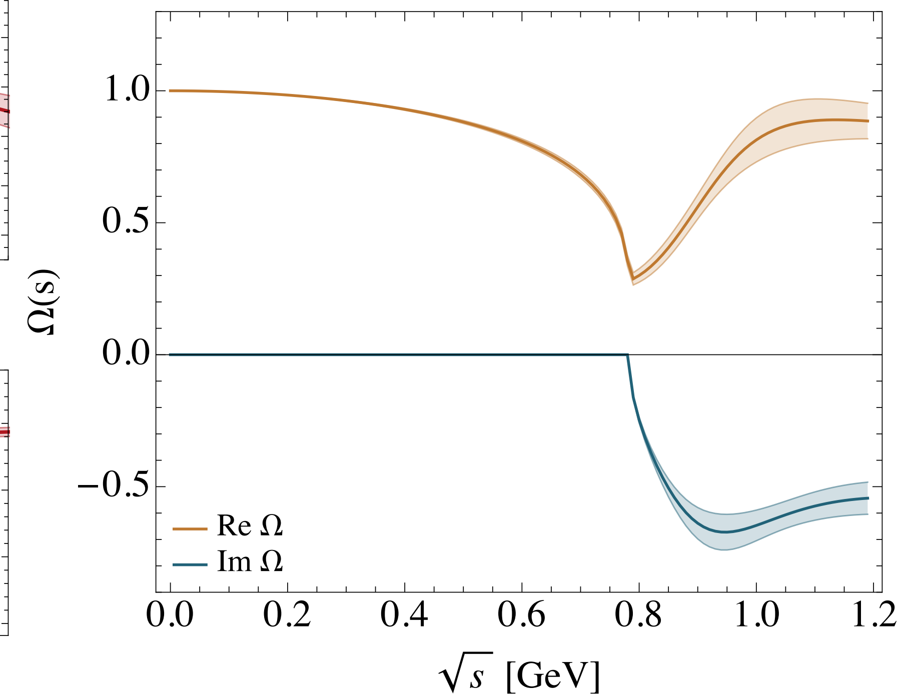








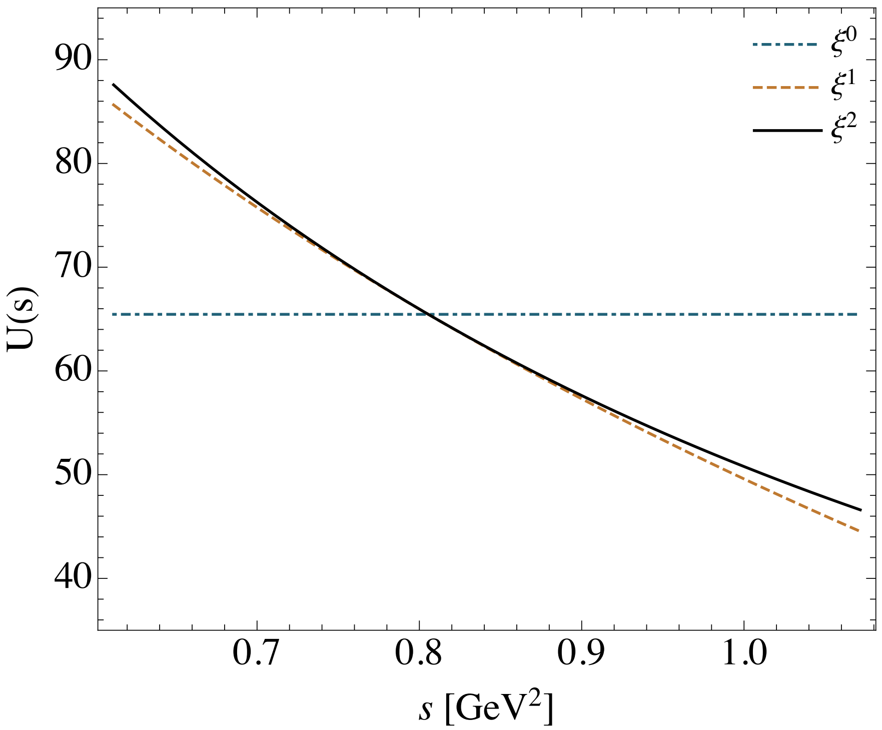
Input: Lattice data 3 parameters fit

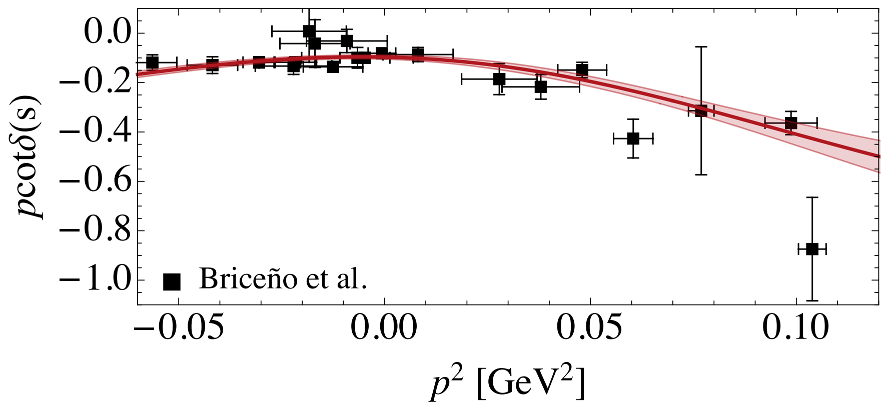
N/D analysis of the lattice data: $\sqrt{s_{B,ND+Latt}} = 758(5)(0)$ MeV

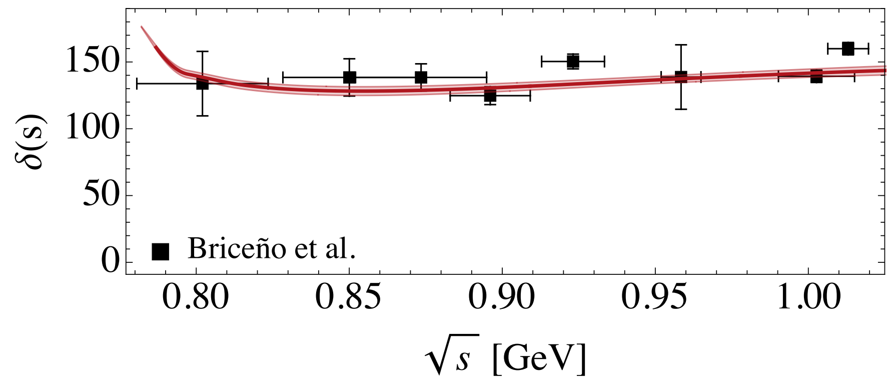
K-matrix analysis of the lattice data: $\sqrt{s_{B,Kmat+Latt}} = 758(4)$ MeV — Briceño *et al.* Phys.Rev.Lett. 118 (2017) 2, 022002

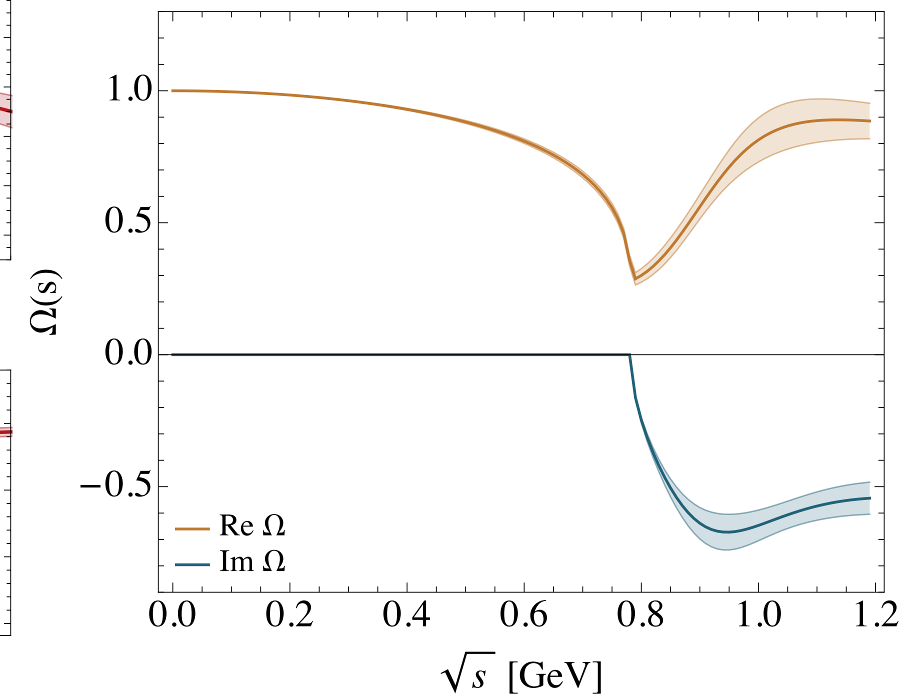
Prediction: $\sqrt{s_{B,mIAM_{NNLO}}} = 765$ MeV — Peláez, Rios Phys.Rev.D 82 (2010) 114002

Single channel $\pi\pi$ scattering for $m_\pi = 391$ MeV









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N/D analysis of the lattice data: $\sqrt{s_{B,ND+Latt}} = 758(5)(0)$ MeV

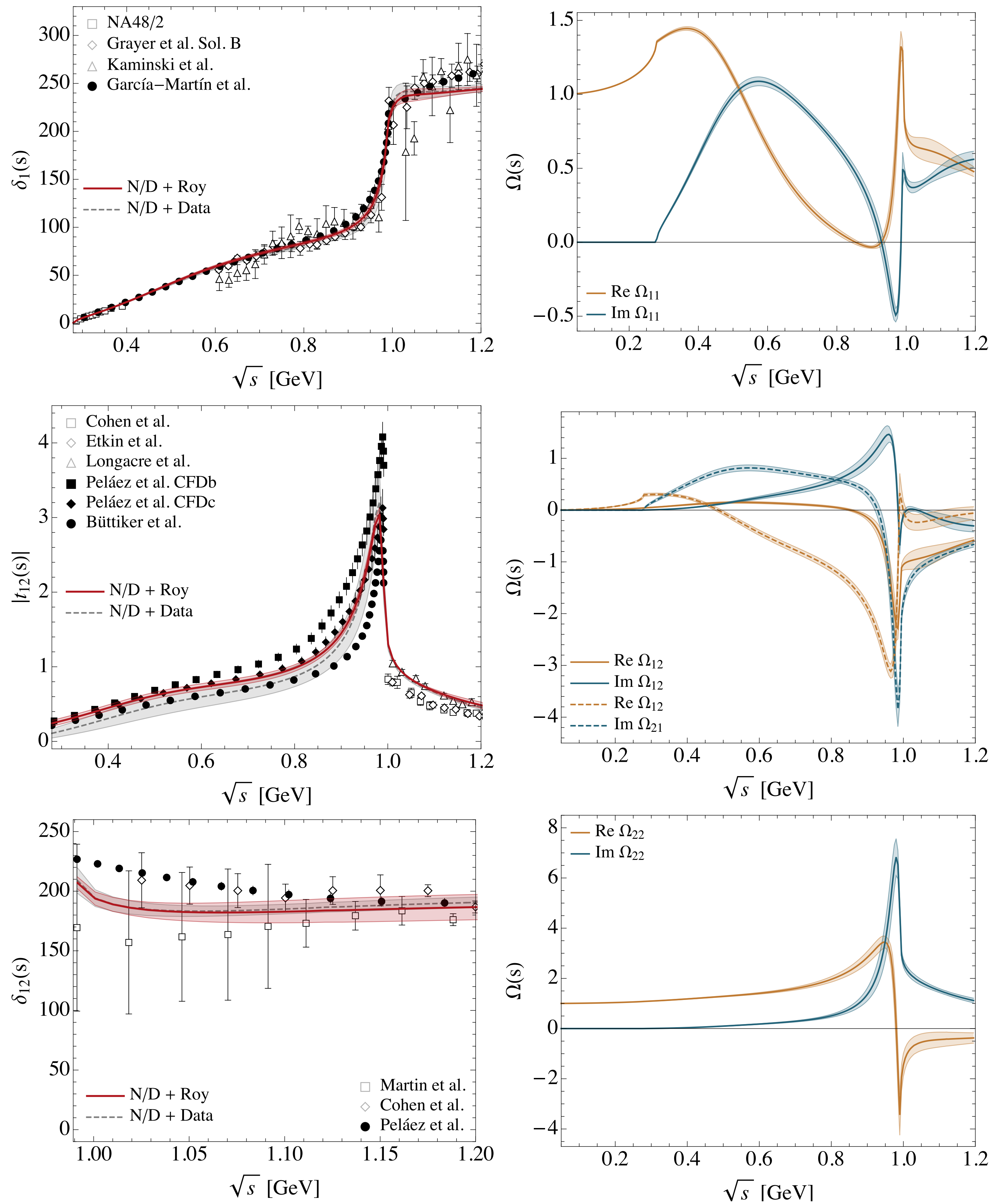
K-matrix analysis of the lattice data: $\sqrt{s_{B,Kmat+Latt}} = 758(4)$ MeV   Briceño *et al.* Phys.Rev.Lett. 118 (2017) 2, 022002

Prediction: $\sqrt{s_{B,mIAM_{NNLO}}} = 765$ MeV   Peláez, Rios Phys.Rev.D 82 (2010) 114002

$$\det(D_{ab}(s_B)) = 0 \quad s_B < s_{th}$$

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{s_B}\frac{g_{Ba}g_{Bb}}{s_B - s} + \frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\text{Disc } t_{ab}(s')}{s' - s}$$

Coupled channel $\pi\pi$ scattering



$$t(s) = \begin{pmatrix} \dfrac{\eta(s)\, e^{2i\delta_1(s)} - 1}{2\, i\, \rho_1(s)} & |t_{12}(s)|\, e^{\delta_{12}(s)} \\ |t_{12}(s)|\, e^{\delta_{12}(s)} & \dfrac{\eta(s)\, e^{2i\delta_2(s)} - 1}{2\, i\, \rho_2(s)} \end{pmatrix}$$

$$\delta_{12}(s) = \delta_1(s) + \delta_2(s)\, \theta(s > 4m_K^2)$$

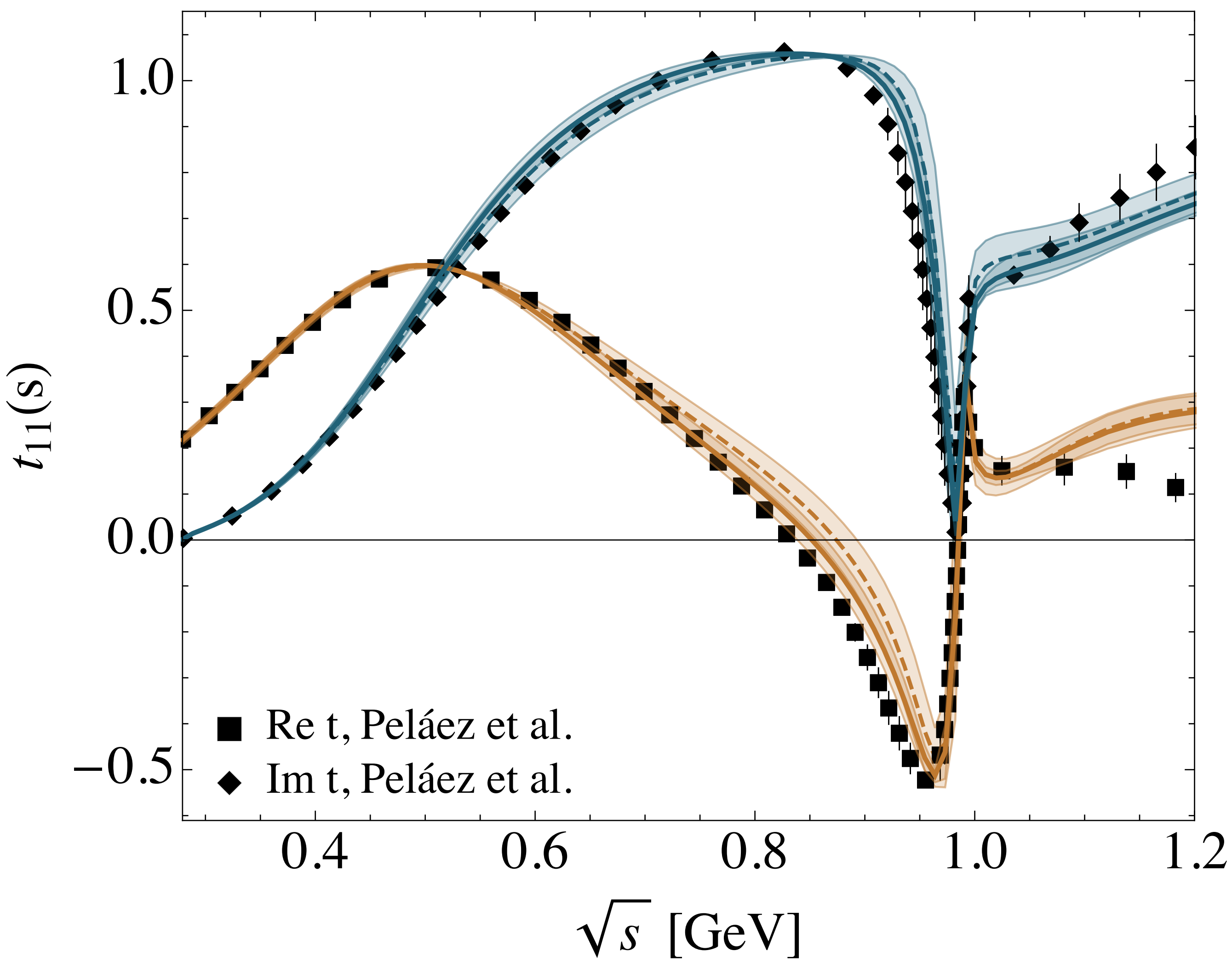
$$\eta(s) = \sqrt{1 - 4\, \rho_1(s)\, \rho_2(s)\, |t_{12}(s)|^2}$$

4+2+3 parameters fit

García-Martín et al. Phys.Rev.D 83 (2011) 074004

Peláez, Rodas arXiv: 2010.11222 [hep-ph]

Coupled channel $\pi\pi$ scattering



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4+2+3 parameters fit

$$\sqrt{s_{f_0(500),ND+exp}} = 454(12)^{+6}_{-7} - i262(12)^{+8}_{-12} \text{ MeV}$$

$$\sqrt{s_{f_0(500),ND+Roy}} = 458(10)^{+7}_{-15} - i\,256(9)^{+5}_{-8} \text{ MeV}$$

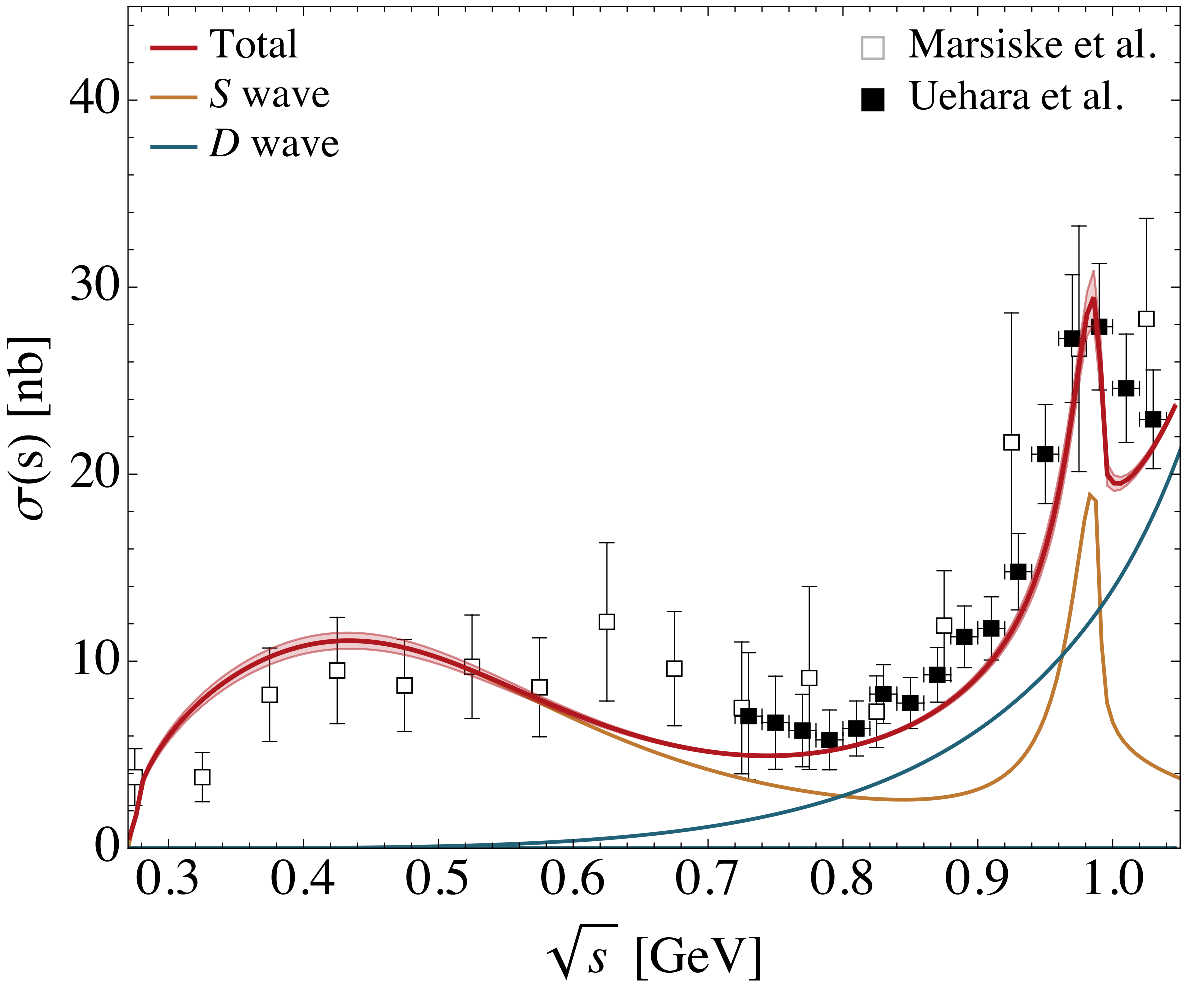
$$\sqrt{s_{f_0(500),Roy}} = 449^{+22}_{-16} - i\,245(15) \text{ MeV}$$

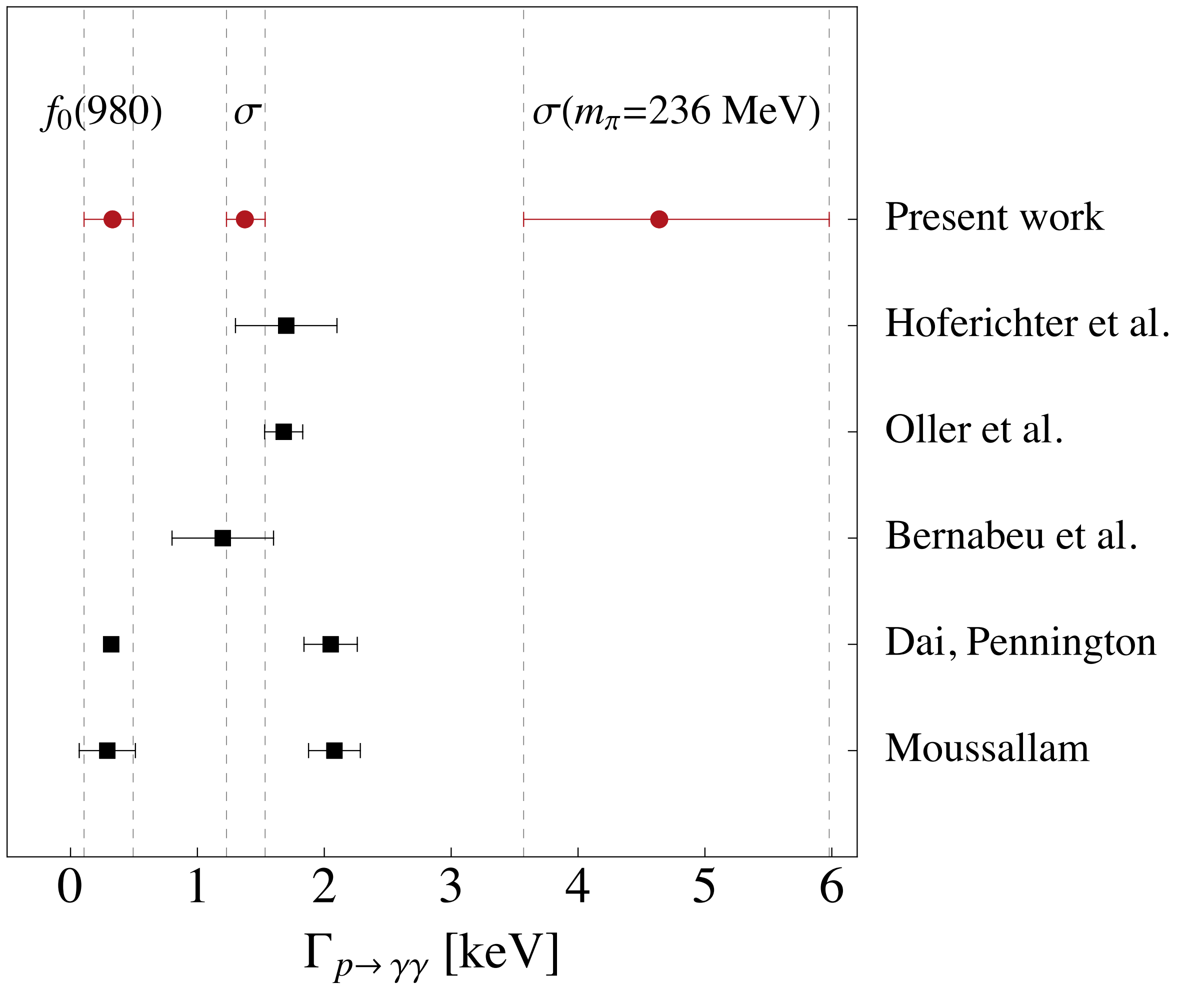
$$\sqrt{s_{f_0(980),ND+exp}} = 990(7)^{+2}_{-4} - 17(7)^{+4}_{-1} \text{ MeV}$$

$$\sqrt{s_{f_0(980),ND+Roy}} = 993(2)^{+2}_{-1} - i\,21(3)^{+2}_{-4} \text{ MeV}$$

$$\sqrt{s_{f_0(980),Roy}} = 996^{+7}_{-14} - i\,25^{+11}_{-6} \text{ MeV}$$

Application to $\gamma\gamma \to \pi\pi$ process



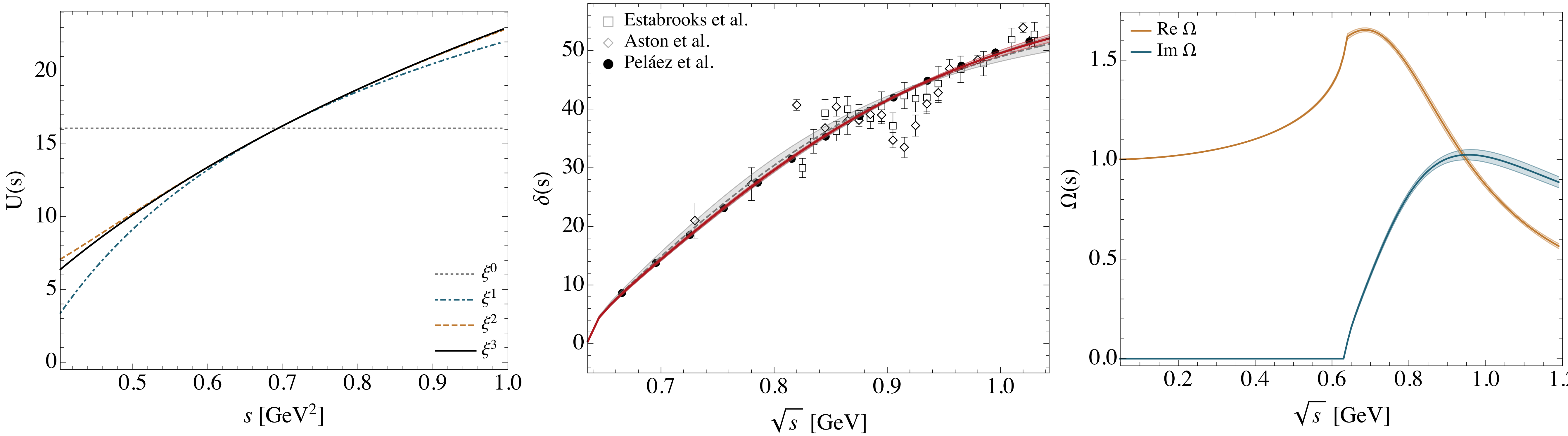


$$\Gamma_{\sigma\to\gamma\gamma} = 1.37(13)^{+0.09}_{-0.06} \left[1.38(9)^{+0.01}_{-0.01}\right] \text{ keV}$$

$$\Gamma_{f_0(980)\to\gamma\gamma} = 0.33(16)^{+0.04}_{-0.16} \text{ keV}$$

$$\Gamma^{m_\pi=236}_{\sigma\to\gamma\gamma} = 4.64(1.01)^{+0.88}_{-0.35} \text{ keV}$$

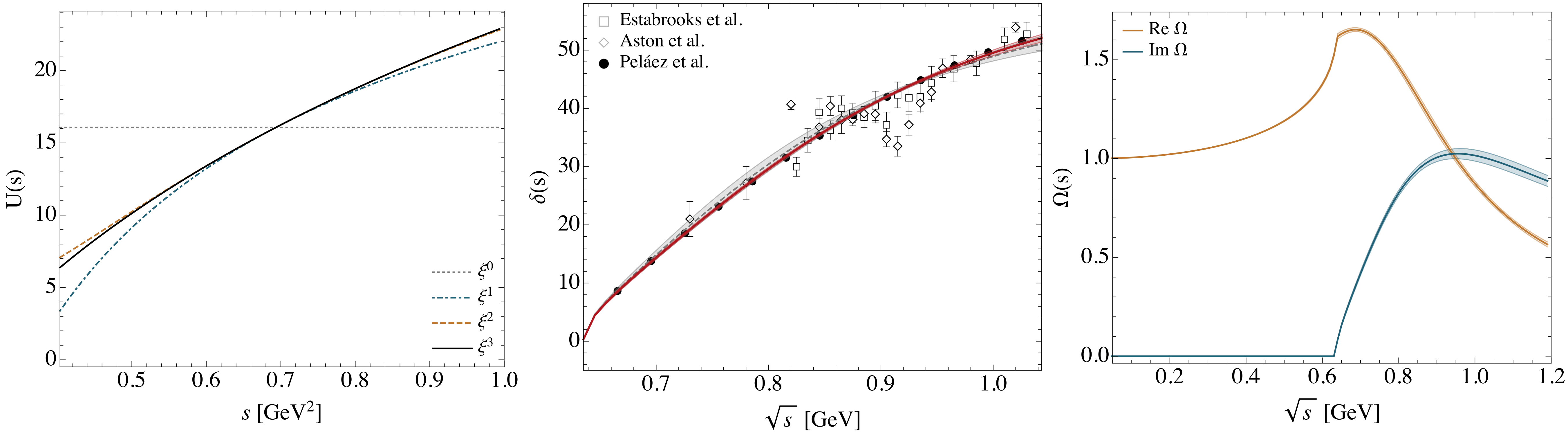
Single channel πK scattering



Input: experimental data/Roy analysis + NNLO χPT a + NLO χPT Adler zero

4 parameters fit

Single channel πK scattering



Input: experimental data/Roy analysis + NNLO χPT a + NLO χPT Adler zero

4 parameters fit

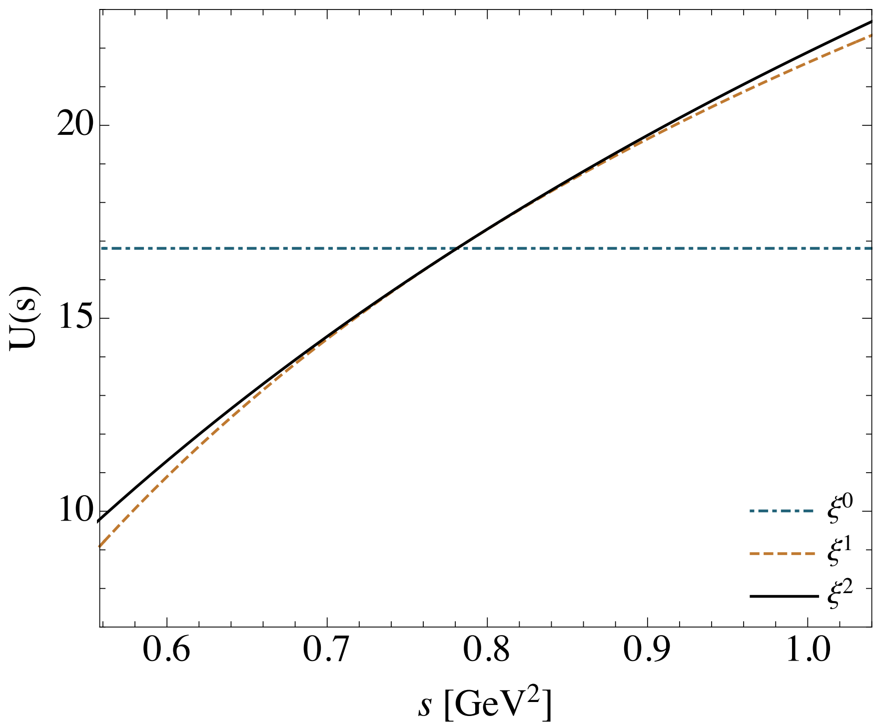
$$\sqrt{s_{K_0^*(700),ND+exp}} = 689(24)^{+3}_{-2} - i\,263(33)^{+5}_{-8}\ \text{MeV}$$

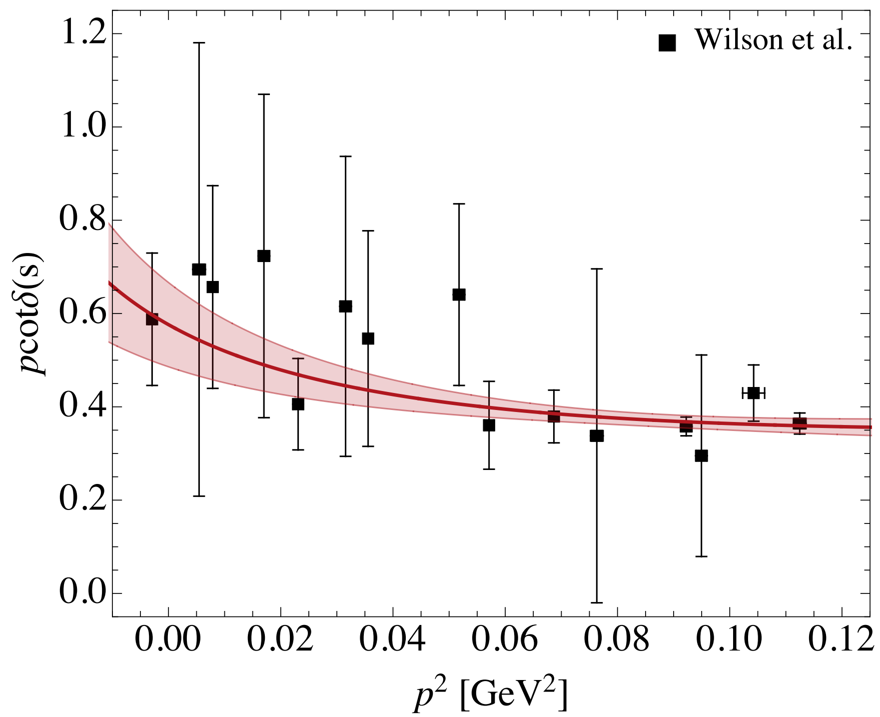
$$\sqrt{s_{K_0^*(700),ND+Roy}} = 702(12)^{+4}_{-5} - i\,285(16)^{+8}_{-13}\ \text{MeV}$$

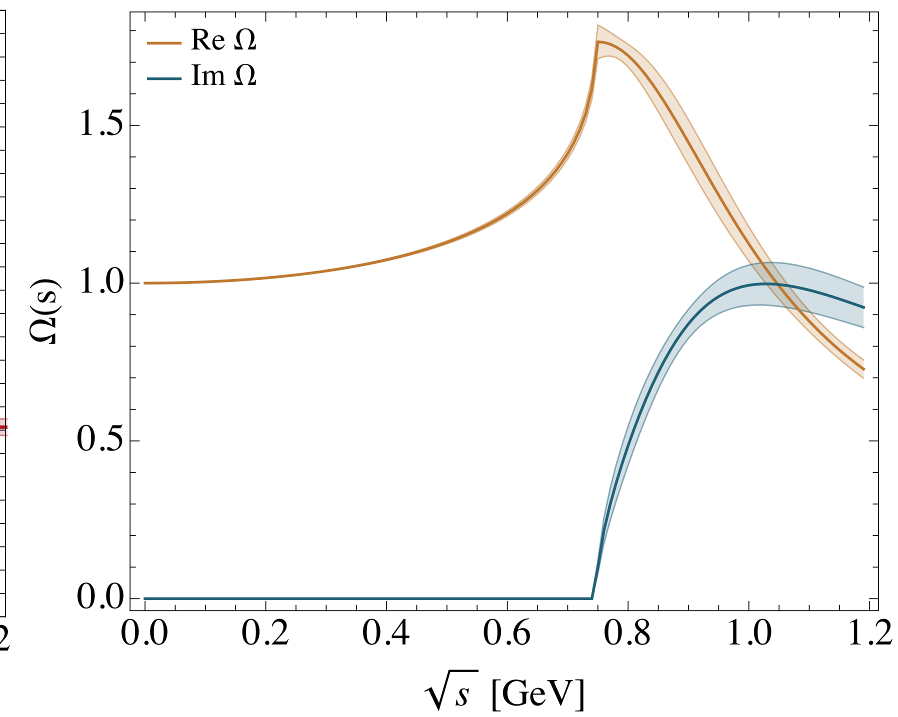
$$\sqrt{s_{K_0^*(700),Roy}} = 653^{+18}_{-12} - i\,280(16)\ \text{MeV}$$

Peláez, Rodas Rhys. Rev. Lett. 124 (2020) 17, 172001

Single channel πK scattering for $m_\pi = 239$ MeV

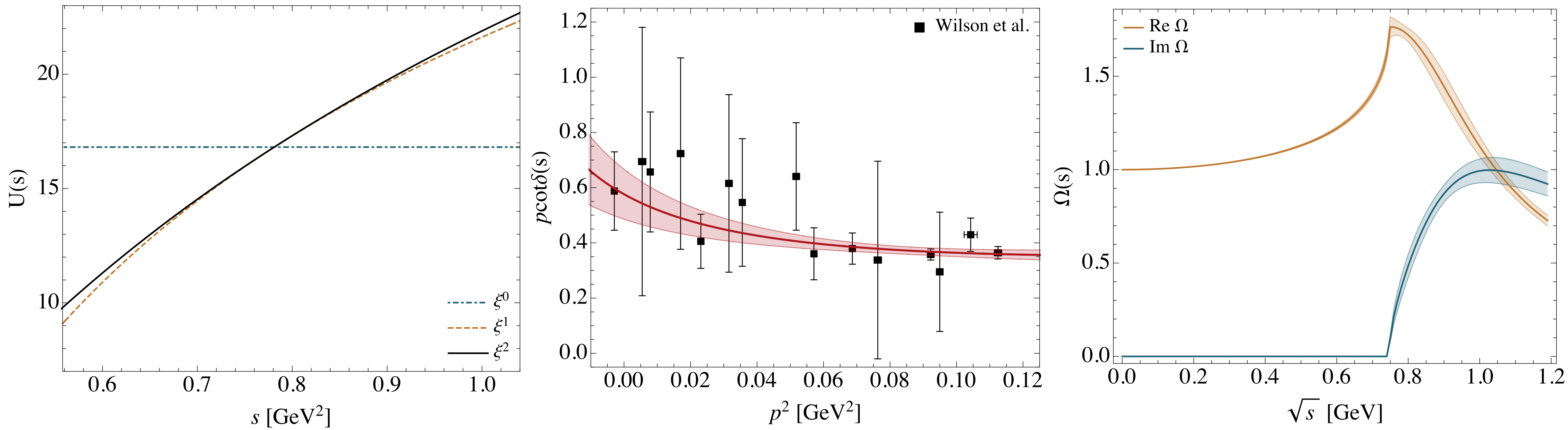






Input: lattice data + NLO χPT Adler zero   3 parameters fit

Single channel πK scattering for $m_\pi = 239$ MeV



Input: lattice data + NLO χPT Adler zero

3 parameters fit

$$\sqrt{s_{f_0(500),ND}} = 747(39)^{+2}_{-0} - i\,265(16)^{+7}_{-6} \text{ MeV}$$

Wilson *et al.* Phys. Rev. Lett. 123, 042002 (2019)

This result is compatible qualitatively with the prediction from UχPT

Nebreda, Peláez Phys. Rev.D81, 054035 (2010).

Summary and outlook

- We constructed a data driven dispersive approach based on the N/D ansatz, which can be applied both to the experimental and lattice data.
- We are able to describe $\sigma/f_0(500)$, $f_0(980)$, $\kappa/K^*(700)$ resonances. Our analysis provides consistent result whether the data or Roy/analysis input is used which allows to extend the range of applicability to the processes, where no Roy analysis is available.
- We offer a consistent tool, which fully respects unitarity and analyticity and can be an alternative to the K-matrix approach for the analysis of the lattice data
- The current result for the $f_0(980)$ resonance has been used for the calculation of its contribution to muon $(g-2)$
- This method can also be extended for the coupled-channel analysis of the lattice data
- There are many exciting processes to which the method can be applied, e.g $J/\psi J/\psi$ scattering, for which the data from LHC is available

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Thank you!

Supplementary