Pion and Kaon valence structure in a confining NJL model

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A bit of advertising

Parts of this presentation are based on

- "Valence-quark distributions of pions and kaons in a nuclear medium" Phys.Rev. D100 (2019) no.9, 094011, arXiv:1908.02406 [hep-ph]

In collaboration with

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Motivation

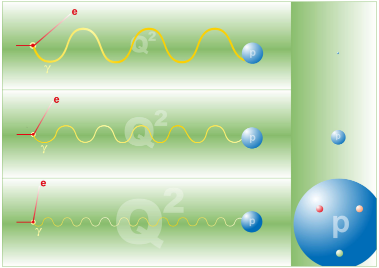
- The first evidence that triggered the interest for hadron properties in a nuclear medium is the EMC effect [PLB 123, 275 (1983)]–where the DIS data revealed that parton distributions of bound nucleons in nuclei are different from those of free nucleons.
- Thus, we expect that the internal structure of mesons will be modified in a nuclear medium and affect, for example, meson production in heavy-ion collisions [PRC 77, 024903 (2008)], the quark distributions of mesons, and hadronization phenomena in HIC.
- A deeper understanding of the in-medium modifications of the quark distributions of mesons can provide us with useful information.
- There is a recently proposed experimental measurements of the structure functions of pions and kaons at CERN SPS [Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER), arXiv: 1808.00848]

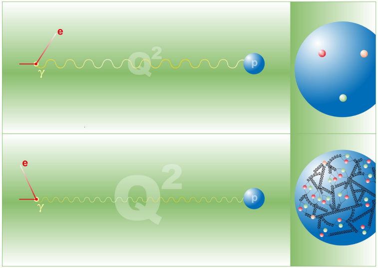
Motivation

- It is widely accepted that the vacuum expectation of $\langle \overline{q}q \rangle$ is non-zero due to the spontaneous breaking of chiral symmetry of the vacuum
- This $\langle \overline{q}q \rangle$-condensation is the major source of masses of low-lying hadrons such as protons, neutrons, and pions (more than 99% of the mass of the visible Universe).
- The $\langle \overline{q}q \rangle$ expectation value (chiral order parameter) is a function of temperature and density (chemical potential), so that various experimental studies have been performed to detect the restoration of chiral symmetry
- In 2007, the KEK-PS E325 experiment reported a 3.4% mass reduction of the ϕ meson mass–this result points towards the a possible restorarion of chiral symmetry in a nuclear medium.
- Future experiments at J-PARC and JLab will be searching for a ϕ-nucleus bound states as a signal for the partial restoration of chiral symmetry.

Quarks, Gluons, and the modern picture of the Hadrons

- Hadrons (meson, baryons, exotics) are strongly-interacting, complex bound-state objects of quarks and gluons.
- The quarks and gluons have always being there, we just did not have enough energy to find them.

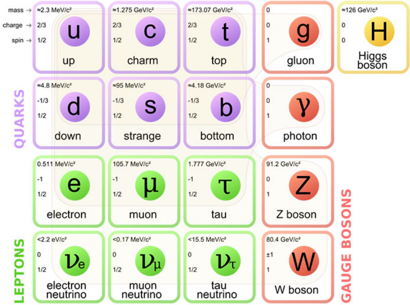




- Hadrons are bound states of elementary quarks and gluons—This modern picture is valid for any hadron (meson, baryon, exotic)

Quarks, Gluons, QCD and the Standard Model

- It turns out there are 6 flavours of quarks (u, d, s, c, b, t); each one comes in three colors; and there are 8 gluons.
- Ordinary matter is made of light quarks (u and d quarks) and gluons



- $\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{QCD}} \otimes \mathcal{L}_{\mathrm{Weak}} \otimes \mathcal{L}_{QED}$
- More about $\mathcal{L}_{\mathrm{QCD}}$ later

- QCD is the strongly interacting part of the SM.
- QCD is the accepted theory of the Strong Interactions
- Can we go back and describe the properties of hadrons in terms of quarks, gluons, and their interactions?

QCD's ingredients: Feynman rules

- QCD is the fundamental theory of quarks (spin 1/2 fermions), gluons (spin 1 gauge bosons), and their interactions; a consistent QFT, based entirely on the invariance under the local $SU_{\mathrm{color}}(3)$ gauge group and renormalisability.
- QCD is a powerful tool in the description of large momentum transfer experiments due to its property of asymptotic freedom —This is the realm of pQCD.
- Over the years QCD has become the accepted theory of the strong interactions at the fundamental level.

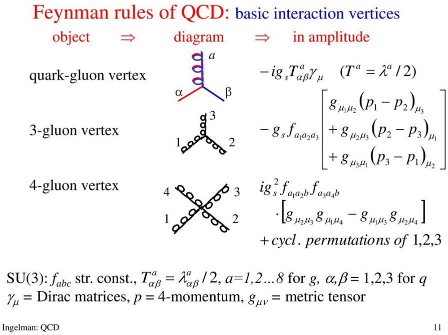
$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \overline{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\mu\nu},$$

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\mu - \partial_\nu \mathcal{A}^A_\mu - g_s f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu .$$

- Parameters: current quark masses m_q; Coupling constant $\alpha_S = \frac{g_s^2}{4\pi}$

QCD's ingredients: Feynman rules and pQCD

- The Feynman rules and pQCD



Feynman rules of QCD: basic interaction vertices

object ⇒ diagram ⇒ in amplitude

quark-gluon vertex $\quad -ig_s T^a_{\alpha\beta}\gamma_\mu \quad (T^a = \lambda^a/2)$

3-gluon vertex $\quad -g_s f_{a_1a_2a_3}\left[\begin{array}{l} g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3} \\ +g_{\mu_2\mu_3}(p_2-p_3)_{\mu_1} \\ +g_{\mu_3\mu_1}(p_3-p_1)_{\mu_2} \end{array}\right]$

4-gluon vertex $\quad ig_s^2 f_{a_1a_2b} f_{a_3a_4b} \cdot \left[g_{\mu_2\mu_3}g_{\mu_1\mu_4} - g_{\mu_1\mu_3}g_{\mu_2\mu_4}\right] + cycl.\ permutations\ of\ 1,2,3$

SU(3): f_{abc} str. const., $T^a_{\alpha\beta} = \lambda^a_{\alpha\beta}/2$, $a=1,2...8$ for g, $\alpha,\beta = 1,2,3$ for q
γ_μ = Dirac matrices, p = 4-momentum, $g_{\mu\nu}$ = metric tensor

G. Ingelman: QCD 11

- $\alpha_S = \frac{g_s^2}{4\pi}$ runs with the energy of the process under consideration;

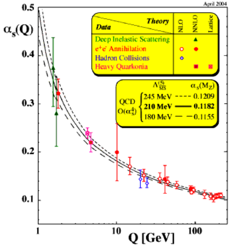


FIG. 1: QCD running coupling constant

- (some) QCD processes can be computed in pQCD using the Feynam rules and all that, due to asymptotic freedom.
- In contrast, at large distances the strong- coupling constant becomes large and pQCD cannot be applied–and thus other approaches have to be developed.

Strong QCD

- Strange predicament: One actually has at one's disposal a definite Lagrangian in which all the dynamics is contained but it's not (easily) able to extract useful information for many physical processes of interest, such as the properties of hadrons and their interactions, or the behaviour of hadronic matter at high density.
- Two approaches to Strong QCD founded on the QCD Lagrangian are: Lattice QCD and the SDEs-BSEs of QCD, both with their own strengths and weaknesses.
- Still one is strongly motivated to look for a simple model Lagrangian that displays one or more essential features of QCD.
- The model we shall introduce is the Nambu–Jona-Lasinio model (Ca 1961). In its original form, this model was constructed as a pre-QCD theory of nucleons that interact via an effective two-body interaction.
- This model today is reinterpreted as theory with quarks degrees of freedom.

The NJL model of QCD

- We are looking for a model that displays one or more essential features of QCD.
- Of primary importance is the fact that the Lagrangian of this model is constructed such as that the symmetries of QCD that are also observed in nature are part of it.
- One of the most important of these is chiral symmetry, which is essential to the undestanding of light hadrons.
- QCD is distinguished not only by its symmetries but also by the breaking of these–The NJL model is particularly useful for observing how these things happen.
- In particular, the dynamical generation of quark masses brought about by the breaking of chiral symmetry is one of the features of the NJL model.

Symmetries of QCD

- The term $\mathcal{L}_{\text{chiral}} = \overline{\psi} \mathrm{i} \not{\partial} \psi$ in $\mathcal{L}_{\text{QCD}}$ has some symmetries:

Symmetry	Transformation	Current	Name	Manifestation
$SU_V(2)$	$\psi \to \mathrm{i}^{-\mathrm{e}\vec{\tau}\cdot\vec{\omega}/2}\psi$	$J^k_\mu = \overline{\psi}\gamma_\mu\tau^k\psi$	Isospin	Approx. conserved
$U_V(1)$	$\psi \to \mathrm{e}^{-\mathrm{i}\alpha}\psi$	$J_\mu = \overline{\psi}\gamma_\mu\psi$	Baryonic	Always conserved
$SU_A(2)$	$\psi \to \mathrm{i}^{-\mathrm{e}\vec{\tau}\cdot\vec{\theta}/2}\psi$	$J^k_{5\mu} = \overline{\psi}\gamma_\mu\gamma_5\tau^k\psi$	Chiral	CSB; Goldstone mode
$U_A(1)$	$\psi \to \mathrm{i}^{-\mathrm{e}\beta\gamma_5}\psi$	$J_{5\mu} = \overline{\psi}\gamma_\mu\gamma_5\psi$	Axial	$U_A(1)$ "puzzle"

- $SU_V(2)$ and $U_V(1)$ correspond to baryon and isospin conservation.
- The $SU_A(2)$ symmetry is manifested in the Goldstone mode via chiral symmetry breaking and the associated Goldstone bosons the pions $m_\pi = 140\text{MeV}$)
- $U_A(1)$ requires the existence of $I = 0$ pseudoscalar meson having roughly the same mass as the pion (the η'; but $m_{\eta'} = 958\text{MeV}$). No such candidate was observed, and thus arose the $U_A(1)$ puzzle: where is the Goldstone boson? The problem was resolved by 't Hooft, who showed that, due to instanton effects, the $U_A(1)$ symmetry should not result in physical manifestations.

SU(3) flavour NJL

- The overall continuous symmetries of $\mathcal{L}_{\text{QCD}}$ are

$$\mathcal{G} = SU_V(3) \otimes SU_A(3) \otimes U_V(1) \otimes U_A(1)$$

- $\mathcal{L}_6$ (below) explicitly breaks $U_A(1)$ symmetry; removes the $U_A(1)$ problem in the pseudoscalar meson; and produces flavor mixing for quarks and mesons
- We will study the following extended NJL model

$$\mathcal{L}_{NJL} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_4 + \mathcal{L}_6; \quad \mathcal{L}_{\text{Dirac}} = \sum_{f=1}^{3} \overline{\psi}_f (\mathrm{i}\not{\partial} - m_f)\psi_f$$

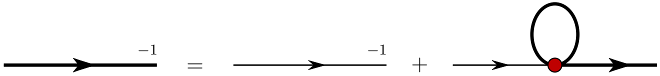
where
$$\begin{aligned}
\mathcal{L}_4 &= G_\pi \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 - (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] \\
&\quad - G_\rho \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \\
\mathcal{L}_6 &= -K\left[\det \bar{\psi}(1+\gamma_5)\psi + \det \bar{\psi}(1-\gamma_5)\psi\right]
\end{aligned}$$

The NJL model of QCD: shortcomings

- The interaction between quarks is assumed to be point-like, with the result that the model is not a renormalizable field theory.
- Hence, to define the NJL model completely, as an effective model, a regularisation scheme must be specified to deal with the divergent integrals that occur.
- A second shortcoming of the NJL model is that the local interaction does not confine quarks. For many questions, however, the issue of confinement may not be important.

SU(3) flavour NJL Gap equations

- The gap equations for the constituent quark masses $M_q (q = u, d, s)$ are

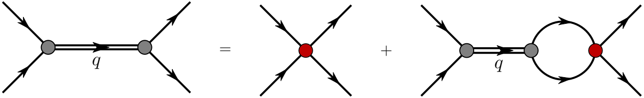


$$M_q = m_q - 4G_\pi \langle \bar{q}q \rangle \tag{1}$$

- $\langle \bar{q}q \rangle = \mathrm{tr} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{1}{\not{p} - M_q + \mathrm{i}\epsilon}$ is the quark condensate
- A dynamical mass is generated for the quarks through a nonperturbative interaction with the vacuum, $\langle \bar{q}q \rangle \neq 0$
- In the chiral limit the NJL Lagrangians has $SU(3)_L \otimes SU(R)_R$ symmetry (this is by construction)
- However, a nontrivial nonperturbative solution $M_q \neq 0$ exists provided $G_\pi > G_{\text{critical}}$, which is a signature of DCSB.

Hadron physics—real-world QCD

- In the NJL model the quark-antiquark T-matrix is given by



$$\mathcal{T}(q) = \mathcal{K} + \int \frac{\mathrm{d}^4 k}{\mathrm{i}(2\pi)^4} \mathcal{K} S(k+q) \mathcal{T}(q) S(k)$$

- The solution to the above inhomogeneous equation for the pseudoscalar channels $\alpha = \pi, K$ is given in terms of the reduced t_α-matrix

$$[T_\alpha(p)]_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab} t_\alpha(p) [\gamma_5 \lambda_\alpha^\dagger]_{cd} \tag{2}$$

- In the pseudoscalar channel we have

$$t_\alpha(p) = \frac{-2i\, G_\pi}{1 + 2\, G_\pi\, \Pi_\alpha(p^2)}, \tag{3}$$

- The quark propagators $S(k)$ are solns to the previous gap equations.

Hadron physics—real-world QCD

- The solution to the inhomogeneous equation for the pseudoscalar channels $\alpha = \pi$, K is given in terms of the reduced t_α-matrix

$$[T_\alpha(p)]_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab} t_\alpha(p) [\gamma_5 \lambda_\alpha^\dagger]_{cd} \tag{4}$$

- In the pseudoscalar channel we have

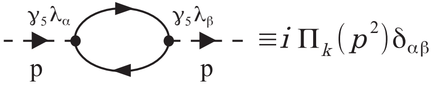
$$t_\alpha(p) = \frac{-2i\, G_\pi}{1 + 2\, G_\pi\, \Pi_\alpha(p^2)}, \tag{5}$$

- Flavor matrices: λ_α are the appropriate flavour matrices, for example, $\lambda_{\pi^0} = \lambda_3$, $\lambda_{\pi^\pm} = \frac{1}{\sqrt{2}}(\lambda_1 \pm i\lambda_2)$ and $\lambda_{K^\pm} = \frac{1}{\sqrt{2}}(\lambda_4 \pm i\lambda_5)$.
- From here follow Bethe-Salpeter equations for scalar, pseudoscalar, vector and axial-vector mesons, which we can solve to determine, for example, the masses and amplitudes of these mesons "directly" from the NJL Lagrangian and therefore study the impact of the QCD symmetries.

Hadron physics—real-world QCD

- Polarization propagator: [PRC 91, 025202 (2015)]

$$t_\alpha(p) = \frac{-2i\, G_\pi}{1 + 2\, G_\pi\, \Pi_\alpha(p^2)},$$



- The self energy part, $\Pi_\alpha(p^2)$, for $\alpha = \pi$, K are given by

$$\Pi_\pi(q^2) = 6i \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}_D\left[\gamma_5\, S_\ell(k)\gamma_5\, S_\ell(k+q)\right], \qquad (6)$$

$$\Pi_K(q^2) = 6i \int \frac{d^4k}{(2\pi)^4}\, \mathrm{Tr}_D\left[\gamma_5\, S_\ell(k)\gamma_5\, S_s(k+q)\right], \qquad (7)$$

- The meson masses are identified by the pole positions in the corresponding t-matrices.

$$\begin{aligned} 1 + 2\, G_\pi\, \Pi_\pi(p^2 = m_\pi^2) &= 0, \\ 1 + 2\, G_\pi\, \Pi_K(p^2 = m_K^2) &= 0. \end{aligned} \qquad (8)$$

Hadron physics—Goldstone bosons

- The meson masses are identified by the pole positions in the corresponsing t-matrices. The pion and kaon masses are given, respectively, by the solutions of the equations,

$$1 + 2\,G_\pi\,\Pi_\pi(p^2 = m_\pi^2) = 0,$$
$$1 + 2\,G_\pi\,\Pi_K(p^2 = m_K^2) = 0. \tag{9}$$

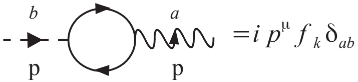
- These can be rearranged to give the pion and kaon masses as

$$m_\pi^2 = \frac{m_l}{M_l}\frac{2}{G_\pi\,\mathcal{I}_{ll}(m_\pi^2)},$$
$$m_K^2 = \left(\frac{m_s}{M_s} + \frac{m_l}{M_l}\right)\frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2)} + (M_s - M_l)^2, \tag{10}$$

- These equations make evident that chiral symmetry and its breaking pattern are embedded in the NJL model and the pion and kaon become massless in the chiral limit, being realised as Goldstone bosons.

Hadron physics—decay constants

- The pion and kaon weak decay constants are determined from the meson to vacuum transition matrix elements $\langle 0|J_a^{5\mu}|\alpha(p)\rangle$ with $J_a^{5\mu}$ being the quark weak axial vector current operator for a flavor quantum number a.



$$
\begin{aligned}
\langle 0|J_a^{5\mu}|\alpha_b(p)\rangle &= \mathrm{i} p^{\mu} f_P \delta_{ab}, \\
&= -\mathrm{i} g_{\alpha qq} \int \frac{\mathrm{d}^4 q}{\mathrm{i}(2\pi)^4} \mathrm{Tr}\left[\frac{1}{2}\gamma^{\mu}\gamma_5 \lambda_a S(q_+)\lambda_b \gamma_5 S[q_-]\right]
\end{aligned}
$$

- The meson (α)-quark coupling constant $g_{\alpha qq}$ is the residue at a pole in the t_α-matrix defines as

$$
g_{\alpha qq}^{-2} = -\left.\frac{\partial \Pi_\alpha(p^2)}{\partial p^2}\right|_{p^2=m_\alpha^2}, \quad \alpha = \pi, K. \tag{11}
$$

Proper time regularization

- The integral in the gap eqn is divergent and thus needs to be regularized.

$$M_q = m_q - 4G_\pi\langle\bar{q}q\rangle, \ \ \langle\bar{q}q\rangle = \mathrm{Tr}\int \frac{\mathrm{d}^4p}{\mathrm{i}(2\pi)^4}\frac{1}{\not{p} - M_q + \mathrm{i}\epsilon}$$

- We use proper time regularization

$$\begin{aligned}\langle\bar{q}q\rangle &= \mathrm{Tr}\int \frac{\mathrm{d}^4p}{\mathrm{i}(2\pi)^4}\frac{1}{\not{p} - M_q + \mathrm{i}\epsilon} = \frac{N_c}{4\pi^2}M_q\int_0^\infty \mathrm{d}k^2\frac{1}{k^2+M_q^2} \\ &= \frac{N_c}{4\pi^2}M_q\int_0^\infty \mathrm{d}k^2\int_0^\infty \mathrm{d}\tau \mathrm{e}^{(k^2+M_q^2)\tau} \\ &\to \frac{N_c}{4\pi^2}M_q\int_0^\infty \mathrm{d}k^2\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} \mathrm{d}\tau \mathrm{e}^{-(k^2+M_q^2)\tau}\end{aligned}$$

- The regularization procedure then becomes part of the model.
- Only the ultraviolet cutoff parameters Λ_{UV} is needed to regularize the integrals.

Proper time regularization

- Only Λ_{UV} is needed to regularize the integrals.
- We use proper time regularization

$$\langle \bar{q}q \rangle = \mathrm{Tr} \int \frac{\mathrm{d}^4 p}{\mathrm{i}(2\pi)^4} \frac{1}{\not{p} - M_q + \mathrm{i}\epsilon} = \frac{N_c}{4\pi^2} M_q \int_0^\infty \mathrm{d}k^2 \frac{1}{k^2 + M_q^2}$$
$$= \frac{N_c}{4\pi^2} M_q \int_0^\infty \mathrm{d}k^2 \int_0^\infty \mathrm{d}\tau \mathrm{e}^{(k^2+M_q^2)\tau}$$
$$\rightarrow \frac{N_c}{4\pi^2} M_q \int_0^\infty \mathrm{d}k^2 \int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} \mathrm{d}\tau \mathrm{e}^{-(k^2+M_q^2)\tau}$$

- The NJL model does not *a priori* contain quark confinement.
- However, one important aspect can be incorporated into the NJL model by introducing an infrared cutoff ($\Lambda_{IR} \neq 0$).
- This additional cutoff eliminates unphysical thresholds for the decays of hadrons into quarks [Cloet et al, PRC 91, 025202 (2015)], and thus plays the role of simulating confinement in the NJL model. $\Lambda_{IR} \approx \Lambda_{QCD}$.

Proper time regularization expressions

- Dynamical quark masses: $M_i = m_i - 4G_\pi \langle \bar{q}_i q_i \rangle$, $i = u, d, s$.
- Pion and Kaon masses: $1 + 2G_\pi \Pi_\alpha(p^2 = m_\alpha^2)$, $\alpha = \pi, K$.

$$\Pi_\pi(p^2) = 2\Pi^{ud}(p^2), \quad \Pi_K(p^2) = 2\Pi^{us}(p^2),$$

$$\Pi^{ij}(p^2) = \frac{-\langle \bar{q}_i q_i \rangle}{2M_i} + \frac{-\langle \bar{q}_j q_j \rangle}{2M_j} + \frac{1}{2}\left[p^2 - (M_i - M_j)^2\right] I_{ij}(p^2)$$

PT regularized integrals ($\Delta_{ij}(p^2) = xM_i + (1-x)M_j - x(1-x)p^2$)

$$\langle \bar{q}_i q_i \rangle = \frac{3}{4\pi^2} \int \mathrm{d}\tau \tau^{-2} \mathrm{e}^{-M_i^2 \tau}, \quad I_{ij}(p^2) = \frac{3}{4\pi^2} \int_0^1 \mathrm{d}x \int \mathrm{d}\tau \tau^{-1} \mathrm{e}^{-\Delta_{ij}(p^2)\tau}$$

- Pion and Kaon decay constants: $g_\alpha^{-2} = \left(\partial \Pi_\alpha(p^2)/\partial p^2\right)|_{p^2 = m_\alpha^2}$

$$f_K = \frac{3}{4\pi^2} g_K \int_0^1 \mathrm{d}x \int \mathrm{d}\tau \tau^{-1} \left[xM_u + (1-x)M_s\right] \mathrm{e}^{-\Delta_{us}(m_K^2)\tau}$$

(f_π is obtained with $M_s \to M_d$, $g_K \to g_K$, and $m_K \to m_\pi$)

Confining NJL model parameters

- The NJL model by itself has a few parameters, these being the coupling constants: four- and six-fermion coupling constants G_π, G_ρ, and K, respectively. For this work we will set $G_\rho = 0 = K$.
- Quark masses: $m_u = m_d$ and m_s
- The confining proper time regularisation scheme introduced two more paramters: Λ_{UV} and Λ_{IR}. However, we choose $\Lambda_{IR} \approx \Lambda_{QCD}$.
- Here the model parameters are G_π, $m_u = m_d$ and m_s, and Λ_{UV}).
- These are fixed by low-energy static properties of the pion and kaon: The coupling G_π and Λ_{UV} are fixed by the physicak pion mass ($m_\pi = 0.140$GeV) and pion leptonic decay constant ($f_\pi = 0.0934$GeV); M_s is fixed by the physical kaon mass ($m_K = 0.494$GeV). This gives $M_u = 0.400$GeV, $M_s = 0.611$MeV, $G_\pi = 19.04\text{GeV}^{-2}$, and $\Lambda_{UV} = 0.645$GeV.

Valence quark distributions in a confining NJL model for the pion and kaon

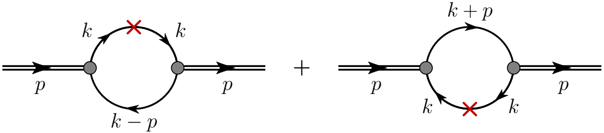
Valence quark distributions

- We start with the twist-2 quark distribution in a hadron α defined by

$$q_\alpha(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle\alpha|\bar{\psi}_q(0)\gamma^+\psi_q(\xi^-)|\alpha\rangle_c, \tag{12}$$

- c denotes the connected-diagram matrix element and $x = k^+/p^+$ is the Bjorken scaling variable with p^+ (k^+) being the plus-component of the hadron (struck quark) momentum.
- In the NJL model, gluons are “integrated out” and the gauge-link, which should appear in Eq. (12), is unity.

- The valence-quarks PDFs are calculated from the diagrams



- The operator insertion is $\gamma^+\delta\left(p^+x - k^+\right)\hat{P}_q$, with $\hat{P}_q$ being the projection operator for a quark q defined as

$$\hat{P}_{u/d} = \frac{1}{2}\left(\frac{2}{3}\,\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\,\lambda_8\right), \qquad \hat{P}_s = \frac{1}{3}\,\mathbb{1} - \frac{1}{\sqrt{3}}\,\lambda_8. \tag{13}$$

- Using the relation $\bar{q}(x) = -q(-x)$ the valence quark and anti-quark distributions in the pion or kaon are given by

$$q_\alpha(x) = ig_\alpha^2 \int \frac{d^4k}{(2\pi)^4}\delta\left(p^+ - xk^+\right)\mathrm{Tr}\left[\gamma_5\lambda_\alpha^\dagger S(k)\gamma^+\hat{P}_q S(k)\gamma_5\lambda_\alpha S(k_-)\right],$$

$$\bar{q}_\alpha(x) = -ig_\alpha^2 \int \frac{d^4k}{(2\pi)^4}\delta\left(k^+ + xp^+\right)\mathrm{Tr}\left[\gamma_5\lambda_\alpha S(k)\gamma^+\hat{P}_q S(k)\gamma_5\lambda_\alpha^\dagger S(k_+)\right.$$

Valence quark distributions

- To evaluate $q_\alpha(x)$ and $\bar{q}_\alpha(x)$ we take the moments defined by

$$\mathcal{A}_n = \int_0^1 dx\, x^{n-1}\, q(x), \quad n = 1, 2, \ldots$$

- Using the Ward-like identity, $S(k)\gamma^+ S(k) = -\partial S(k)/\partial k_+$

$$u_{K^+}(x) = \frac{3\, g_{Kqq}^2}{4\pi^2} \int d\tau \mathrm{e}^{-\tau\left[x(x-1)\, m_K^2 + x\, M_s^2 + (1-x)\, M_l^2\right]} \times \left[\frac{1}{\tau} + x(1-x)\left[m_K^2 - (M_l - M_s)^2\right]\right],$$

$$\bar{s}_{K^+}(x) = \frac{3\, g_{Kqq}^2}{4\pi^2} \int d\tau \mathrm{e}^{-\tau\left[x(x-1)\, m_K^2 + x\, M_l^2 + (1-x)\, M_s^2\right]} \times \left[\frac{1}{\tau} + x(1-x)\left[m_K^2 - (M_l - M_s)^2\right]\right].$$

- Valence-quark distributions of the π^+ can be obtained by replacing $M_s \to M_l$ and $g_{Kqq} \to g_{\pi qq}$, which leads to $u_{\pi^+}(x) = \bar{d}_{\pi^+}(x)$.

Valence quark distributions: sum rules

- The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx \left[u_{K^+}(x) - \bar{u}_{K^+}(x)\right] = \int_0^1 dx \left[\bar{s}_{K^+}(x) - s_{K^+}(x)\right] = 1,$$

  for the number sum rule and at the model scale the momentum sum rule is give by

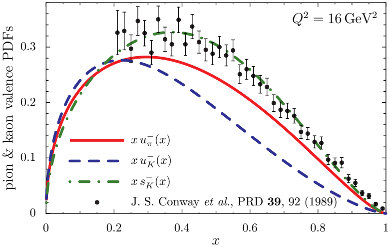
$$\int_0^1 dx\ x \left[u_{K^+}(x) + \bar{u}_{K^+}(x) + s_{K^+}(x) + \bar{s}_{K^+}(x)\right] = 1.$$

- Analogous results hold for the remaining kaons and the pions.

Numerical results for the valence quark distributions

Valence quark distributions: numerical results

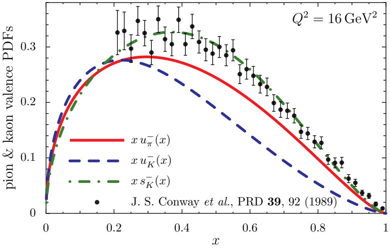
- Valence quark distribution of the π^+ and K^+, evolved from the model scale Q_0^2 to $Q^2 = 16\text{GeV}^2$ using NLO DGLAP eqns



- Note that there is no data for the kaon
- At the model scale, the momentum fraction carried by the valence u and $\bar{s}$ quarks in the K^+ are, $\langle xu \rangle_{K^+} = 0.42$ and $\langle x\bar{s} \rangle_{K^+} = 0.42$.

Valence quark distributions: numerical results

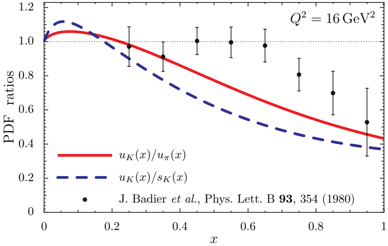
- Valence quark distribution of the π^+ and K^+, evolved from the model scale Q_0^2 to $Q^2 = 16\text{GeV}^2$ using NLO DGLAP eqns



- For the pion, the momentum fraction carried by the valence u and $\bar{d}$ quarks in the π^+ are the same (0.50).
- However, at the scale of $Q^2 = 4\text{GeV}^2$ $2\langle x\rangle_\pi \approx 0.46$, $2\langle x^2\rangle_\pi \approx 0.22$. These are consistent with other analyses at the same scale, such as Aicher et al [PRL 105, 252003 (2010)] $2\langle x\rangle_\pi \approx 0.55$, $2\langle x^2\rangle_\pi \approx 0.18$; Sutton et al [PRD 45, 2349 (1992)] $2\langle x\rangle_\pi \approx 0.40 \pm 0.02$, $2\langle x^2\rangle_\pi \approx 0.16 \pm 0.01$

Valence quark distributions: numerical results

- Ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO DGLAP evolution to $Q^2 = 16\text{GeV}^2$



- The ratio $u_K/u_\pi \to 0.43 \approx M_u/M_s$ as $x \to 1$, which is in good agreement with existing data.
- The x-depence differs much from the data in the valence region.
- This disagreement may lie with absence of the momentum dependence in the NJL Bethe-Salpeter vertices and momentum-independent dynamical mass.
- We are interested in the in-medium changes of these distributions

Medium effects in the Valence quark distributions in a confining NJL model for the pion and kaon

Nuclear medium effects: QMC model

- The nuclear medium effects on the properties of the pion and kaon will be calculated by combining the NJL model with the quark-meson coupling (QMC) model
- The in-medium dressed quark propagator takes the form [G.A. Miller eta al, PRL 101, 082301 (2009)]

$$S(k) \to S^*(k^*) = \frac{\not{k}^* + M_l^*}{k^{*\,2} - M_l^{*\,2}}$$

- The in-medium modifications enter as a shift of the light-quark momenta: $k^\mu \to k^{*\,\mu} = k^\mu + V^\mu$,
- The vector mean-field potential $V^\mu = (V^0, 0)$ felt by the light quarks in the nuclear medium is calculated self-consistently in the QMC model.
- In the present approach, the strange quark does not couple to the nuclear medium, so its properties are the same as in vacuum.

The quark meson coupling model [PPNP 58, 1 (2007)]

- The QMC model is a quark-based, relativistic mean field model of nuclear matter and nuclei.
- Here the relativistically moving confined light quarks in the nucleon bags (MIT bag) self-consistently interact directly with the scalar-isoscalar σ, vector-isoscalar ω, and vector-isovector ρ mean fields (Hartree approximation) generated by the light quarks in the other nucleons.
- The meson mean fields are responsible for nuclear binding.
- The self-consistent response of the bound light quarks to the mean field σ field leads to novel saturation mechanism for nuclear matter.
- The model has opened tremendous opportunities for studies of the structure of finite nuclei and hadron properties in a nuclear medium (nuclei) with a model based on the underlying quarks dof.

Nuclear medium effects: static properties

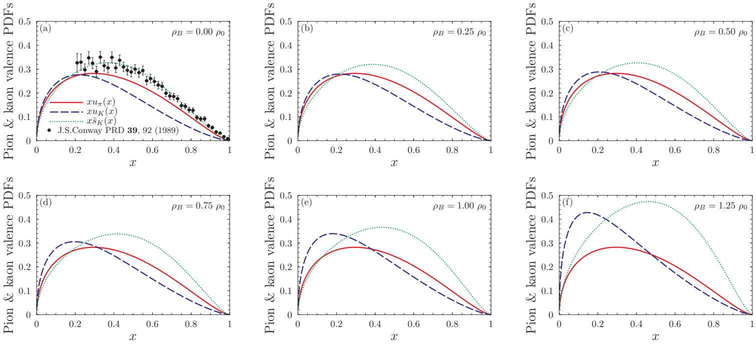
- The nuclear medium effects on the properties of the pion and kaon are calculated as a function of the nuclear matter density:

ρ_B/ρ_0	M_u^*	m_K^*	f_K^*	m_π^*	f_π^*
0.0	0.400	0.495	0.097	0.140	0.093
0.25	0.370	0.465	0.091	0.136	0.092
0.50	0.339	0.437	0.090	0.134	0.089
0.75	0.307	0.411	0.089	0.132	0.086
1.00	0.270	0.386	0.088	0.131	0.081
1.25	0.207	0.359	0.084	0.136	0.069

- Here we have $M_s^* = M_s = 0.611\text{GeV}$.
- The dynamical u quark mass decreases by about 30%; while its condensate decreases by 13%.
- These results indicate that chiral symmetry is partially restored in a nuclear medium

Nuclear medium effects: valence quark distribution

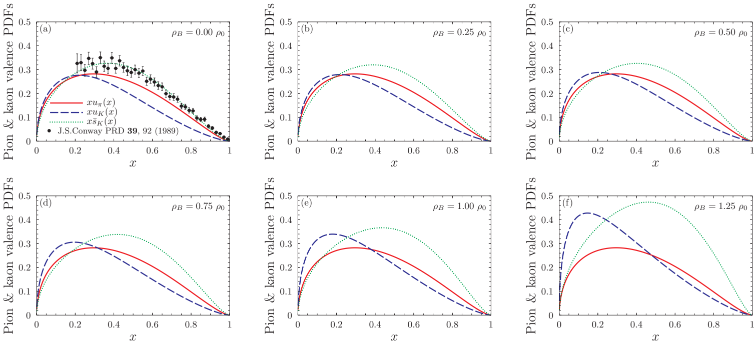
- The in-medium valence-quark distributions satisfy the baryon number and momentum sum rules as in they did in vacuum.



- Plots (b)-(f) show that the density dependence of the quark distributions in the pion is rather weak in agreement with Suzuki, PLB 368, 1 (1996)

Nuclear medium effects: valence quark distributions

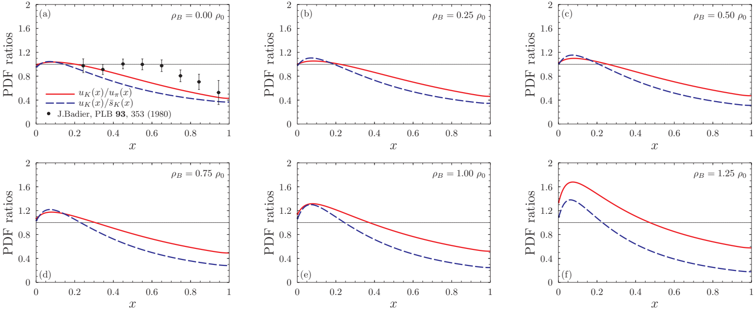
- The valence PDFs for the K^+ (dashed and dotted curves) show a different density depence compared to the pion.



- Plots (b)-(f) show that the PDFs in the K^+ change in shape and in magnitude. In particular, the peak position of $x\bar{s}$ (xu) is displaced to the right (left) as density increases.
- Experimental measurements are required to verify these predictions.

Nuclear medium effects: valence quark distributions

- Ratios of PDFs: u_K/u_π (solid lines) and $u_K/\bar{s}_K$ (dashed lines).



- The ratio u_K/u_π is enhanced at higher densities and, in particular, for small x.
- For large x, $u_K/\bar{s}_K$ is suppressed when nuclear density increases, which is opposite to u_K/u_π.
- The deviations of these ratios from 1 show the pattern of the flavor symmetry breaking. Thus, flavor symmetry breaking effects become larger as density increases.

Summary

- We have studied a kaon and pion structure via valence quark distributions in a confining NJL model. Static properties are very well described.
- We have studied the valence distributions of quarks in the pion and kaon in vacuum and in nuclear matter
- Our predicted pion valence quark distribution follows the general trend of the data at the high energy scale in vacuum. For the kaon there is no data. The EIC will, possibly, produce some.
- We have extend our study to nuclear matter in order to explore its influence on the PDFs of the pion and kaon. Here, experimental measurements are required to verify our findings.
- This can be extended to other pseudoscalar and vector mesons. In particular, η and η' pseudoscalars and the ϕ vector meson are more interesting since these will require the introduction of the Kobayashi-Maskawa-'t Hooft term.