



# Exclusive vector meson production as a tool to probe QCD low $x$ evolution

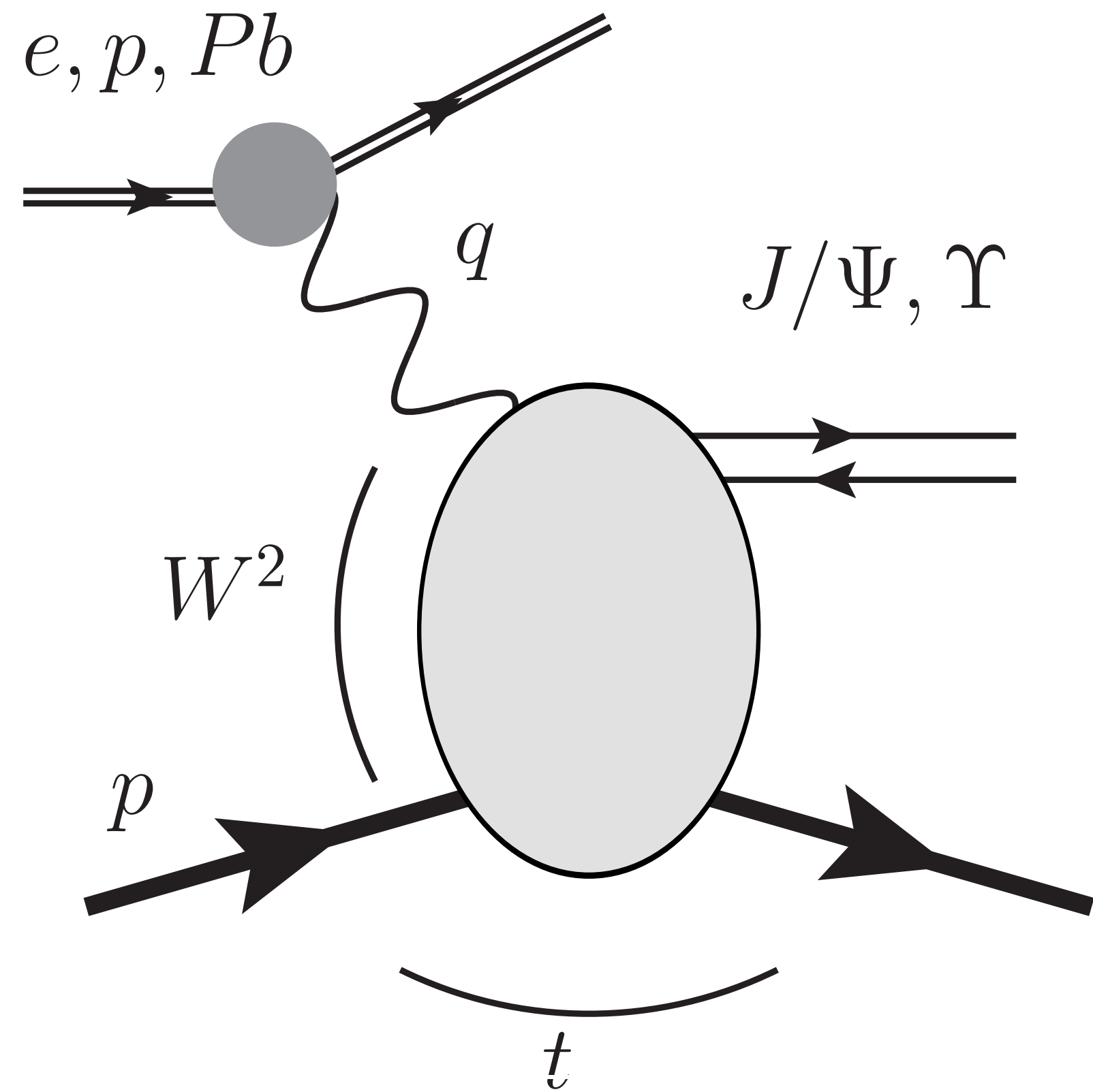
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based on:

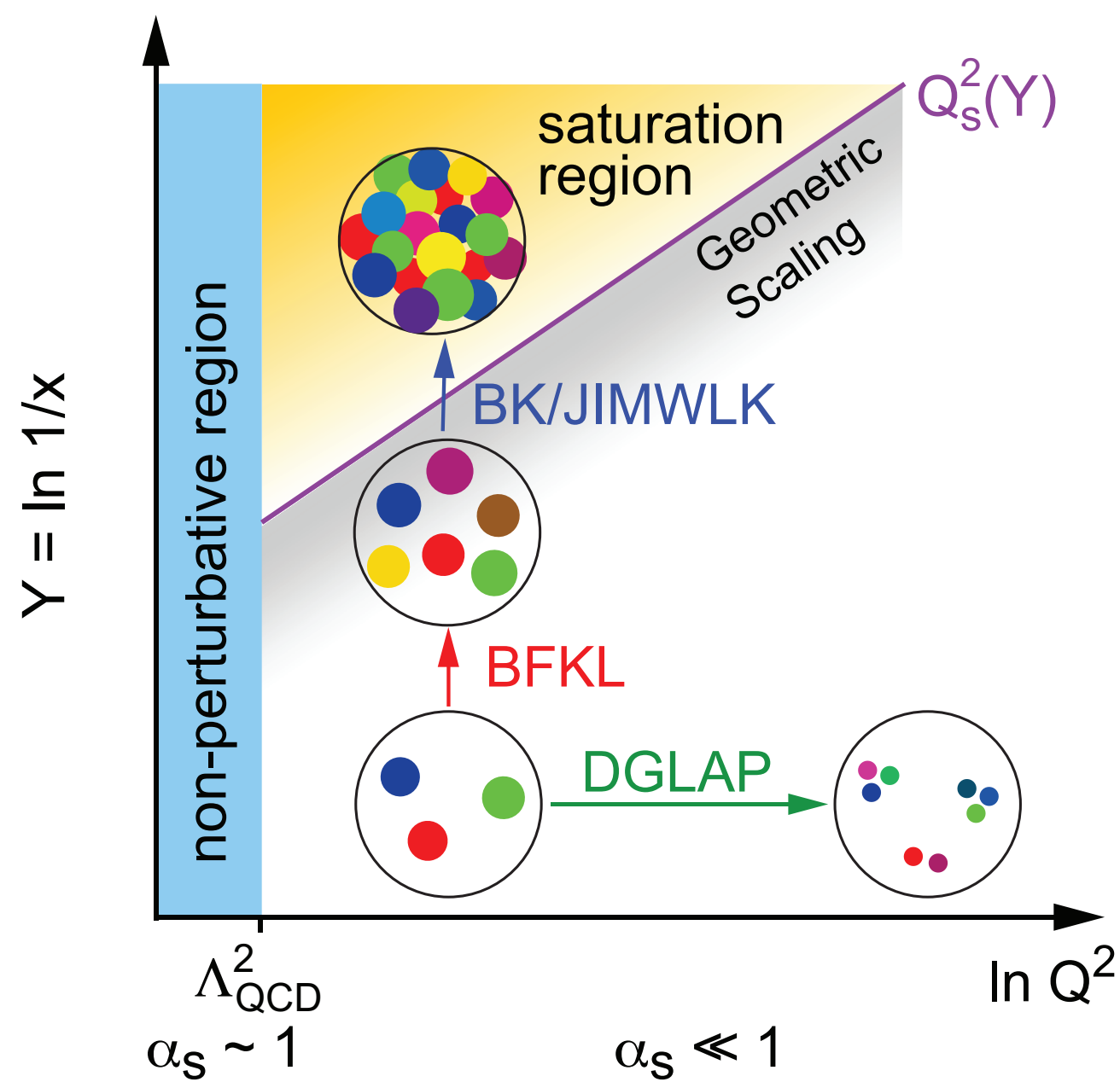
- I. Bautista, Fernandez Tellez, MH, PRD 94 (2016) 5, 054002, arXiv:1607.05203
- A. Arroyo Garcia, MH, K.Kutak, PLB 795 (2019) 569-575, arXiv:1904.04394
- MH, E. Padron Molina, Phys.Rev.D 103 (2021) 7, 074008, arXiv:2011.02640

A process to explore the low  $x$  gluon in the proton at the LHC:  
exclusive photo-production of  $J/\Psi$  and  $\Psi(2s)$



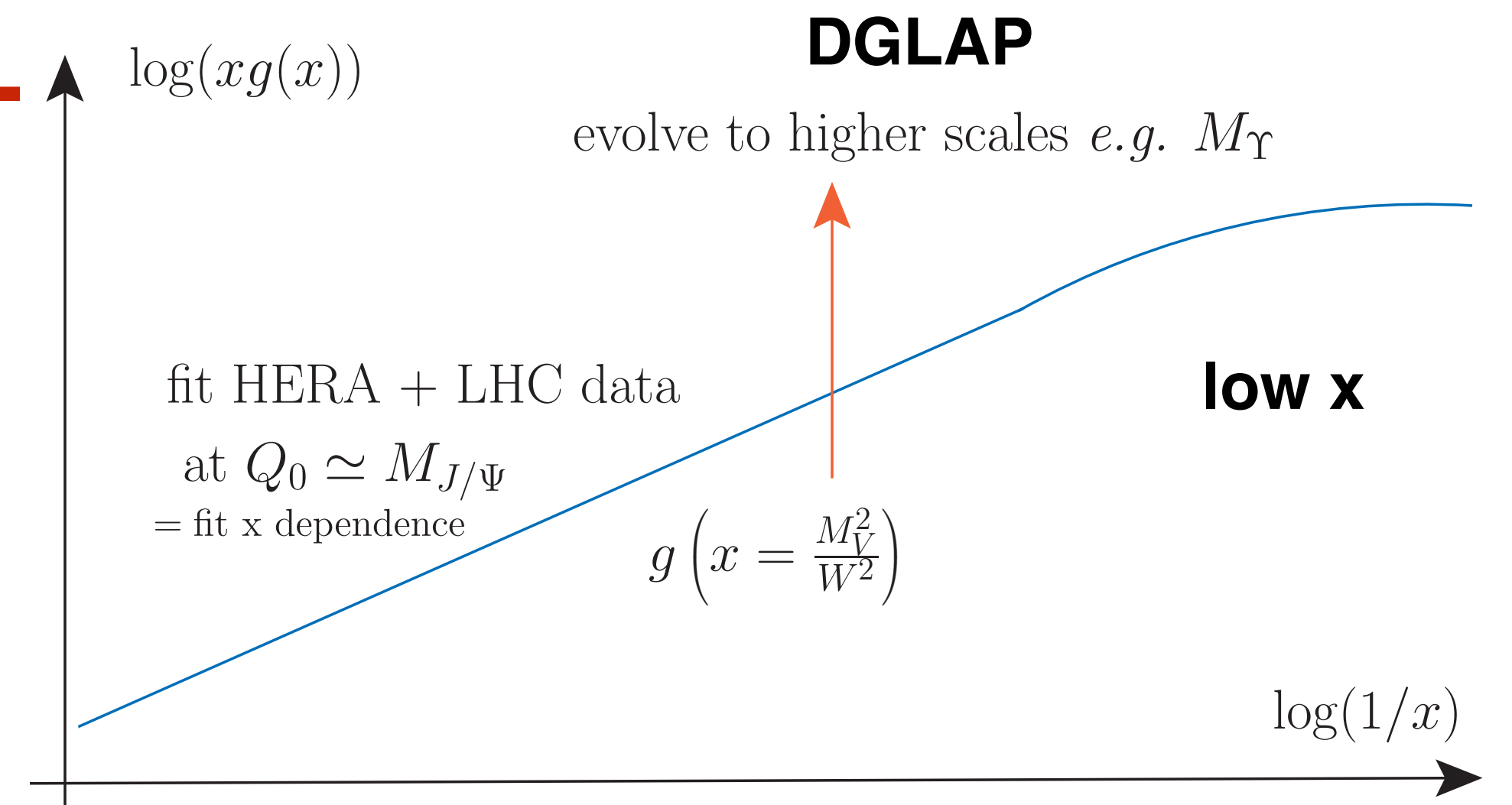
- hard scale: charm mass (small, but perturbative)
- reach up to  $x \gtrsim .5 \cdot 10^{-6}$
- perturbative cross-check:  $\Upsilon$  (b-mass)
- measured at **LHC** (LHCb, ALICE, CMS) & **HERA** (H1, ZEUS)

the challenge:  
describe correctly  
charmonium  
production



our study:

- instead of DGLAP vs low x
- linear low x (BFKL) vs. non-linear low x (BK)
- failure of BFKL = sign for BK → high & saturated gluon



**details:**

BK evolution for dipole amplitude  $N(x,r) \in [0,1]$  [related to gluon distribution]

kernel calculated in pQCD

$$\frac{dN(x,r)}{d \ln \frac{1}{x}} = \int d^2 \mathbf{r}_1 K(\mathbf{r}, \mathbf{r}_1) [N(x, r_1) + N(x, r_2) - N(x, r) - N(x, r_1)N(x, r_2)]$$

linear BFKL evolution = subset of complete BK

non-linear term relevant for  $N \sim 1$  (=high density)

linear low  $x$  evolution as benchmark  $\rightarrow$  requires precision

(updated version desirable, work has started; not expected too soon)

use: HSS NLO BFKL fit [MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

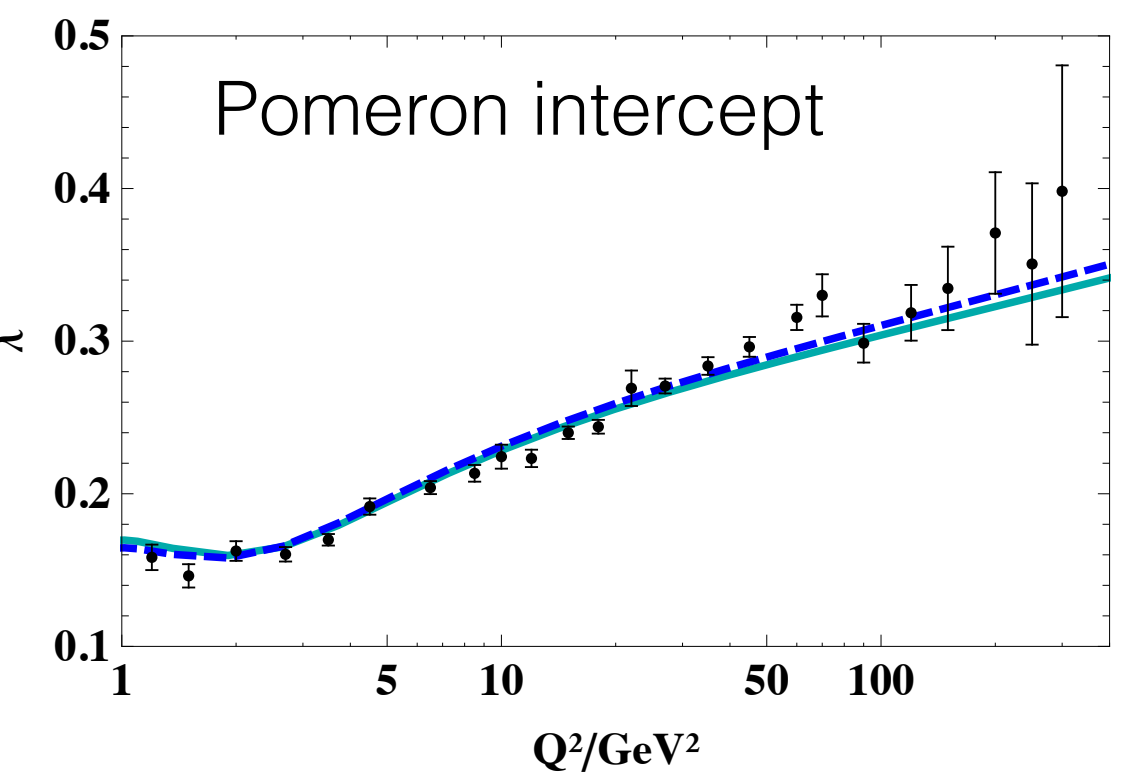
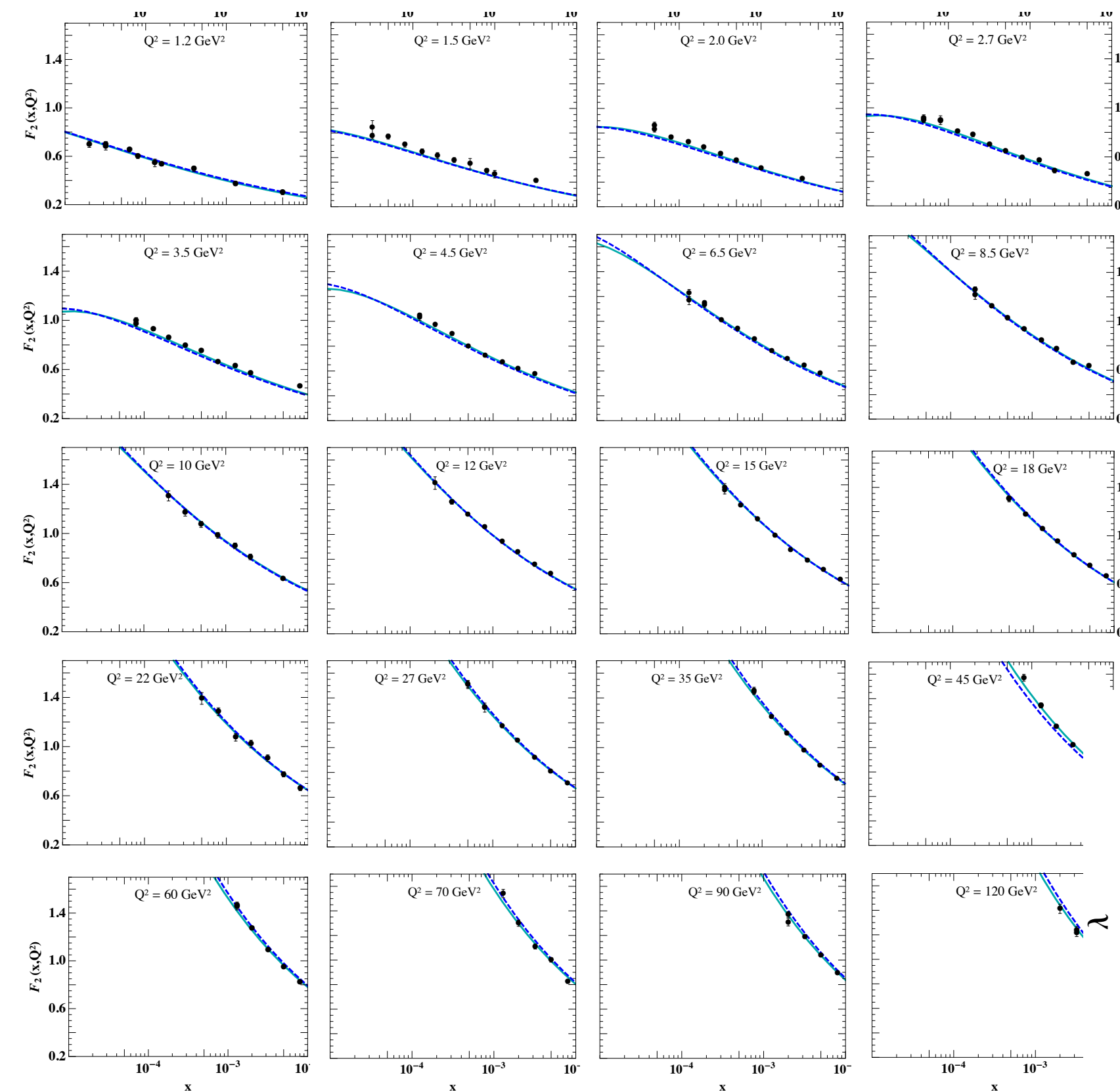
- uses NLO BFKL kernel

[Fadin, Lipatov; PLB 429 (1998) 127]

+ resummation of collinear logarithms

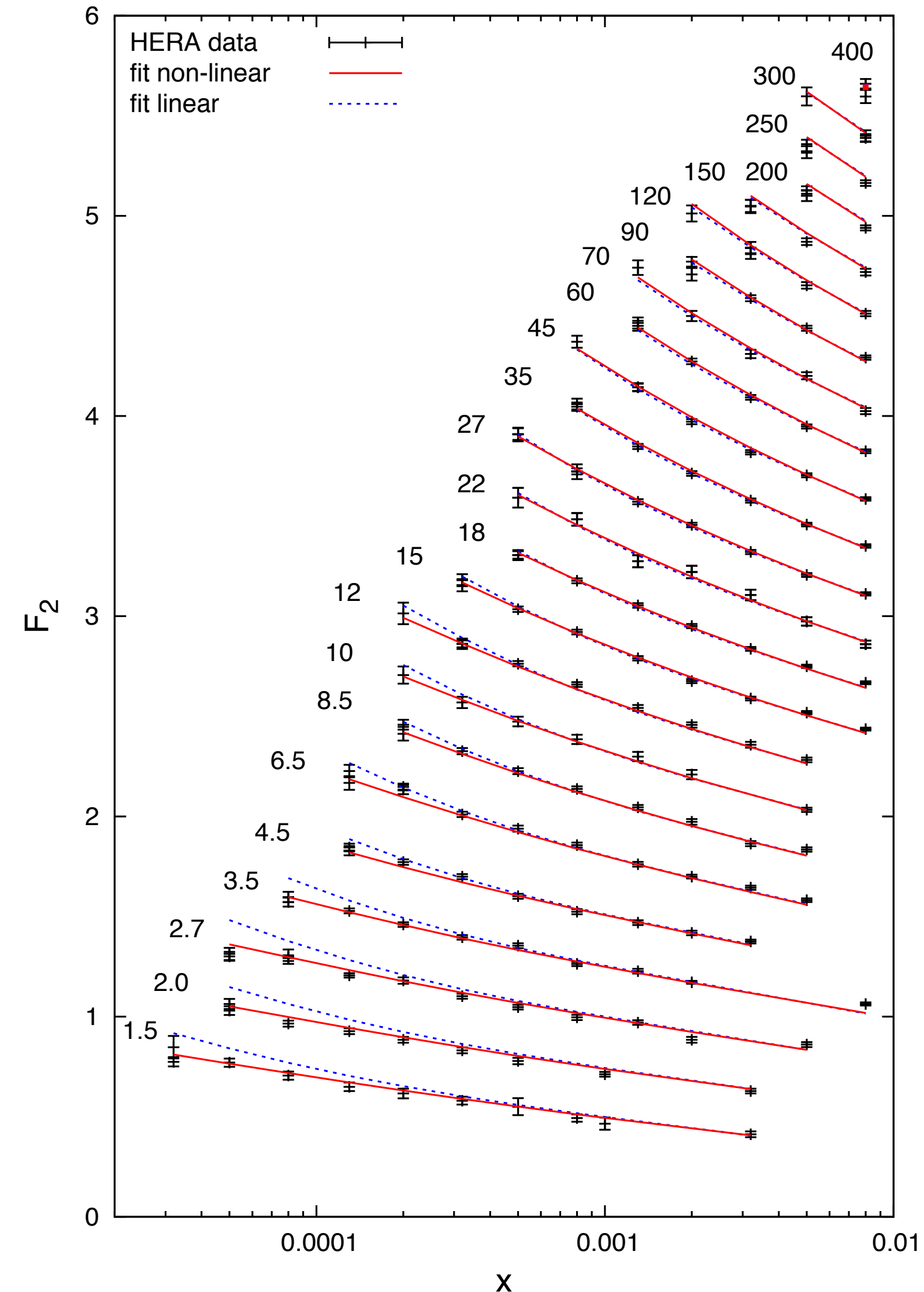
- initial  $k_T$  distribution from fit to combined HERA data

[H1 & ZEUS collab. 0911.0884]



# gluon with non-linear terms: KS gluon [Kutak, Sapeta; 1205.5035]

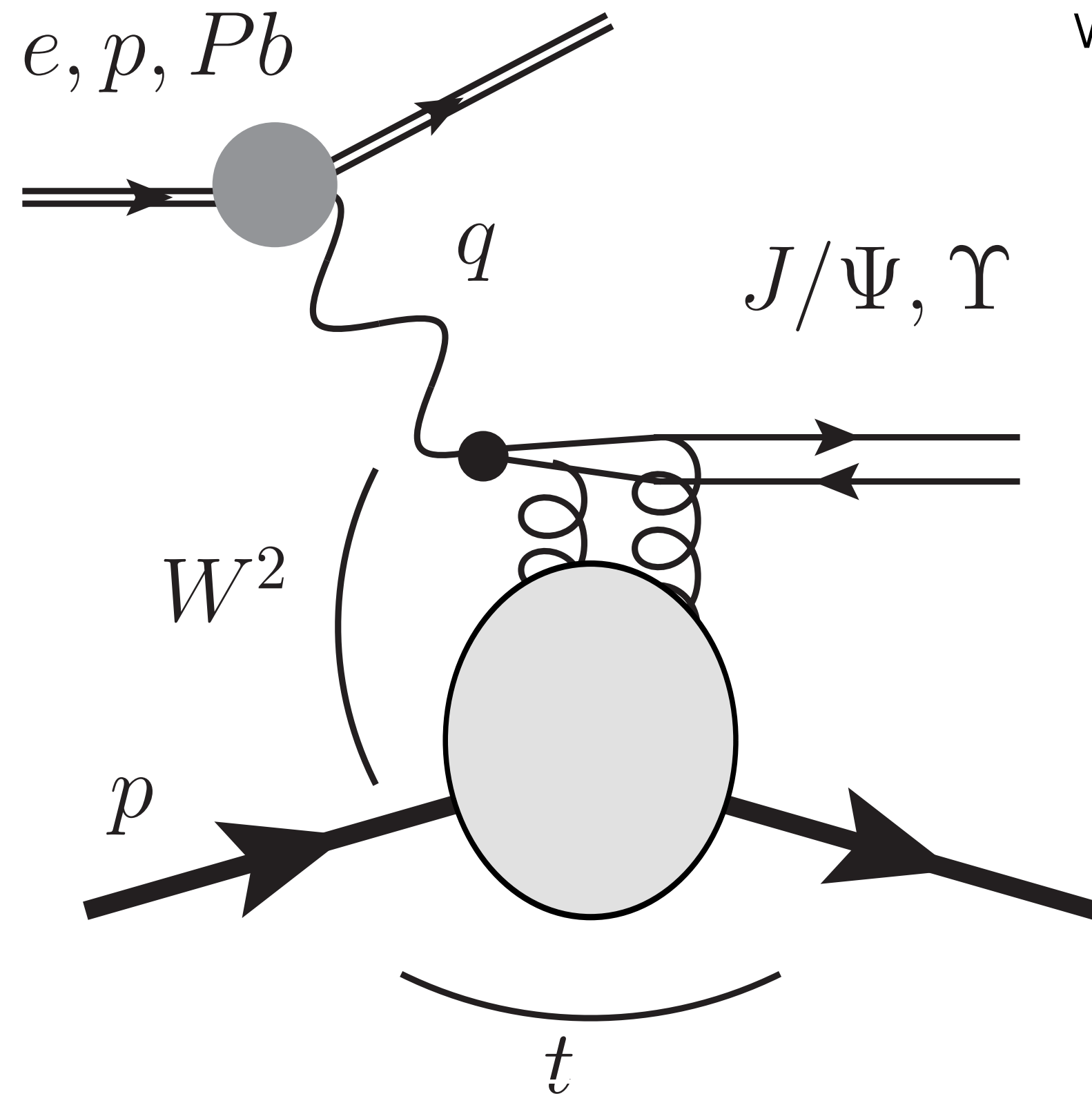
- based on unified (leading order) DGLAP+BFKL framework [Kwieciński, Martin, Stasto, PRD 56(1997) 3991]
- combined with leading order BK evolution [Kutak, Kwiecinski; hep-ph/0303209][Kutak, Stasto; hep-ph/0408117]
- initial conditions: fit to combined HERA data [H1 & ZEUS collab. 0911.0884]
- both non-linear and linear version available (= non-linearity switched off)



# The photo-production Xsection

part I

simple approach: Gaussian light-front  
wave functions [[1607.05203](#), [1904.04394](#)]



$$\Im m \mathcal{A}^{\gamma p \rightarrow V p}(x, t=0) = \int_0^\infty dr W(r) \sigma_{q\bar{q}}(x, r)$$

elements:

r: transverse size of quark-antiquark dipole

$$W(r) = 2\pi r \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_T(r, z)$$

integrated light-front wave function overlap =  
transition photon → dipole → vector meson

$$\sigma_{q\bar{q}}(x, r) = \frac{4\pi}{N_c} \int \frac{d^2 \mathbf{k}}{\mathbf{k}^2} \left(1 - e^{i\mathbf{k} \cdot \mathbf{r}}\right) \alpha_s \mathcal{F}(x, \mathbf{k}^2)$$

explore inclusive gluon → can only calculate  
imaginary part of scattering amplitude at t=0

= diffraction process

dipole cross-section from unintegrated gluon  $\mathcal{F}$   
= **object of interest**

# how to compare to experiment?

(sort of standard procedure for comparing inclusive gluon to exclusive data)

a) analytic properties of scattering amplitude  $\rightarrow$  real part

$$\mathcal{A}^{\gamma p \rightarrow V p}(x, t=0) = \left( i + \tan \frac{\lambda(x)\pi}{2} \right) \cdot \Im \mathcal{A}^{\gamma p \rightarrow V p}(x, t=0)$$

with intercept

$$\lambda(x) = \frac{d \ln \Im \mathcal{A}(x, t)}{d \ln 1/x}$$

b) differential Xsection at  $t=0$ :

$$\left. \frac{d\sigma}{dt} (\gamma p \rightarrow V p) \right|_{t=0} = \frac{1}{16\pi} \left| \mathcal{A}^{\gamma p \rightarrow V p}(W^2, t=0) \right|^2$$

c) from experiment:

$$\frac{d\sigma}{dt} (\gamma p \rightarrow V p) = e^{-B_D(W) \cdot |t|} \cdot \left. \frac{d\sigma}{dt} (\gamma p \rightarrow V p) \right|_{t=0}$$

$$\sigma^{\gamma p \rightarrow V p}(W^2) = \frac{1}{B_D(W)} \left. \frac{d\sigma}{dt} (\gamma p \rightarrow V p) \right|_{t=0} \quad \text{extracted from data}$$

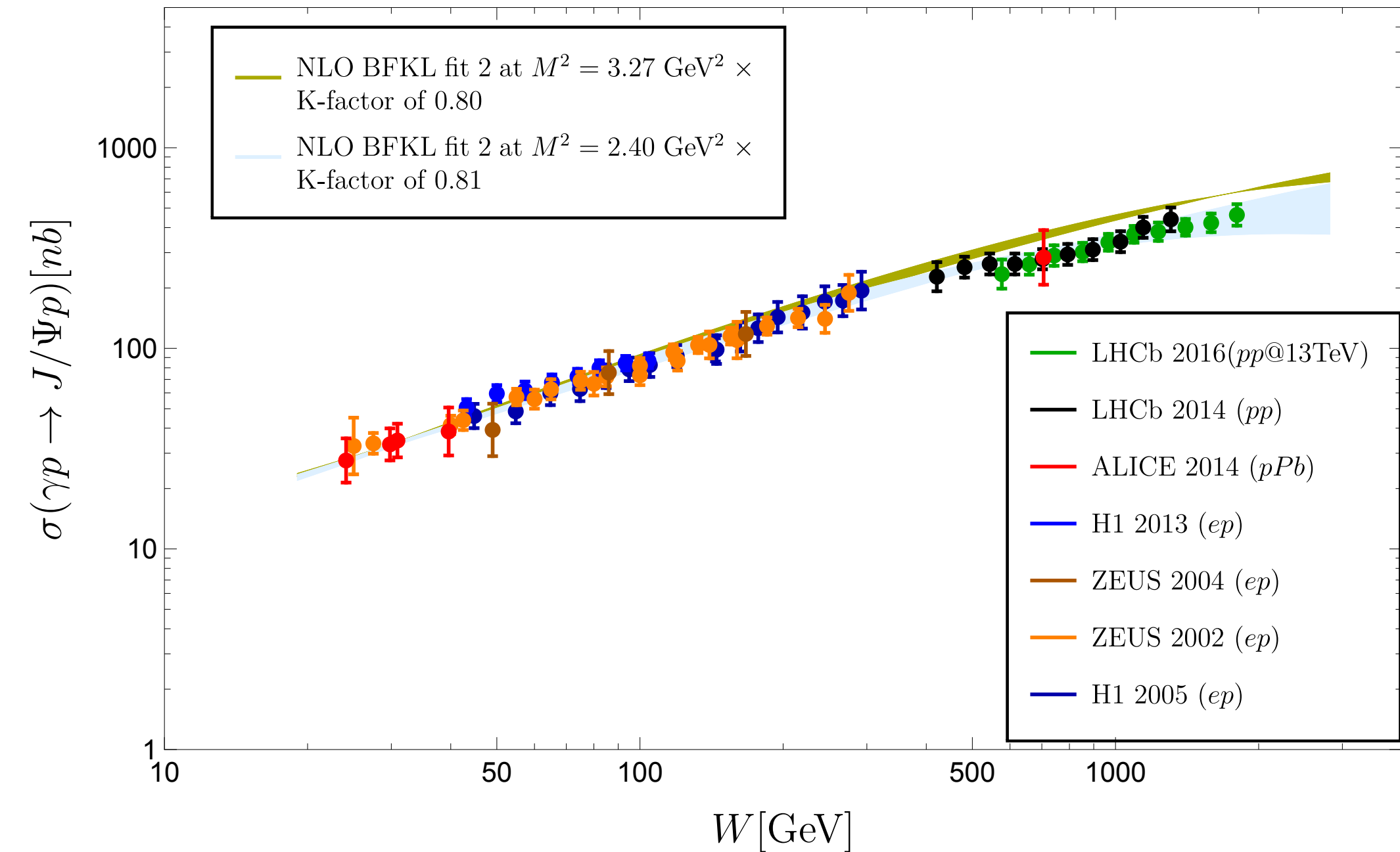
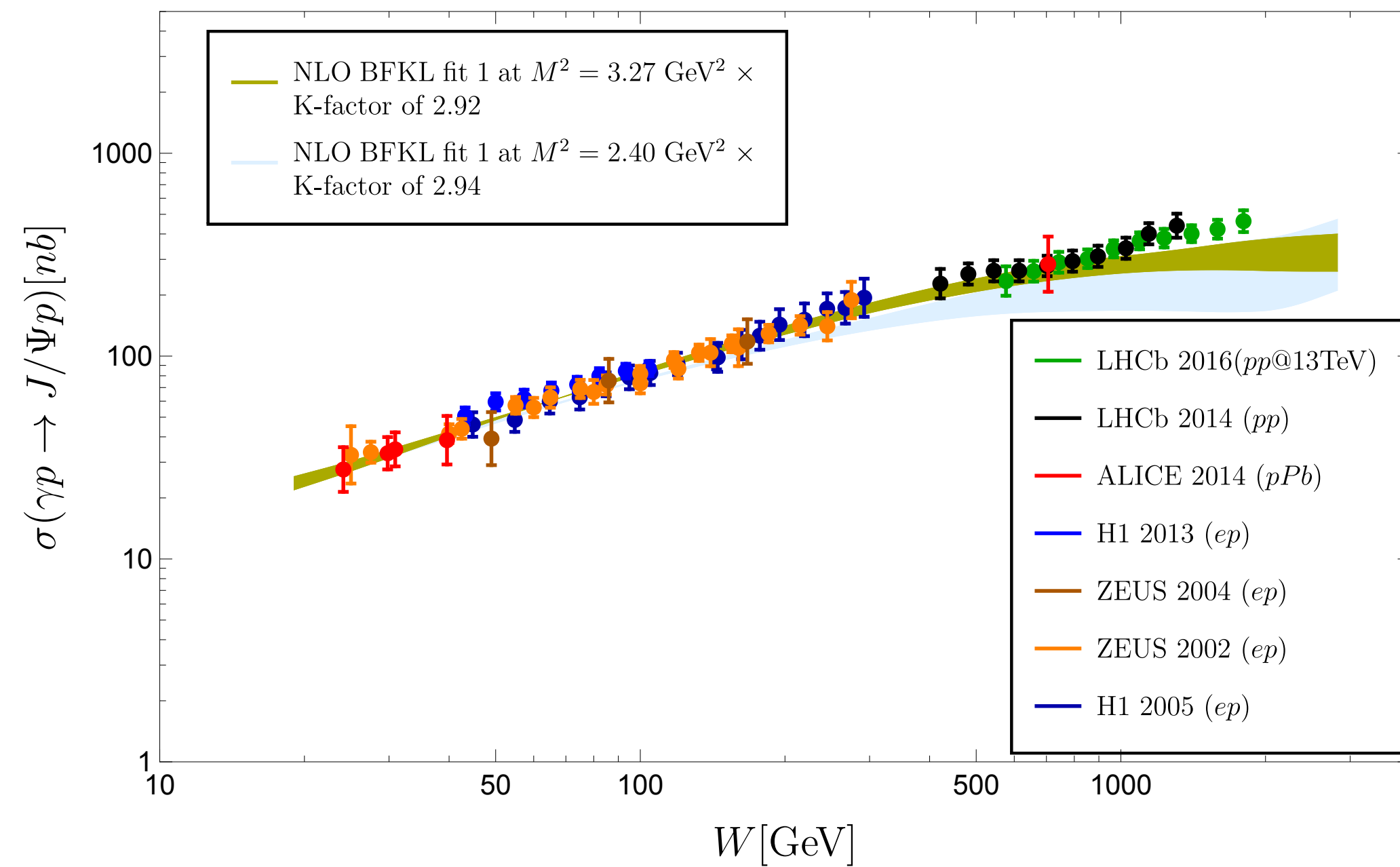
weak energy dependence from  
slope parameter

$$B_D(W) = \left[ b_0 + 4\alpha' \ln \frac{W}{W_0} \right] \text{GeV}^{-2}.$$

# First study (BFKL only, also for $\Upsilon$ )

[Bautista, MH, Fernandez-Tellez;1607.05203]

NLO BFKL describes energy dependence, but .....



error band: variation of renormalization scale  
 → in general pretty small = stability

...but error blows up for highest energies

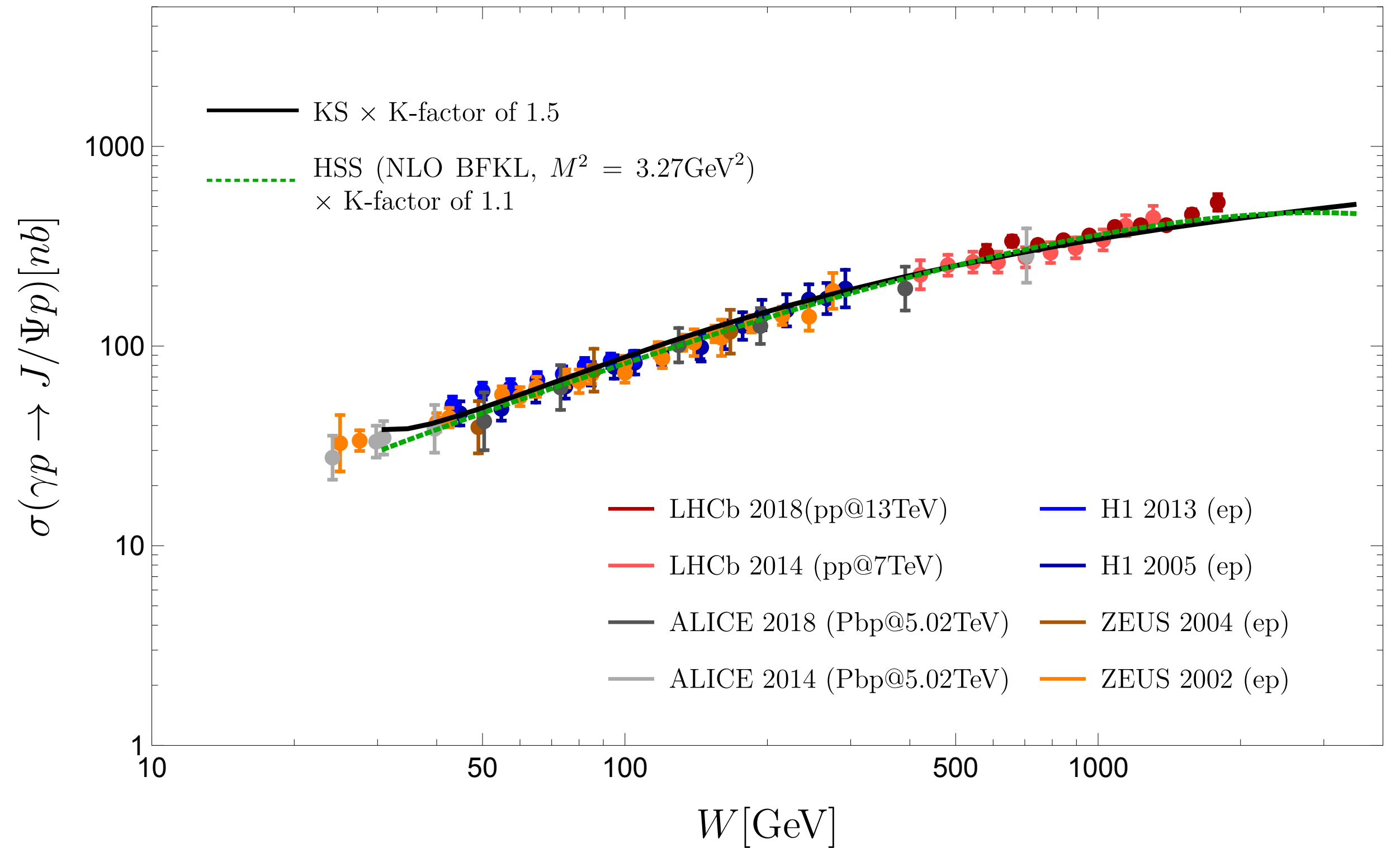
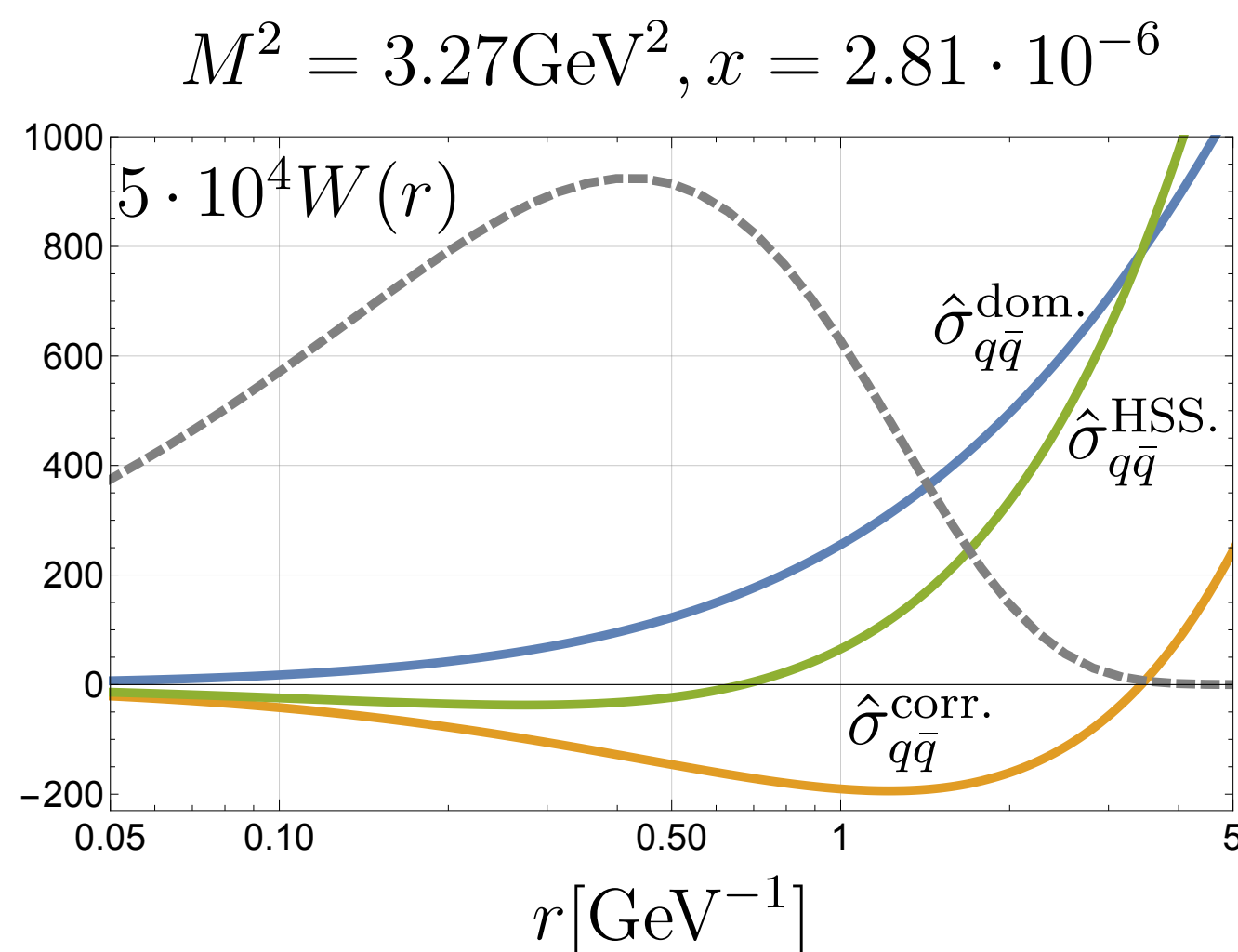
does it mean something?



# Second Study

[Arroyo, MH, Kutak;1904.04394]

- linear vs. nonlinear
- with standard scale choice for NLO BFKL gluon, both distribution describe energy dependence with equal quality



but find:

- with standard scale choice, HSS gluon is unstable for largest energies

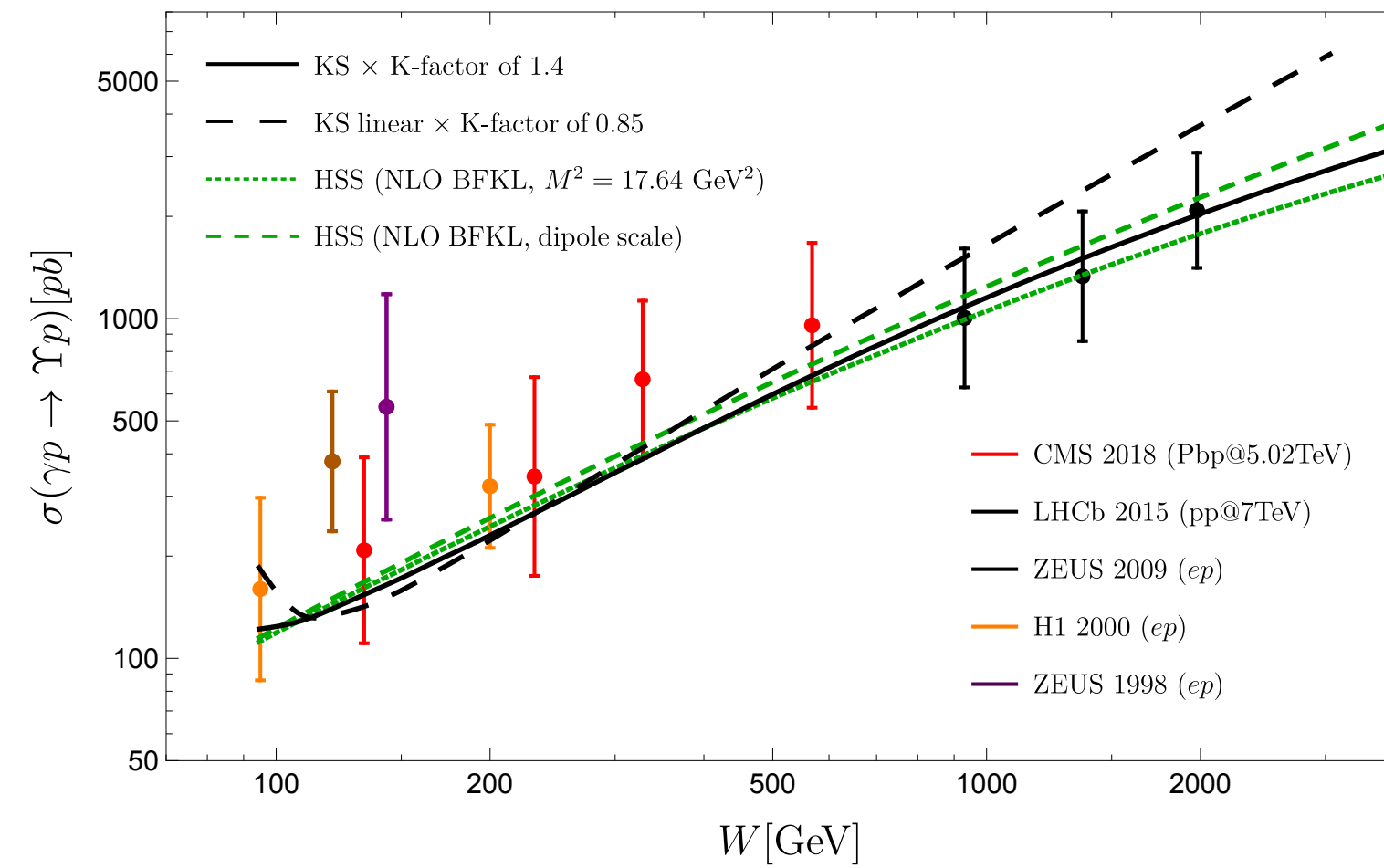
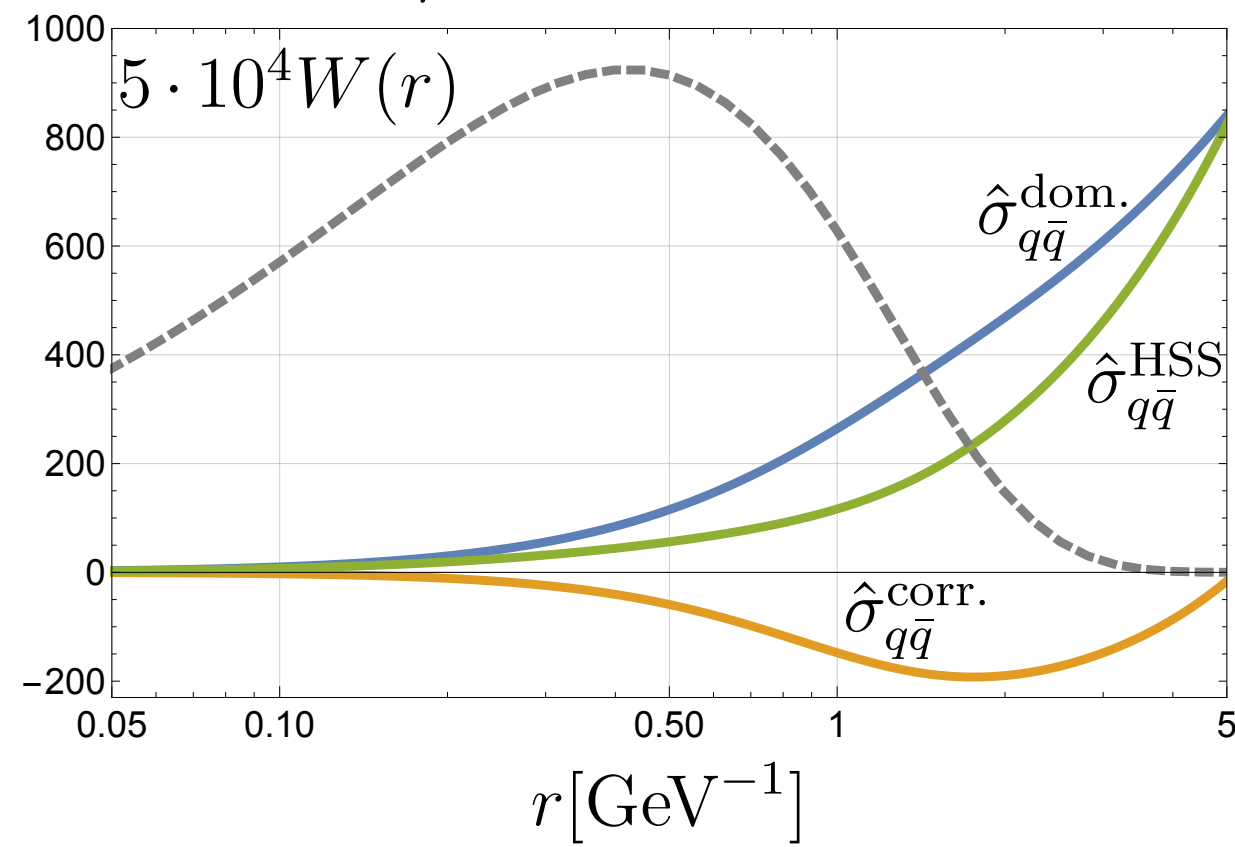
$$\hat{\sigma}_{q\bar{q}}^{(\text{HSS})}(x, r) = \hat{\sigma}_{q\bar{q}}^{(\text{dom.})}(x, r) + \hat{\sigma}_{q\bar{q}}^{(\text{corr.})}(x, r),$$

- fix this through dipole size dependent renormalization scale

$$M^2 = \frac{4}{r^2} + \mu_0^2 \text{ with } \mu_0^2 = 1.51 \text{ GeV}^2$$

→ stabilize perturbative expansion through resummation

$$M^2 = \frac{4}{r^2} + \mu_0^2, x = 2.81 \cdot 10^{-6}$$

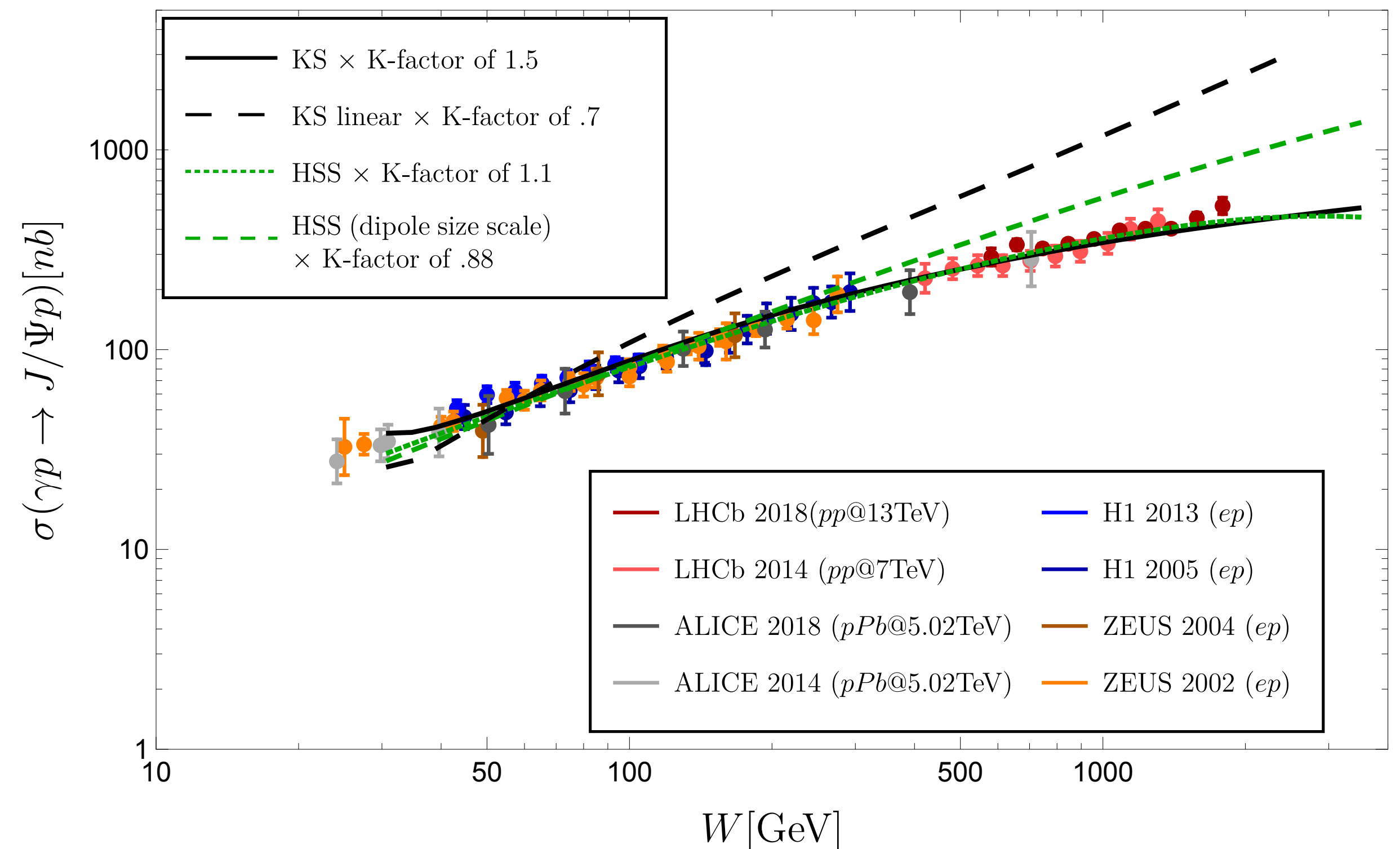


- still describe  $\Upsilon$  production  
→ perturbative cross-check
- not true for high precision HERA data

stabilizes perturbative expansion → stable NLO BFKL evolution at highest  $W$

BUT:

- resulting growth too strong for  $J/\Psi$  production
- classical sign for onset of high density effects/transition towards saturated regime?



# Shortcomings of our 2nd study

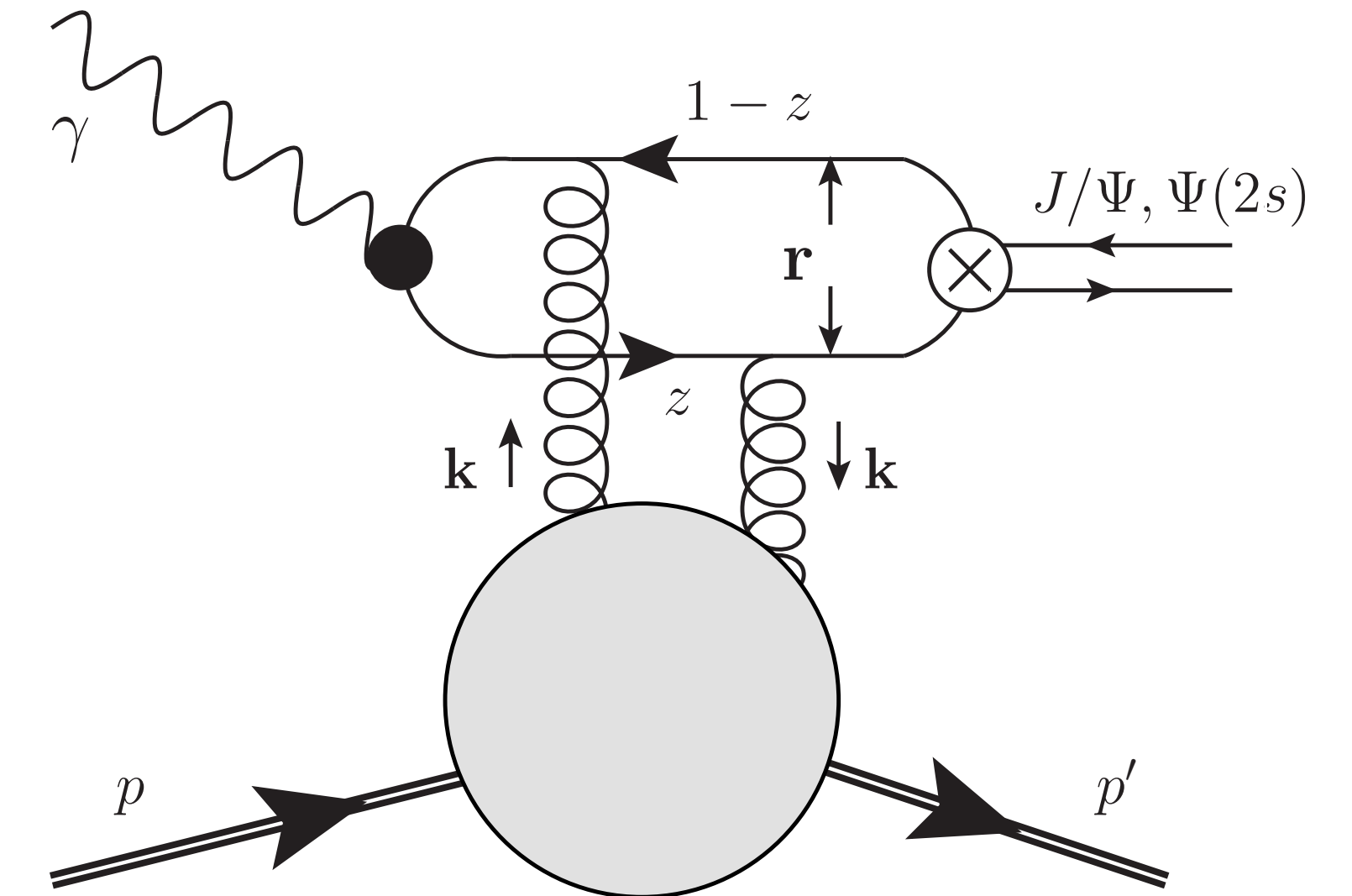
- Vector meson wave functions use (conventional) boosted Gaussian model → what about more refined descriptions?
- We do not address excited states  $\Psi(2s)$  → different  $r$ -shape of the transition due to nodes in the wave function
- refit of NLO BFKL gluon → desirable, but beyond this study; project for future
- estimate uncertainties (scale variation) → how stable is our observation?

# Transition amplitude $\gamma \rightarrow VM$

includes relativistic spin rotation effects + (more) realistic  $c\bar{c}$  potential both for  $J/\Psi$  and  $\Psi(2s)$

[Hufner, Y. Ivanov, B. Kopeliovich, A. Tarasov; [hep-ph/0007111](#)],

[M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)]



$$\Im \mathcal{A}_T(W^2, t=0) = \int d^2\mathbf{r} \left[ \sigma_{q\bar{q}} \left( \frac{M_V^2}{W^2}, r \right) \bar{\Sigma}_T^{(1)}(r) + \frac{d\sigma_{q\bar{q}} \left( \frac{M_V^2}{W^2}, r \right)}{dr} \bar{\Sigma}_T^{(2)}(r) \right]$$

- depends both on dipole cross-section and its derivative
- wave functions have been obtained in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)] through numerical solution to corresponding Schrödinger equation
- transition function factorizes for real photon ( $Q = 0$ )
- problem: need boosted wave function

$$\bar{\Sigma}_T^{(i)}(r) = \hat{e}_f \sqrt{\frac{\alpha_{e.m.} N_c}{2\pi^2}} K_0(m_f r) \Xi^{(i)}(r), \quad i = 1, 2$$

$$\Xi^{(1)}(r) = \int_0^1 dz \int \frac{d^2\mathbf{p}}{2\pi} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{m_T^2 + m_T m_L - 2p_T^2 z(1-z)}{m_T + m_L} \Psi_V(z, |\mathbf{p}|),$$

$$\Xi^{(2)}(r) = \int_0^1 dz \int \frac{d^2\mathbf{p}}{2\pi} e^{i\mathbf{p}\cdot\mathbf{r}} |\mathbf{p}| \frac{m_T^2 + m_T m_L - 2\mathbf{p}^2 z(1-z)}{2m_T(m_T + m_L)} \Psi_V(z, |\mathbf{p}|),$$

- $\Psi_V(z, \mathbf{p})$  provided as table by authors of [\[1812.03001; 1901.02664\]](#)

$$m_T^2 = m_f^2 + \mathbf{p}^2, \quad m_L^2 = 4m_f^2 z(1-z),$$

# potentials for wave functions:

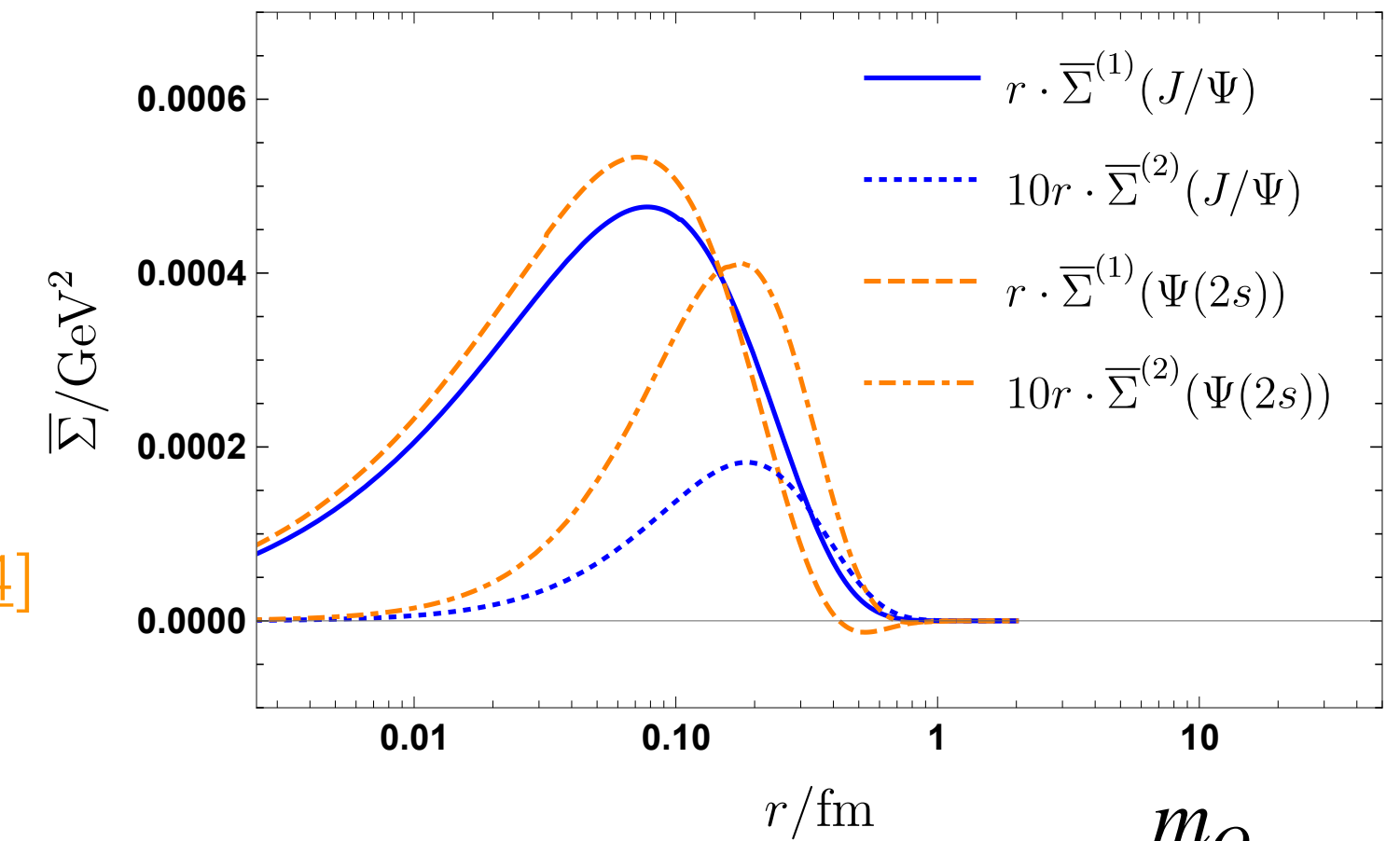
as implemented in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)]

Note:

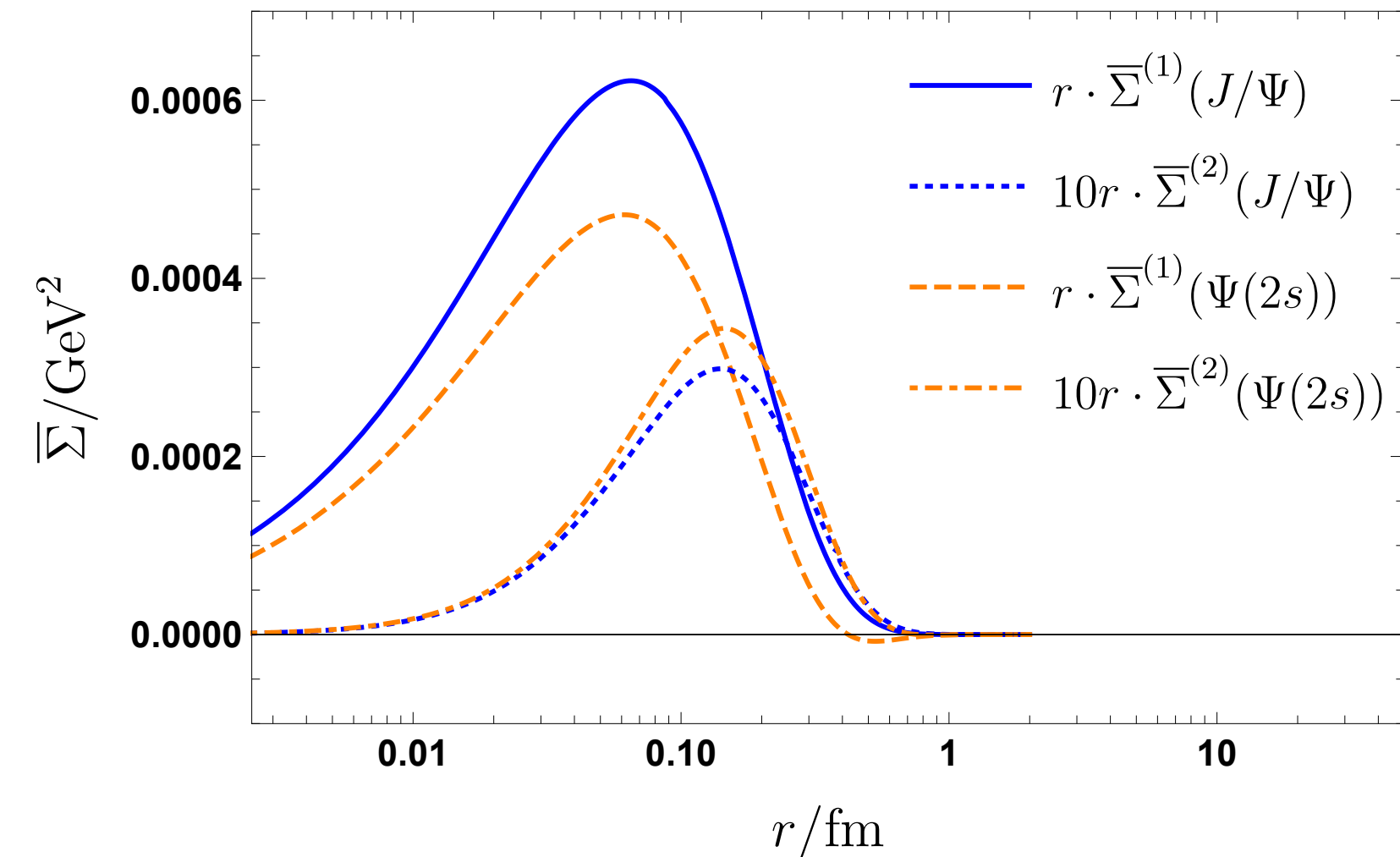
- plots show transition function  $\gamma \rightarrow VM$ , not wave function
- $\Psi(2s)$ : node structure of wave function absent in transition after integration over photon momentum fraction  $z$
- $\bar{\Sigma}^{(2)}(r)$  enhanced for  $\Psi(2s)$ , but still considerable smaller

→  $\Psi(2s)$  gives access to a (slightly) different region in  $r$  than  $J/\Psi$

→ requires separate diffractive slopes  $B_D(W)$  as obtained in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)]

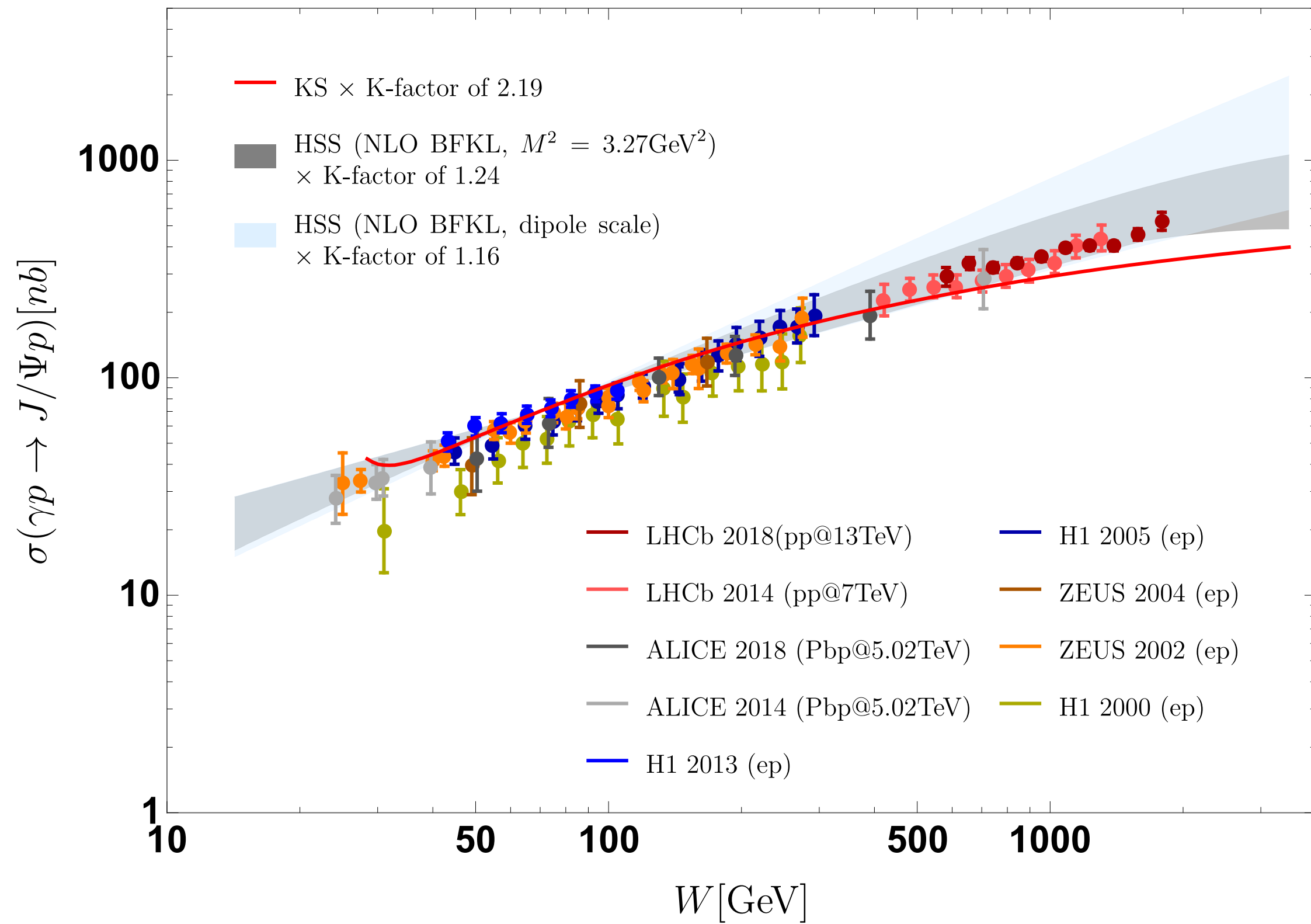


harmonic oscillator (HO):  $U(r) = \frac{m_Q}{2} \omega^2 r^2$   
 $\omega = 0.3 \text{ GeV} \rightarrow$  Gaussian shape

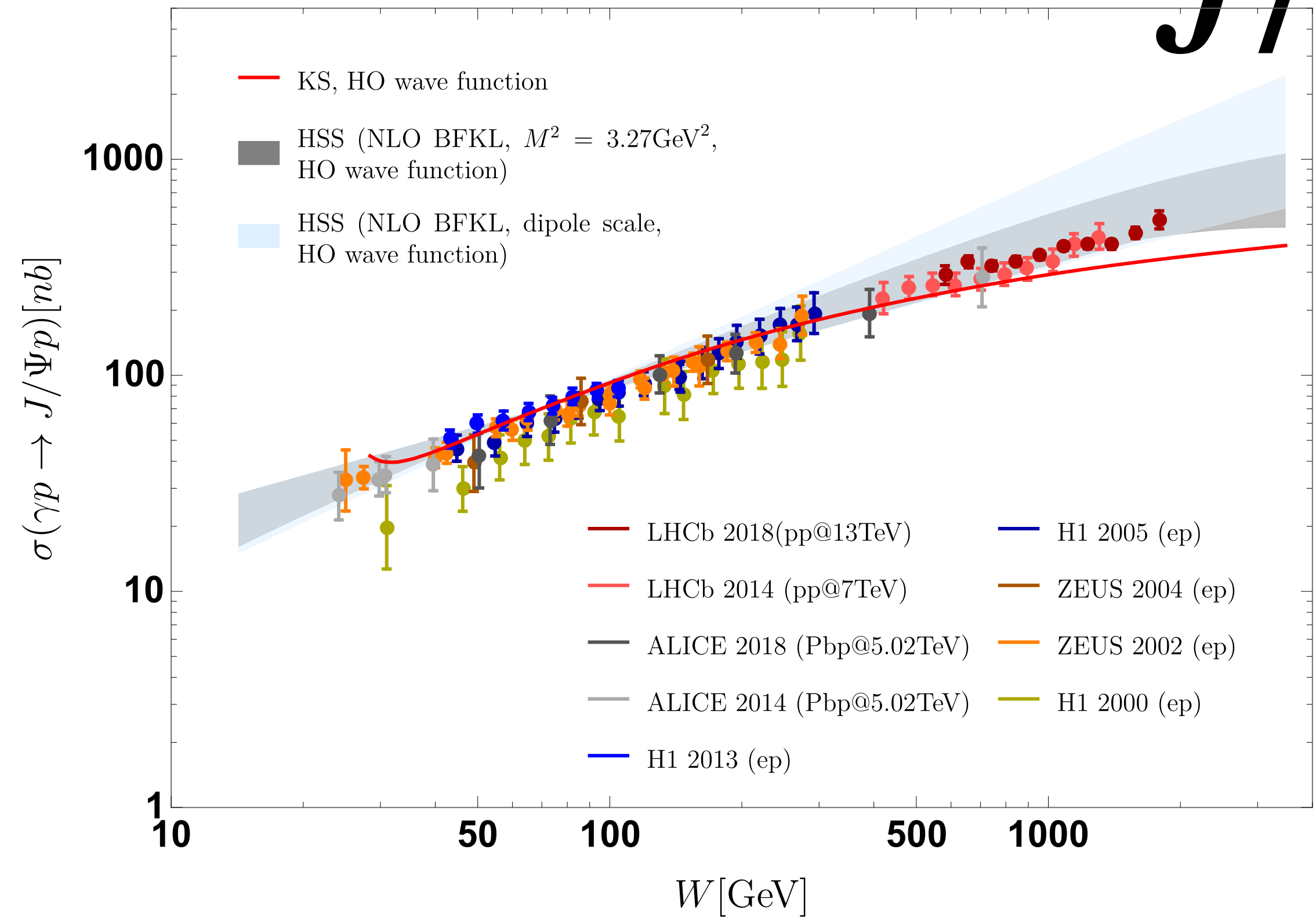


Buchmüller-Tye Potential: Coulomb-like behavior at small  $r$  and a string-like behavior at large  $r$  [Buchmüller, Tye; PRD24, 132 (1981)]

## Buchmüller-Tye potential



## Harmonic Oscillator potential

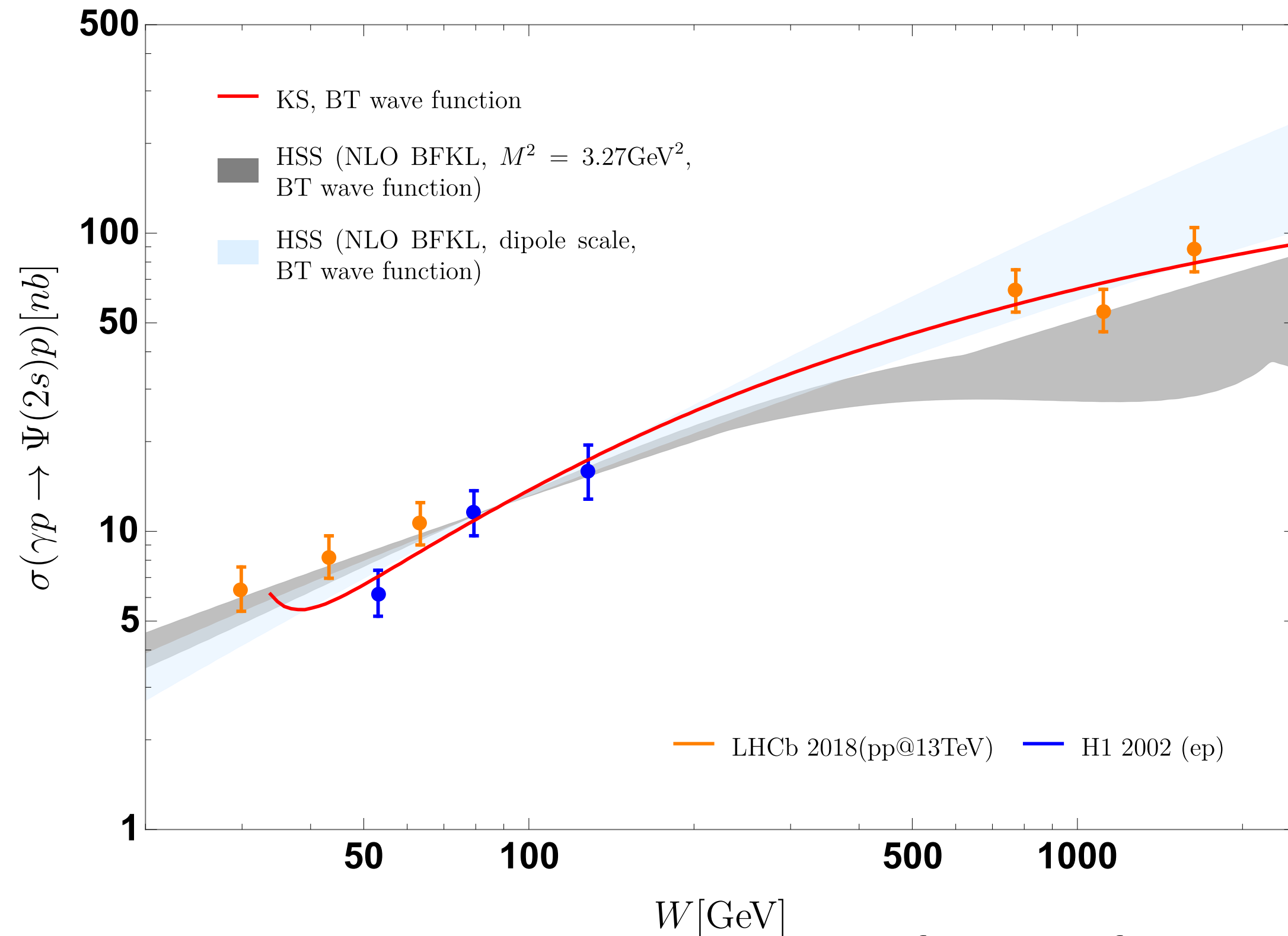


# J/ψ

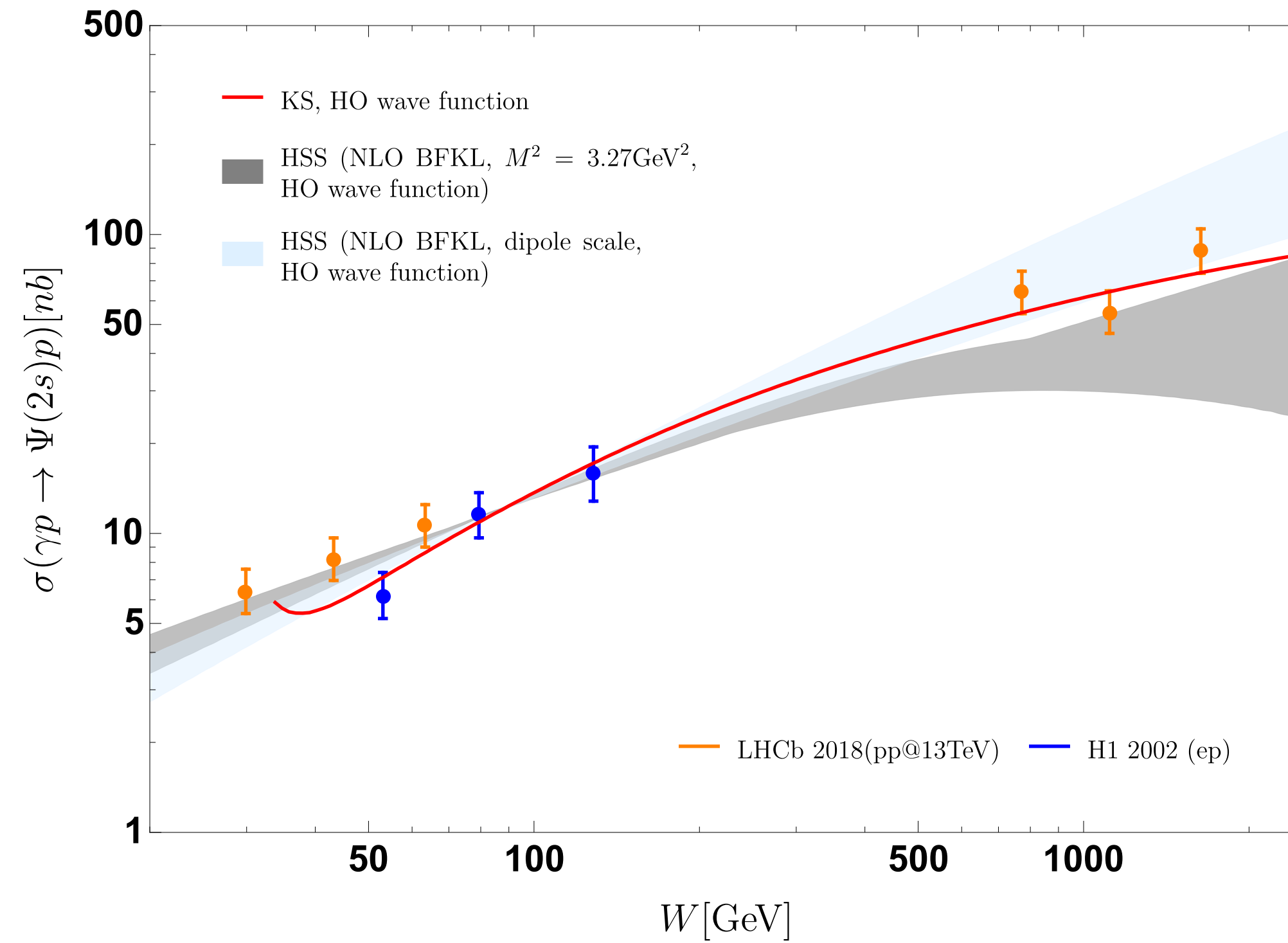
- Fix normalization with low energy data point (HERA); offset in normalization also seen in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)]
- Uncertainty band = variation of renormalization scale  $\bar{M} \in [M/\sqrt{2}, M\sqrt{2}]$
- Difference between linear & non-linear persists, but scale uncertainty too large to distinguish them clearly

# $\Psi(2s)$

## Buchmüller-Tye potential



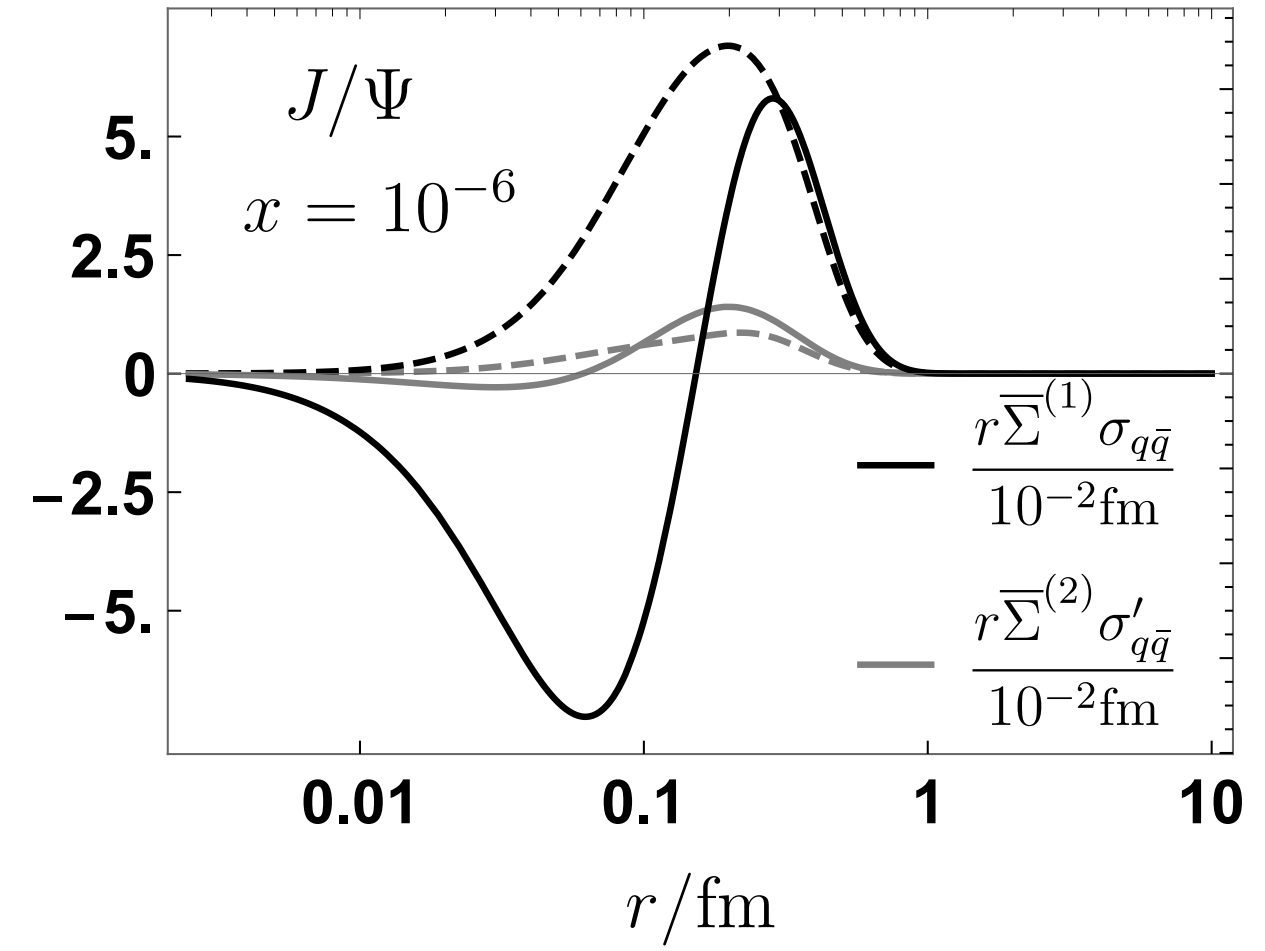
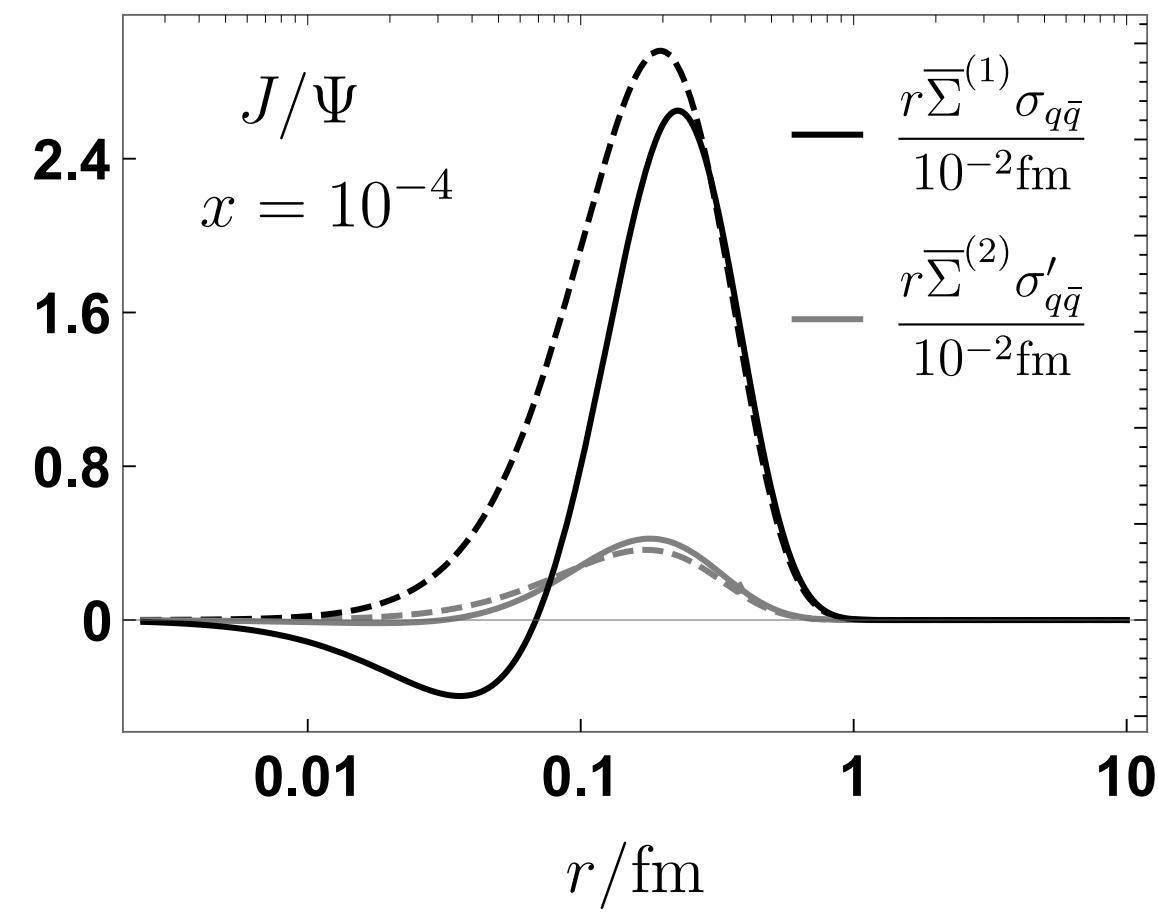
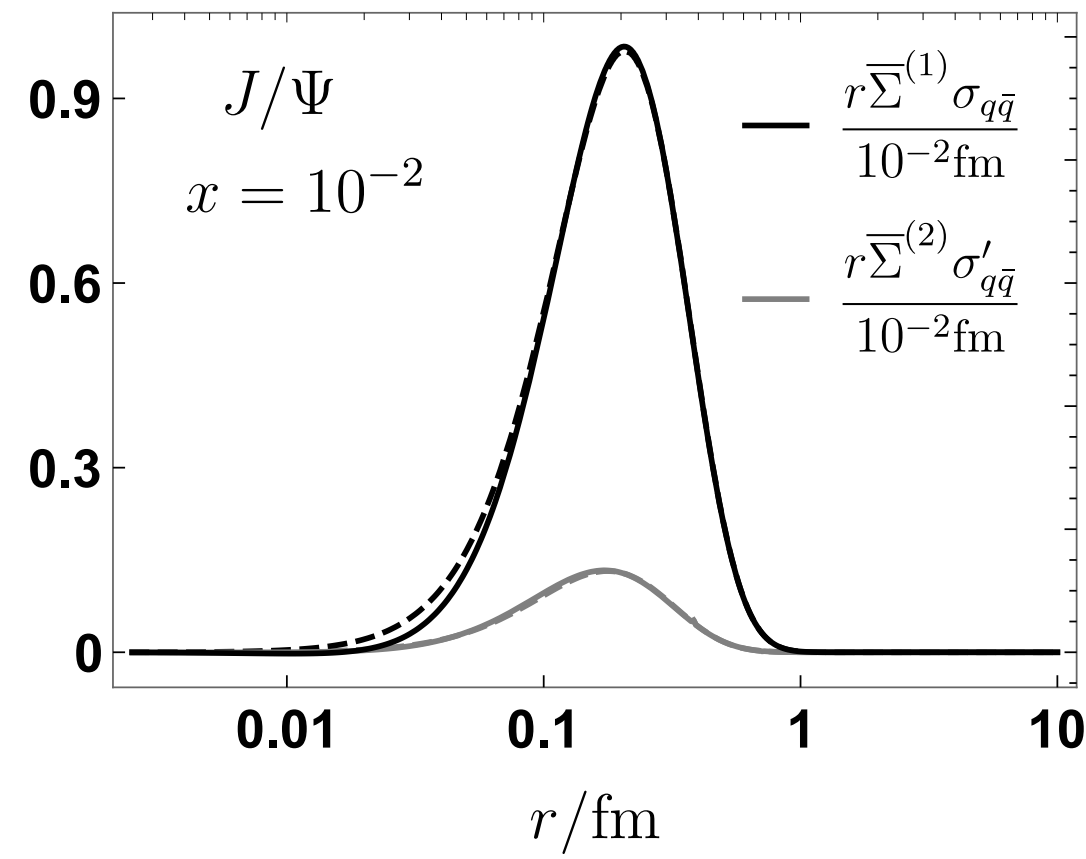
## Harmonic Oscillator potential



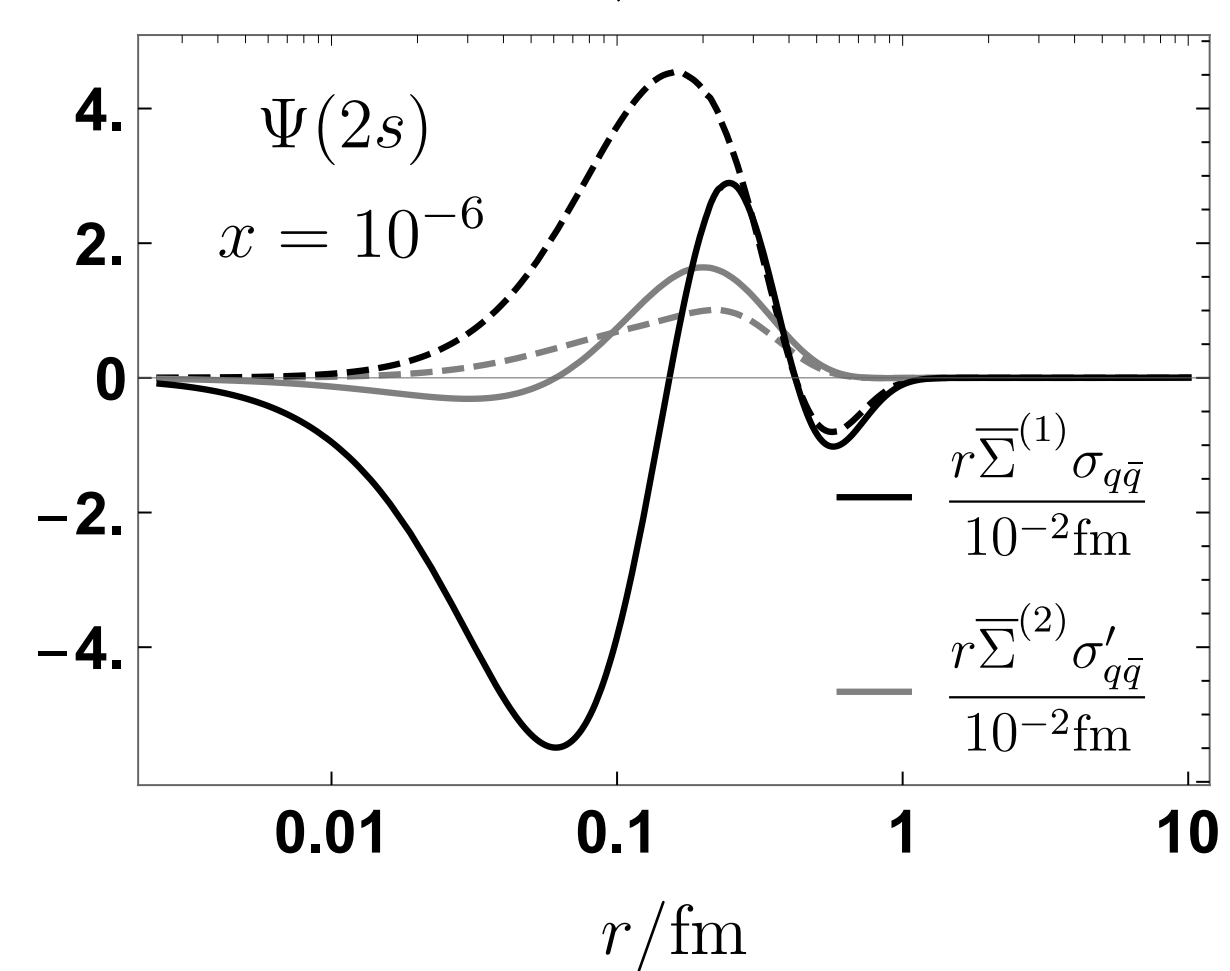
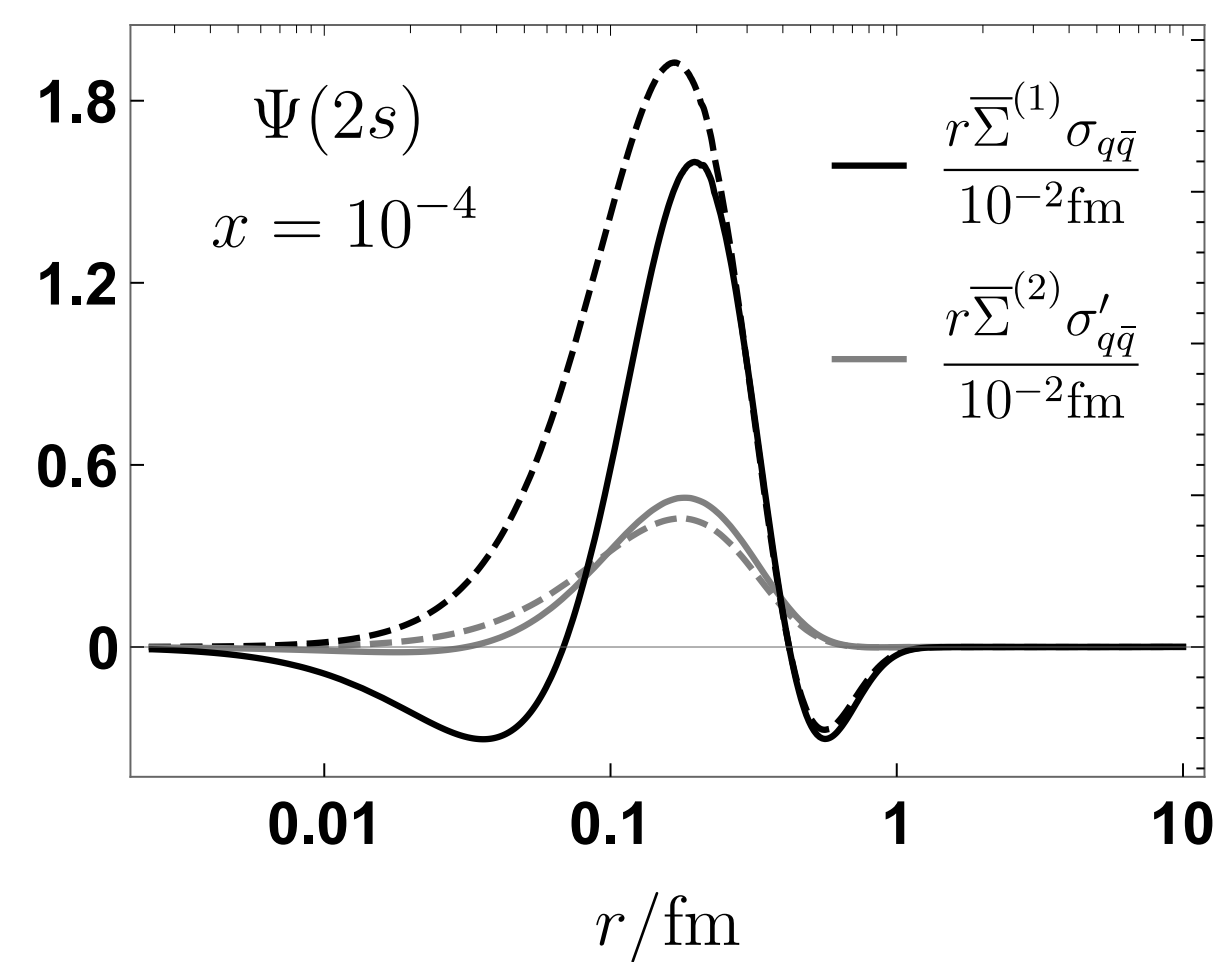
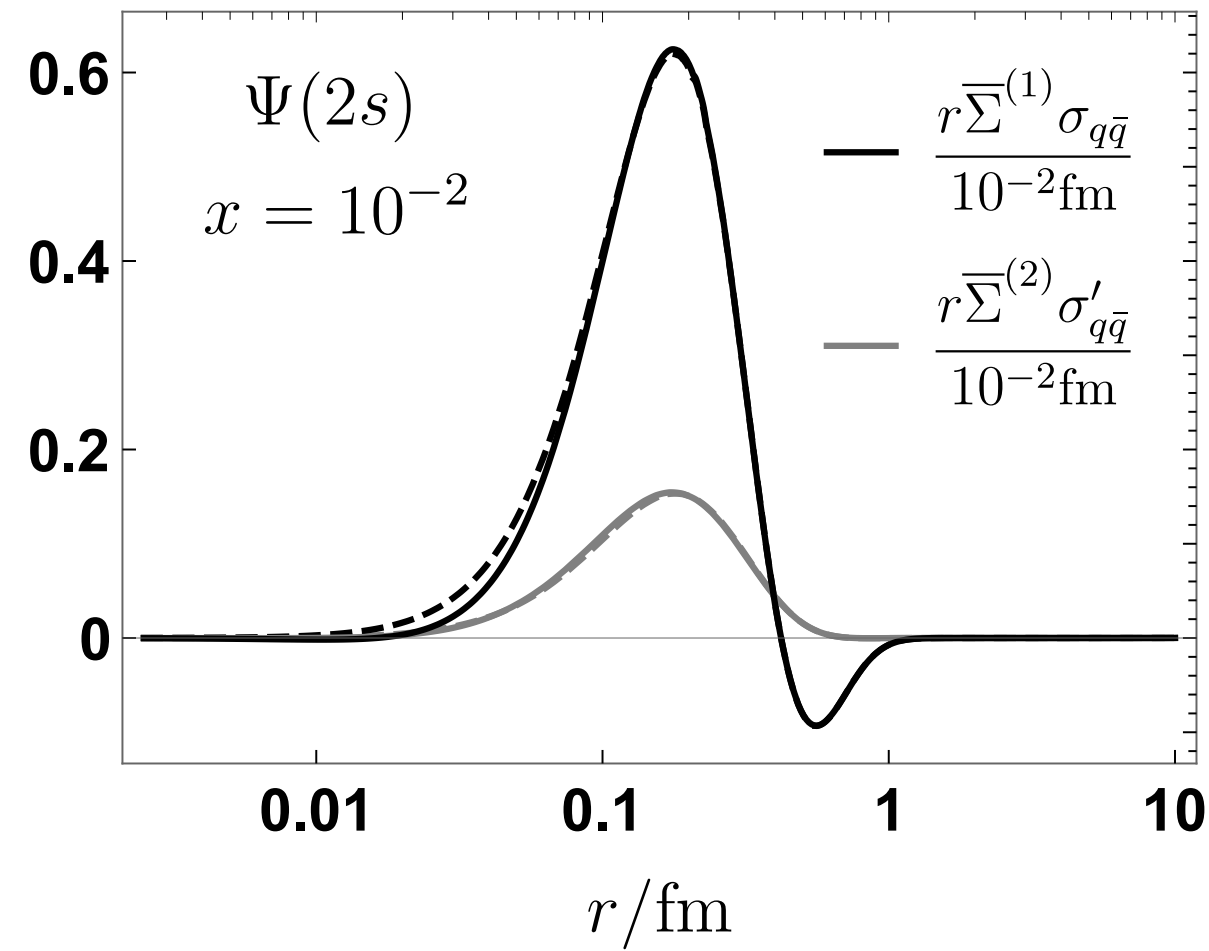
- Relative large uncertainties for the fixed scale HSS (NLO BFKL) gluon  $\rightarrow$  BFKL dipole + node structure;

- stabilized BFKL and non-linear evolution appear closer than for  $J/\Psi$

[M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)] steeper (perturbative) energy dependence for  $\Psi(2s)$   $\rightarrow$  attributed to reduced cancellation below and above  $\Psi(2s)$  node at higher energies



$J/\Psi$



$\Psi(2s)$

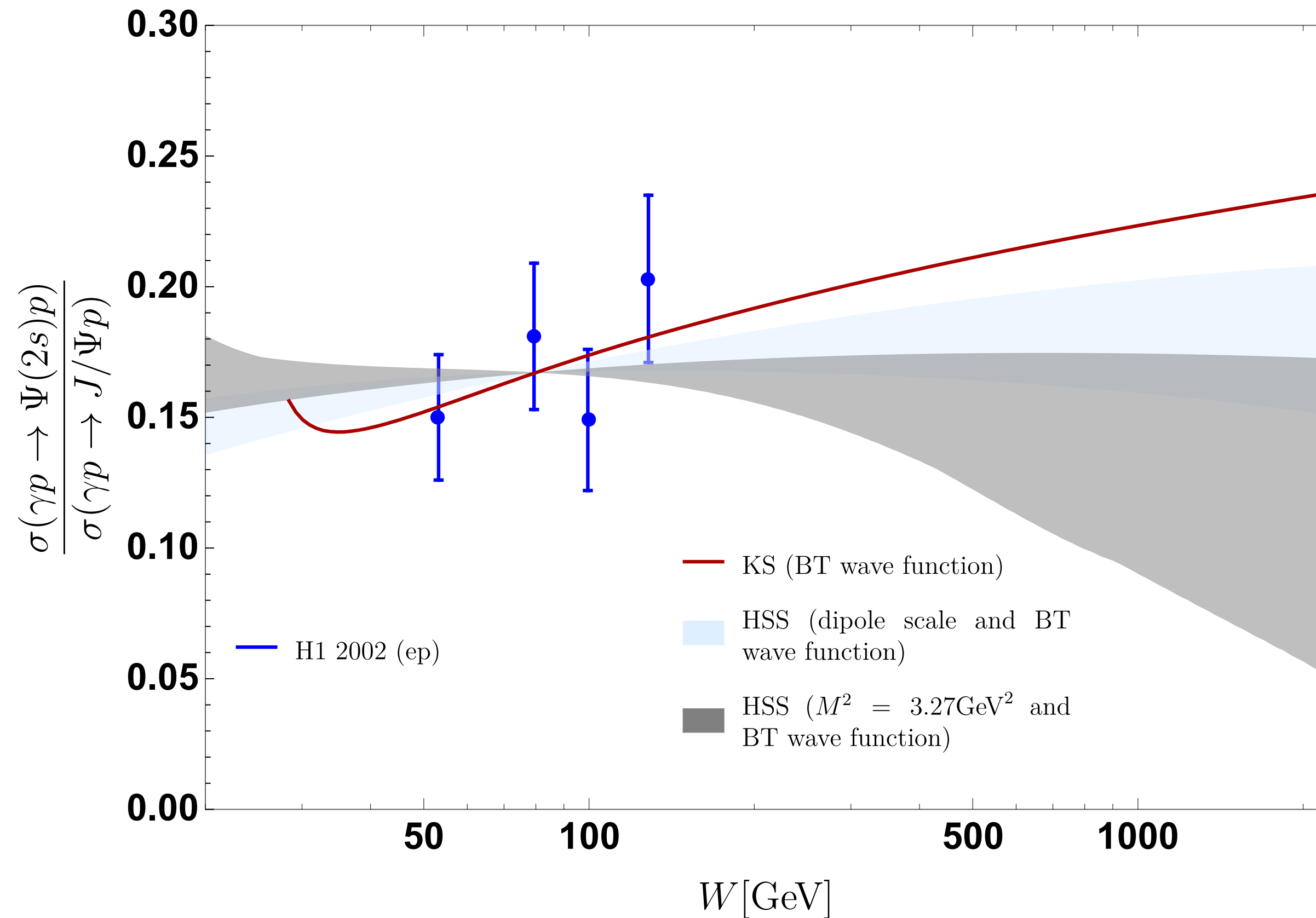
— HSS ( $M^2 = 3.27\text{GeV}^2$ )  
 - - - HSS ( $M^2 = 4/r^2 + \mu_0^2$ )

- in both cases the fixed scale dipole develops a negative region with decreasing  $x$
- $\Psi(2s)$ : node structure of wave function does not allow to compensate for it  $\rightarrow$  instability

advantage:  $\Psi(2s)$  challenges our dipole cross-sections



# More interesting: the ratio $\sigma[\Psi(2s)]/\sigma[J/\Psi]$



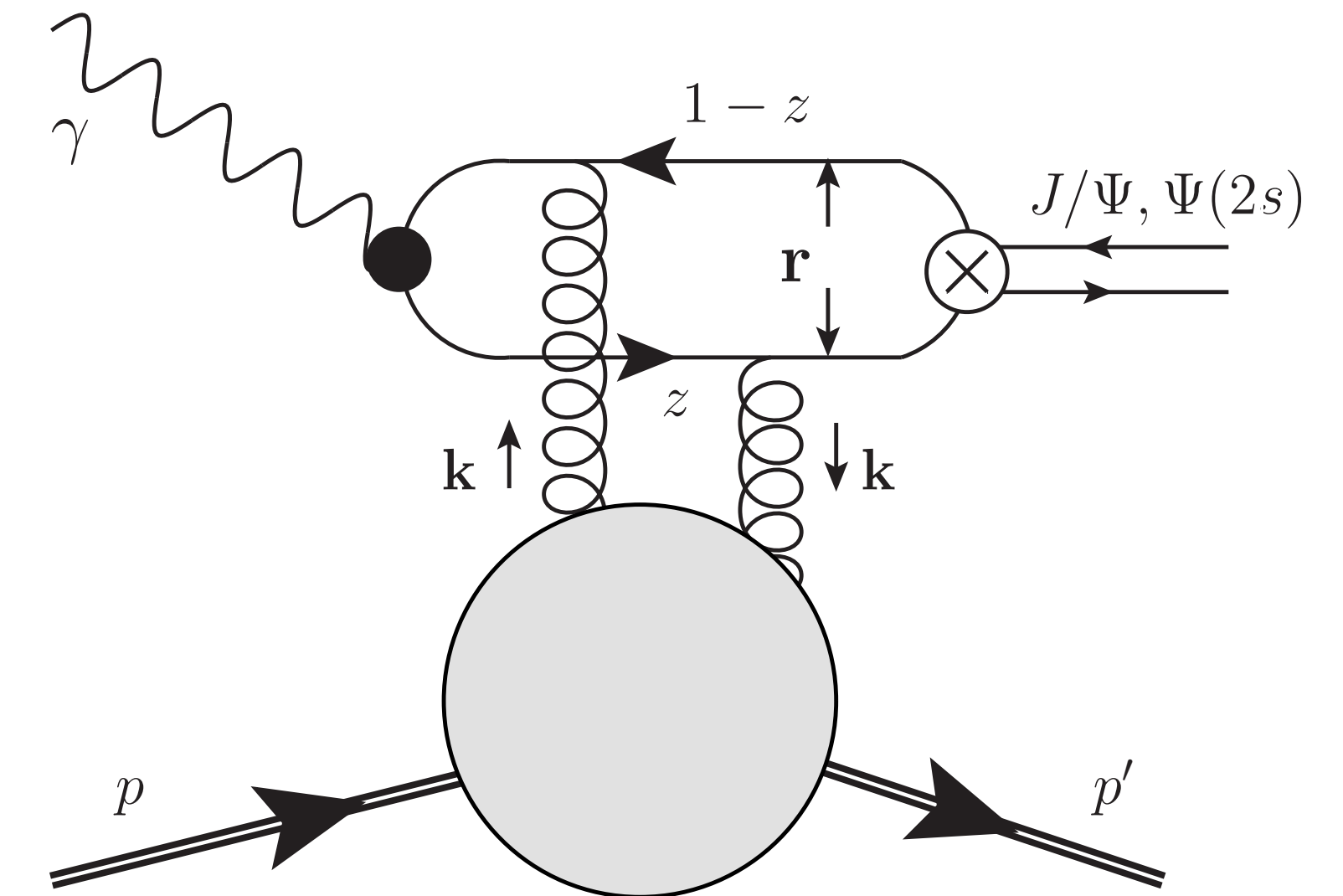
problem: no data at high energies

( $J/\Psi$  and  $\Psi(2s)$  LHCb data in different  $W$ -bins)

- rise of non-linear gluon also observed in [M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; [1812.03001](#); [1901.02664](#)] → KST dipole X-section [Kopeliovich, Schäfer, Tarasov, [hep-ph/9908245](#)]
- here: confirmed for KS (BK) gluon
- rise is not present for HSS (NLO BFKL) gluon (stabilized version)
- both slope & curvature differ
- general feature of perturbative QCD evolution?
- Hope: a tool to distinguish linear vs. non-linear evolution?

# Conclusions:

- $J/\Psi$ : theory uncertainty bands due not allow to clearly distinguish between linear (stabilized) and non-linear evolution  $\rightarrow$  reduction of uncertainty bands is needed
- $\Psi(2s)$ : fixed scale HSS gluon suffers instability; stabilized HSS and KS gluon too close to distinguish them
- ratio: find different energy dependence for BFKL and BK gluon  
[\[M. Krelina, J. Nemchik, R. Pasechnik, J. Cepila; 1901.02664\]](#) see decreasing ratio for  $\Upsilon$  at the level of dipole models



- despite of all of its challenges: VM production remains a useful observable to quantify presence of non-linear effects in low  $x$  evolution equations
- probes different aspects (& suffers different uncertainties) than e.g. angular de-correlation dihadron or dijet  $\rightarrow$  complementary observables

# Appendix

the transition photon  $\rightarrow$  quark-antiquark dipole  $\rightarrow$  vector meson: Gaussian model

$$W(r) = 2\pi r \int_0^1 \frac{dz}{4\pi} (\Psi_V^* \Psi)_T(r, z)$$

$$(\Psi_V^* \Psi)_T(r, z) = \frac{\hat{e}_f e N_c}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r) \phi_T(r, z) - [z^2 + (1-z)^2] \epsilon K_1(\epsilon r) \partial_r \phi_T(r, z) \right\}$$

boosted Gaussian scalar wave function using Brodsky-Huang-Lepage prescription

$$\phi_{T,L}^{1s}(r, z) = \mathcal{N}_{T,L} z(1-z) \exp \left( -\frac{m_f^2 \mathcal{R}_{1s}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}_{1s}^2} + \frac{m_f^2 \mathcal{R}_{1s}^2}{2} \right)$$

parameters from normalization and decay width [\[Armesto, Rezaeian; 1402.4831\]](#) ( $J/\Psi$ ) and [\[Goncalves, Moreira, Navarra; 1408.1344\]](#) ( $\Upsilon$ )

Meson	$m_f/\text{GeV}$	$\mathcal{N}_T$	$\mathcal{R}^2/\text{GeV}^{-2}$	$M_V/\text{GeV}$
$J/\psi$	$m_c = 1.4$	0.596	2.45	3.097
$\Upsilon$	$m_b = 4.2$	0.481	0.57	9.460