Photoproduction of hidden-charm pentaquarks

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Outline

Introduction

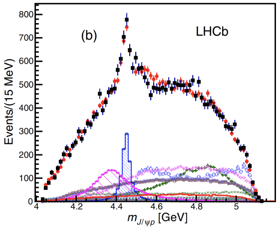
Pentaquarks $qqqQ\bar{Q}$
- Quark Model
- Classification of states

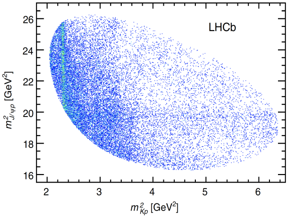
Photoproduction of hidden-charm pentaquarks
- Harmonic Oscillator
- Hyper-Coulomb

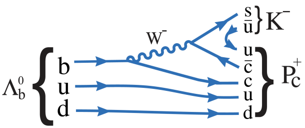
Conclusions

LHCb

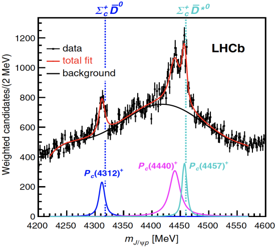
Pentaquarks $P_c(4380)^+$ and $P_c(4450)^+$, with $J^P = 3/2^{\mp}, 5/2^{\pm}$. [1]

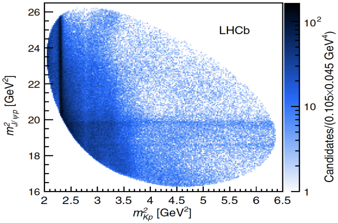


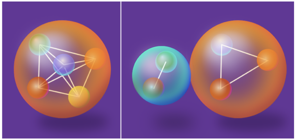




Pentaquarks $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$. [2]







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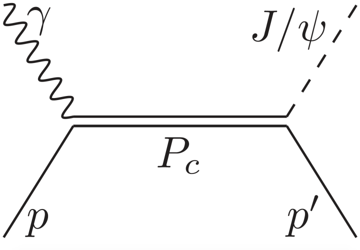
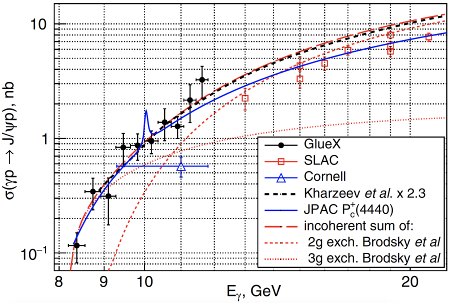
[1] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **115**, 072001 (2015).

[2] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **122**, 222001 (2019).

Jefferson Lab

One of the proposals is to confirm the nature of the pentaquark, through photoproduction.





In 2019, The GlueX Collaboration published their first results, in which it was reported that **they do not see evidence for a pentaquark structure** by J/ψ photoproduction [3].

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[3] A. Ali *et al.* (GlueX Collaboration), Phys. Rev. Lett. **123**, 072001 (2019).

Pentaquarks $qqqQ\bar{Q}$

Pentaquark wave functions

- Quark Model

Hadrons → Multiquark systems→ Degrees of freedom

$$\psi = \psi^{\mathrm{o}} \chi^{\mathrm{s}} \phi^{\mathrm{f}} \psi^{\mathrm{c}}$$

(i) Since quarks are fermions, the total pentaquark wave function should be antisymmetric under any permutation of the three light quarks.

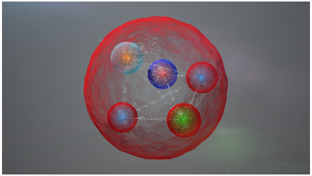
(ii) As all physical states, the pentaquark wave function should be a color singlet.

Harmonic Oscillator

$$H = \frac{{p_1}^2}{2m} + \frac{{p_2}^2}{2m} + \frac{{p_3}^2}{2m} + \frac{{p_4}^2}{2m'} + \frac{{p_5}^2}{2m'} + \frac{1}{2}C\sum_{i<j}^{5} |\vec{r_i} - \vec{r_j}|^2$$

- Jacobi coordinates

$$\begin{cases} \vec{\rho} \equiv \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \\ \vec{\lambda} \equiv \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \\ \vec{\eta} \equiv \frac{1}{\sqrt{2}}(\vec{r_4} - \vec{r_5}) \\ \vec{\zeta} \equiv \sqrt{\frac{6}{5}}\left[\frac{1}{3}(\vec{r_1} + \vec{r_2} + \vec{r_3}) - \frac{1}{2}(\vec{r_4} + \vec{r_5})\right] \\ \vec{R} \equiv \frac{m(\vec{r_1}+\vec{r_2}+\vec{r_3})+m'(\vec{r_4}+\vec{r_5})}{3m+2m'} \end{cases}$$



$$H = \frac{P^2}{2M} + \frac{{p_\rho}^2}{2m_\rho} + \frac{{p_\lambda}^2}{2m_\lambda} + \frac{{p_\eta}^2}{2m_\eta} + \frac{{p_\zeta}^2}{2m_\zeta} + \frac{5}{2}C\rho^2 + \frac{5}{2}C\lambda^2 + \frac{5}{2}C\eta^2 + \frac{5}{2}C\zeta^2$$

$$M = 3m + 2m', \qquad m_\rho = m_\lambda \equiv m, \qquad m_\eta \equiv m', \qquad m_\zeta \equiv \frac{5mm'}{3m+2m'},$$

$$\alpha_i^2 = (5Cm_i)^{\frac{1}{2}}, \qquad \omega_i = \sqrt{\frac{5C}{m_i}}, \qquad i = \{\rho, \lambda, \eta, \zeta\}.$$

- The orbital wave function of the pentaquark and the proton

$$\psi_{Pc}(\vec{p_1}, \vec{p_2}, \vec{p_3}, \vec{p_4}, \vec{p_5}) = \delta^3(\vec{P} - \vec{K_{Pc}})\sqrt{5\sqrt{5}}\frac{1}{\pi^{\frac{3}{4}}\alpha_\rho^{\frac{3}{2}}}\frac{1}{\pi^{\frac{3}{4}}\alpha_\lambda^{\frac{3}{2}}}\frac{1}{\pi^{\frac{3}{4}}\alpha_\eta^{\frac{3}{2}}}\frac{1}{\pi^{\frac{3}{4}}\alpha_\zeta^{\frac{3}{2}}} e^{-\frac{1}{2\alpha_\rho^2}p_\rho^2} e^{-\frac{1}{2\alpha_\lambda^2}p_\lambda^2} e^{-\frac{1}{2\alpha_\eta^2}p_\eta^2} e^{-\frac{1}{2\alpha_\zeta^2}p_\zeta^2}$$

$$\psi_p(\vec{p_1}, \vec{p_2}, \vec{p_3}) = \delta^3(\frac{3m}{M}\vec{P} + \sqrt{\frac{6}{5}}\vec{p_\zeta} - \vec{K_p})\sqrt{3\sqrt{3}}\left(\frac{1}{\pi^{\frac{3}{4}}\beta^{\frac{3}{2}}}\right)^2 e^{-\frac{1}{2\beta^2}(p_\rho^2+p_\lambda^2)}$$

The wave functions

- Ground state pentaquarks[4] with $J^P = 3/2^-$ and $L^\pi = 0^+$

$$\psi = \left[\psi^{\rm o}_{A_1} \times \psi^{\rm csf}_{A_2}\right]_{A_2} \quad \Rightarrow \quad \left\{ \begin{array}{l} \psi^{\rm csf}_{A_2} = \left[\psi^{\rm c}_{A_2} \times \psi^{\rm sf}_{A_1}\right]_{A_2} \\ \psi^{\rm csf}_{A_2} = \left[\psi^{\rm c}_{E} \times \psi^{\rm sf}_{E}\right]_{A_2} \end{array} \right.$$

5 configurations

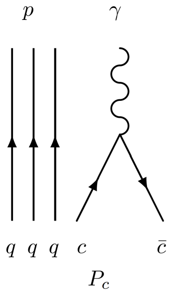
- Pentaquarks with one quantum of orbital excitation $J^P = 3/2^+$ y $L^\pi = 1^-$

$$\begin{array}{l} \psi = \left[\psi^{\rm o}_{A_1} \times \psi^{\rm csf}_{A_2}\right]_{A_2} \\ \psi = \left[\psi^{\rm o}_{E} \times \psi^{\rm csf}_{E}\right]_{A_2} \end{array} \quad \Rightarrow \quad \left\{ \begin{array}{l} \psi^{\rm csf}_{A_2} = \left[\psi^{\rm c}_{A_2} \times \psi^{\rm sf}_{A_1}\right]_{A_2} \\ \psi^{\rm csf}_{A_2} = \left[\psi^{\rm c}_{E} \times \psi^{\rm sf}_{E}\right]_{A_2} \\ \psi^{\rm csf}_{E} = \left[\psi^{\rm c}_{A_2} \times \psi^{\rm sf}_{E}\right]_{E} \\ \psi^{\rm csf}_{E} = \left[\psi^{\rm c}_{E} \times \psi^{\rm sf}_{A_2}\right]_{E} \\ \psi^{\rm csf}_{E} = \left[\psi^{\rm c}_{E} \times \psi^{\rm sf}_{A_1}\right]_{E} \\ \psi^{\rm csf}_{E} = \left[\psi^{\rm c}_{E} \times \psi^{\rm sf}_{E}\right]_{E} . \end{array} \right.$$

19 configurations

[4] E. Ortiz-Pacheco, R. Bijker, and C. Fernández-Ramírez, J. Phys. G: Nucl. Part. Phys. **46**, 065104 (2019).

Photoproduction of pentaquarks



- Interaction Hamiltonian [5]

$$H = e \int d^3x \, \underbrace{\bar{q}(\vec{x}) e_q \gamma^\mu q(\vec{x})}_{J^\mu \text{ quark current}} A_\mu(\vec{x})$$

$$A_\mu(\vec{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \frac{1}{\sqrt{2k^0}} \left[a_\mu(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + a_\mu^\dagger(\vec{k}) e^{-i\vec{k}\cdot\vec{x}} \right]$$

$$q(\vec{x}) = \int \frac{d^3p}{(2\pi)^{\frac{3}{2}}} \left(\frac{m}{p^0}\right)^{\frac{1}{2}} \sum_s \left[u_s(\vec{p}) b_s(\vec{p}) e^{i\vec{p}\cdot\vec{x}} + v_s(\vec{p}) d_s^\dagger(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} \right]$$

$$J_3^\mu(\vec{x}) = \int \frac{d^3p}{(2\pi)^{\frac{3}{2}}} \left(\frac{m}{p^0}\right)^{\frac{1}{2}} \int \frac{d^3p'}{(2\pi)^{\frac{3}{2}}} \left(\frac{m}{p'^0}\right)^{\frac{1}{2}} \sum_{qss'} \bar{v}_{s'}(\vec{p'}) e_q \gamma^\mu u_s(\vec{p}) d_{s'}(\vec{p'}) b_s(\vec{p}) e^{i(\vec{p}+\vec{p'})\cdot\vec{x}}$$

[5] A. Le Yaouanc *et al.* *Hadron Transitions in the Quark Model* (Gordon and Breach, NY, USA, 1988).

Fotoproduction of pentaquarks

- Non relativistic approximation

$$\bar{v}_{s'}(\vec{p'})\gamma^0 u_s(\vec{p}) \rightarrow \chi^\dagger_{s'} \frac{\vec{\sigma}\cdot(\vec{p}+\vec{p'})}{2m}\chi_s$$
$$\bar{v}_{s'}(\vec{p'})\gamma^k u_s(\vec{p}) \rightarrow \chi^\dagger_{s'}\sigma^k\chi_s$$

- Decay width $\Gamma(P_c \rightarrow p+\gamma) = 2\pi\rho\,\frac{2}{2J+1}\sum_{\nu>0}|A_\nu(k)|^2$

  For a frame with the pentaquark at rest:

$$\rho = 4\pi\frac{E_p k^2}{m_{P_c}}, \qquad E_p = \sqrt{m_p^2+k^2}.$$

The four-momentum $(k_0,\vec{k})$ of the photon is $Q^2 = Q^\mu Q_\mu = k_0^2 - k^2$

$$k^2 = \left(\frac{Q^2 - m_{P_c}^2 - m_p^2}{2m_{P_c}}\right)^2 - m_p^2 \quad \xrightarrow{Q^2=0} \quad k = \frac{m_{P_c}^2 - m_p^2}{2m_{P_c}}$$

- The helicity amplitude and the form factor

$$A_\nu(k) = \langle\gamma p|H_3|P_c\rangle = \frac{e}{(2\pi)^{\frac{3}{2}}2\sqrt{k_0}}\langle\psi_p^{csf}1/2,\nu-1|e_c\sigma_-|\psi_{P_c}^{csf}3/2,\nu\rangle\underbrace{F(k)}_{\text{Orbital contribution}}$$

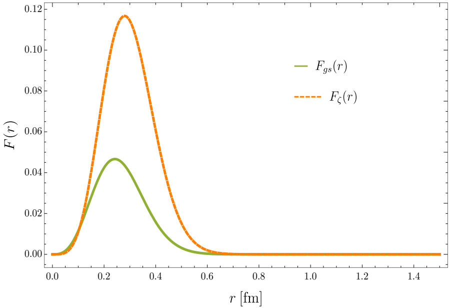
Form factor $F(k)$ in the Harmonic Oscillator

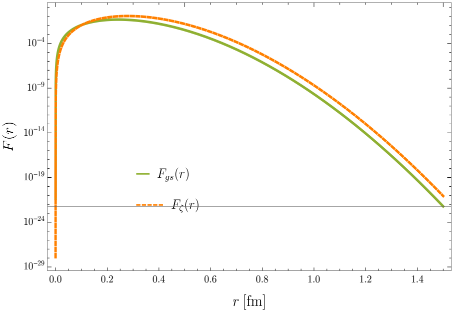
$$F_{gs}(k) = \left(\frac{\sqrt{5(2a+3)}}{3}\right)^{\frac{3}{4}} \left(\frac{2\alpha_\rho\beta}{\alpha_\rho^2+\beta^2}\right)^3 e^{-\frac{5}{12\alpha_\rho^2}\sqrt{\frac{2a+3}{5a}}k^2}, \qquad a \equiv \frac{m'}{m}, \qquad k = 2.12\ GeV.$$

Parameters

$m = m_u = m_d$	320	MeV
$m' = m_c$	1500	MeV
$\beta = 1/r_p$	1.19	fm

Pentaquark charge radius: $$r^2 = \frac{1}{\alpha_\rho^2}\left(1 + \frac{1}{5}\left(\frac{5a}{3+2a}\right)^{\frac{3}{2}}\right)$$





Hypercentral Model

$$T = \frac{1}{2m}({p_1}^2 + {p_2}^2 + {p_3}^2) + \frac{1}{2m'}({p_4}^2 + {p_5}^2)$$

- Hamiltonian with the hyper-Coulomb potential

$$H = \frac{P^2}{2M_5} + \frac{1}{\mu}({p_\rho}^2 + {p_\lambda}^2 + {p_\eta}^2 + {p_\zeta}^2) - \frac{\tau}{\sqrt{\rho^2+\lambda^2+\eta^2+\zeta^2}}$$

$$\begin{cases} \vec{\rho} = \vec{r_2} - \vec{r_1} \\ \vec{\lambda} = \frac{1}{\sqrt{N_\lambda}}\left(\vec{r_3} - \frac{\vec{r_1}+\vec{r_2}}{2}\right) \\ \vec{\eta} = \frac{1}{\sqrt{N_\eta}}\left(\vec{r_4} - \frac{\vec{r_1}+\vec{r_2}+\vec{r_3}}{3}\right) \\ \vec{\zeta} = \frac{1}{\sqrt{N_\zeta}}\left(\vec{r_5} - \frac{m(\vec{r_1}+\vec{r_2}+\vec{r_3})+m'\vec{r_4}}{3m+m'}\right) \\ \vec{R} = \frac{m(\vec{r_1}+\vec{r_2}+\vec{r_3})+m'(\vec{r_4}+\vec{r_5})}{3m+2m'} \end{cases} \Rightarrow \begin{cases} x = \sqrt{\rho^2+\lambda^2+\eta^2+\zeta^2} \\ \xi_1 = \arctan\left(\frac{\rho}{\lambda}\right),\ \xi_2 = \arctan\left(\frac{\eta}{\zeta}\right) \\ \xi = \arctan\left(\sqrt{\frac{\rho^2+\lambda^2}{\eta^2+\zeta^2}}\right), \\ \Omega_\rho = (\theta_\rho, \phi_\rho),\ \Omega_\lambda = (\theta_\lambda, \phi_\lambda), \\ \Omega_\eta = (\theta_\eta, \phi_\eta),\ \Omega_\zeta = (\theta_\zeta, \phi_\zeta). \end{cases}$$

Hyper-radial: $\left[-\frac{1}{2\mu}\left(\frac{\partial^2}{\partial x^2} + \frac{11}{x}\frac{\partial}{\partial x} - \frac{\gamma(\gamma+10)}{x^2}\right) - \frac{\tau}{x}\right] R_{\omega\gamma}(x) = E R_{\omega\gamma}(x)$

Hyper-angular: $\left[-\frac{1}{(\cos\xi)^5(\sin\xi)^5}\frac{\partial}{\partial\xi}\left((\cos\xi)^5(\sin\xi)^5\frac{\partial}{\partial\xi}\right) + \frac{\Lambda_2^2(\Omega_2)}{(\cos\xi)^2} + \frac{\Lambda_1^2(\Omega_1)}{(\sin\xi)^2}\right] Y_{[\gamma]}(\Omega) = \gamma(\gamma+10) Y_{[\gamma]}(\Omega).$

$$\psi_{P_c}(\vec{r_1}, \vec{r_2}, \vec{r_3}, \vec{r_4}, \vec{r_5}) = \left[\frac{(2g_0)^{12}}{11!}\right]^{\frac{1}{2}} \frac{2\sqrt{15}}{\pi^3} e^{-g_0 x} \left(\frac{M_2^4 m_3 m_4 m_5}{m_1^3 m_2^3 M_5}\right)^{\frac{3}{4}} \frac{1}{(2\pi)^{\frac{3}{2}}} e^{-i\vec{K}_{P_c}\cdot\vec{R}}.$$

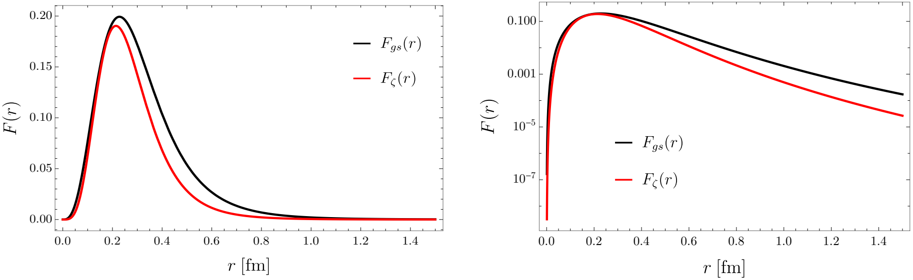
Form factor $F(k)$ in the Hypercentral Model

$$F_{gs}(k) = \left(\frac{2a+3}{3a^2}\right)^{\frac{3}{4}} \left(\frac{a(a+3)}{2a+3}\right)^{\frac{3}{2}} (4\pi)^3 \left[\frac{(2g_3)^6(2g_0)^{12}}{5!11!}\right]^{\frac{1}{2}} \frac{1}{\pi^{\frac{3}{2}}} \frac{2\sqrt{15}}{\pi^3}$$

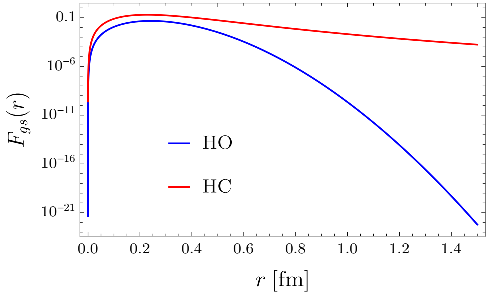
$$\int d\rho d\lambda d\eta \rho^2\lambda^2\eta^2 j_0\left(-\frac{1}{\sqrt{2}}\sqrt{\frac{a+3}{3a}}k\eta\right) e^{-g_3\sqrt{\rho^2+\lambda^2}} e^{-g_0\sqrt{\rho^2+\lambda^2+\left(\frac{2(3+a)}{3+2a}\right)\eta^2}}$$

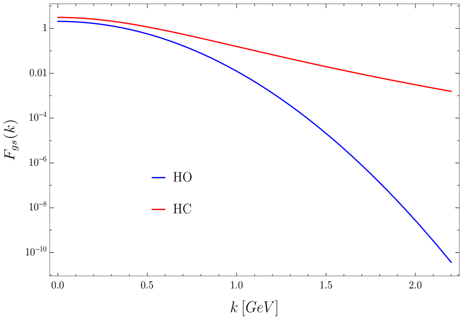
with $a \equiv \frac{m'}{m}$, y $k = 2.12\ GeV$. Parameters: $g_3 = \frac{\tau_3\mu}{\sqrt{2}\frac{5}{2}}$ y $g_0 = \frac{\mu\tau}{\sqrt{2}\frac{11}{2}}$.

Pentaquark charge radius: $r^2 = \frac{13}{4}\left(\frac{\frac{11}{2}\sqrt{2}}{\tau\mu}\right)^2 \frac{3(1+a)}{3+2a} e.$



Comparison of both models





Conclusions

Pentaquarks

- We found that from a large number of pentaquark states, 5 for ground states and 19 for radially excited states, at the end of process there are only 2 contributions; 1 ground state, and 1 radially excited state.

- By calculating the orbital part of the photoproduction of pentaquark, either for $F(k)$ or $F(r)$ and in the HO and HC models, we find that the photoproduction channel $p + \gamma \to P_c(uudc\bar{c})$ is highly suppressed, for both the ground and the excited states.

- For this reason we do not expect it to be feasible to experimentally reproduce the pentaquark state through the photoproduction process.

- The fact that, to date, the pentaquark has not been observed experimentally does not exclude that the signals observed by LHCb belong to compact pentaquark states.

Thanks!