



Dalitz Plot analysis of $D \rightarrow hhh$ @LHCb

Patricia C. Magalhães

Aeronautics Institute of Technology (BR)

p.magalhaes@cern.ch

1st June 2021

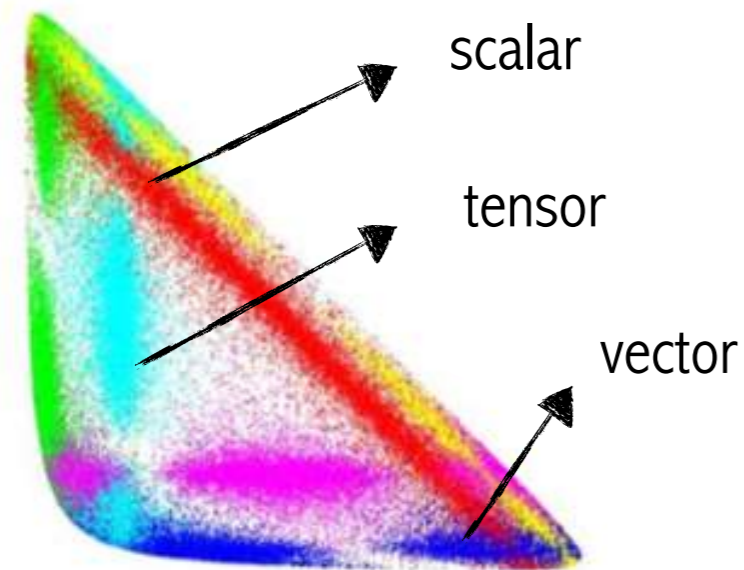
Charm 2020 - Mexico City



Banksy



- D three-body **HADRONIC** decay are dominated by resonances



→ spectroscopy **low energy resonances**

→ underlying strong force behave

↳ meson-meson interactions and resonance structures

• CP-Violation

- $B^\pm \rightarrow h^\pm h^- h^+$ massive localized direct CP asymmetry

- 1st observation in charm 
Angelo and Lorenzo talks

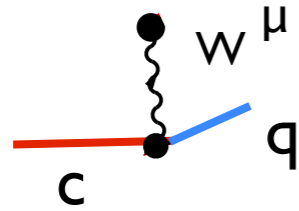
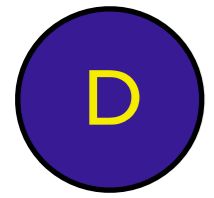
$$A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-)$$

→ CPV on $D \rightarrow hhh$?

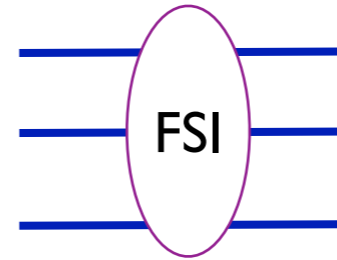
→ searches in many process

→ can lead to new physics

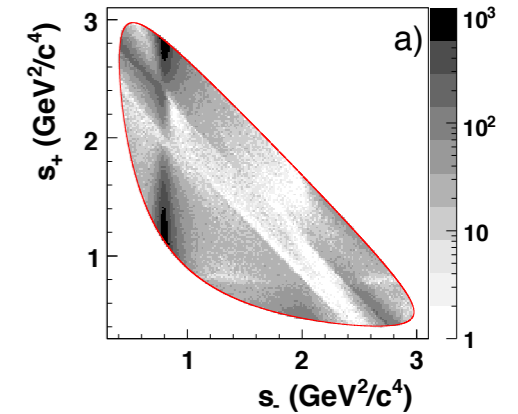
- full description of the underlying structures (amplitude analysis):
 - $D^+ \rightarrow K^+ K^- K^+$ (DCS) [JHEP 04 \(2019\) 063](#) **will discuss**
 - $D^0 \rightarrow K_s K^\pm \pi^\mp$ (SCS) [PRD 93 \(2016\) 052018](#)
 - $D^+ \rightarrow \pi^- \pi^+ \pi^+$ and $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ - on going
- $D^0 \rightarrow K_s h^+ h^-$ ($h = K, \pi$) good sensitivity to measure mixing
 - input for γ determination in $B \rightarrow DK$ [JHEP 08 \(2018\) 176](#)
 - measurement of the mass difference (D^0, \bar{D}^0) in $D^0 \rightarrow K_s \pi^+ \pi^-$ [PRL 122 \(2019\) 231802](#)
Angelo's talk
 - ↳ model dependent - on going
- LHCb data for each $D \rightarrow hhh$ decays include $\sim 10^6 - 10^7$ events
 - **extremely challenging** ➤ claim for precise theoretical models



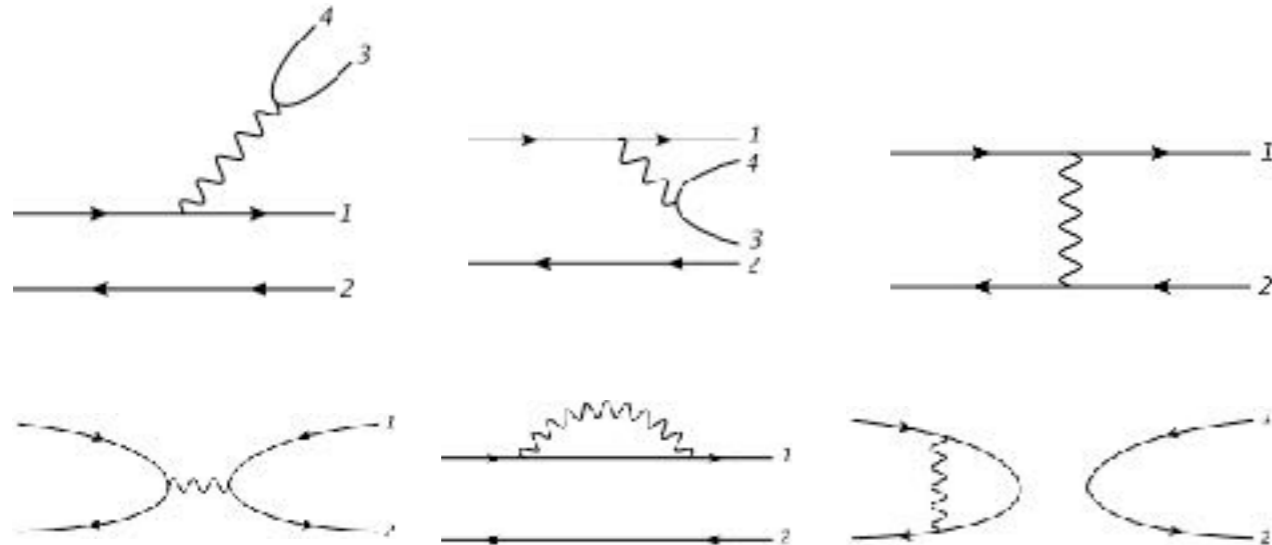
hadronize



observed

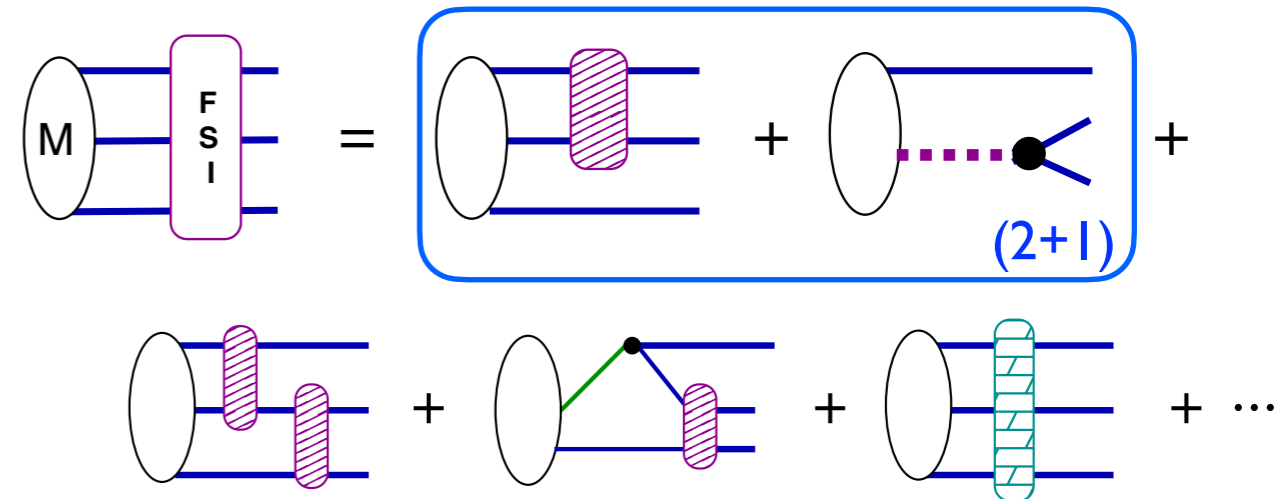


weak transition



QCD, CKM coupling and phase

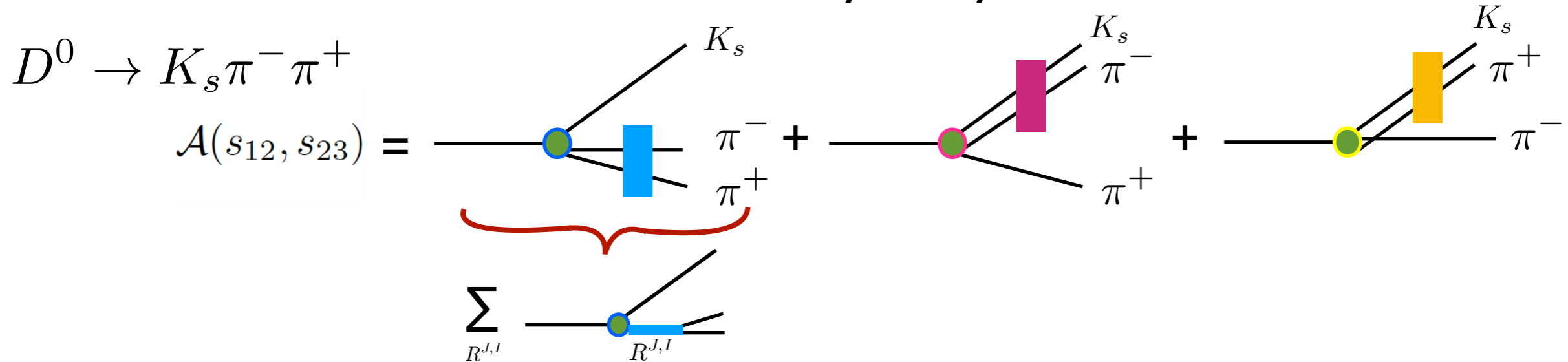
Final State Interactions - strong -



- To extract information from data we need an **amplitude MODEL**

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\mathcal{A}(s_{12}, s_{23})|^2$$

- common cartoon to described 3-body decay



- Standard isobar model widely used by experimentalists:

- $3 = (2+1) \rightarrow$ ignore the interaction with 3rd particle (bachelor)

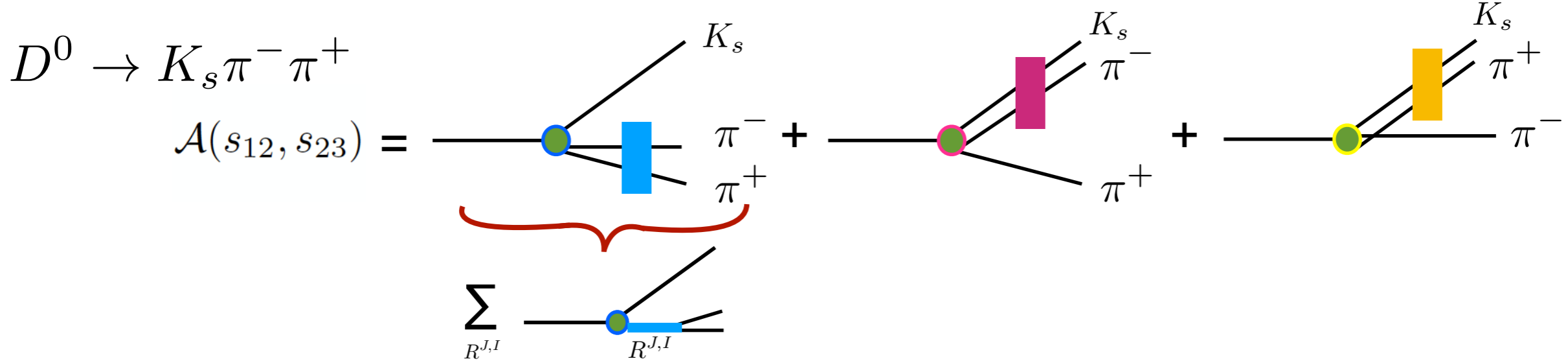
$$A = \sum c_k A_k; + \text{NR} \begin{cases} \text{non-resonant as constant or exponential!} \\ \text{each resonance as Breit-Wigner } \text{BW}(s_{12}) = \frac{1}{m_R^2 - s_{12} - im_R \Gamma(s_{12})}, \end{cases}$$

- always good to remember why we don't like this approach:

- sum of BW violates two-body unitarity (2 res in the same channel);
- do NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !



- common cartoon to described 3-body decay



- one expect to see all 3 channels res:

→ But in reality.....
not all of them are clearly present

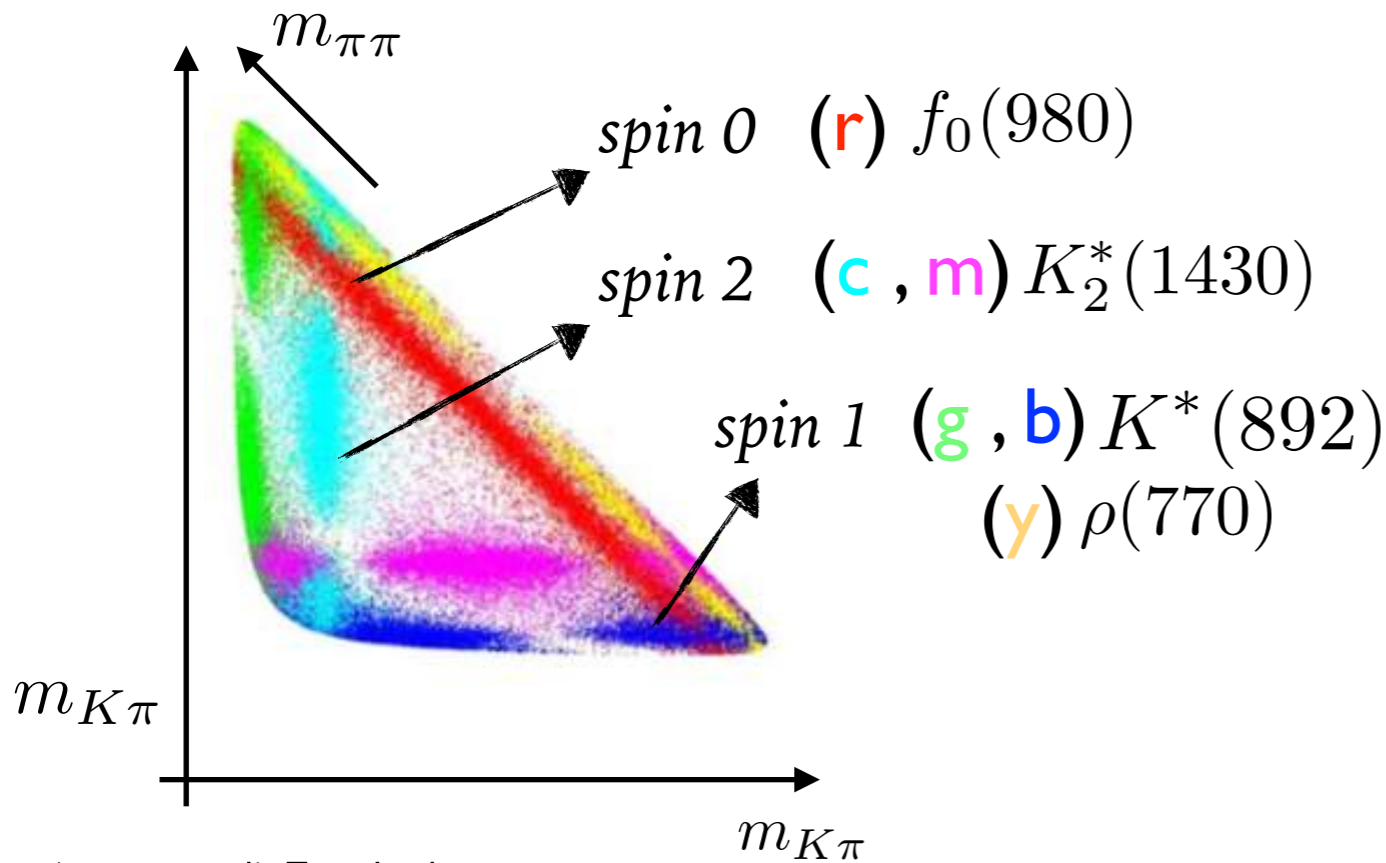
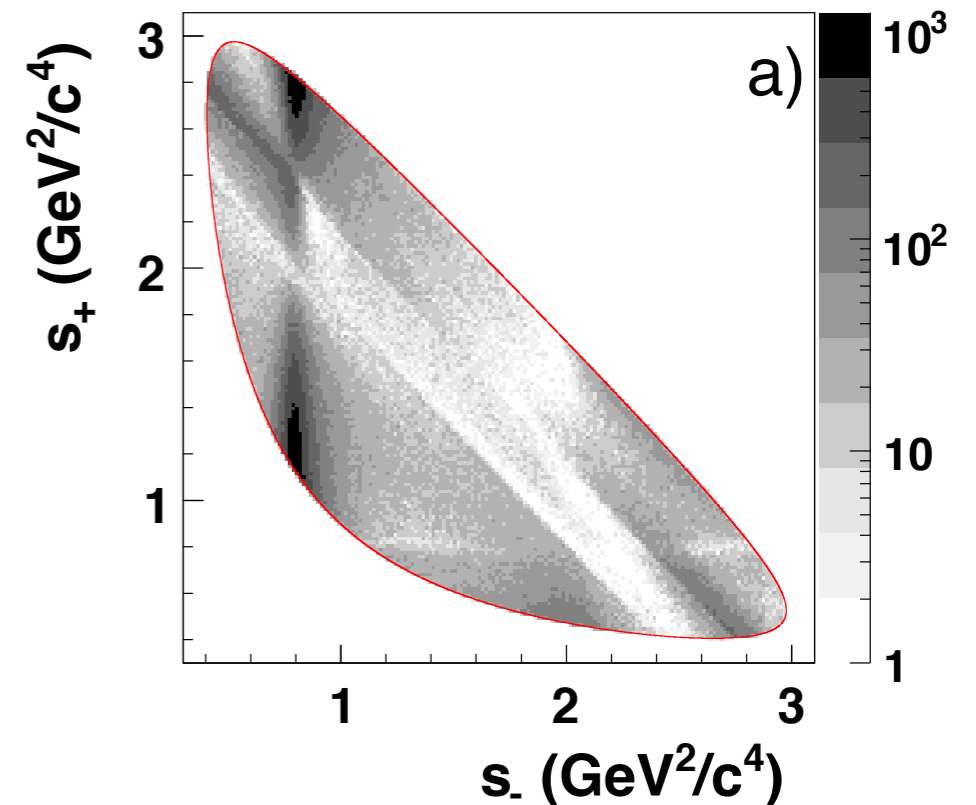
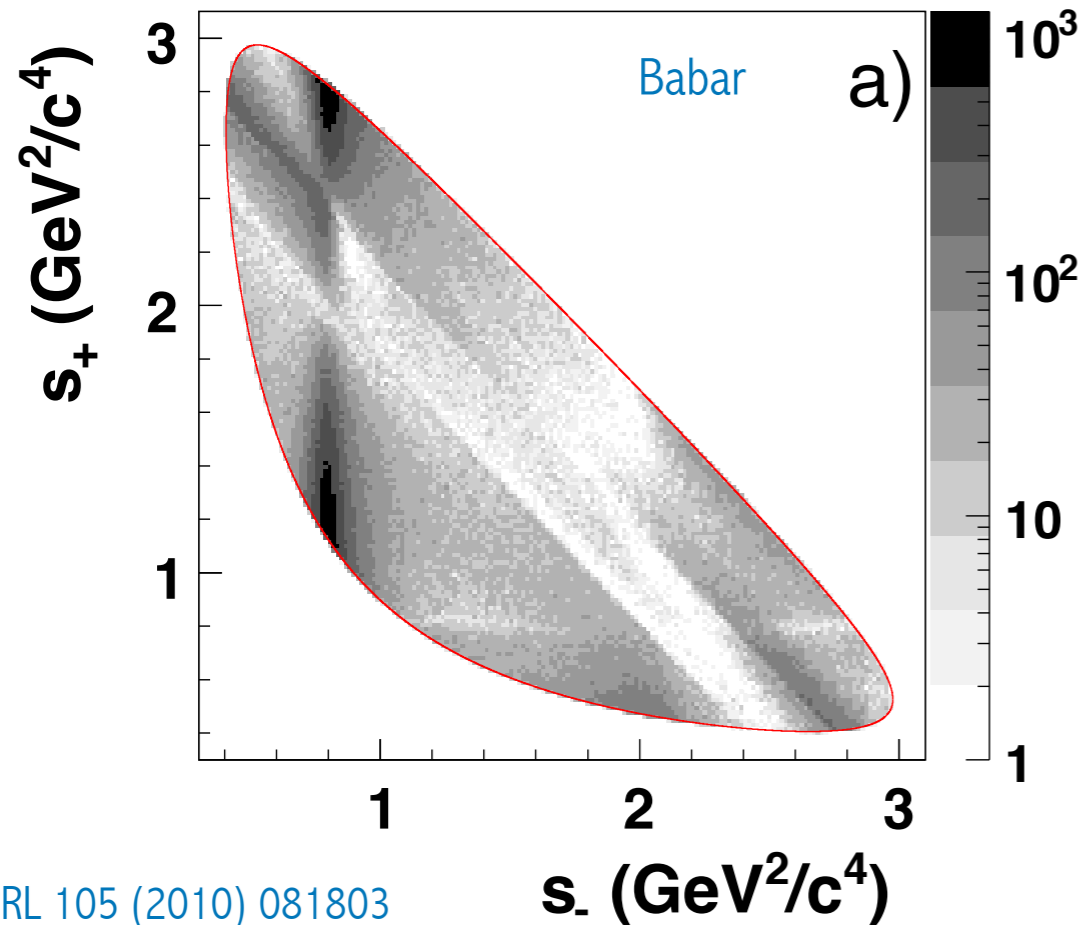


image credit: Tom Latham

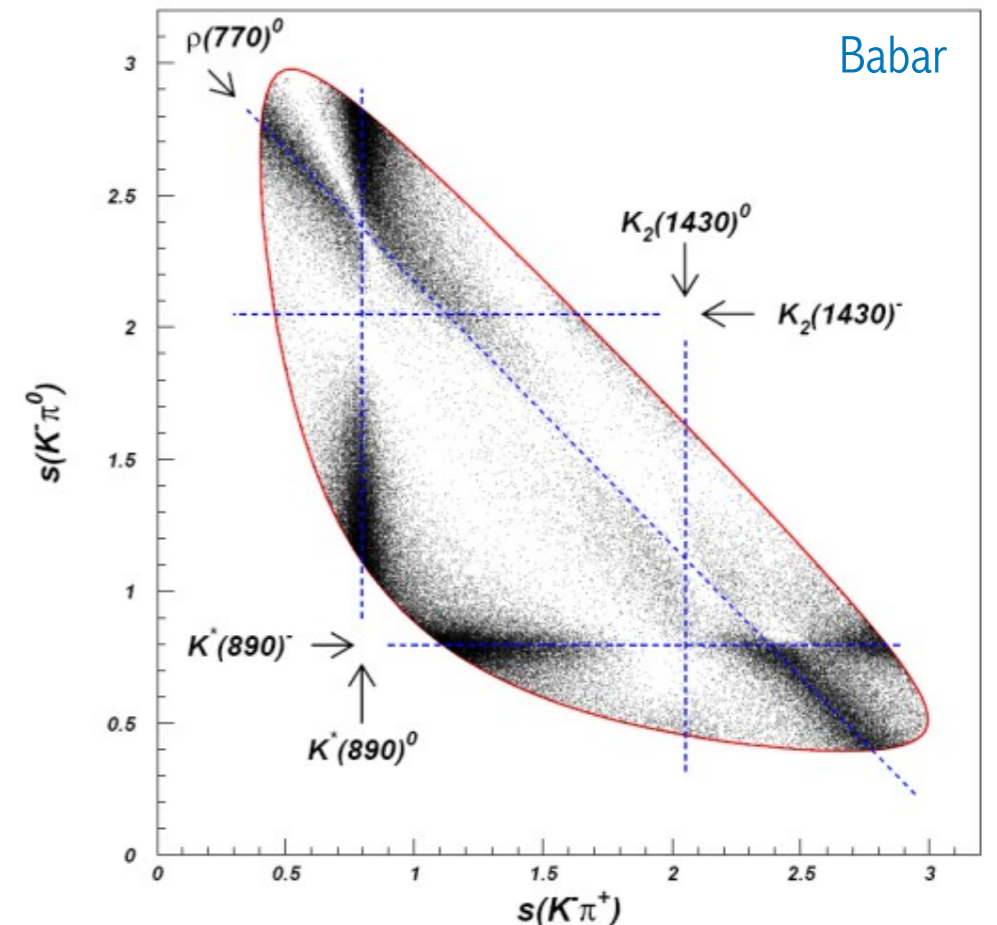


BABAR PRL 105 (2010) 081803

- $D^0 \rightarrow K_s \pi^- \pi^+$



- $D^0 \rightarrow K^- \pi^+ \pi^0$

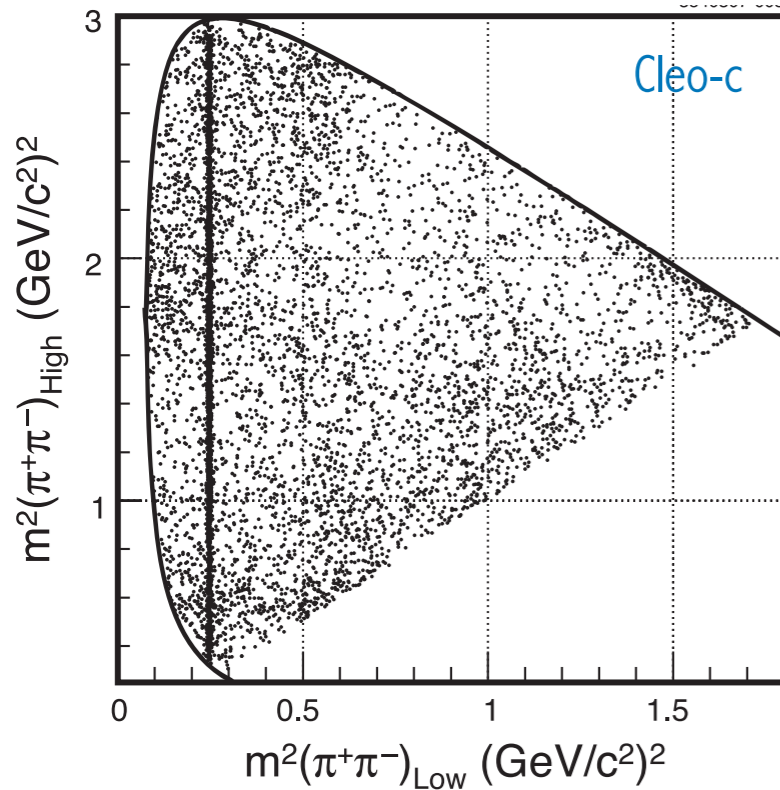


→ Similar final state but different interference pattern

↪ different dynamics to be understood

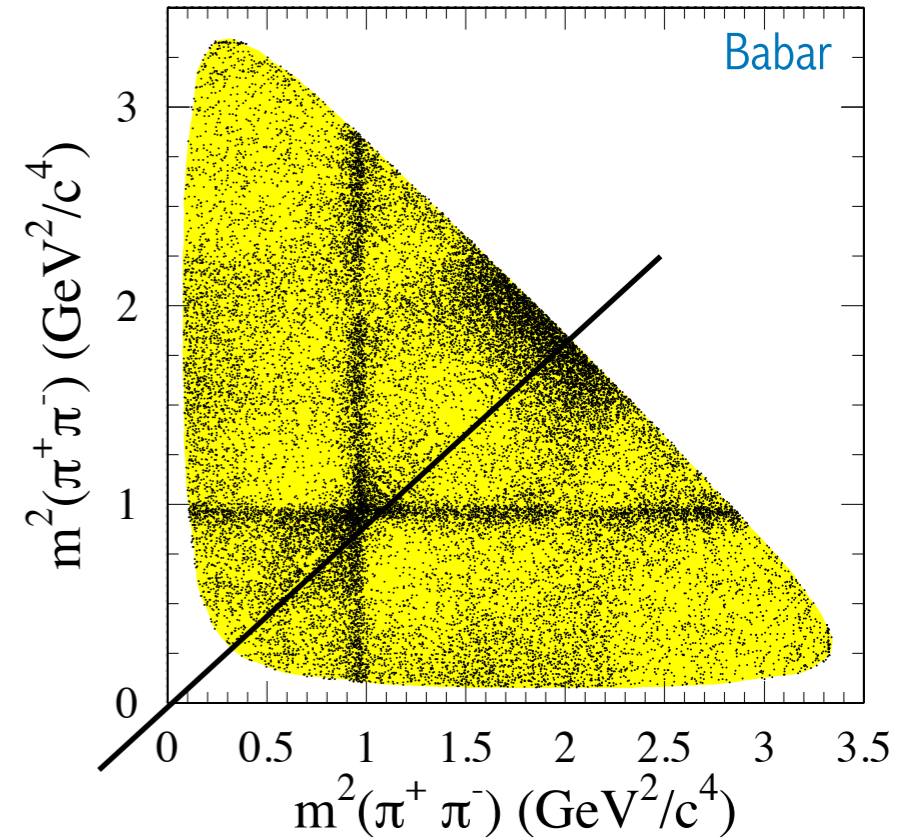
→ to disentangle the interference we need amplitude analysis

• $D^+ \rightarrow \pi^+ \pi^- \pi^+$



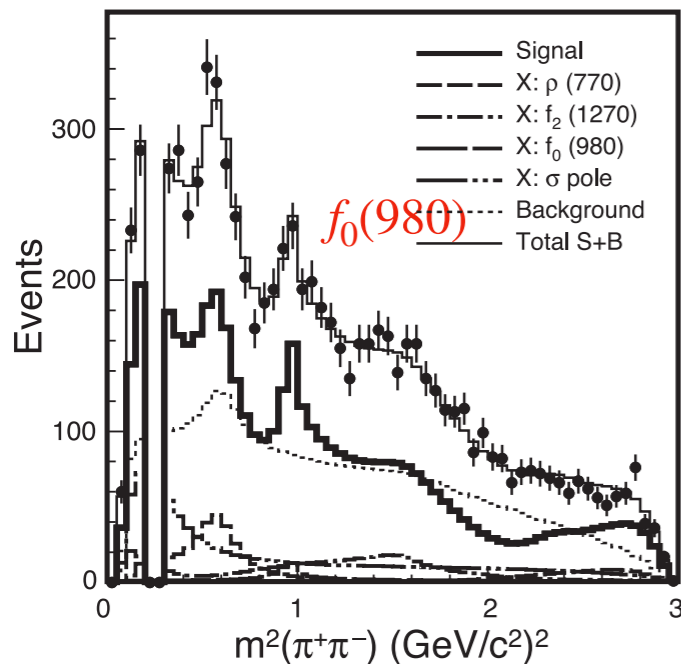
PRD 76 (2007)012001

• $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$



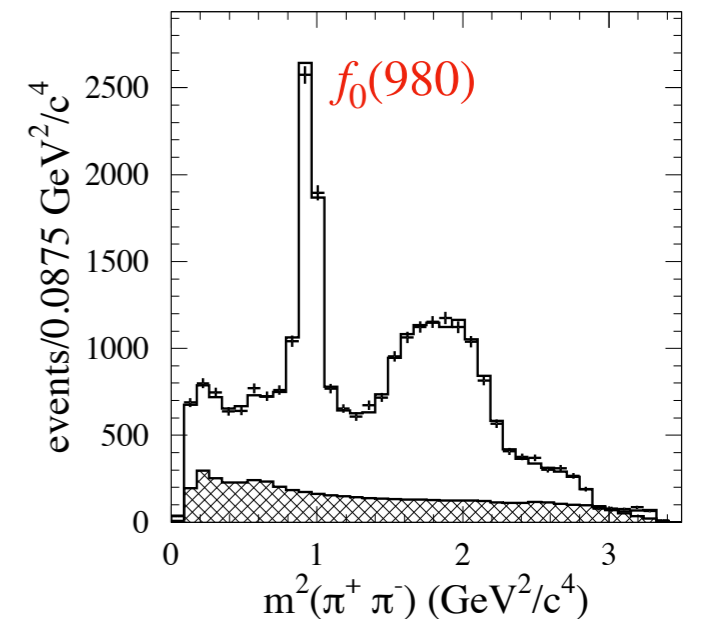
PRD 79 (2009)032003

→ different resonance signature

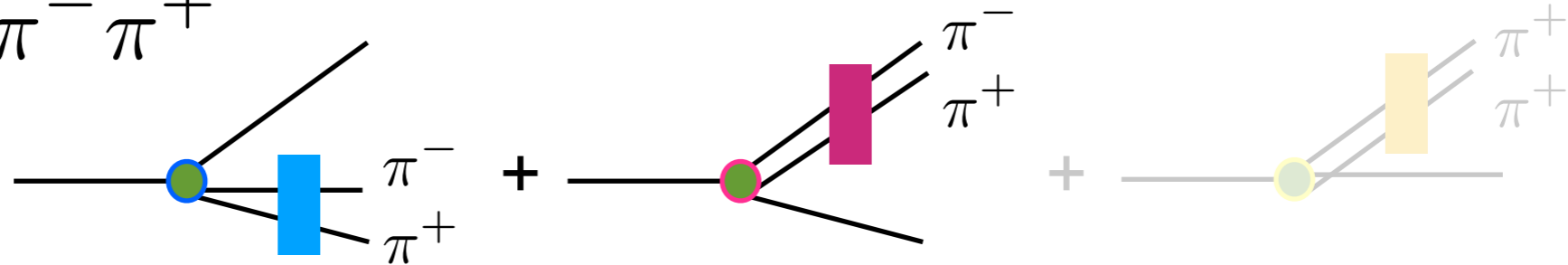


→ projection highlight that S-wave is very different

→ production environment matters



- $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$



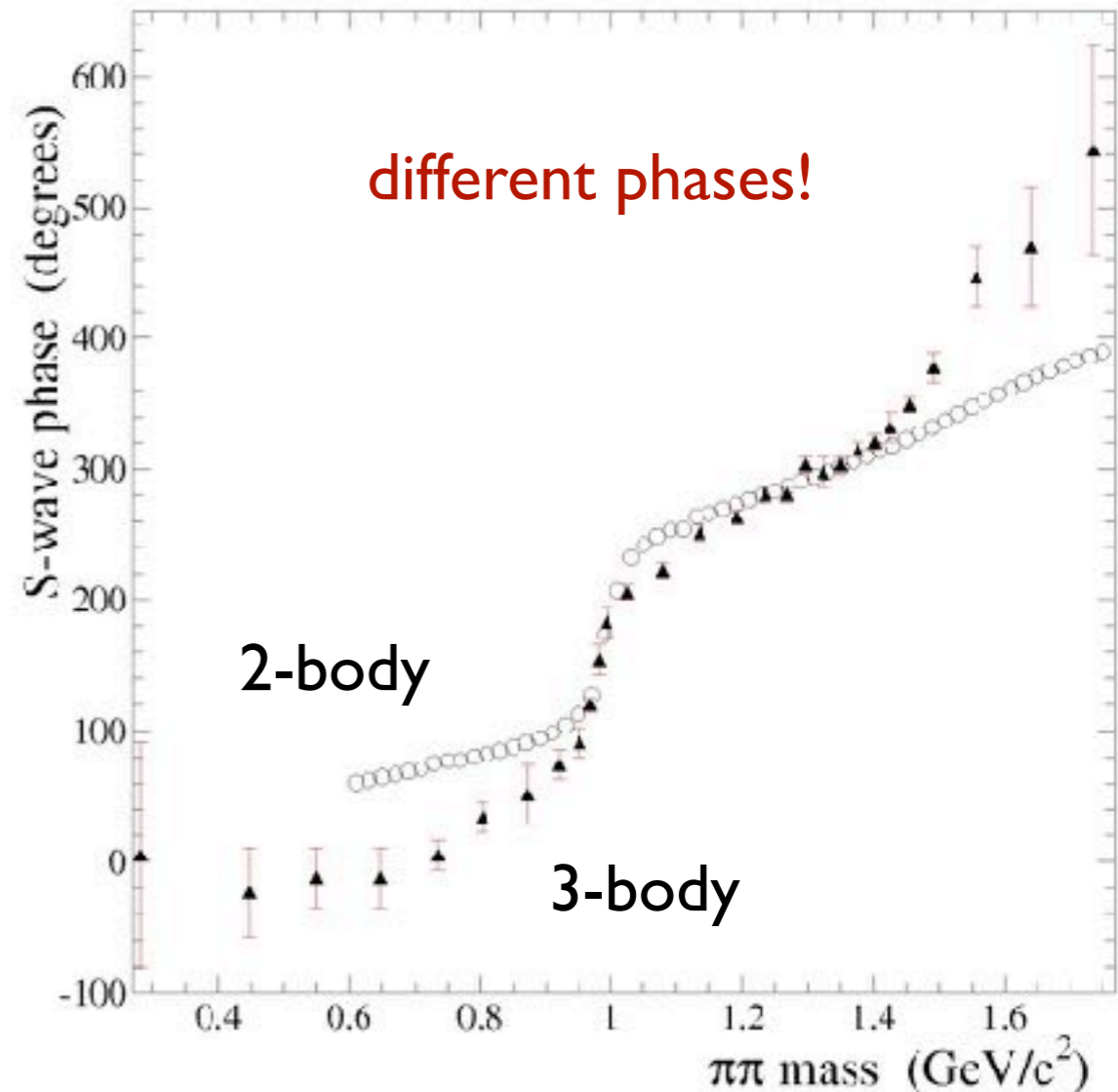
- If this is the “nature” picture → decay **phase** should be the **same** as 2-body

↪ Watson's Theorem

- Quantum numbers:

- 2-body amplitude: spin and isospin well defined!
- 3-body data: only spin! and \neq dynamics

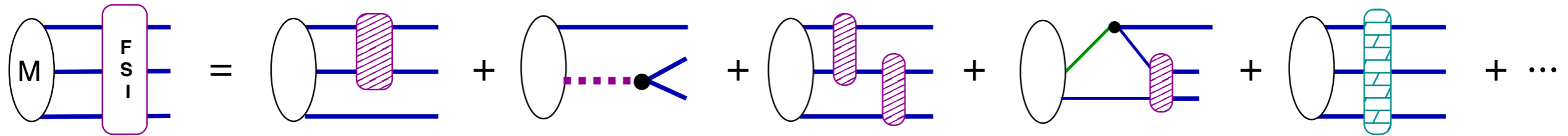
There is more than only 2-body



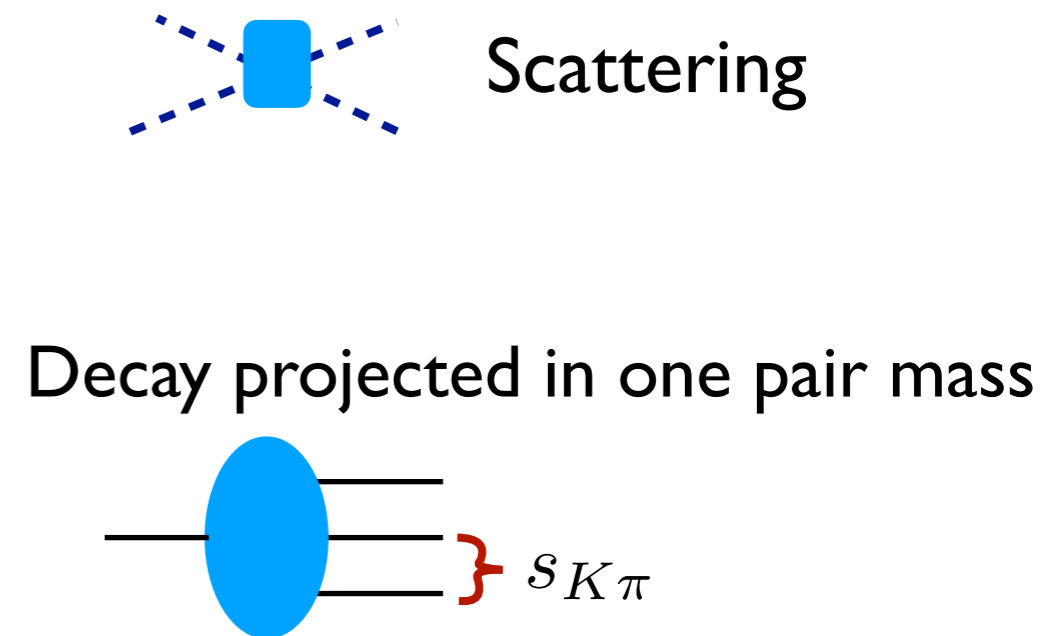
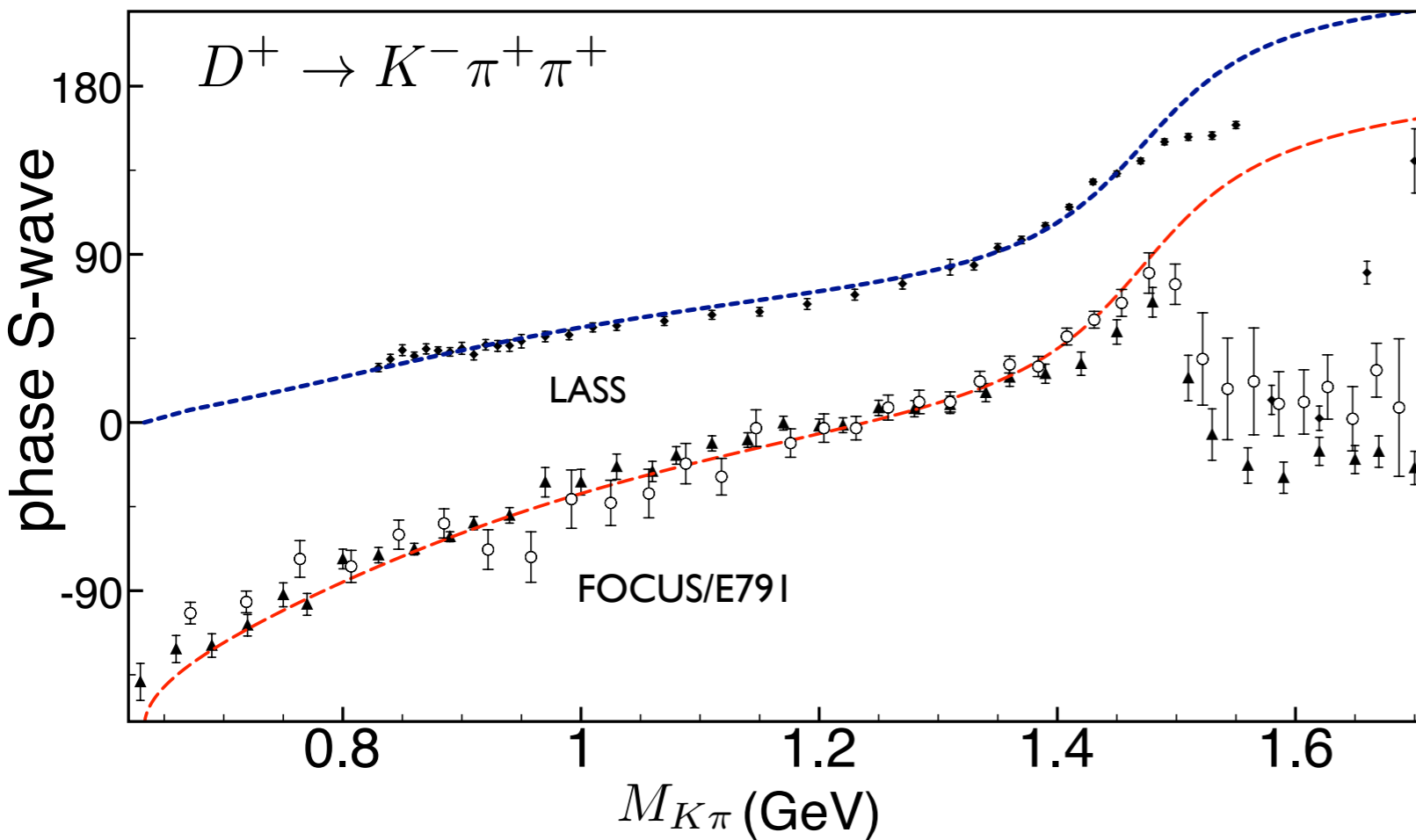
PRD 79 (2009) 032003

p.magalhaes@cern.ch

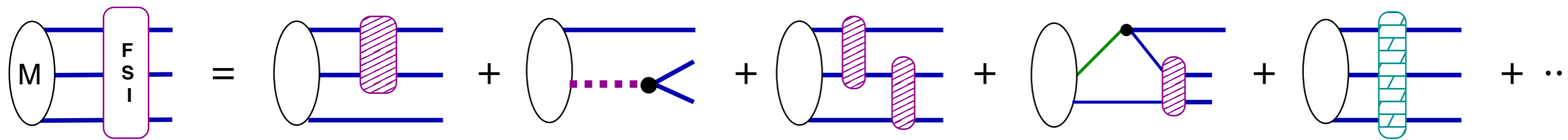
- Three-body FSI (beyond 2+1)



- shown to be relevant on charm sector



- Three-body FSI (beyond 2+1)



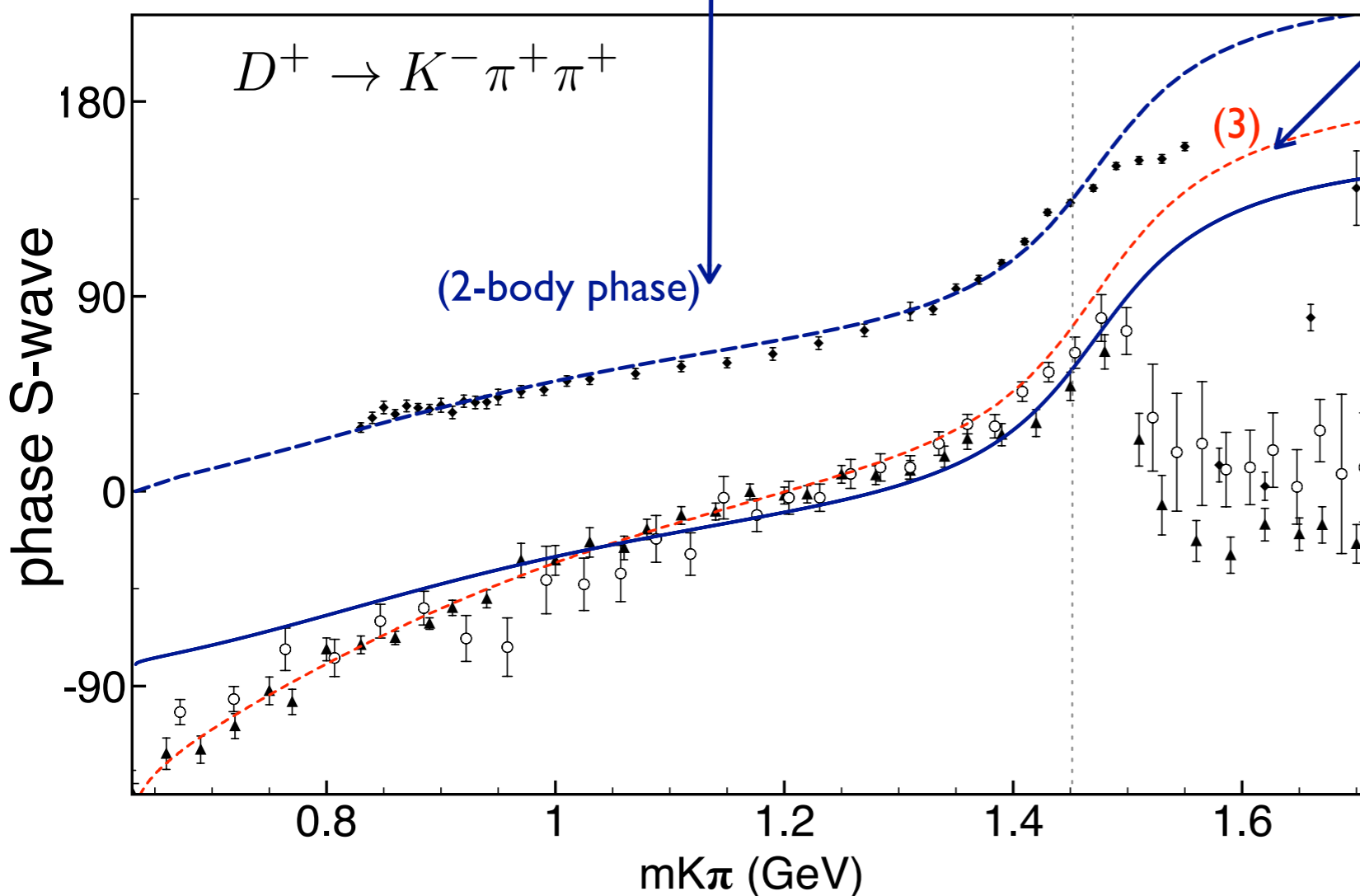
- shown to be relevant on charm sector

PRD92 094005 (2015)

- 3-body approaches

PCM et.al: PRD84 094001 (2011),
S.Nakamura PRD93 014005 (2016)
Niecknig, Kubis, JHEP10 142 (2015)

- 3-body FSI play a role
- data analysis...precision



amplitude analysis @LHCb

$$D^+ \rightarrow K^- K^+ K^+$$



Theoretical model

PHYSICAL REVIEW D **98**, 056021 (2018)

arXiv:1805.11764 [hep-ph]

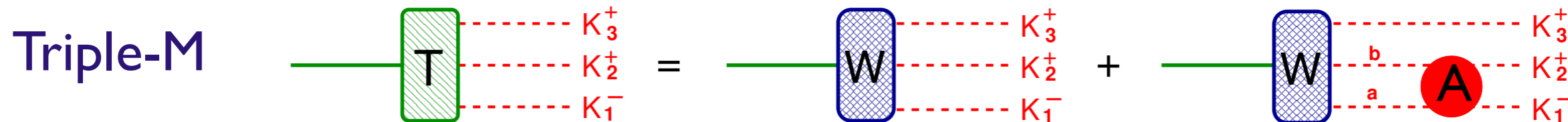
Multimeson model for the $D^+ \rightarrow K^+ K^- K^+$ decay amplitude

R. T. Aoude,^{1,2} P. C. Magalhães,^{1,3,*} A. C. dos Reis,¹ and M. R. Robilotta⁴

fitted to  data

JHEP **1904** (2019) 063

KK scattering
amplitude



- depart from a fundamental theory \longrightarrow ChPT Lagrangian

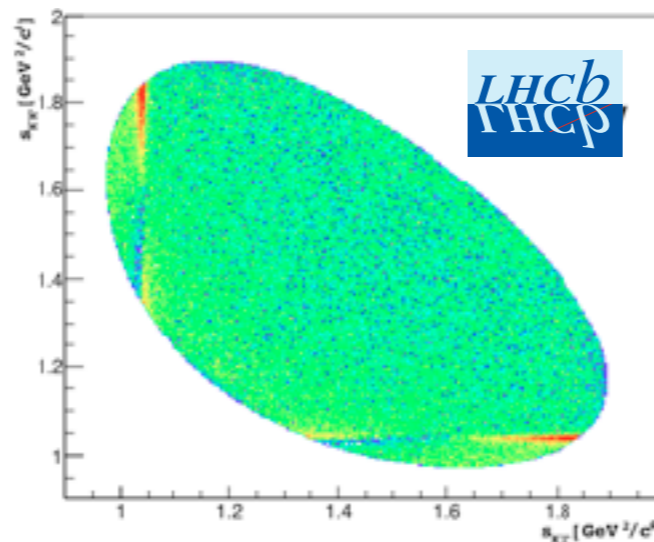
- track the ingredients we include in our model!

- $A_{ab}^{JI} \longrightarrow$ unitary scattering amplitude for $ab \rightarrow K^+ K^-$

- fit the model to LHCb data

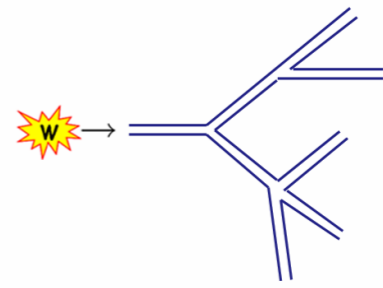
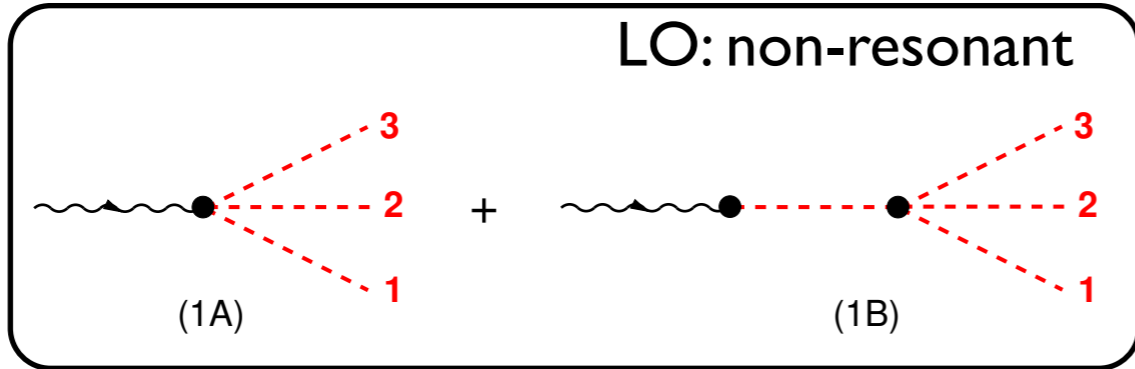
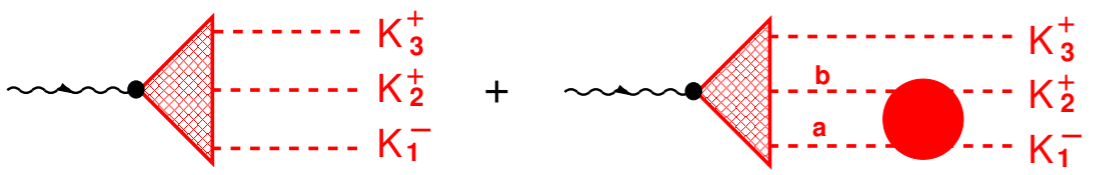
run I (8 TeV CM) $2fb^{-1}$

JHEP 1904 (2019) 063

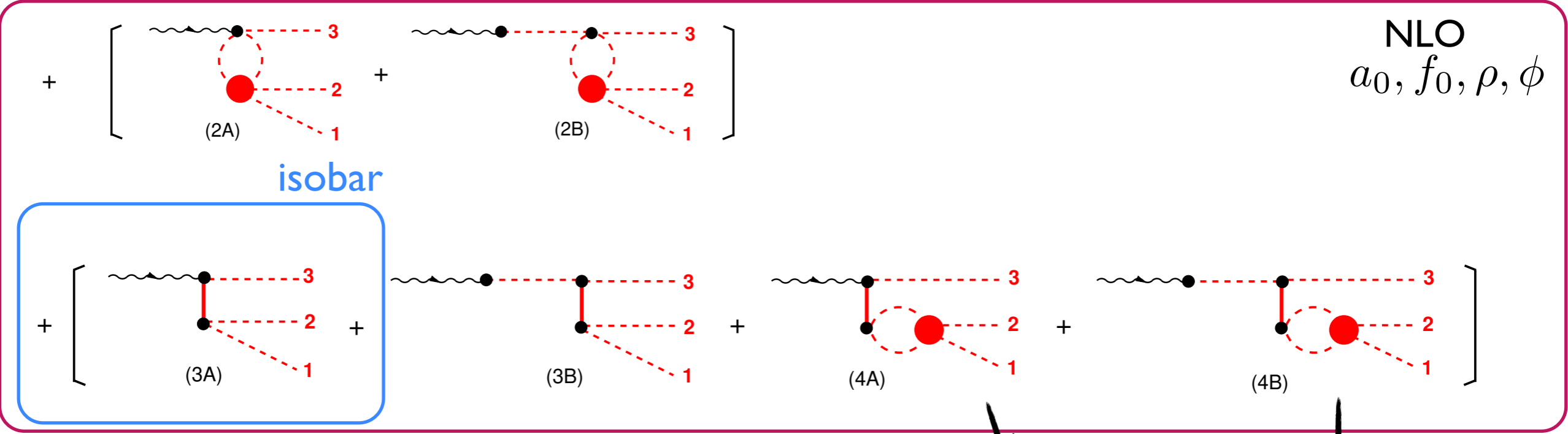


\longrightarrow predict KK scattering amplitude

\longrightarrow parameters have physical meaning: resonance masses and coupling constants



Chiral symmetry



~~•~~ $K\bar{K}$ coupled-channel unitary amplitude
 $\pi\pi, \eta\eta, \pi\eta, \rho\pi$

• isospin decomposition $[J, I = (0, 1), (0, 1)]$
 $\langle K^- K^+ | = (i/2) \langle V_3^{KK} + V_8^{KK} | - (1/2) \langle U_3^{KK} + S^{KK} |$

- Theoretical sound model



$$T^S = T_{NR}^S + T^{00} + T^{01}$$

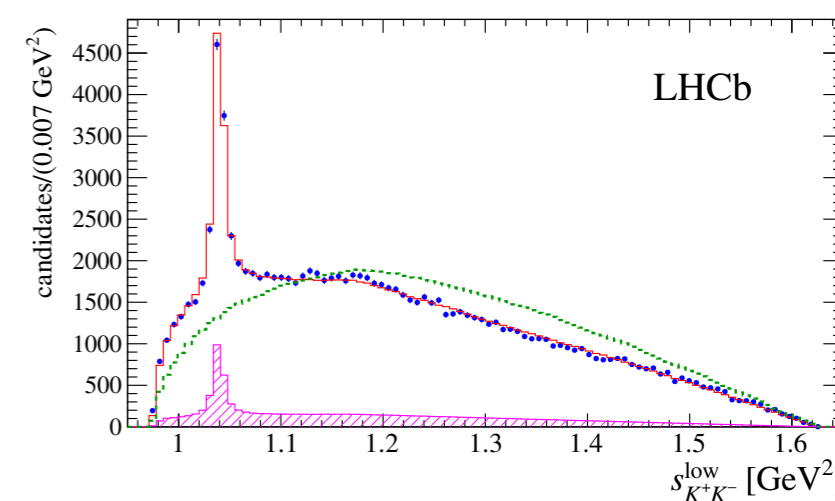
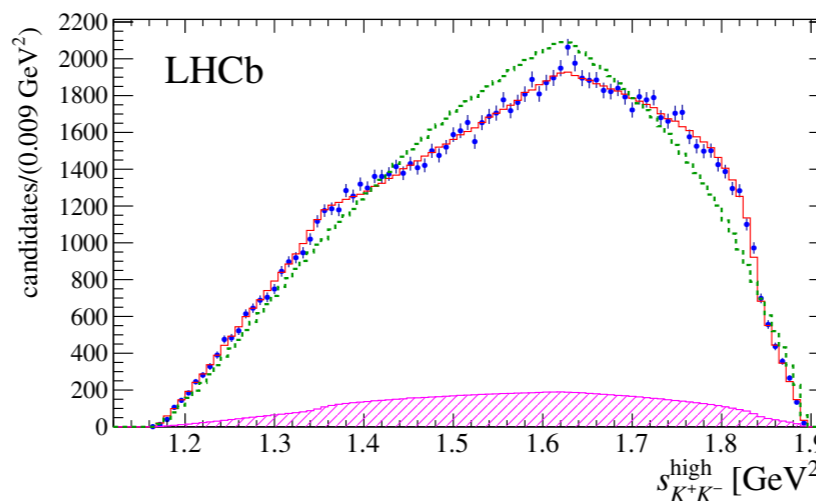
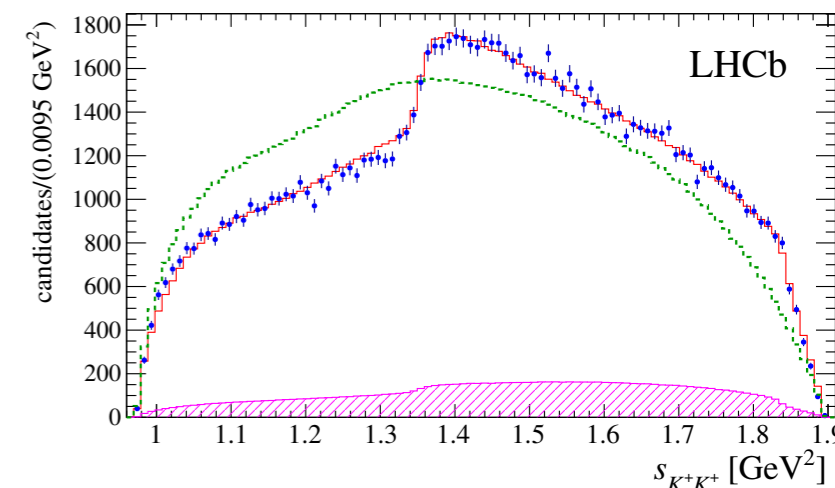
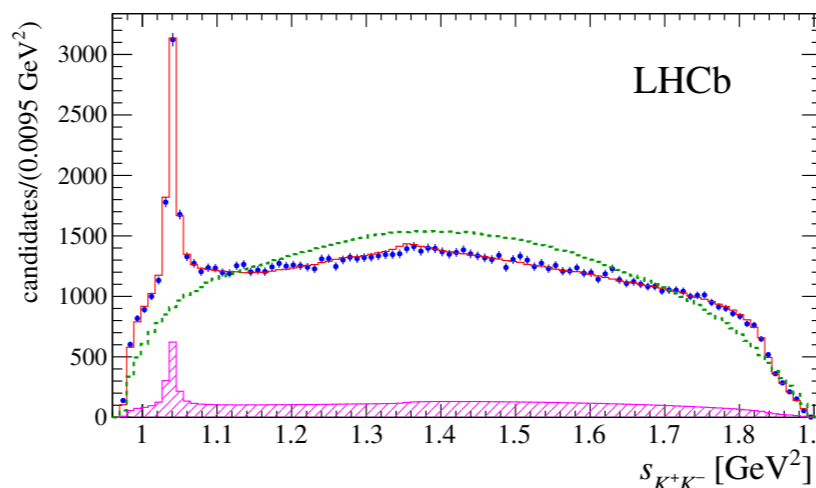
$$T^P = T_{NR}^P + T^{11} + T^{10}$$

FF _{NR}	FF ⁰⁰	FF ⁰¹	FF ¹⁰	FF ¹¹	FF _{S-wave}
14 ± 1	29 ± 1	131 ± 2	7.1 ± 0.9	0.26 ± 0.01	94 ± 1

$\chi^2/\text{ndof} = 1.12$ (Isobar 1.14-1.6)

- free parameters

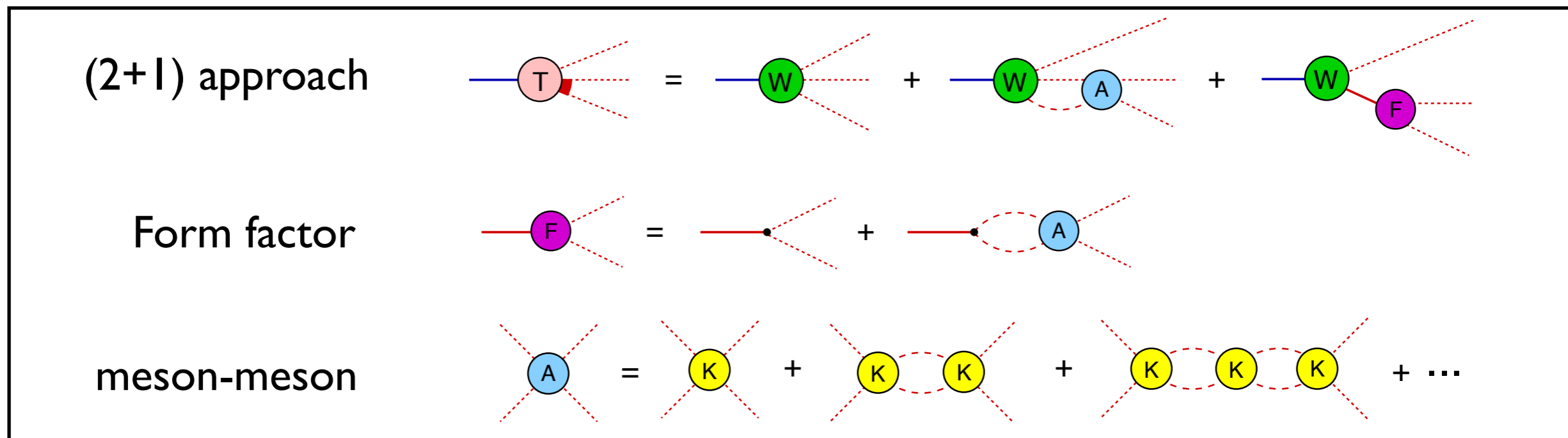
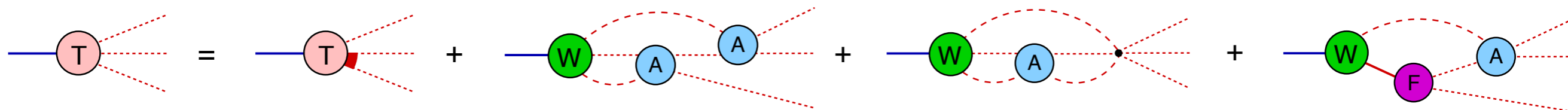
parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5$ MeV
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6$ MeV
m_{S_0}	$992.0^{+8.5}_{-7.5} \pm 8.6$ MeV
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1$ MeV
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51$ MeV
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9$ MeV
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3$ MeV
\tilde{c}_d	$-6.15^{+0.55}_{-0.54} \pm 0.19$ MeV
\tilde{c}_m	$-10.8^{+2.0}_{-1.5} \pm 0.4$ MeV



JHEP 1904 (2019) 063

➔ good fit with fewer parameters than the isobar

- Any 3-body decay amplitude

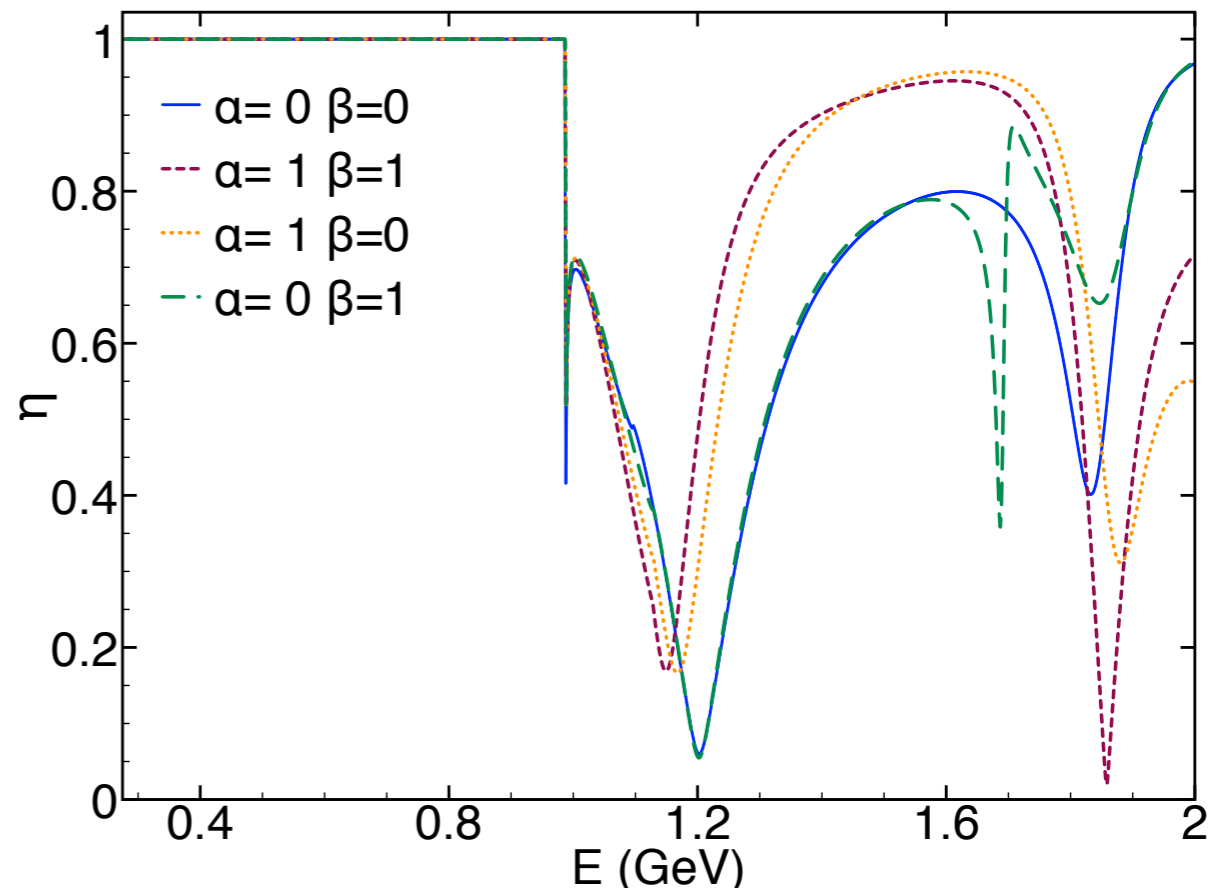
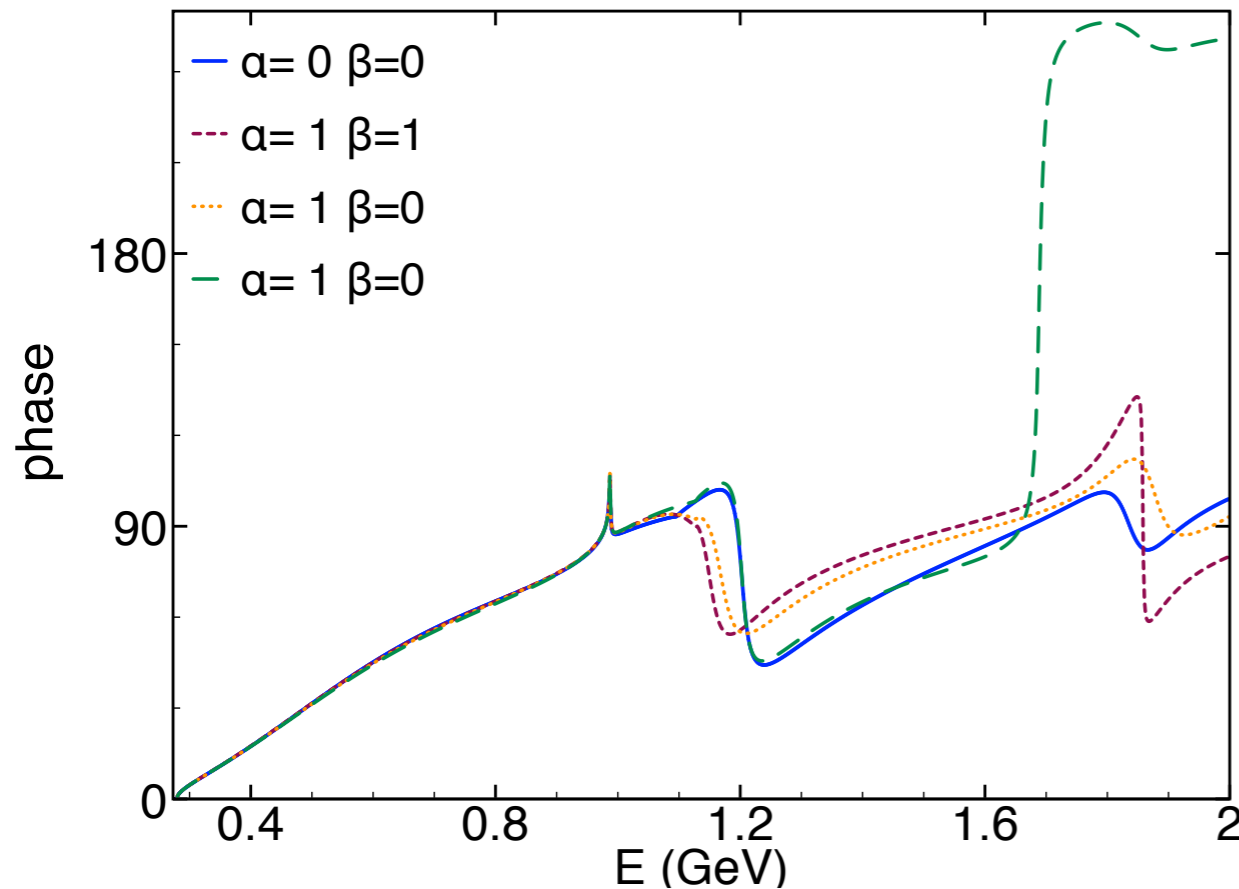
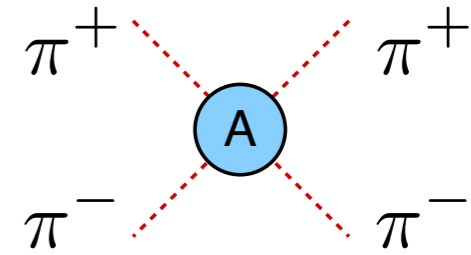


→ provide the building block in SU(3)

- includes multiple resonances in the same channel (as many as wanted)
- free parameter (masses and couplings) to be fitted to data.

→ Available to be implement in data analysis!!

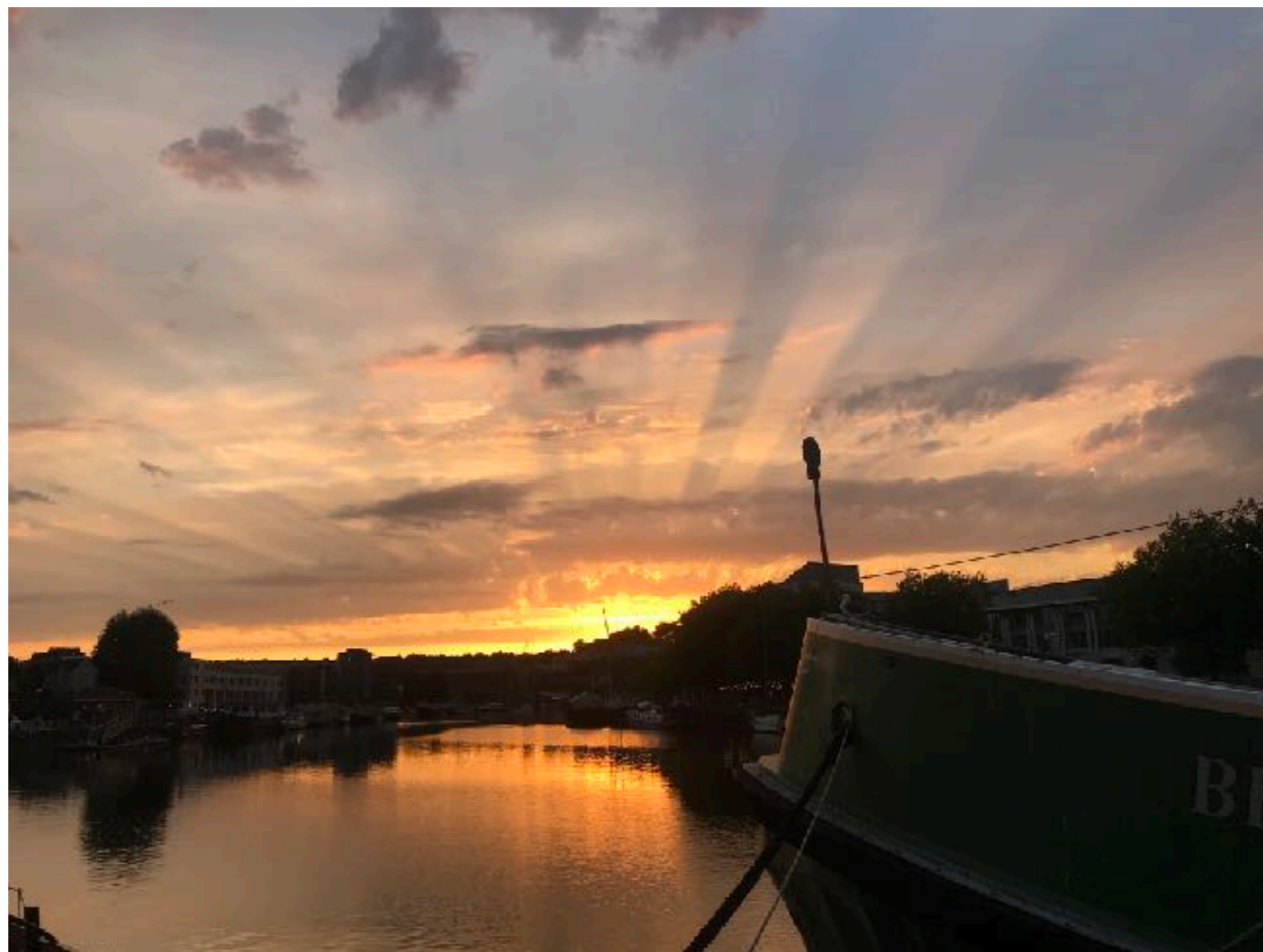
- coupled-channels: $\pi\pi$, KK and $\eta\eta$
 - 3 resonances: $m_x=0.98$, $m_y=1.37$, $m_z=1.7$ GeV
 - ↳ α and β are couplings from m_z



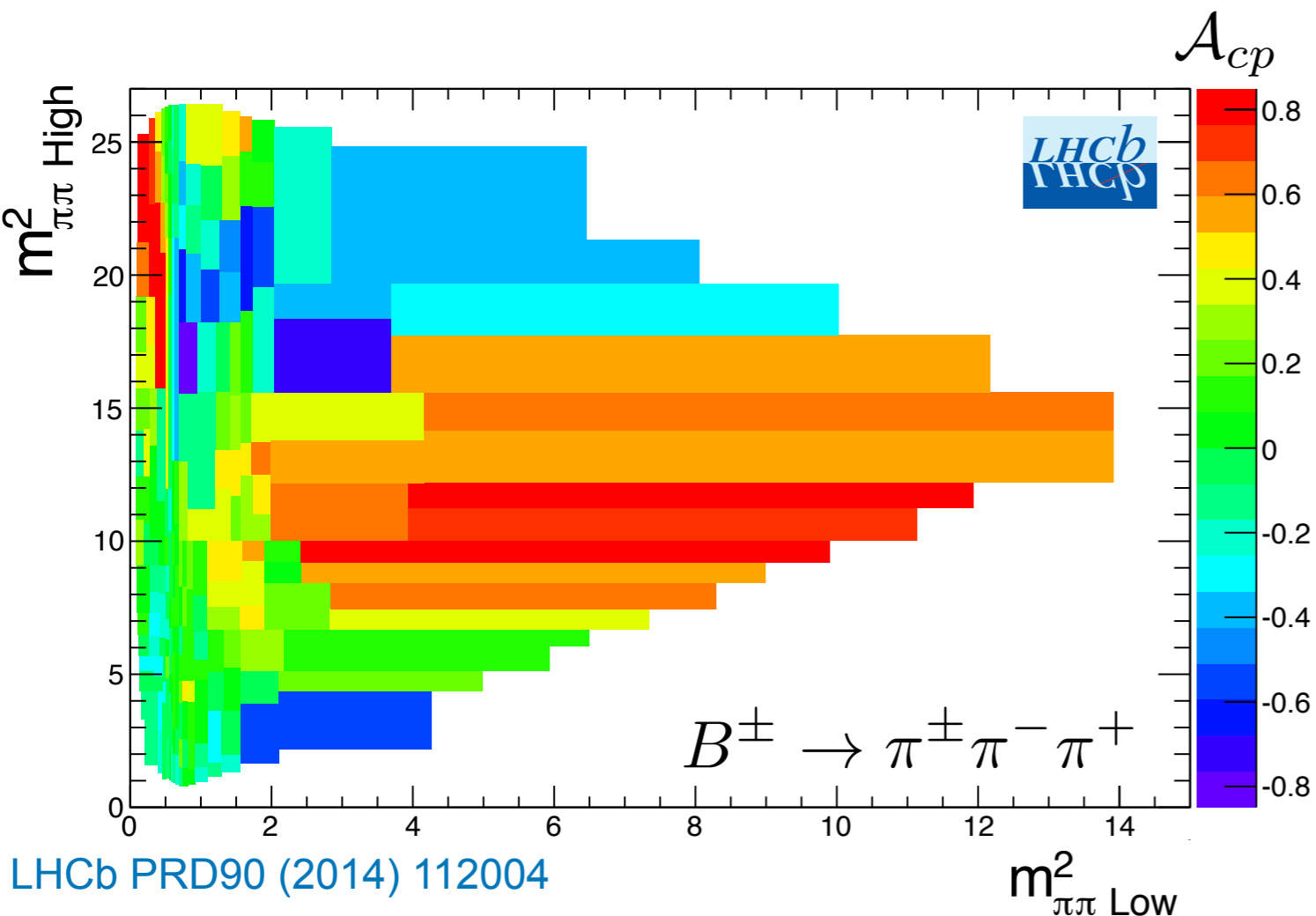
- extra res do not disturb the low-energy!
- parameter should be fixed by data
- will apply this methodology in other $D \rightarrow hhh$

CP violation in charm

Dalitz plot analysis



- CP-Violation directly from Dalitz plot



- massive phase-space **localized** Asymmetry

$$A_{CP} = \frac{\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f})}{\Gamma(M \rightarrow f) + \Gamma(\bar{M} \rightarrow \bar{f})}$$

- condition to CPV:

2 \neq amplitudes, SAME final state,
 \neq strong (δ_i) and weak (ϕ_i) phases

$$\Gamma(M \rightarrow f) - \Gamma(\bar{M} \rightarrow \bar{f}) = -4A_1A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$$

- hadronic Final State Interactions are important sources to generate CPV

- to disentangle CPV mechanisms in data we need amplitude analysis!

- recent LHCb work on $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ PRD101 (2020) 012006; PRL 124 (2020) 031801

↪ CPV on $f_2(1270)$ and rescattering $\pi\pi \rightarrow KK$ in S-wave

- New results on CPV on charm can hint for new searches procedures

↪ $A_{cp}(D^0 \rightarrow K^+K^-) - A_{cp}(D^0 \rightarrow \pi^+\pi^-)$ [PRL 122 \(2019\) 211803](#)

↪ run I + run II

- we expect to see CPV in $D \rightarrow hhh$: similar weak vertices are present !

- To extract CPV from Dalitz plot (as in B decay) we need:

1 - model independent procedure to show localized CP asymmetry ↪ Mirandizing

2 - Amplitude analysis with a theoretical sound model to describe FSI properly

- on going in LHCb for $D_{(s)} \rightarrow \pi\pi\pi$

- A consistent treatment of FSI is crucial to reach precision in $D \rightarrow hhh$ ANA
 - two-body coupled-channels description is mandatory
 - learn much more about the underlying dynamics
 - relevant for CPV search
- New LHCb $D^+ \rightarrow h^+h^-h^+$ amplitude analysis are expected soon (SCS, CF)
 - ↪ huge data samples claim accurate models
- $D^+ \rightarrow KKK$: example of theory/experimental joint work
 - tool kit for amplitude analysis with theoretically sound models

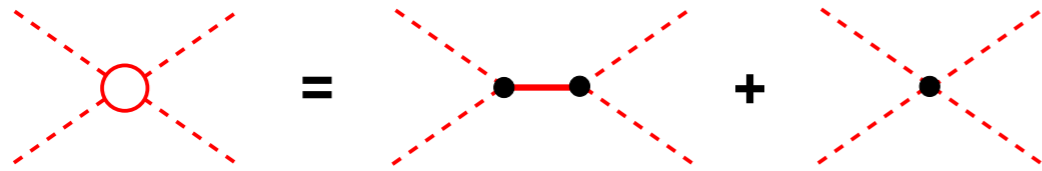
obrigada!!

#staysafe

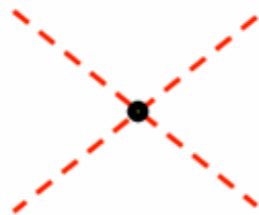


Backup slides !

- solid theory to describe MM interactions at low energy == ChPT



- LO:



$$\mathcal{L}_M^{(2)} = -\frac{1}{6F^2} f_{ijs} f_{kls} \phi_i \partial_\mu \phi_j \phi_k \partial^\mu \phi_l + \frac{B}{24F^2} \left[\sigma_0 \left(\frac{4}{3} \delta_{ij} \delta_{kl} + 2 d_{ijs} d_{kls} \right) + \sigma_8 \left(\frac{4}{3} \delta_{ij} d_{kl8} + \frac{4}{3} d_{ijs} \delta_{kl} + 2 d_{ijm} d_{klm} d_{8mn} \right) \right] \phi_i \phi_j \phi_k \phi_l$$

Gasser & Leutwyler [Nucl. Phys. B250(1985)]

- NLO: include resonances as a field



Ecker, Gasser, Pich and De Rafael [Nucl. Phys. B321(1989)]

scalars

$$\mathcal{L}_S^{(2)} = \frac{2\tilde{c}_d}{F^2} R_0 \partial_\mu \phi_i \partial^\mu \phi_i - \frac{4\tilde{c}_m}{F^2} B R_0 (\sigma_0 \delta_{ij} + \sigma_8 d_{8ij}) \phi_i \phi_j + \frac{2c_d}{\sqrt{2}F^2} d_{ijk} R_k \partial_\mu \phi_i \partial^\mu \phi_i - \frac{4Bc_m}{\sqrt{2}F^2} \left[\sigma_0 d_{ijk} + \sigma_8 \left(\frac{2}{3} \delta_{ik} \delta_{j8} + d_{i8s} d_{jsk} \right) \right] \phi_i \phi_j R_k$$

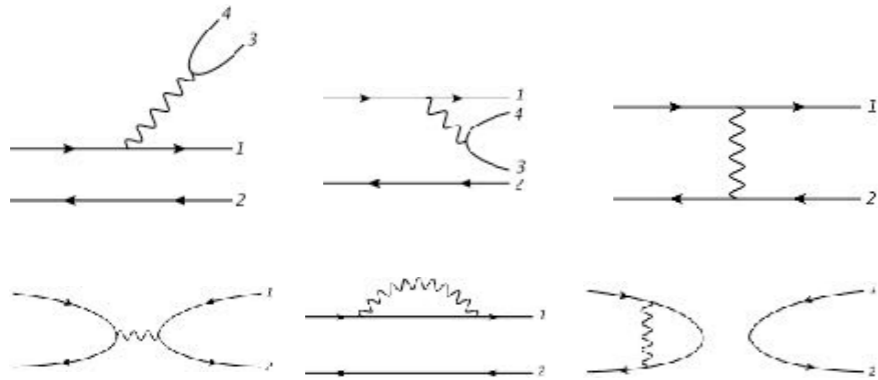
vectors

$$\mathcal{L}_V^{(2)} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

$$\langle V_{\mu\nu} u^\mu u^\nu \rangle = \frac{1}{F^2} V_a^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_j (i f_{aij} + d_{aij})$$

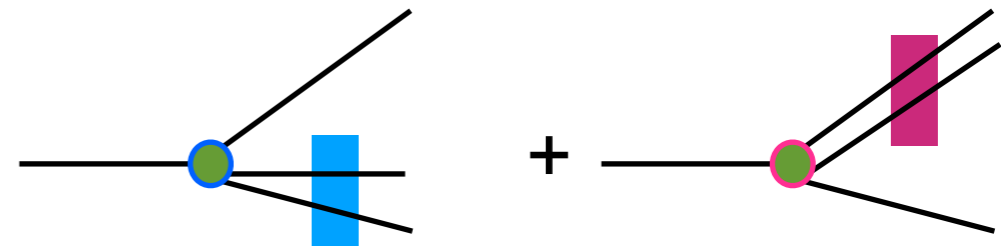
→ because we want to extend this to high E the parameters change meaning and can be free to fit!

- QCD factorization approach → factorize the quark currents



challenging for 3-body
not all FSI and 3-body NR
scale issue with charm !

Chau [Phys. Rep. 95,1 (1983)]



$$\mathcal{H}_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \left[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right] + \text{h.c.},$$

→ ex: $B^+ \rightarrow \pi^+ \pi^- \pi^+$ how to describe it?

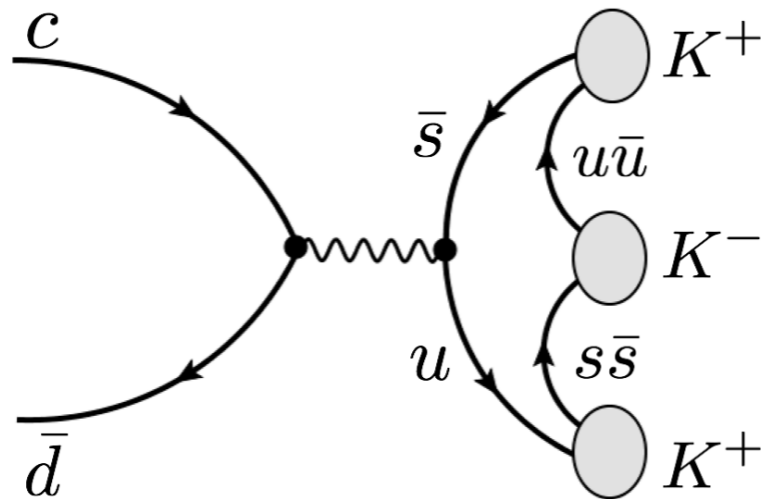
$$A \sim \underbrace{\langle [\pi^+(p_2) \pi^-(p_3)] | (\bar{u}b)_{V-A} | B^- \rangle}_{R} \langle \pi^-(p_1) | (\bar{d}u)_{V-A} | 0 \rangle + \langle \pi^-(p_1) | (\bar{d}b)_{sc-ps} | B^- \rangle \underbrace{\langle [\pi^+(p_2) \pi^-(p_3)] | (\bar{d}d)_{sc+ps} | 0 \rangle}_{FF}$$

- naive factorization {
 - intermediate by a resonance **R**;
 - FSI with scalar and vector form factors **FF**

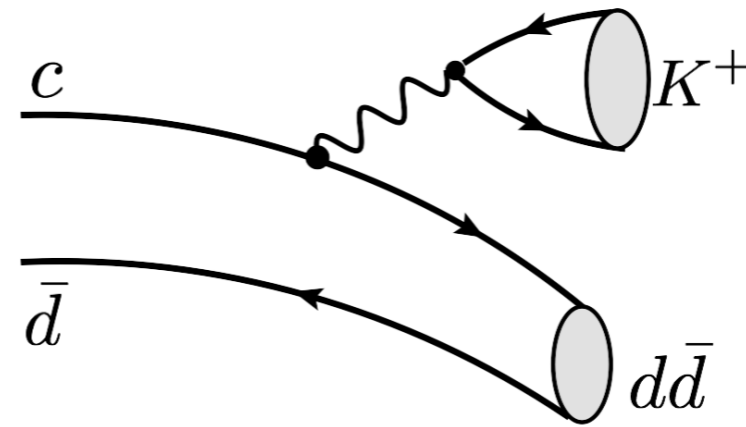
↪ parametrizations for B and D → 3h Boito et al. PRD96 113003 (2017)

- modern QDC factorization: improvement to include “long distance”
Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)

● annihilation

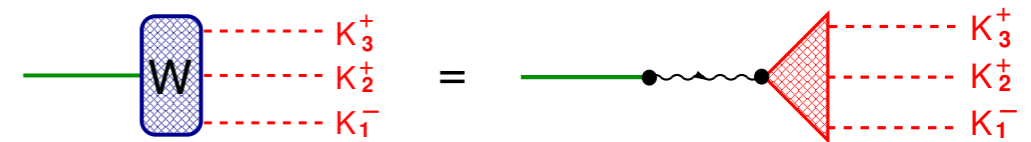


● color allowed



↪ need a rescattering!

- both are doubly Cabibbo-suppressed
- hypotheses that annihilation is dominant



● separate the different energy scales:

$$\mathcal{T} = \langle (KKK)^+ | T | D^+ \rangle = \underbrace{\langle (KKK)^+ | A_\mu | 0 \rangle}_{\text{ChPT}} \langle 0 | A^\mu | D^+ \rangle.$$

↪ $-i G_F \sin^2 \theta_C F_D P^\mu$

➔ know how to calculate everything

- CPT must be preserved

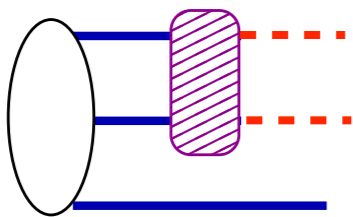
Lifetime $\tau = 1 / \Gamma_{\text{total}} = 1 / \bar{\Gamma}_{\text{total}}$

$$\Gamma_{\text{total}} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \dots$$

$$\bar{\Gamma}_{\text{total}} = \bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \dots$$

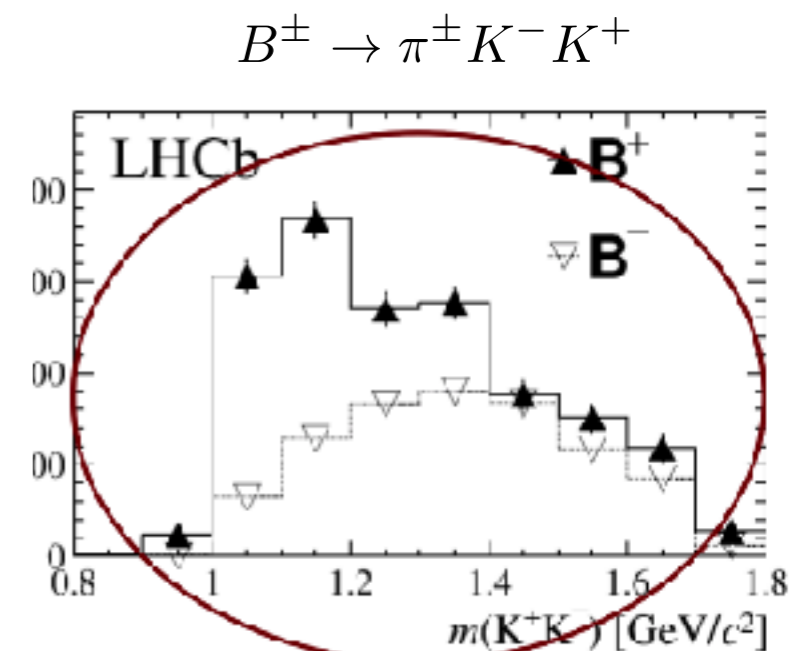
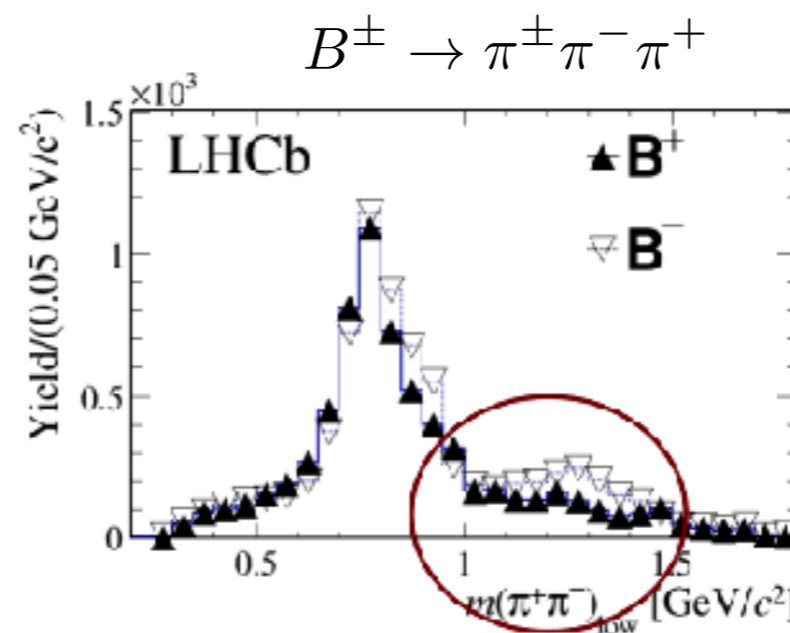
→ CPV in one channel should be compensated by another one with opposite sign

- rescattering $\pi\pi \rightarrow KK$



→ explain CPV at [1 -1.6] GeV

Frederico, Bediaga, Lourenço
PRD89(2014)094013



- confirmed by LHCb Amplitude Analysis $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ and $B^\pm \rightarrow \pi^\pm K^- K^+$

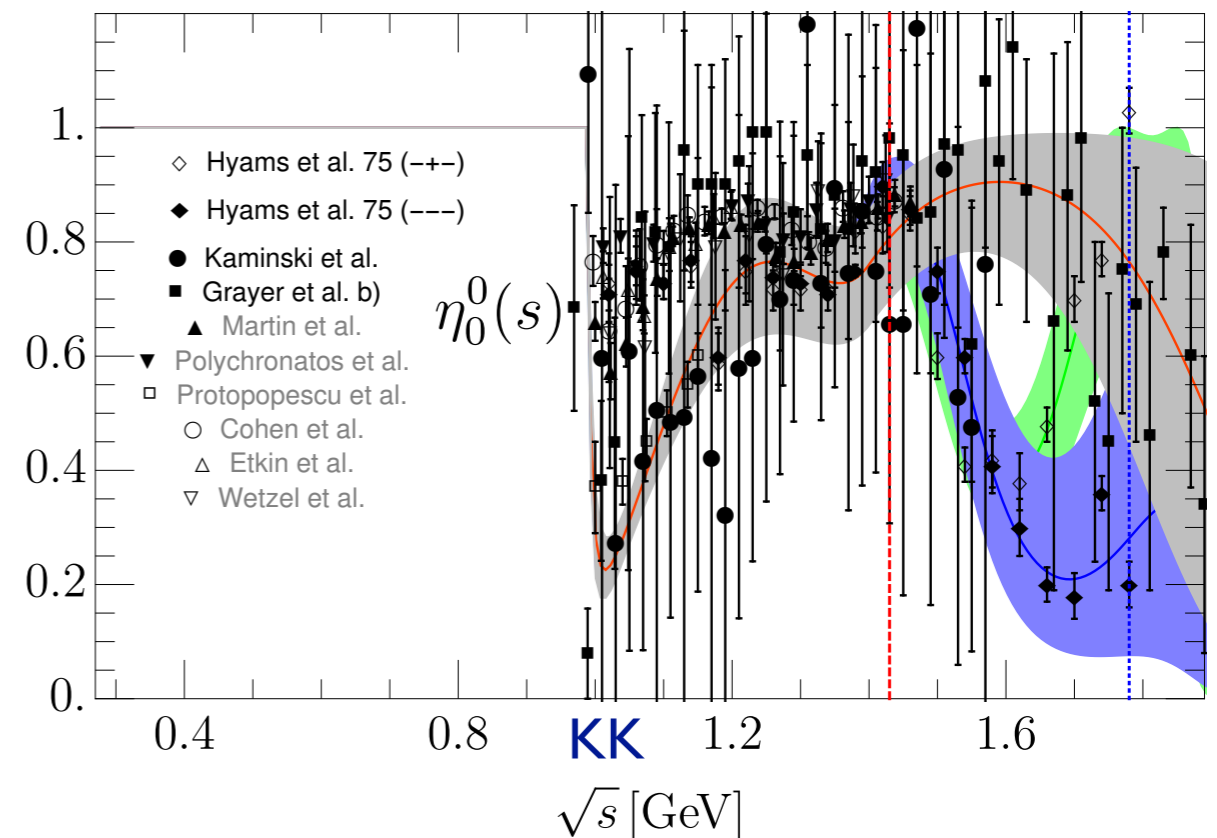
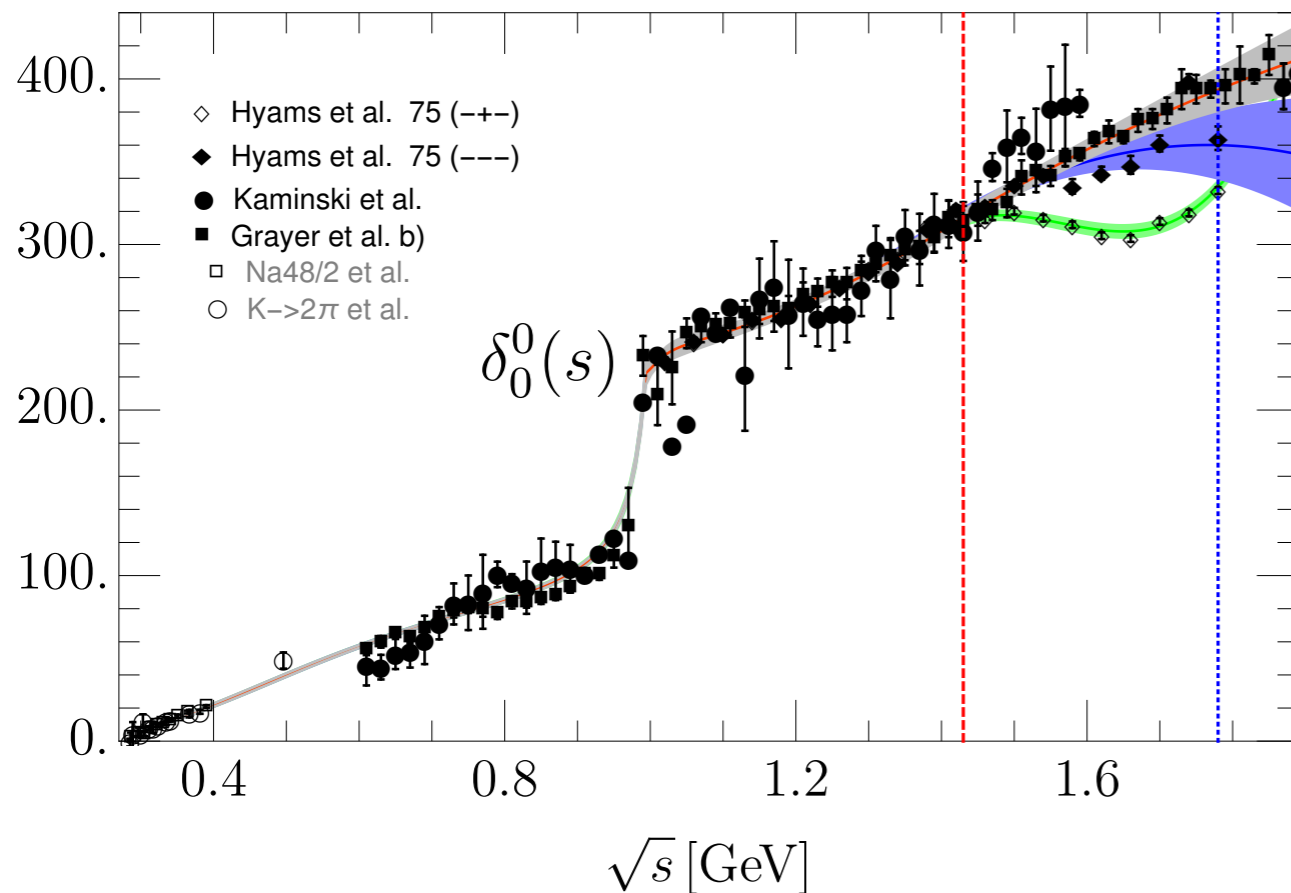
PRD101 (2020) 012006; PRL 124 (2020) 031801 PRL 123 (2019) 231802

● $\pi\pi$ scattering data S-Wave

Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

● amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i} \right]$

● elasticity



$$\sigma_l^{\text{el}} = \frac{1}{2} \left\{ \frac{1 + \eta_l^2}{2} - \eta \cos 2\delta_l \right\}$$

Inelasticity: one minus the probability of losing signal (1=>elastic)