

# Theoretical interpretation of the $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ decay and the nature of $a_0(980)$

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CHINESE ACADEMY OF SCIENCES



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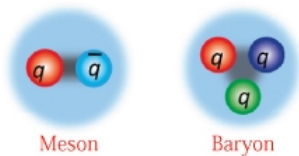
1. Introduction
2. The  $D_s^+ \rightarrow \pi^+ \pi^0 \eta$  decay and the nature of the  $a_0(980)$
3. Conclusions

# Introduction

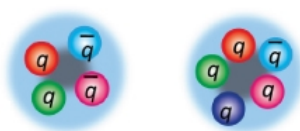
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# Hadrons

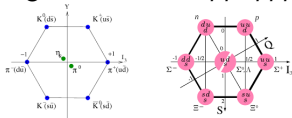
Standard Hadrons



Exotic Hadrons

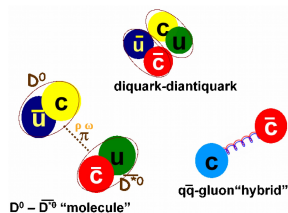


- 'Regular' hadrons:  $q\bar{q}$ ,  $qqq$

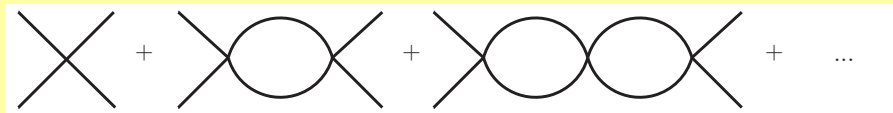


- Exotics:  $q\bar{q}q\bar{q}$ ,  $qqqq\bar{q}$ ,  $qqg$ , ...

Not  $q\bar{q}$ :  $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$



# Dynamically Generated Resonances



## Examples

- $\sigma$ ,  $a_0(980)$ ,  $f_0(980)$ ... Oller, Oset, Pelaez, PRL80(1998)
- $N(1440)$  Krehl, Hanhart, Krewald and Speth, PRC62(2000)
- $\Lambda(1405)$  Ramos, Oset, NPA635(1998); Jido, Meissner, Oller  
NPA725(2003); Hyodo, Weise PRC77(2008)
- $P_c(4450)$  Wu, Molina, Zou, Oset, PRL105(2010)
- **New** state  $X_0(2900)$  Molina, Branz, Oset PRD82(2010)

How to test the nature of a ...

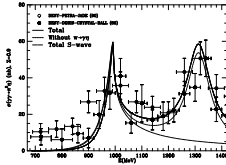
## Dynamically Generated Resonance?

- Production  $B_0 \rightarrow D_0 \sigma(a_0(980))$  Liang, Xie, Oset PRD92(2015)  
Triangle singularity  $D^{*0} \bar{D}^{*0} \rightarrow \gamma X(3872)$  F.K. Guo, PRL122(2019)
- Decays  $Y \rightarrow \gamma \gamma$  Branz, Gutsche, Lyubovitskij PRD82(2010)
- Study of  $N_c$  behavior Pelaez, Phys. Rep. 658(2016)
- Nuclear medium  $X(3872)$  Nieves et al., 2102.08589(2020)
- Weinberg compositeness condition and generalizations Weinberg PR137(1965), Hyodo, Oset, Oller
- Chiral trajectories (EFT) + LatticeQCD data, Molina, Ruiz de Elvira JHEP2011(2020)

**The  $D_s^+ \rightarrow \pi^+ \pi^0 \eta$  decay and the  
nature of the  $a_0(980)$**

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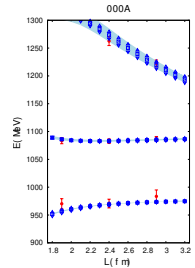
# Is the $a_0(980)$ a threshold effect or a true resonance?



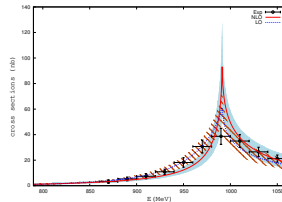
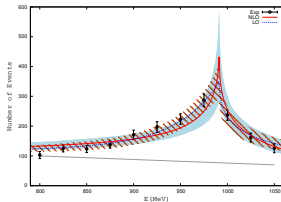
**Figure 1:** Integrated cross section for  $\gamma\gamma \rightarrow \pi^0\eta$ . Data: Oest(1990), Antreasyan(1986).

## UChPT predictions

Channels:  $K\bar{K}$ ,  $\pi\eta$   
 Oller, Oset,  
 NPA629(1998)  
 Guo, Liu, Oller,  
 Rusetski, Meissner  
 PRD95(2017)



**Figure 2:** Fit to HadSpec data.



**Figure 3:** Left:  $\pi\eta$  distribution. Data: WA76(1991). Right. Cross section  $\gamma\gamma \rightarrow \pi\eta$ . Data: Belle(2009).



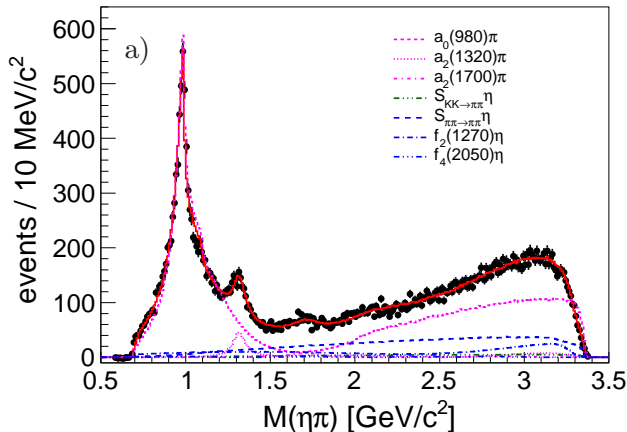
# Is the $a_0(980)$ a threshold effect or a true resonance?

## Large $N_c$ behaviour

“In particular, we have shown that the QCD large  $N_c$  scaling of the unitarized meson-meson amplitudes of chiral perturbation theory is in conflict with a  $\bar{q}q$  nature for the lightest scalars [not so conclusively for the  $a_0(980)$ ]. The  $a_0(980)$  behavior is more complicated. We cannot rule out a possible  $\bar{q}q$  nature, or a sizable mixing], and strongly suggests a  $\bar{q}\bar{q}qq$  or two-meson main component, maybe with some mixing with glue- balls, when possible.”

Pelaez, PRL92(2004)

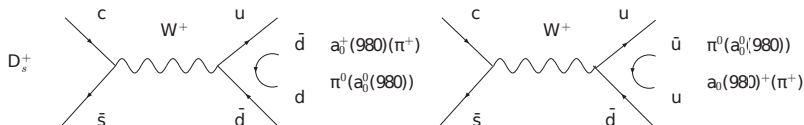
# Amplitude analysis of $\chi_{c1} \rightarrow \eta\pi^+\pi^-$



**Figure 4:** Projections in the (a)  $\eta\pi$ -invariant mass from data, compared with a base-line fit (solid curve) and corresponding amplitudes (various dashed and dotted lines) from PRD95(2017), BESIII.

## BESIII: $D_s^+ \rightarrow \pi^+ \pi^0 \eta$

2019. BESIII has reported the so-called first observation of a pure  $W$ -annihilation decays  $D_s^+ \rightarrow a_0^+(980) \pi^0$  and  $D_s^+ \rightarrow a_0^0(980) \pi^+$



**Figure 5:** Annihilation mechanisms assumed in Ref. for the  $D_s^+ \rightarrow \pi^0 a_0^+(980)$ ,  $\pi^+ a_0^0(980)$ .  $\mathcal{B}[D_s^+ \rightarrow a_0(980)^+ \pi^0, a_0(980)^+ \rightarrow \pi^+ \eta] = (1.46 \pm 0.15 \pm 0.23)\%$

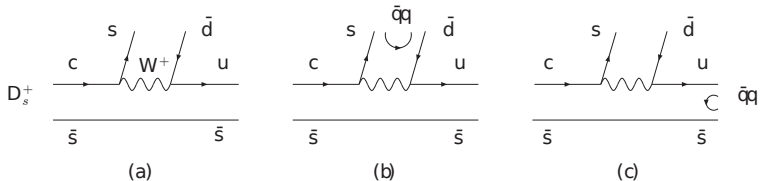
### Topological classification of Weak decays

1. W-external emission
2. W-internal emission
3. W-exchange
4. W-annihilation
5. Horizontal W-loop
6. Vertical W-loop

L.L.Chau. PR(1983), PRD36(1987), PRD39(1989)

(Cabibbo favored) W-external emission?  $D_s^+ \rightarrow \pi^+ \bar{s}s$ , but  $\bar{s}s$  has  $I = 0$ .  
Requires  $f_0(980)$  upon hadronization. **Not good**

# $D_s^+ \rightarrow \pi^+ \pi^0 \eta$ : W-internal emission PLB803(2020)



**Figure 6:**  $D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$ :  $W$  internal emission mechanisms, (a) Primary step; (b) hadronization of the  $s\bar{d}$  pair; (c) hadronization of the  $u\bar{s}$  pair.

## Hadronization

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}, \quad \sum_i s\bar{q}_i q_i \bar{d} = \sum_i M_{3i} \quad M_{i2} = (M^2)_{32},$$

$$\sum_i u\bar{q}_i q_i \bar{s} = \sum_i M_{1i} \quad M_{i3} = (M^2)_{13},$$

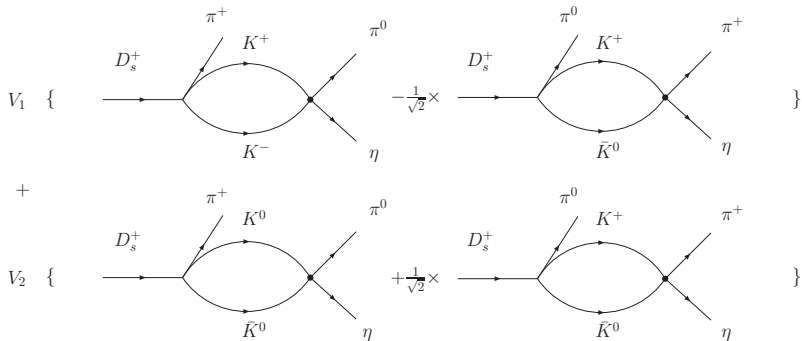
$$(M^2)_{32} = \pi^+ K^- - \frac{1}{\sqrt{2}} \pi^0 \bar{K}^0,$$

$$H_1 = (\pi^+ K^- - \frac{1}{\sqrt{2}} \pi^0 \bar{K}^0) K^+,$$

$$(M^2)_{13} = \frac{1}{\sqrt{2}} \pi^0 K^+ + \pi^+ K^0,$$

$$H_2 = (\frac{1}{\sqrt{2}} \pi^0 K^+ + \pi^+ K^0) \bar{K}^0.$$

$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$



**Figure 7:** Diagrammatic representation of the  $K\bar{K}$  final state interaction of the states  $H_1$  and  $H_2$  leading to  $\pi^+\pi^0\eta$  in the final states.

$$\begin{aligned}
 t = & V_1 [G_{K\bar{K}}(M_{\pi^0\eta}) t_{K^+K^- \rightarrow \pi^0\eta}(M_{\pi^0\eta}) - \frac{1}{\sqrt{2}} G_{K\bar{K}}(M_{\pi^+\eta}) t_{K^+\bar{K}^0 \rightarrow \pi^+\eta}(M_{\pi^+\eta})] \\
 & + V_2 [G_{K\bar{K}}(M_{\pi^0\eta}) t_{K^0\bar{K}^0 \rightarrow \pi^0\eta}(M_{\pi^0\eta}) + \frac{1}{\sqrt{2}} G_{K\bar{K}}(M_{\pi^+\eta}) t_{K^+\bar{K}^0 \rightarrow \pi^+\eta}(M_{\pi^+\eta})] ,
 \end{aligned}$$

$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$

## Chiral Unitary approach

$$T = [1 - VG]^{-1} V$$

Oller, Oset, Pelaez, PRL80(1998)

Xie, Dai, Oset, PLB742(2015)

$q_{\max} = 600 \text{ MeV}$

We obtain,

$$t = \bar{V} \left[ G_{K\bar{K}}(M_{\pi^0\eta}) t_{K\bar{K} \rightarrow \pi\eta}^{I=1}(M_{\pi^0\eta}) - G_{K\bar{K}}(M_{\pi^+\eta}) t_{K\bar{K} \rightarrow \pi\eta}^{I=1}(M_{\pi^+\eta}) \right]$$

with  $\bar{V} = (V_2 - V_1)/\sqrt{2}$ .

Note that, with the isospin multiplets  $(u, d)$ ,  $(-\bar{d}, \bar{u})$ ,

$$|s\bar{d}\rangle = -|1/2, 1/2\rangle$$

$$|u\bar{s}\rangle = |1/2, 1/2\rangle$$

$$|s\bar{d}, u\bar{s}\rangle = -|1, 1\rangle$$

$$|\pi a_0; I=1, I_3=1\rangle = \frac{1}{\sqrt{2}} |\pi^0 a_0^+ - \pi^+ a_0^0\rangle$$

## Isospin

$$t_{K^+K^- \rightarrow \pi^0\eta} = -\frac{1}{\sqrt{2}} t_{K\bar{K} \rightarrow \pi\eta}^{I=1},$$

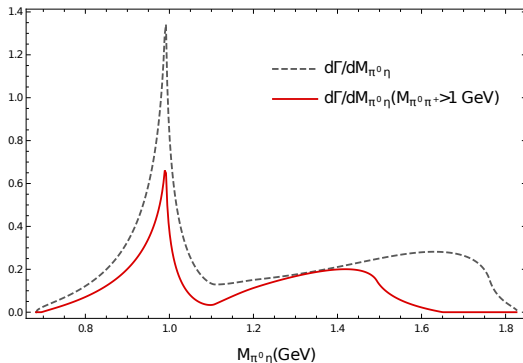
$$t_{K^0\bar{K}^0 \rightarrow \pi^0\eta} = \frac{1}{\sqrt{2}} t_{K\bar{K} \rightarrow \pi\eta}^{I=1},$$

$$t_{K^+\bar{K}^0 \rightarrow \pi^+\eta} = -t_{K\bar{K} \rightarrow \pi\eta}^{I=1},$$

$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$

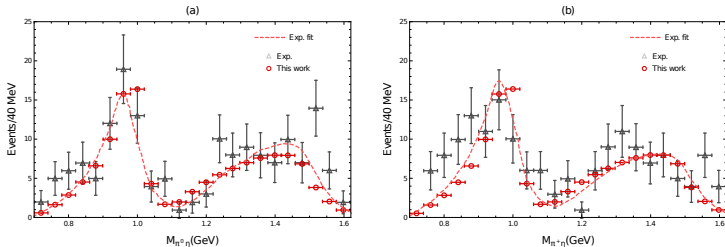
## Invariant mass distribution

$$\frac{d^2\Gamma}{dM_{\pi^0\eta}dM_{\pi^+\eta}} = \frac{1}{(2\pi)^3} \frac{M_{\pi^0\eta}M_{\pi^+\eta}}{8M_{D_s^+}^2} |t|^2$$



**Figure 8:**  $d\Gamma/dM_{\pi^0\eta}$  as a function of  $M_{\pi^0\eta}$ . Dashed line with no  $M_{\pi^+\pi^0}$  restriction. Solid line with the restriction of  $M_{\pi^+\pi^0} > 1$  GeV.

$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$



**Figure 9:** Event distribution in 40 MeV bins of  $d\Gamma/dM_{\pi\eta}$  compared with experiment with  $M_{\pi^+\pi^0} > 1$  GeV. (a) for  $\pi^0\eta$  distribution; (b) for  $\pi^+\eta$  distribution. The dashed lines are taken from [1] after the non  $\pi a_0$  events are removed. **Molina, Xie, Liang, Geng and Oset, PLB803(2020)**

[1] BESIII Collaboration, PRL123(2019)

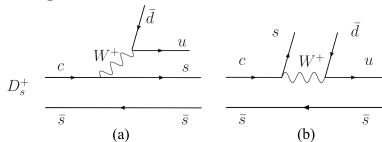
**But...this is not the end of the story!**



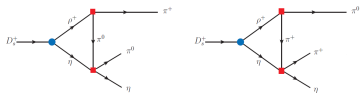
$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$

**Arxiv: 2102.0534, Ling, Liu, Lu, Geng and Xie**

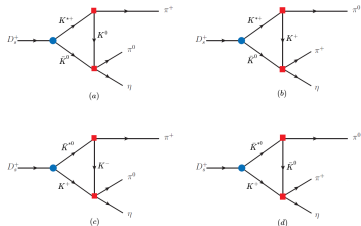
Inspired in the work of Hsiao et al., EPJC80(895), the authors find that both mechanisms, internal and external- $W$  emission through triangle diagrams are relevant.



**Figure 10:** a) External  $W$ -emission mechanism for  $D_s^+ \rightarrow \rho^+ \eta$  and b) internal  $W$ -conversion mechanisms.



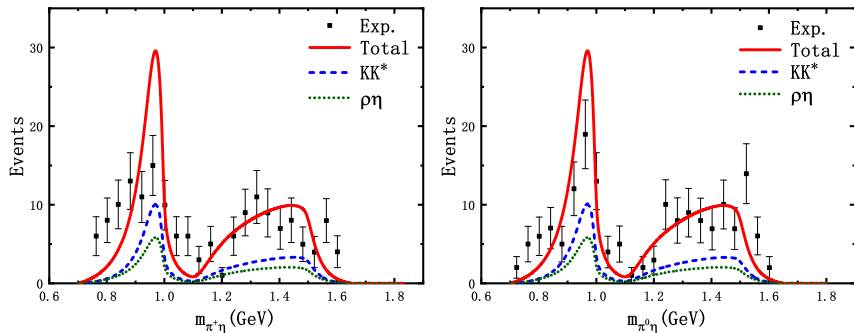
**Figure 11:**  $D_s^+ \rightarrow (\rho^+ \eta \rightarrow) \pi^+ \pi^0 \eta$ .



**Figure 12:** Triangle rescattering diagrams for  $D_s^+ \rightarrow (K^{*0} \bar{K}^0 \rightarrow) \pi^+ \pi^0 \eta$  and  $D_s^+ \rightarrow (K^+ \bar{K}^{*0} \rightarrow) \pi^+ \pi^0 \eta$ .

$$D_s^+ \rightarrow \pi^0 a_0^+(980), \pi^+ a_0^0(980)$$

Arxiv: 2102.0534, Ling, Liu, Lu, Geng and Xie



**Figure 13:** Invariant mass distributions of  $\pi\eta$  with the cut of  $m_{\pi^+\pi^0} > 1$  GeV for the decay  $D_s^+ \rightarrow \pi(a_0(980) \rightarrow \pi^0)\pi^+\eta$ , in comparison with the BESIII data.

## Conclusions

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# Conclusions

- The  $D_s^+ \rightarrow \pi^+ \pi^- \eta$  through the  $a_0(980)$  proceeds via W-internal/external emission and not W-annihilation.
- The new BESIII data and the analysis shown here supports the  $a_0(980)$  as a dynamically generated resonance from the  $\pi\eta$  and  $K\bar{K}$  channels.