

# $V_{cs}$ determination from $D \rightarrow K\ell\nu$

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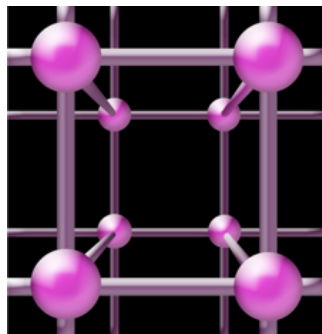
B. Chakraborty, C. Bouchard, C.T.H. Davies, J. Koponen, G.P. Lepage



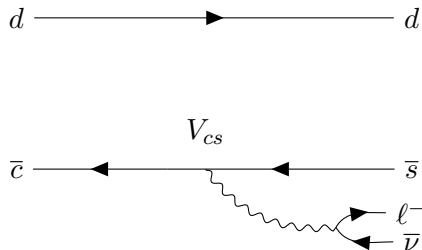
# Overview

I will summarise the results  
in arXiv:2104.09883

- ▶ Motivation.
- ▶ Calculation of hadronic form factors.
- ▶  $D \rightarrow K$  form factor results.
- ▶ Sub 1% accurate determination of  $V_{cs}$ .



# Motivation



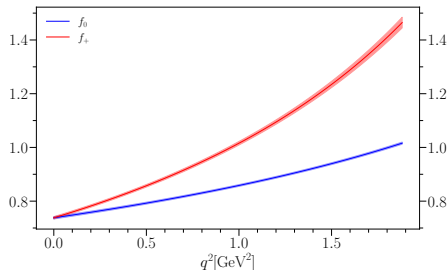
- ▶ Flavour changing weak decays of heavy mesons can test the SM.
- ▶  $D \rightarrow K \ell \nu$  depends on CKM element:  $V_{cs}$ . We calculate hadronic part of decay.
- ▶ CKM unitarity is a good place to look for new physics.
- ▶ We need very precise CKM element determinations to test SM.



# Form factors

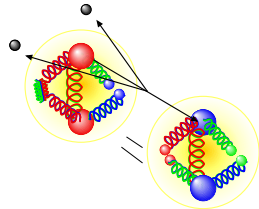
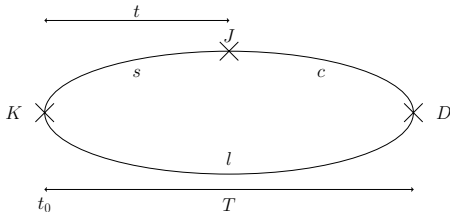
$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} |\vec{p}_K|^3 |f_+(q^2)|^2$$

- ▶ Parameterise the ‘QCD bit’ in the differential decay rate.
- ▶ Interested in  $f_+(q^2)$  and  $f_0(q^2)$  form factors for  $D \rightarrow K$ .
- ▶ Encode meson structure.
- ▶ Describe the shape in  $q^2 = (p_{\text{mother}} - p_{\text{daughter}})^2$  space.



# $D \rightarrow K$ form factors

- ▶ Want meson form factors over the full range of  $q^2$  values.
- ▶ Require three-point correlators with scalar and vector current insertions for  $f_0$  and  $f_+$ .
- ▶ Calculate these using lattice QCD - averaging over gluon fields.



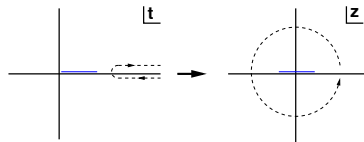
## $D \rightarrow K$ form factors

- ▶ MILC HISQ 2+1+1 ensembles. All valence quarks HISQ.
- ▶ 5 lattice spacings in range 0.15-0.045fm. Three with physical light quark masses - others with heavier light quarks as they are very expensive.
- ▶ Charm mass easy to reach on all ensembles.
- ▶ Cover whole physical  $q^2$  range.



# $D \rightarrow K$ form factors

Convert to  $z$  space to perform  
standard continuum-chiral  
extrapolation, guided by effective  
theories:



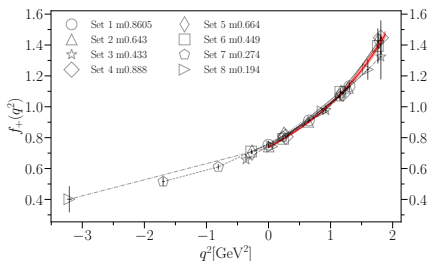
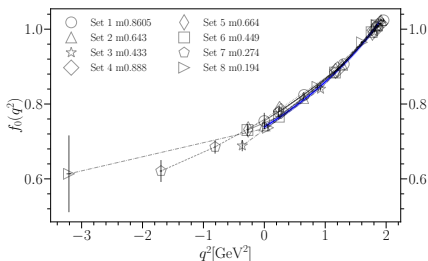
$$N = 3$$

$$f_0(q^2) = \frac{\log s}{1 - \frac{q^2}{M_{D_s^0}^2}} \sum_{n=0}^{N-1} a_n^0 z^n, \quad (1)$$

$$f_+(q^2) = \frac{\log s}{1 - \frac{q^2}{M_{D_s^*}^2}} \sum_{n=0}^{N-1} a_n^+ \left( z^n - \frac{n}{N} (-1)^{n-N} z^N \right).$$



# $D \rightarrow K$ form factor results

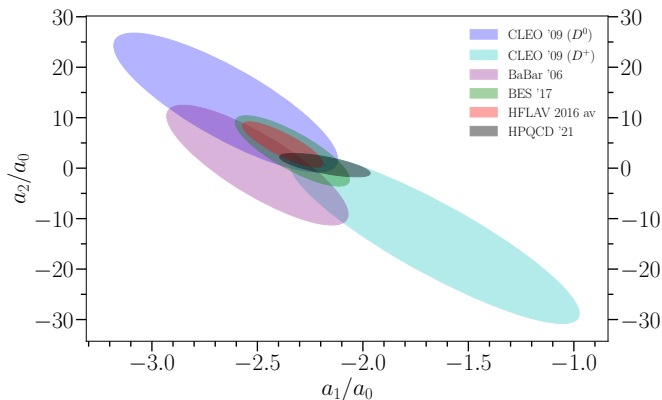


- ▶ We obtain the full  $q^2$  range  $\Rightarrow$  can compare bin by bin with exp. partial decay rate data to extract  $V_{cs}$ .
- ▶ Experiments also use  $z$  expansion, so we can compare coefficients to test SM using shape in  $z$ .





# $D \rightarrow K$ form factor results



Ratios of  $f_+$   $z$  expansion coefficients  $a_n$ , directly compare form factor shape with experiment. 68% confidence ellipses show good agreement



## $D \rightarrow K$ form factor results

Full expression includes previously neglected electroweak and EM corrections, as well as terms in  $\epsilon = \frac{m_\ell^2}{q^2}$ ,

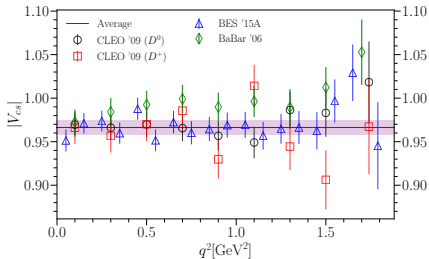
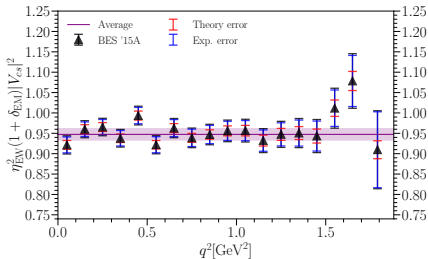
$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 (\eta_{EW} |V_{cs}|)^2}{24\pi^3} (1 - \epsilon)^2 (1 + \delta_{EM}) \times \left[ |\vec{p}_K|^3 \left(1 + \frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\vec{p}_K| M_D^2 \left(1 - \frac{M_K^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right], \quad (2)$$

where  $\eta_{EW} = 1.009(2)$ . We take  $\delta_{EM}$  as a 0.5% (1%) error for  $K^0$  ( $K^\pm$ ). Final state interactions dominate  $\delta_{EM}$  - hence  $K^0$  smaller.

We can use this to extract  $V_{cs}$  in three different ways:



Method 1: Using  $q^2$  binned differential decay rates,

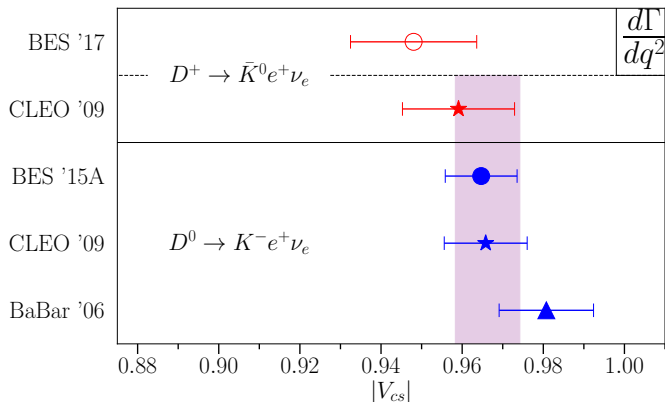


- Experimental error dominates each bin, theory dominates final result.

$$\Delta_i \Gamma = \int_{q_i^2}^{q_{i+1}^2} \frac{d\Gamma}{dq^2} dq^2$$



# $V_{cs}$ results

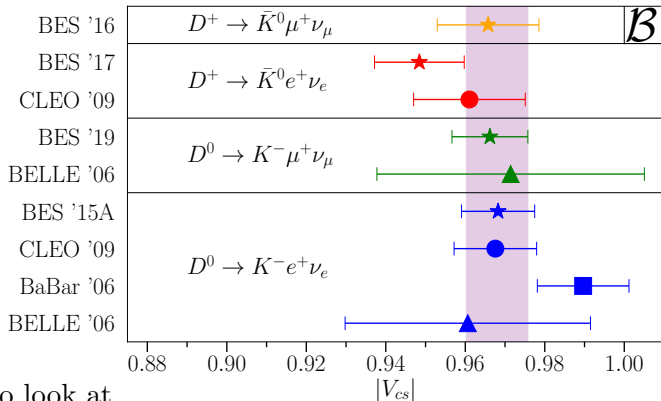


- Preferred method using whole  $q^2$  range and multiple experiments.
- Theory still (just) dominates error, but can also be improved with future binned experimental data (with correlations).



# $V_{cs}$ results

Method 2: Using total branching fraction for all 4 decay modes,



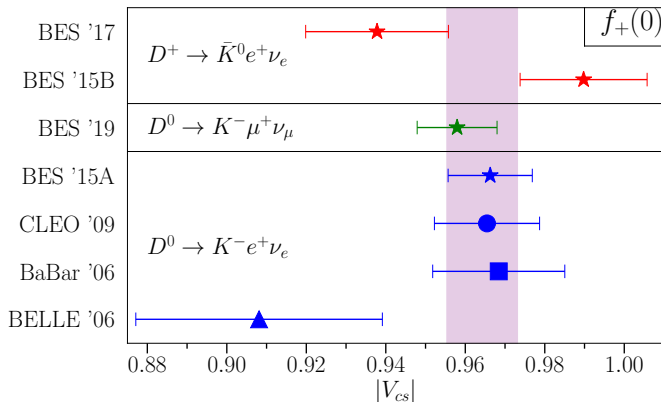
Can also look at

$$R_{\mu/e} = \mathcal{B}_{\mu}/\mathcal{B}_e$$

$$\mathcal{B} = \tau_D \int_0^{q_{\max}^2} \frac{d\Gamma}{dq^2} dq^2$$



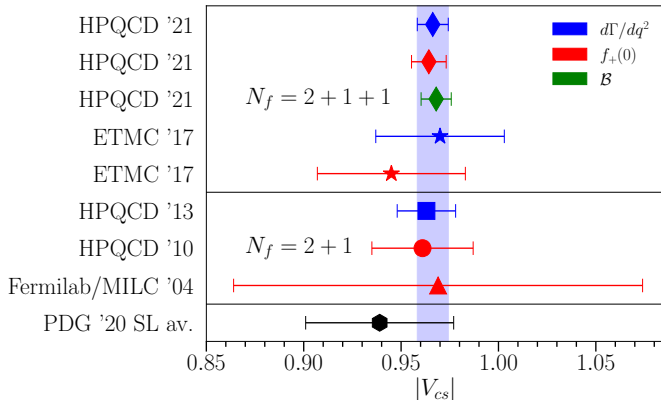
## Method 3: Using $|V_{cs}|f_+(0)$



$$f_+(0)\eta_{EW}\sqrt{(1+\delta_{EM})}|V_{cs}| = 0.7180(33) \quad (HFLAV, arXiv:1612.07233)$$



# $V_{cs}$ results



HPQCD '21 results:

$$|V_{cs}|^{\Gamma} = 0.9663(53)_{\text{latt}}(39)_{\text{exp}}(19)_{\eta_{\text{EW}}}(40)_{\delta_{\text{EM}}}$$

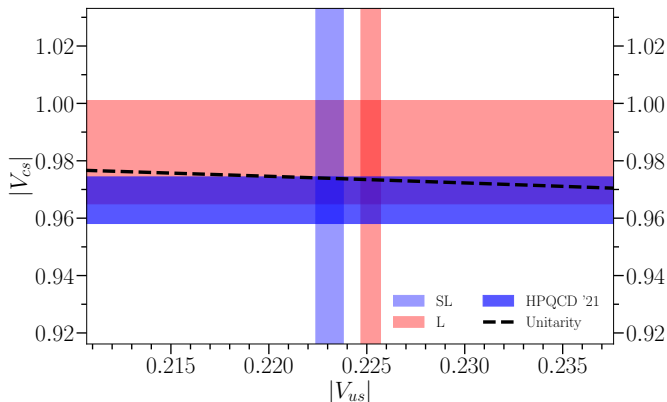
$$|V_{cs}|^{\mathcal{B}} = 0.9680(54)_{\text{latt}}(42)_{\text{exp}}(19)_{\eta_{\text{EW}}}(30)_{\delta_{\text{EM}}}$$

$$|V_{cs}|^{f_+(0)} = 0.9643(57)_{\text{latt}}(44)_{\text{exp}}(19)_{\eta_{\text{EW}}}(48)_{\delta_{\text{EM}}}$$



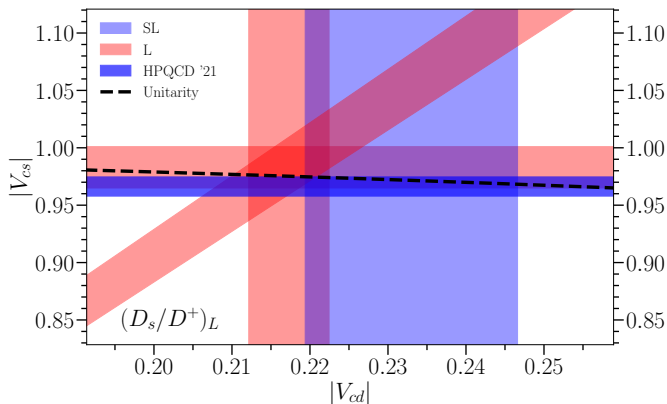
## $V_{cs}$ results

$|V_{cs}| = 0.9663(80)$ , big improvement on current PDG sl value of  $0.939(38)$





# $V_{cs}$ results



Additional constraint from leptonic  $D_s$  and  $D^+$  decays, combined with lattice decay constants.



# Conclusions

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.9826(22)_{V_{cd}}(155)_{V_{cs}}(1)_{V_{cb}}$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 0.9859(2)_{V_{us}}(155)_{V_{cs}}(1)_{V_{ts}}$$

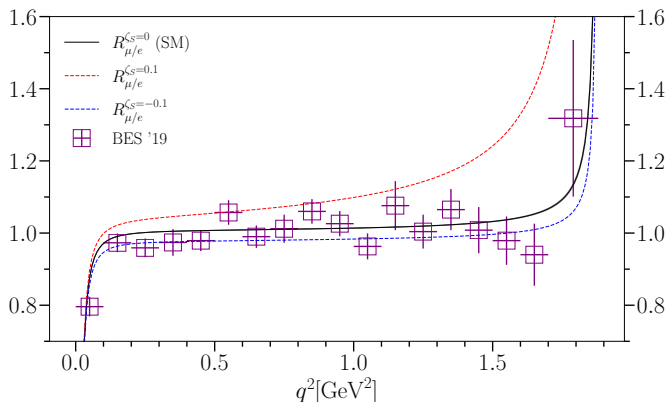
- ▶  $|V_{cs}| = 0.9663(80)$  determination from  $D \rightarrow K\ell\nu$  using bin by bin comparisons with experimental differential decay rate.
- ▶ First determination showing  $V_{cs}$  to be significantly lower than 1 and first sub 1% uncertainty.
- ▶ Agrees well with determinations from 2 other methods.
- ▶ Theory and experimental error similar. EM error also a large contribution.
- ▶ New generation of lattice calculation, more stats, or EM work needed for theory improvement.
- ▶ Can also look for BSM physics in  $R_{\mu/e}$ .

Thanks for listening. Any questions?

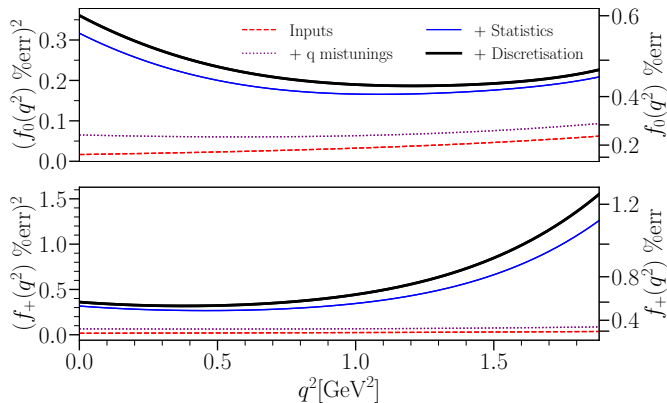


## Extra Slides

$R_{\mu/e} = \frac{\mathcal{B}_{\mu}}{\mathcal{B}_e}$ .  $\zeta$  allows for NP scalar coupling in  $\mu$  which modifies the  $f_0$  coefficient (see arXiv:1502.07488).



# Extra Slides



$$\begin{aligned} Z_V \langle K | V^0 | \hat{H} \rangle = \\ f_+(q^2) \left( p_H^0 + p_K^0 - \frac{M_H^2 - M_K^2}{q^2} q^0 \right) \\ + f_0(q^2) \frac{M_H^2 - M_K^2}{q^2} q^0, \end{aligned} \quad (5)$$

$$\langle K | S | H \rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2), \quad (6)$$

$$Z_T \langle \hat{K} | T^{i0} | \hat{H} \rangle = \frac{2i M_H p_K^i}{M_H + M_K} f_T(q^2), \quad (7)$$



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad (8)$$

$$\frac{m_l}{m_s} \approx \frac{M_\pi^2}{M_{\eta_s}^2} \quad (9)$$

