# In medium Langevin dynamics of heavy particles

## Workshop on Charm Physics

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# Motivation

use heavy quarks and their bound states to probe the strongly coupled medium formed in heavy ion collisions

- high mass of charm (and bottom) quarks and the short formation time of their bound states make them ideal probes of the quark gluon plasma (QGP)
- experimental observables including nuclear modification factor ($R_{AA}$) and elliptic flow ($v_2$) provide insights into nature of QGP
- need to describe nonequilibrium evolution of heavy particles in strongly coupled medium

# Method

### open quantum systems

allows for rigorous treatment of a quantum system of interest (quark/quarkonium) coupled to a bath or reservoir (QGP)

### effective field theories

potential non-relativistic QCD (pNRQCD) to describe the interaction of a non-relativistic bound state (heavy quarkonium) with medium gluons

### Advantages

- minimal assumptions on medium, i.e., strongly ($T \sim m_D \sim gT$) or weakly ($T \gg m_D \sim gT$) coupled
- heavy quark number conserving
- explicitly account for non-Abelian nature of QCD, i.e., singlet and octet states
- account for both dissociation and recombination

# Scales of Problem

## System

- bound state of heavy quark and heavy antiquark characterized by heavy quark mass $M$, Bohr radius $a_0$, binding energy $E$
- for charmonium (bottomonium):

$$M = 1.67\,(4.78)\ \text{GeV}, \quad \frac{1}{a_0} = 0.839\,(1.49)\ \text{GeV}, \quad |E| = 0.421\,(0.461)\ \text{GeV},$$

- hierarchical ordering of scales: $M \gg 1/a_0 \gg E$

## Medium

- QGP characerized by temperature: $(\pi)T \sim (\pi)\mathcal{O}(100)\,\text{MeV}$

## Combined System

- system time scale $\tau_S$, environment time scale $\tau_E$, relaxation time $\tau_R$:

$$\tau_S \sim \frac{1}{E}, \quad \tau_E \sim \frac{1}{\pi T}, \quad \tau_R \sim \frac{1}{\Sigma_s} \sim \frac{1}{a_0^2(\pi T)^3}$$

# Form of Evolution

- **Markov Approximation**: for $\tau_R \gg \tau_E$, state of system independent of history, i.e., evolution equations local in time
- **Quantum Brownian Motion**: for $\tau_S \gg \tau_E$, system realizes quantum Brownian motion
- **Langevin dynamics**:
  - description of Brownian motion, i.e., a particle moving randomly due to uncorrelated interactions with its environment
  - ansatz for description of heavy quarkonium interacting with thermal gluons in the QGP

# Langevin Dynamics

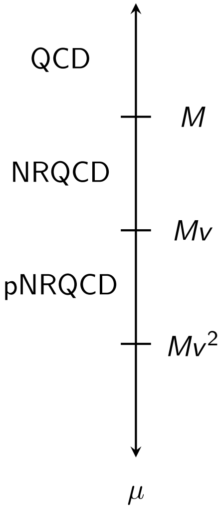
- Langevin equations

$$\frac{\mathrm{d}p_i}{\mathrm{d}t} = -\eta_D p_i + \xi_i(t),\ \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t'),\ \eta_D = \frac{\kappa}{2MT},$$

  where $p_i$ is the momentum of the particle (heavy quark), $\eta_D$ is the drag coefficient, and $\xi_i$ encodes the random, uncorrelated interactions of the particle with the medium
- $\kappa$ is the heavy quark momentum diffusion coefficient
- as shown by Casalderrey-Solana and Teaney, for an in medium heavy quark, integration of force-force correlator along the Schwinger-Keldysh contour gives $\kappa$ in terms of a chromo electric correlator[1]

[1] Phys. Rev. D 74, 085012 (2006).

# potential Non-Relativistic QCD (pNRQCD)




- effective theory of the strong interaction obtained from full QCD via non-relativistic QCD (NRQCD) by successive integrating out of the hard ($M$) and soft ($Mv$) scales
- degrees of freedom are singlet and octet heavy-heavy bound states and ultrasoft gluons
- small bound state radius and large quark mass allow for double expansion in $r$ and $M^{-1}$

# In Medium Evolution Equations

evolution equations given by[2]

$$\frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = -\, i\,[h_s, \rho_s(t)] - \Sigma_s \rho_s(t) - \rho_s(t)\Sigma_s^\dagger + \Xi_{so}(\rho_o(t)),$$

$$\frac{\mathrm{d}\rho_o(t)}{\mathrm{d}t} = -\, i\,[h_o, \rho_o(t)] - \Sigma_o \rho_o(t) - \rho_o(t)\Sigma_o^\dagger + \Xi_{os}(\rho_s(t)) + \Xi_{oo}(\rho_o(t))$$

- $\rho_{s,o}(t)$: density matrix of color singlet, octet bound state
- $h_{s,o} = \frac{\mathrm{p}^2}{M} + V_{s,o}$: singlet, octet Hamiltonian
  - $V_s = -\frac{C_f \alpha_s(1/a0)}{r}$: singlet potential
  - $V_o = \frac{\alpha_s(1/a0)}{2N_c r}$: octet potential
- $\Sigma$, $\Xi$: encode medium interactions in correlators of the form

$$\langle \tilde{E}^{a,j}(0,\mathbf{0}) \tilde{E}^{a,j}(s,\mathbf{0}) \rangle, \quad \tilde{E}^{a,i}(s,\mathbf{0}) = \Omega(s) E^{a,i}(s,\mathbf{0}) \Omega(s)^\dagger,$$

$$\Omega(s) = \exp\left[-ig \int_{-\infty}^{s} \mathrm{d}s' A_0(s',\mathbf{0})\right]$$

[2] Phys. Rev. D 97, 074009 (2018).

# Master Equation

evolution equations can be rewritten as master equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H,\rho(t)] + \sum_{n,m} h_{nm}\left(L_i^n \rho(t) L_i^{m\dagger} - \frac{1}{2}\left\{L_i^{m\dagger}L_i^n, \rho(t)\right\}\right),$$

where

$$\rho(t) = \begin{pmatrix} \rho_s(t) & 0 \\ 0 & \rho_o(t) \end{pmatrix}, \quad H = \begin{pmatrix} h_s + \mathrm{Im}(\Sigma_s) & 0 \\ 0 & h_o + \mathrm{Im}(\Sigma_o) \end{pmatrix},$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i, \quad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}, \quad L_i^2 = \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} \\ 1 & 0 \end{pmatrix} r^i,$$

$$L_i^3 = \begin{pmatrix} 0 & \frac{1}{\sqrt{N_c^2-1}} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}, \quad h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$A_i^{uv} = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s\, e^{-ih_u s} r^i e^{ih_v s} \langle \tilde{E}^{a,j}(0,\mathbf{0}) \tilde{E}^{a,j}(s,\mathbf{0}) \rangle$$

# Lindblad Form

- for $(\pi)T \gg E$, i.e., $e^{-ih_{s,o}s} \approx 1$, medium interactions encoded in transport coefficients

$$\kappa = \frac{g^2}{6N_c}\int_0^\infty \mathrm{d}t \Big\langle \left\{ E^{a,i}(t,0), E^{a,i}(0,0) \right\} \Big\rangle,$$
$$\gamma = -\frac{ig^2}{6N_c}\int_0^\infty \mathrm{d}t \Big\langle \left[ E^{a,i}(t,0), E^{a,i}(0,0) \right] \Big\rangle$$

- as shown by Casalderrey-Solana and Teaney, $\kappa$ is the heavy quark momentum diffusion coefficient occurring in a Langevin equation[3]; $\gamma$ is its dispersive counterpart
- evolution equation can be written as Lindblad equation

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -i[H(t),\rho] + \sum_n \left( C_i^n \rho(t) C_i^{n\dagger} - \frac{1}{2}\left\{ C_i^{n\dagger} C_i^n, \rho(t) \right\} \right)$$

[3] Phys. Rev. D 74, 085012 (2006).

# Langevin Form

- taking $e^{-ih_{s,o}s} \approx 1 - ih_{s,o}s$, medium interactions take more complicated form as Hamiltonian term gives rise to terms suppressed by $E/T$, i.e.,

$$A_i^{uv} = \frac{r_i}{2}(\kappa - i\gamma) + \left(-\frac{ip_i}{2MT} + \frac{\Delta V_{uv} r_i}{4T}\right)\kappa$$

- evolution equation can no longer be written as a Lindblad equation without introducing subleading corrections
- following the procedure of Blaizot and Escobedo[4], we project the evolution equations onto eigenstates of the bound state radius $\langle r|$ and $|r'\rangle$ corresponding to the radius pre- and post-, respectively, interaction with the medium; we work in the system of coordinates

$$\mathrm{r}_+ = \frac{\mathrm{r} + \mathrm{r}'}{2}, \quad \mathrm{r}_- = \mathrm{r} - \mathrm{r}'$$

[4] JHEP 06 (2018) 034.

# Scaling

- the projected evolution parameters depend on the operators/quantities $r_+$, $r_-$, $\nabla_+$, $\nabla_-$, $V_{s,o}$, $\kappa$, and $\gamma$
- we assign a scaling to extract leading order evolution
  - bound state is Coulombic

$$r_+ \sim 1/\sqrt{EM}, \quad \nabla_+ \sim \sqrt{EM}$$

  - potential scales as the binding energy

$$V_{s,o} \sim E$$

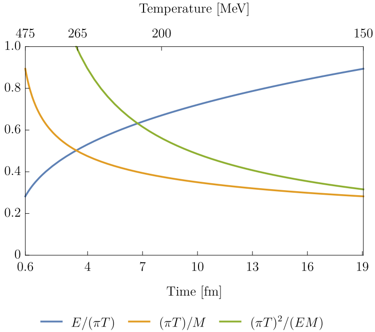
  - $\kappa$, $\gamma$ are thermal quantities of dimension 3

$$\kappa,\ \gamma \sim (\pi T)^3$$

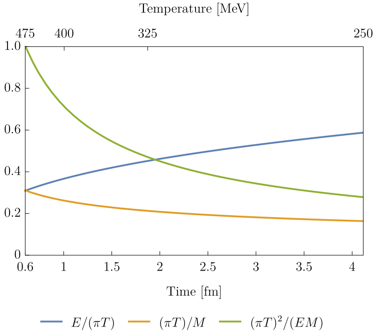
  - interaction with medium thermalizes bound state

$$r_- \sim 1/\sqrt{\pi TM}, \quad \nabla_- \sim \sqrt{\pi TM}$$

# Evolution of Scales




Evolution of relevant scales for charmonium assuming Bjorken evolution.




Evolution of relevant scales for bottomonium assuming Bjorken evolution.

# Leading Order Evolution

- as $M \gg \pi T \gg E$, there are two small parameters in which to expand; for $(\pi T)/M \sim E/(\pi T)$, the leading order evolution operators are of order $\pi T$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\rho_s^{\mathbf{rr}'}\\ \rho_o^{\mathbf{rr}'}\end{pmatrix} = \begin{pmatrix} -r_+^2\kappa & \frac{1}{N_c^2-1}r_+^2\kappa \\ r_+^2\kappa & -\frac{1}{N_c^2-1}r_+^2\kappa \end{pmatrix}\begin{pmatrix}\rho_s^{\mathbf{rr}'}\\ \rho_o^{\mathbf{rr}'}\end{pmatrix} + \cdots,$$

  where $\rho_{s,o}^{\mathbf{rr}'} = \langle \mathrm{r}|\rho_{s,o}(t)|\mathrm{r}'\rangle$ and the ellipsis indicates terms suppressed by addition powers of $(\pi T)/M \sim E/(\pi T)$
- evolution matrix has eigenvalues

$$\{\lambda_0,\, \lambda_8\} = \left\{0, -r_+^2\kappa\frac{N_c^2}{N_c^2-1}\right\}$$

# Corrections to Leading Order Evolution I

- à la Blaizot and Escobedo, move to basis in which LO evolution is diagonal

$$\rho_0 = \frac{\rho_s + \rho_o}{N_c^2}, \quad \rho_8 = \frac{(N_c^2 - 1)\rho_s - \rho_o}{N_c^2},$$

- include terms suppressed by powers of $(\pi T)/M \sim E/(\pi T)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\rho_0^{\mathbf{rr}'}\\ \rho_8^{\mathbf{rr}'}\end{pmatrix} = \begin{pmatrix}\ell_{00}^{(1)} + \ell_{00}^{(2)} & \ell_{08}^{(1)} + \ell_{08}^{(2)}\\ \ell_{80}^{(1)} + \ell_{80}^{(2)} & \ell_{88}^{(0)} + \ell_{88}^{(1)} + \ell_{88}^{(2)}\end{pmatrix}\begin{pmatrix}\rho_0^{\mathbf{rr}'}\\ \rho_8^{\mathbf{rr}'}\end{pmatrix} + \cdots,$$

where superscripts in parenthesis indicate degree of suppression in $\sqrt{(\pi T)/M} \sim \sqrt{E/(\pi T)}$ with respect to LO evolution and the ellipsis indicates further suppressed terms

# Corrections to Leading Order Evolution II

- evolution matrix has eigenvalues $\{\lambda_0',\, \lambda_8'\}$ which reduce to $\{\lambda_0,\, \lambda_8\}$ in limit $(\pi T)/M \sim E/(\pi T) \to 0$
- $\lambda_0'$ given by

$$\lambda_0' = \ell_{00}^{(1)} + \ell_{00}^{(2)} - \frac{\ell_{08}^{(1)}\,\ell_{80}^{(1)}}{\ell_{88}^{(0)}} + \cdots\,,$$

where

$$\ell_{00}^{(1)} = \frac{2i}{M}\nabla_+ \cdot \nabla_- - \frac{i}{2}\mathrm{r}_+ \cdot \mathrm{r}_-\gamma, \quad \ell_{00}^{(2)} = -\kappa\left(\frac{\mathrm{r}_- \cdot \nabla_-}{2MT} + \frac{\mathrm{r}_-^2}{4}\right),$$

$$\ell_{08}^{(1)} = -\frac{1}{N_c^2 - 1}\frac{i}{2}\mathrm{r}_+ \cdot \mathrm{r}_-\gamma, \quad \ell_{80}^{(1)} = -\frac{i}{2}\mathrm{r}_+ \cdot \mathrm{r}_-\gamma, \quad \ell_{88}^{(0)} = -\frac{N_c^2}{N_c^2 - 1}\mathrm{r}_+^2\kappa$$

# Fokker Planck Equation

Wigner transforming the evolution equation of the state evolved by $\lambda_0'$ gives the Fokker Planck equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_+\right) \tilde{\rho}_0(t) = \Bigg[\frac{\kappa}{4}\nabla_{\mathbf{p}}^2 + \frac{M}{2}\eta \nabla_{\mathbf{p}} \cdot \mathbf{v} + \frac{\gamma}{2}\mathbf{r}_+ \cdot \nabla_{\mathbf{p}} + \left(\frac{\gamma}{\sqrt{\kappa}} \frac{\mathrm{r}_+ \cdot \nabla_{\mathbf{p}}}{2N_c|\mathrm{r}_+|}\right)^2\Bigg]\tilde{\rho}_0(t),$$

where $\tilde{\rho}_0(t)$ is the Wigner transform of the state evolved by $\lambda_0'$, v is the relative velocity of the quark and antiquark and $\mathrm{p} = M\mathrm{v}/2$

# Langevin Equation

the corresponding Langevin equations are

$$\frac{\mathrm{d}\mathsf{r}_i^+}{\mathrm{d}t} = \frac{2\mathsf{p}_i}{M}, \quad \frac{M}{2}\frac{\mathrm{d}^2 r_i^+}{\mathrm{d}t^2} = -F_i(\mathsf{r}^+) - \eta_{ij} p_j + \xi_i(t, \mathsf{r}^+) + \theta_i(t, \mathsf{r}^+).$$

where

- $\langle \xi_i(t, \mathsf{r}^+) \xi_j(t', \mathsf{r}^+) \rangle = \delta(t - t') \delta_{ij} \, \kappa$: $\xi_i$ encodes random, uncorrelated interactions with medium; $\kappa$ is heavy quark momentum diffusion coefficient
- $\eta_{ij}(\mathsf{r}^+) = \frac{\kappa}{2MT} \delta_{ij}$: Einstein relation between $\kappa$ and drag coefficient $\eta$
- $\langle \theta_i(t, \mathsf{r}^+) \theta_j(t', \mathsf{r}^+) \rangle = \delta(t - t') \frac{r_i^+ r_j^+ \gamma^2}{4 N_c^2 \kappa \, r_+^2}$: $\theta_i$ is second random force due to fluctuations in force between quark and antiquark which are, on average, 0
- $F_i(\mathsf{r}^+) = -\gamma \frac{r_i^+}{2}$: correction to quark-antiquark potential

# Summary and Future Directions

- heavy quarkonia and their bound states are excellent probes of QGP formed in HICs
- heavy quark mass $M$ and scale hierarchy $1/a_0 \gg \pi T$make problem ideally suited for use of pNRQCD; furthermore, $M \gg \pi T$ makes Langevin equation natural candidate for description of dynamics
- evolution equations of in medium Coulombic bound state depend on chromo electric-electric correlators; in limit $\pi T \gg E$, correlectors reduce to linear combination of $\kappa$ and $\gamma$ [5]
- inclusion of higher order corrections à la Blaizot and Escobedo[6] allows for derivation of Langevin equation containing $\kappa$ from first principles
- future work: similar calculation for single quark

[5] Phys. Rev. D 97, 074009 (2018).
[6] JHEP 06 (2018) 034.

Thank you!