Medium evolution of a static quark-antiquark pair in the large N_c limit

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Outline

1. Introduction
2. Non-relativistic Effective Field Theories
3. The static limit
4. Conclusions

Heavy quarkonium in heavy-ion collisions

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- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

Quarkonium as an Open quantum system

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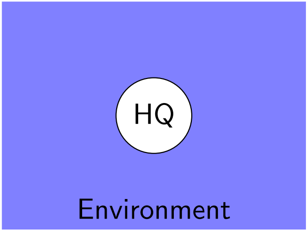

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

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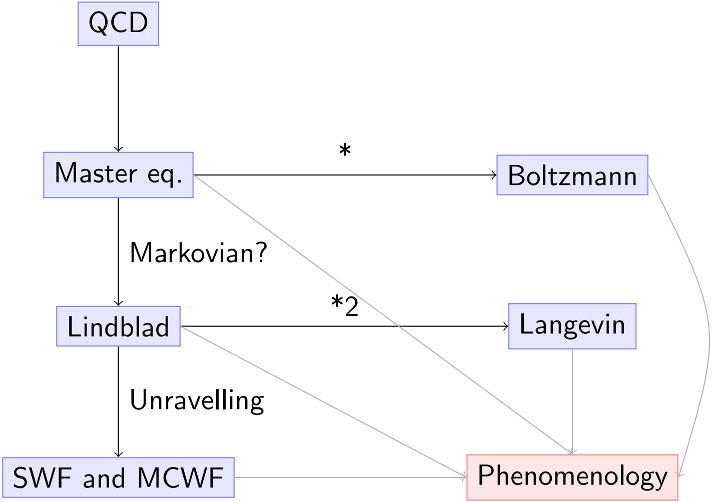
In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

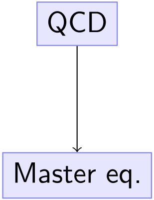
Roadmap for OQS approach to quarkonium suppression



* Thermal effects are slow compared to the inverse of the binding energy.

*2 Heavy quarks have a well-defined (classical) position.

Motivation



- The master equation has been derived using Hard Thermal Loop perturbation theory and pNRQCD in the $\frac{1}{r} \gg T$.

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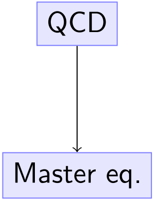

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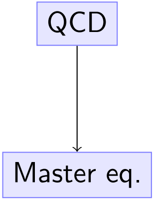

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- Can we learn something using the large N_c limit?

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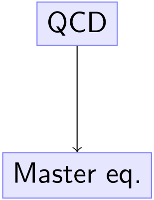

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- Can we say something about the regime $\frac{1}{r} \sim T$? Incorporating non-perturbative information?
- Can we learn something using the large N_c limit?
- We can try to get some insights from studying the static limit.

Plan

The use of Effective Field Theories to study heavy quarks

Reminder

- The mass of a heavy quark m is much bigger than Λ_{QCD}. The production or annihilation of heavy quarks is a perturbative process.
- The temperature T of the medium is much smaller than m.
- In the case of quarkonium, other energy scales appear. The inverse of the typical radius $\frac{1}{r} \sim mv$ and the binding energy $E \sim mv^2$.

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Effective Field Theories

The appearance of different and very separated energy scales in a system can be a problem.

- Breaking of naive perturbation theory.
- All the relevant scales need to fit in the lattice. Large lattices, small lattice step.

This can be solved using EFTs.

Integrating out the heavy quark mass

- Integrating out the scale m can be useful both to study heavy quark diffusion and quarkonium suppression.
- This step can always be done perturbatively and is not affected by the presence of the medium. $m \gg \Lambda_{QCD}, T$.

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Classification of gluons

- Hard gluons, with energy and momentum of order m.
- Soft gluons, with energy and momentum of order mv.
- Potential gluons, with energy of order mv^2 and momentum of order mv.
- Ultrasoft gluons, with energy and momentum of order mv^2.

NRQCD

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{d_2}{m_Q^2} F^a_{\mu\nu} D^2 F^{\mu\nu a} + d_g^3 \frac{1}{m_Q^2} g f_{abc} F^a_{\mu\nu} F^{\mu b}_{\alpha} F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + c_2 \frac{\mathrm{D}^2}{2m_Q} + c_4 \frac{\mathrm{D}^4}{8m_Q^3} + c_F g \frac{\sigma \mathrm{B}}{2m_Q} + c_D g \frac{\mathrm{DE}-\mathrm{ED}}{8m_Q^2} \right.$$
$$\left. + i c_S g \frac{\sigma(\mathrm{D}\times\mathrm{E}-\mathrm{E}\times\mathrm{D})}{8m_Q^2} \right) \psi$$

$$\mathcal{L}_\chi = c.c \text{ } of \text{ } \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2} \psi^\dagger \chi \chi^\dagger \psi + \frac{f_1(^3S_1)}{m_Q^2} \psi^\dagger \sigma \chi \chi^\dagger \sigma \psi + \frac{f_8(^1S_0)}{m_Q^2} \psi^\dagger T^a \chi \chi^\dagger T^a \psi$$
$$+ \frac{f_8(^3S_1)}{m_Q^2} \psi^\dagger T^a \sigma \chi \chi^\dagger T^a \sigma \psi$$

potential NRQCD Lagrangian at T=0

Brambilla, Pineda, Soto and Vairo, NPB566 (2000) 275

Starting from NRQCD and integrating out the scale $\frac{1}{r}$.

$$\mathcal{L}_{pNRQCD} = \int d^3\mathrm{r}\, Tr\left[S^\dagger\left(i\partial_0 - h_s\right)S\right.$$
$$\left.+O^\dagger\left(iD_0 - h_o\right)O\right] + V_A(r)Tr(O^\dagger \mathrm{r}g\mathrm{E}S + S^\dagger \mathrm{r}g\mathrm{E}O)$$
$$+\frac{V_B(r)}{2}Tr(O^\dagger \mathrm{r}g\mathrm{E}O + O^\dagger O\mathrm{r}g\mathrm{E}) + \mathcal{L}_g + \mathcal{L}_q$$

- Degrees of freedom are singlet and octets.
- Allows to obtain manifestly gauge-invariant results. Simplifies the connection with Lattice QCD.
- If $1/r \gg T$ we can use this Lagrangian as starting point. In other cases the matching between NRQCD and pNRQCD will be modified.

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Strategy

- We are going to use NRQCD in the static limit.

$$\mathcal{L}_{NRQCD} = \mathcal{L}_{QCD} + \psi^\dagger iD_0\psi + \chi^\dagger iD_0\chi$$

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- We study the evolution of operators that transform like singlet or octets (interpolating fields).
- We expect these fields to be close to the *real* pNRQCD singlet and octets. The precise matching needs to be worked out in the future.

Birdtrack notation

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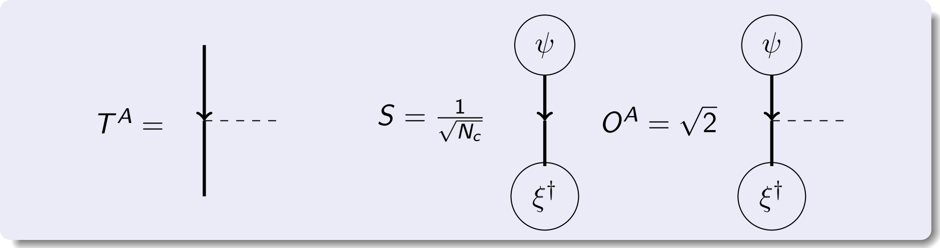

The evolution equations

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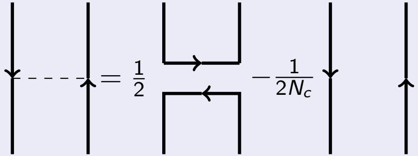
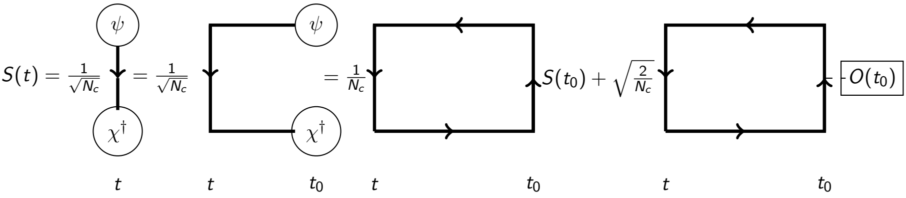
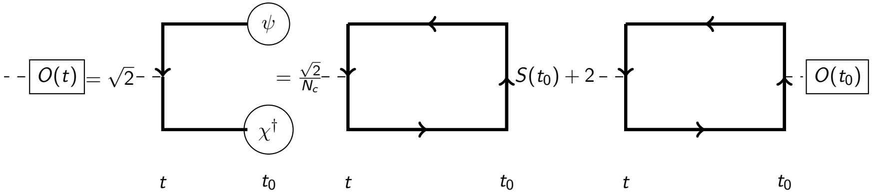
- We can use the Fierz identity



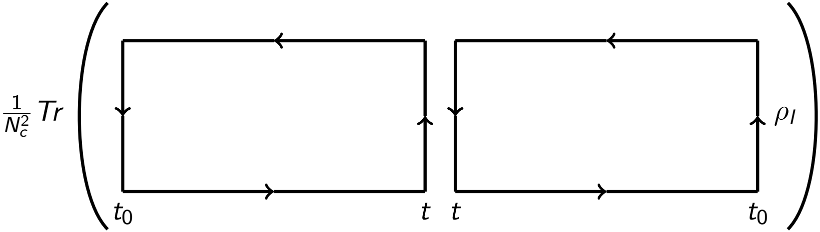
The backwards evolution of the singlet and the octet





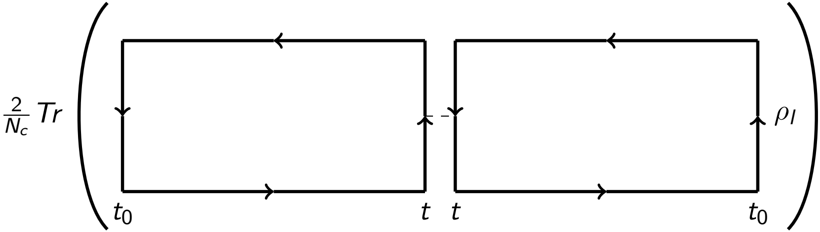
Singlet to singlet transition

Probability of finding a singlet at time t if there was a singlet at time t_0.



Singlet to octet transition

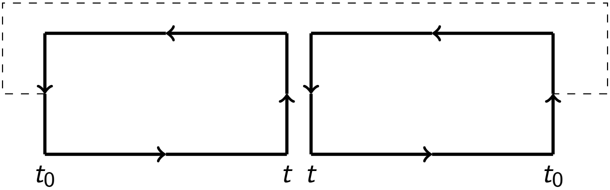
Probability of finding an octet at time t if there was a singlet at time t_0.



Octet to singlet transition

Probability of finding a singlet at time t if there was an octet at time t_0.

$\frac{2}{N_c(N_c^2-1)} Tr\Bigg($



$\tilde{\rho}_I \Bigg)$

Octet to octet transition

Probability of finding an octet at time t if there was an octet at time t_0.

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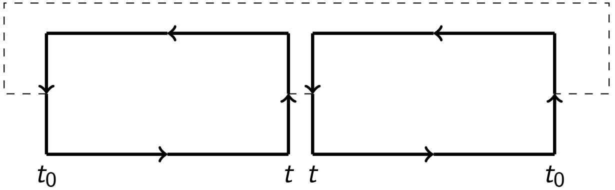

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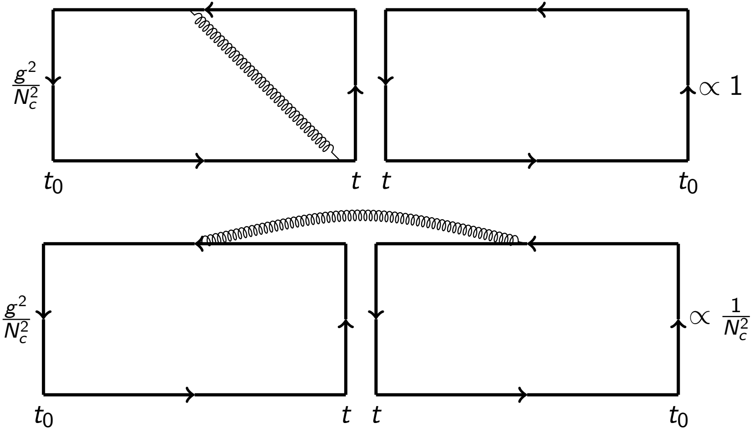
Singlet to singlet transition in the large N_c limit



Singlet to singlet transition in the large N_c limit

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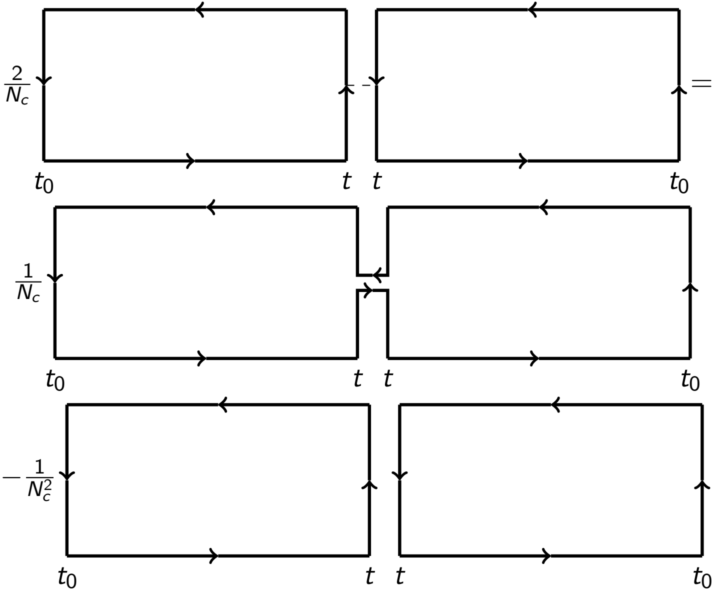
Singlet to singlet transition in the large N_c limit

- Diagrams in which a gluon connects Wilson lines in different paths of the Schwinger-Keldysh contour are suppressed.
- At large N_c, what we get is

$$Tr(W_{SS}^{\dagger}(\mathrm{R},\mathrm{r};t,t_0)W_{SS}(\mathrm{R},\mathrm{r};t,t_0)\rho_I) = \left|Tr(W_{SS}(\mathrm{R},\mathrm{r};t,t_0)\rho_I)\right|^2$$

These results suggest that, in this approximation, the survival probability of a singlet can be encoded in an effective Hamiltonian. $Tr(W_{SS}(\mathrm{R},\mathrm{r};t,t_0)\rho_I)$ is the Wilson loop from which the static potential is obtained in lattice QCD computations (Burnier and Rothkopf (2016)).

Singlet to octet transition in the large N_c limit



Singlet to octet transition in the large N_c limit

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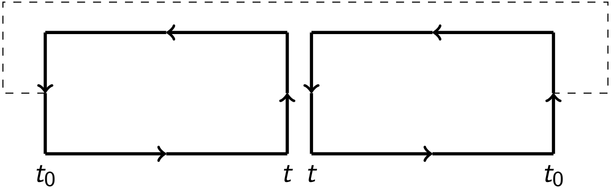
Singlet to octet transition in the large N_c limit

- Intuitively, the probability to get an octet is the probability to get anything minus the probability to get a singlet. This is the Fierz identity.
- Both terms are of the same size. The first term is a quadrupole (expectation value of four Wilson lines) and the second term is the square of a dipole.

Octet to singlet transition in the large N_c limit

$\frac{2}{N_c(N_c^2-1)}$



$=$

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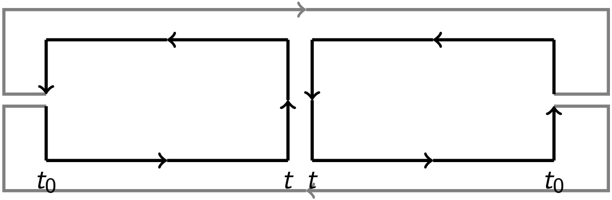
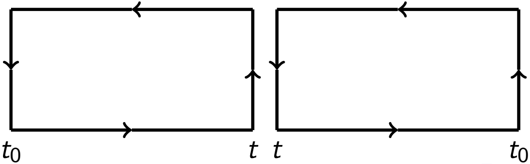


$-\frac{1}{N_c^2(N_c^2-1)}$



$= \mathcal{O}\left(\frac{1}{N_c^2}\right)$

Octet to singlet transition in the large N_c limit

- Qualitatively similar to the singlet to octet transition.

Octet to singlet transition in the large N_c limit

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- Now, both terms are a factor of $\frac{1}{N_c^2}$ suppressed.

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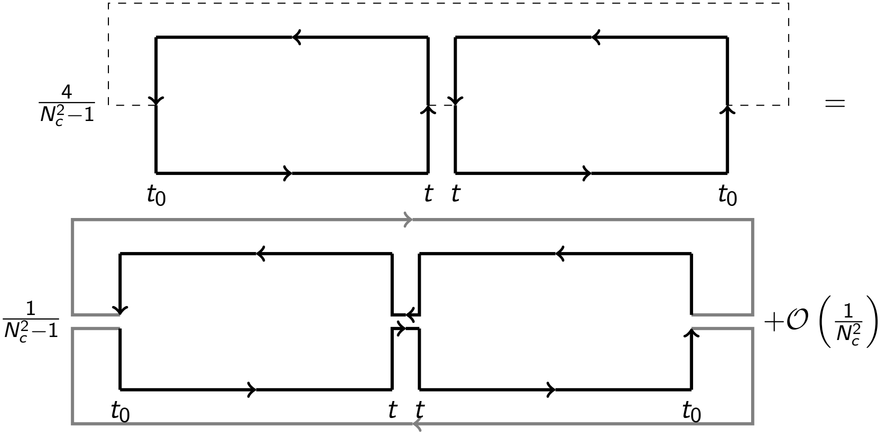

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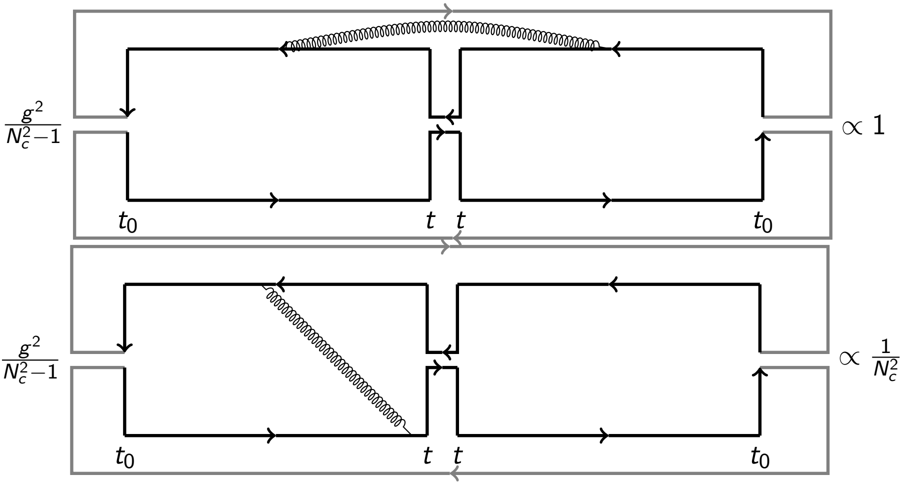

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- Terms involving gluons connecting the quark with the antiquark are suppressed. Therefore, the octet evolves similarly to a pair of uncorrelated particles.

Evolution equations

$$D_s(t) = Sd(t-t_0)D_s(t_0) + \frac{Iq(t-t_0) - Sd(t-t_0)}{N_c^2-1}D_o(t_0)$$

$$D_o(t) = (Q(t-t_0) - Sd(t-t_0))D_s(t_0) + \frac{N_c^2 Qd(t-t_0) - Q(t-t_0) - Iq(t-t_0) + Sd(t-t_0)}{N_c^2-1}D_o(t_0)$$

Sd, Iq, Q and Qd are each of the Wilson loops that appear after applying the Fierz identity.

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- The singlet decays into octets with an evolution that can be encoded in an effective Hamiltonian.
- The octet evolves like a pair of uncorrelated particles plus a source coming of the decay of singlets.
- This in maintained until we reach the point in which $D_s \sim \frac{1}{N_c^2}$.

Evolution equations in the large N_c limit when $D_s \sim \frac{1}{N_c^2}$ and $D_o \sim 1$

$$D_s(t) = Sd(t-t_0)D_s(t_0) + \frac{Iq(t-t_0) - Sd(t-t_0)}{N_c^2-1}D_o(t_0)$$

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Plan

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- Regarding the octet, they evolve approximately like a pair of uncorrelated particle. Support for the molecular chaos hypothesis?