Charm quark transport in a viscous QGP : colliding and radiating

(arXiv : 2105.14296)

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10th INTERNATIONAL WORKSHOP ON CHARM PHYSICS

Outline

1. Introduction
2. Heavy quark (HQ) transport coefficients (in ideal QGP)
   - Collision and radiative energy loss of HQ
3. Deviation from ideal QGP
   - Thermal medium interaction and EQPM (Effective fugacity Quasi-particle Model)
   - Viscous hydrodynamics corrections to the in-medium particle distribution functions
4. Results
5. Summary and Outlook

Introduction

What is Quark-Gluon Plasma (QGP)?

- Deconfined phase of quarks and gluons as effective degrees of freedom.
- Formed in heavy-ion collision at time scale of ~ 1 fm.
- Exist at high temperature and/or high baryon density.

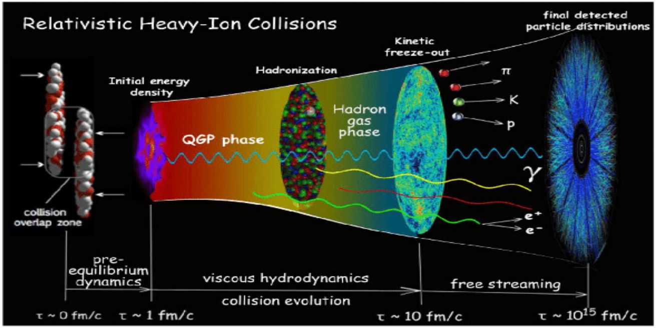


Image source: Nuclear Physics News, Vol. 25, No. 2, 2015

Introduction

Why study heavy quarks (charm and bottom)?

- Heavy quark mass : $m_{HQ} >> \Lambda_{QCD}$ ($\sim$ 200 MeV)
- Small coupling : perturbative expansion in α_s, non-relativistic treatment of bound state.
  $\alpha_s^2(m_c = 1.3\text{ GeV}) \approx 0.3$
  $\alpha_s^2(m_b = 4.5\text{ GeV}) \approx 0.1$

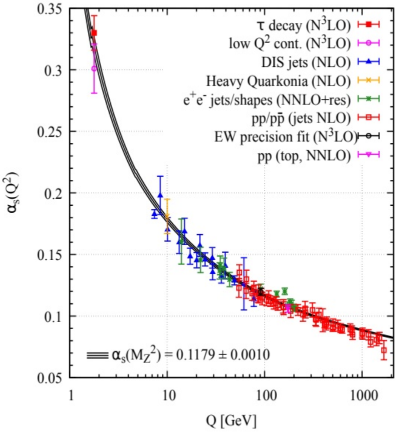


Image source: pdg.lbl.gov/2019/reviews/rpp2019-rev-qcd

Introduction

What about heavy quarks in QGP?

- Hard energy scale : $(m_Q) >> T$
- Formed in initial stage at time scale of $\sim 0.02 - 0.07$ fm
- Witness entire evolution of QGP : From creation to hadronization
- Important probes to study the QCD medium properties

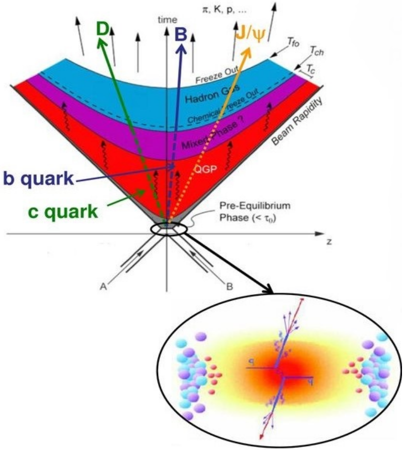


Image source: F. Prino talk, EMMI-RRTF workshop, GSI, 2016

HQ transport

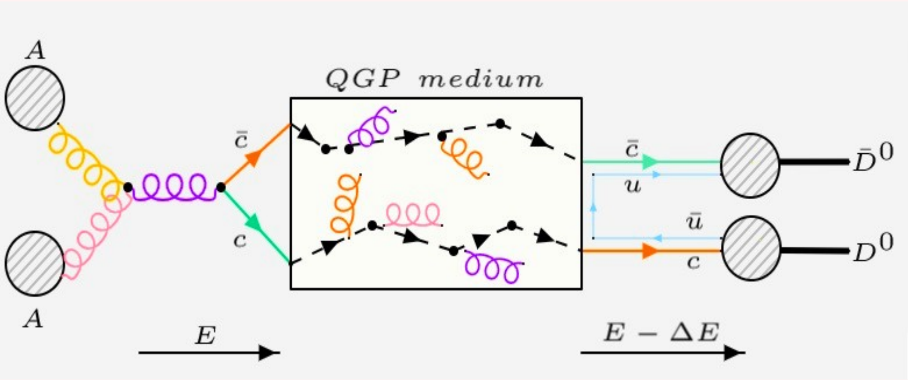
Energy loss and transport coefficients

- Charm quark moving in QGP loose its energy.
  1. Collision ($2 \rightarrow 2$): Elastic scattering with the medium constituents
  2. Radiation ($2 \rightarrow 3$): Medium induced gluon emission

- Transport coefficients in small momentum transfer limit.
  1. **Drag**
     $\rightarrow$ Resistance to the motion by plasma particles
     i.e. light quarks (u, d, s), anti-quarks $(\bar{u}, \bar{d}, \bar{s})$ and gluons (g).
     $\rightarrow$ LO coefficient in the series expansion of soft momentum transfer
  2. **Diffusion**
     $\rightarrow$ Diffusion in momentum space along transverse and longitudinal direction.
     $\rightarrow$ NLO coefficient in the the series expansion of soft momentum transfer

HQ transport (Ideal QGP)

Charm quark fate within ideal QGP



HQ transport (Ideal QGP)

Collisional energy loss

- Non-equilibrated heavy quark traversing equilibrated plasma.
- Brownian motion of heavy quark within QGP medium
- *Boltzmann transport equation* for the phase space density $f(\mathbf{x}, \mathbf{p}, t)$ of the heavy quark.
- Homogeneous plasma ($\partial f / \partial \mathbf{x} = \mathbf{0}$) with no external force ($\mathbf{F} = 0$)

B. Svetitsky, Phys. Rev. D, 37(9), 1988

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E_p}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}\right) f(\mathbf{x}, \mathbf{p}, t) = \left(\frac{\partial f}{\partial t}\right)_{col} \implies \frac{\partial f(\mathbf{p}, t)}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{col}$$

- *Landau's soft scattering approximation* (small momentum transfer) $\implies$ *Fokker-Planck equation*

$$\frac{\partial f}{\partial t} \approx \frac{\partial}{\partial p_i}\left(A_i(\mathbf{p}) f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})] f\right)$$

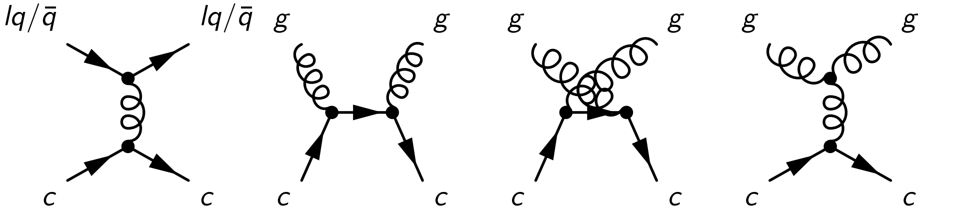
HQ transport (Ideal QGP)

Collisional energy loss

- $HQ(p) \; + \; lq/l\bar{q}/g(q) \rightarrow HQ(p') \; + \; lq/l\bar{q}/g(q')$

$$\rightarrow \textbf{Drag :} \; A_i = \frac{1}{2E_p} \int \frac{d^3q}{(2\pi)^3(2E_q)} \int \frac{d^3q'}{(2\pi)^3(2E_{q'})} \int \frac{d^3p'}{(2\pi)^3(2E_{p'})} \frac{1}{\gamma_c}$$
$$\times \sum |\mathcal{M}_{2\rightarrow 2}|^2 (2\pi)^4 \delta(p + q - p' - q') f(\mathbf{q})[1 \pm f(\mathbf{q}')] \; [(p - p')_i]$$

$$A_i \equiv \frac{1}{2} \langle (p - p')_i \rangle$$



HQ transport(Ideal QGP)

Collisional energy loss

- A_i and B_{ij} depends only on the initial momentum $(\mathbf{p})$

$$A_i = p_i A(p^2) \implies A(p^2) = \frac{p_i A_i}{p^2} = \langle \mathbf{1} \rangle - \frac{\langle \mathbf{p} \cdot \mathbf{p}' \rangle}{p^2}$$

$$\rightarrow \textbf{Diffusion :}\ B_{ij} \equiv \frac{1}{2} \langle (p - p')_i (p - p')_j \rangle$$

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_0(p^2) + \left(\frac{p_i p_j}{p^2} \right) B_1(p^2)$$

$$B_0(p^2) = \frac{1}{2} \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{4} \left[\langle p'^2 \rangle - \frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} \right]$$

$$B_1(p^2) = \left(\frac{p_i p_j}{p^2} \right) B_{ij} = \frac{1}{2} \left[\frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} - 2 \langle \mathbf{p} \cdot \mathbf{p}' \rangle + p^2 \langle \mathbf{1} \rangle \right]$$

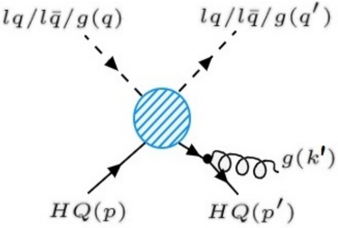
HQ transport (Ideal QGP)

Radiative energy loss

- Soft gluon emission by the charm quark induced by the QGP medium (after scattering by light quarks, antiquarks and gluons).
- $HQ(p)\ +\ lq/l\bar{q}/g(q) \rightarrow HQ(p')\ +\ lq/l\bar{q}/g(q')\ +\ g(k')$
- For soft gluon emission, $k' = (E_{k'}, \mathbf{k}'_\perp, k'_z) \rightarrow 0$,

$$|\mathcal{M}|^2_{2\rightarrow 3} = |\mathcal{M}|^2_{2\rightarrow 2} * \frac{12 g_s^2}{{k'_\perp}^2}\left(1 + \frac{m_{HQ}^2}{s} e^{2y_{k'}}\right)^{-2}$$

R. Abir et al., Phys. Rev. D 85, 054012 (2012)



HQ transport (Ideal QGP)

Radiative energy loss

$$X(p) = \int (\text{phase space}) \times (\text{interaction}) \times (\text{transport})$$

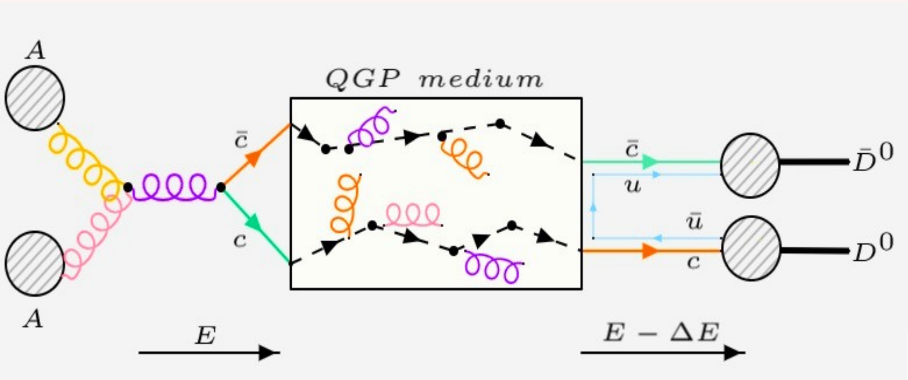
S. Mazumdar et al., Phys. Rev. D, 89 (014002), 2014

$$X_{col}(p) = \frac{1}{2E_p\gamma_{HQ}} \int \frac{d^3q}{(2\pi)^3(2E_q)} \int \frac{d^3q'}{(2\pi)^3(2E_{q'})} \int \frac{d^3p'}{(2\pi)^3(2E_{p'})}$$
$$\times (2\pi)^4 \delta(p + q - p' - q')\; f(\mathbf{q})\; [1 \pm f(\mathbf{q}')]$$
$$\times \sum |\mathcal{M}|^2_{2\to 2} \times \; F(p)$$

$$X_{rad}(p) = \frac{1}{2E_p\gamma_{HQ}} \int \frac{d^3q}{(2\pi)^3(2E_q)} \int \frac{d^3q'}{(2\pi)^3(2E_{q'})} \int \frac{d^3p'}{(2\pi)^3(2E_{p'})}$$
$$\times \int \frac{d^3k'}{(2\pi)^3(2E_{k'})} (2\pi)^4 \delta(p + q - p' - q' - k')\; f(\mathbf{q})\; [1 \pm f(\mathbf{q}')]\; [1 + f(\mathbf{k}')]$$
$$\times\, \theta(\tau - \tau_F)\; \theta(E_p - E_{k'}) \times \sum |\mathcal{M}|^2_{2\to 3} \times \; F(p)$$

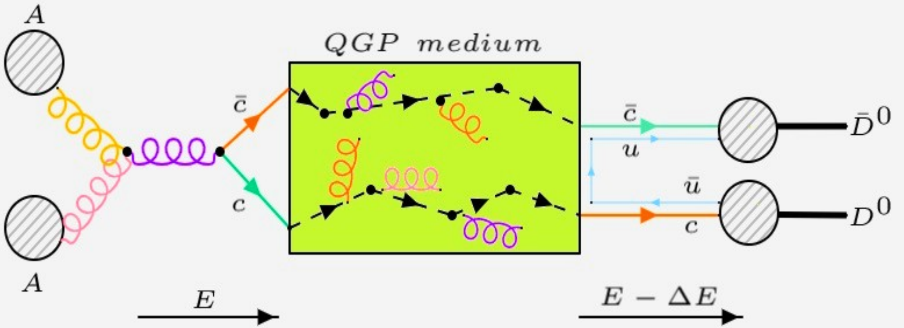
HQ transport (Ideal QGP)

Charm quark fate within ideal QGP



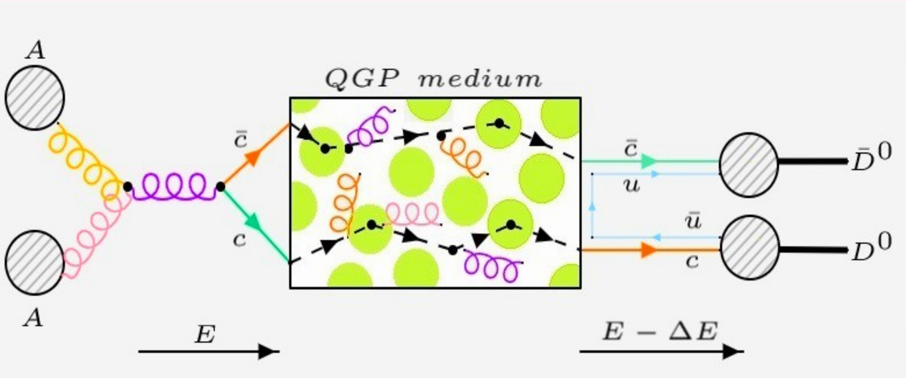
Deviation from Ideal QGP

Charm quark fate within thermally interacting QGP



Deviation from Ideal QGP

Charm quark fate with EQPM



Deviation from ideal QGP

Interacting QGP and EQPM

- **EQPM** : **E**ffective fugacity **Q**uasi-**P**article **M**odel

  V. Chandra et al., Phys. Rev. C, 76 (054909), 2007

- In-medium interactions of QGP encoded into particle : quasi-particle
  $\rightarrow$ Model based on mapping of the EoS with lattice QCD EoS
- Introduction of temperature dependent effective fugacity z_k in the distribution functions of quasi-particles $k \equiv (lq, l\bar{q}, g)$.

$$f_k^0 = \frac{z_k \exp\{-\beta\, E_k\}}{1 \pm z_k \exp\{-\beta\, E_k\}}$$

- Quasi-particle dispersion relation: $\tilde{q}_k{}^\mu = q_k^\mu + \delta\omega_k\, u^\mu$
- Collective excitations of quasi-partons: $\delta\omega_k = T^2\, \partial_T\{\ln(z_k)\}$

$$\omega_{q/g} = E_{q/g} + T^2\, \partial_T\{\ln(z_k)\}$$

Deviation from ideal QGP

Interacting QGP and EQPM

- Temperature dependence of effective fugacity $z_{k=q,g}$ is parametrized using (2+1) flavor LQCD EoS at T_c =170 MeV as follows,

$$z_k = a_k \ \exp\left\{-\frac{b_k}{(T/T_c)^2} - \frac{c_k}{(T/T_c)^4} - \frac{d_k}{(T/T_c)^6}\right\}$$

- Effective strong coupling constant $\alpha_{s(eff)}$ is introduced through EQPM based Debye mass.

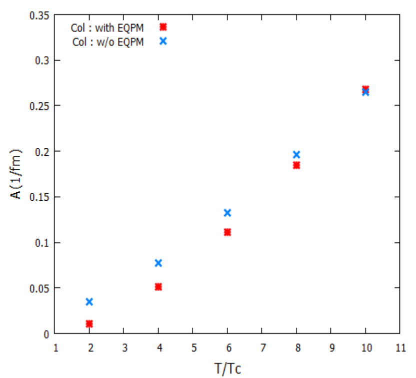
S. Mitra et al., Phys. Rev. D, 96 (094003), 2017

- The effective QCD coupling constant in EQPM is,

$$\alpha_{s(eff)}(T) = \alpha_s(T)\frac{\left\{\frac{2N_c}{\pi^2}\text{PolyLog}[2, z_g] - \frac{2N_f}{\pi^2}\text{PolyLog}[2, -z_q]\right\}}{\left\{\frac{N_c}{3} - \frac{N_f}{6}\right\}}$$

Deviation from ideal QGP

Interacting QGP and EQPM



Deviation from ideal QGP

Viscous hydrodynamic corrections to the distribution functions

- Viscous hydrodynamic evolution of QGP to study its transport properties with EQPM using covariant kinetic theory approach.

  S. Bhadury et al., J. Phys. G 47 (2020) 8, 085108

- Near local thermal equilibrium, the quasi-parton distribution function becomes,

$$f_k = f_k^0 + \delta f_k \text{ where } \delta f_k / f_k^0 << 1$$

- Energy-momentum tensor for the dissipative (viscous) hydrodynamics,

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Considering longitudinal boost-invariant Bjorken expansion of QGP.
  $\rightarrow$ In the fluid rest frame $\implies u^\mu \equiv (1, \overrightarrow{0})$

Deviation from ideal QGP

Viscous hydrodynamic corrections to the distribution functions

- Relativistic Boltzmann equation with RTA for quasiparticles,

$$\tilde{q}_k^{\mu}\,\partial_{\mu} f_k(x, \tilde{q}_k) + F_k^{\mu}\,(u\cdot\tilde{q}_k)\,\partial_{\mu}^{(q)} f_k = -\,(u\cdot\tilde{q}_k)\,\frac{\delta f_k}{\tau_R}$$

- Viscous corrections contribute to the thermal distribution function as,

A. Shaikh et al., arXiv : 2105.14296

$$\delta f_k = f_k^0(1 \pm f_k^0)\{\phi_k(bulk) + \phi_k(shear)\}$$

$$\phi_k(bulk) = \frac{s}{\beta_{\Pi}\omega_k T\tau}\left(\frac{\zeta}{s}\right)\left[\omega_k^2 c_s^2 - \frac{|\overrightarrow{q_k}|^2}{3} - \omega_k \delta\omega_k\right]$$

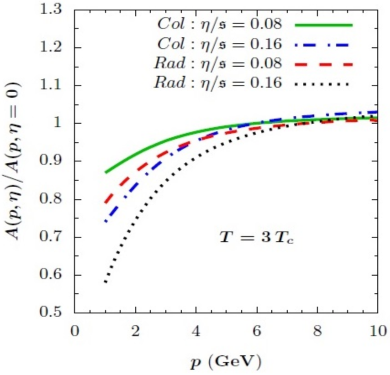
$$\phi_k(shear) = \frac{s}{\beta_{\pi}\omega_k T\tau}\left(\frac{\eta}{s}\right)\left[\frac{|\overrightarrow{q_k}|^2}{3} - (q_k)_z^2\right]$$

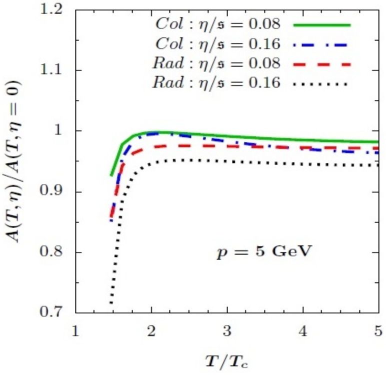
Results (Shear)

Drag coefficient $A(p, T)$

$$N_c = 3, \quad N_f = 3, \quad m_{lq} = 0, \quad \mu_{lq} = 0,$$

$$m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$





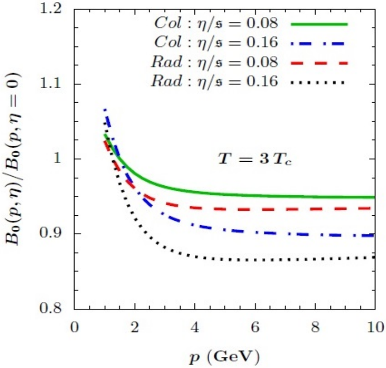
* Similar trend in the past: **Das, Chandra, Alam (2014)** and **Singh, Mishra (2019)**

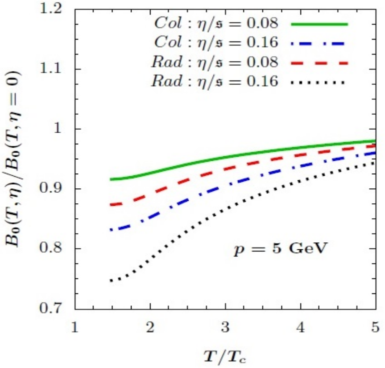
Results (Shear)

Transverse momentum diffusion coefficient $B_0(p, T)$

$$N_c = 3, \quad N_f = 3, \quad m_{lq} = 0, \quad \mu_{lq} = 0,$$

$$m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$



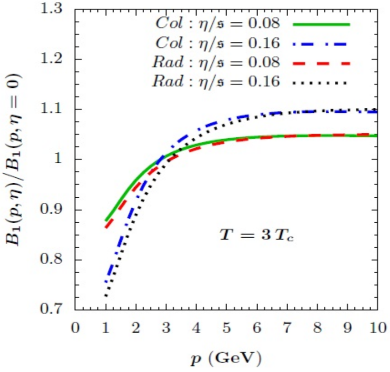


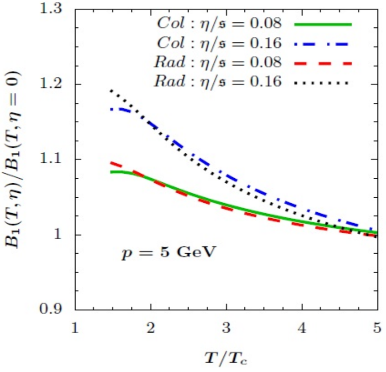
Results (Shear)

Longitudinal momentum diffusion coefficient $B_1(p, T)$

$$N_c = 3, \quad N_f = 3, \quad m_{lq} = 0, \quad \mu_{lq} = 0,$$

$$m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

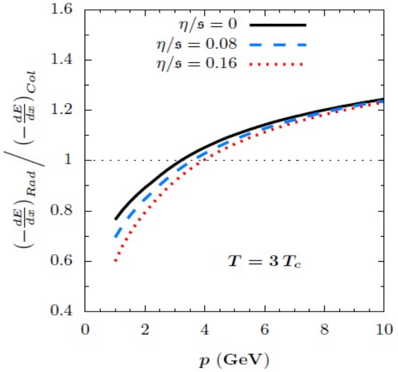


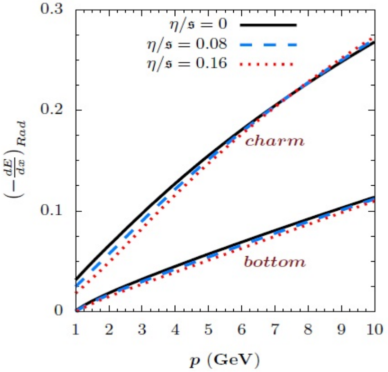


Results (Shear)

Differential energy loss

$$-\frac{dE}{dx} = p\,A(p,T)$$



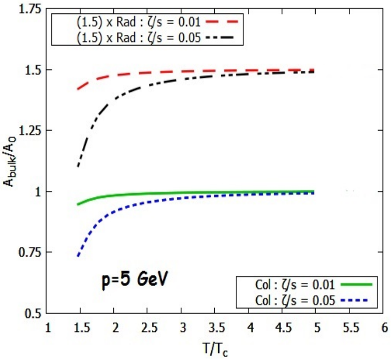


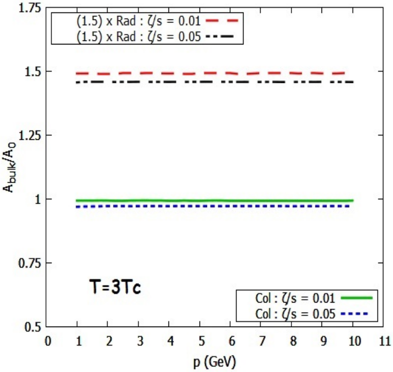
Preliminary Results (Bulk)

Drag coefficient $A(p, T)$

$$N_c = 3, \quad N_f = 3, \quad m_{lq} = 0, \quad \mu_{lq} = 0,$$

$$m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$





Summary and Outlook

1. Heavy quark transport coefficient have been studied for collisional and radiative processes in a thermally interacting viscous QGP.
2. The thermal medium interactions are incorporated using EQPM and the viscosity effects as leading order correction to the distribution function of QGP particles using covariant kinetic theory.
3. Shear viscosity effects are prominent for slow moving charm and at low temperatures.
4. Inclusion of (shear) viscosity affect the drag and the transverse momentum diffusion more for radiative process in comparison to the collision.
5. Collisional energy loss is dominant at $p \lesssim 3$ GeV and radiative energy loss at $p \gtrsim 3$ GeV.
6. Bottom quark radiation suppressed due to dead-cone effect.
7. Bulk viscous corrections to the HQ transport coefficients needs further analysis...

$\mathscr{Thank\ you}$!