Revisiting the Charm-meson lifetimes

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Lifetimes and ratios: experiment

- Lifetimes of charmed hadrons are measured precisely in experiments

[PDG-2021], [BESIII, 2104.07311]*

	D^0	D^+	D_s^+
$\tau[\mathrm{ps}]$	0.4101(15)	1.040(7)	0.504(4)
$\Gamma_{\mathrm{tot}}[\mathrm{ps}^{-1}]$	2.44(1)	0.96(1)	1.98(2)
$\frac{\tau(D_X)}{\tau(D^0)}$	1	2.54(2)	1.20(1)
$\mathrm{Br}(D_X \to X e^+ \nu_e)[\%]$	6.49(11)	16.07(30)	6.30(16)*
$\frac{\Gamma(D_X \to X e^+ \nu_e)}{\Gamma(D^0 \to X e^+ \nu_e)}$	1	0.977(26)	0.790(26)*

Lifetimes and ratios: theory

- The total width of a hadron H is given by

$$\Gamma(H) = \frac{1}{2m_H} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_H - p_X) \, |\langle X(p_X)|\mathcal{H}_{\rm eff}|H(p_H)\rangle|^2$$

Optical Theorem

$$= \frac{1}{2m_B} \,\mathrm{Im} \, \langle H(p_H)| \, i \int d^4x \, T\left\{\mathcal{H}_{\rm eff}(x), \mathcal{H}_{\rm eff}(0)\right\} |H(p_H)\rangle$$

- The lifetimes ratio $\tau(H_1)/\tau(H_2)$

$$\frac{\tau(H_1)}{\tau(H_2)} = \frac{\Gamma_Q + \delta\Gamma_{H_2}}{\Gamma_Q + \delta\Gamma_{H_1}} = 1 + \left(\delta\Gamma_{H_2} - \delta\Gamma_{H_1}\right) \tau(H_1)$$

- May be sensitive to New Physics contributions

$$\frac{\tau(H_1)}{\tau(H_2)} = \frac{\Gamma_Q + \delta\Gamma_{H_2}}{\Gamma_Q + \delta\Gamma_{H_1}} = 1 + \left(\delta\Gamma^{\rm SM}_{H_2} - \delta\Gamma^{\rm SM}_{H_1}\right) \tau(H_1) + \left(\delta\Gamma^{\rm NP}_{H_2} - \delta\Gamma^{\rm NP}_{H_1}\right) \tau(H_1)$$

HQE: Bottom vs Charm

- HQE: $p_Q = m_Q v + k, \quad k \sim \Lambda \qquad v = p_H/m_H$
- HQE in the bottom sector

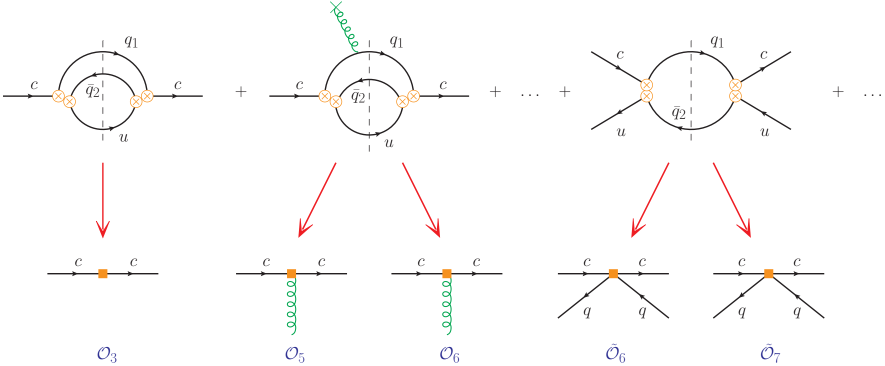
$$\frac{\Lambda}{m_b} \sim 0.12 \qquad \alpha_s(m_b) \sim 0.22, \quad \frac{\alpha_s(m_b)}{\pi} \sim 0.07$$

- HQE in the charm sector

$$\frac{\Lambda}{m_c} \sim 0.39 \qquad \alpha_s(m_c) \sim 0.38, \quad \frac{\alpha_s(m_c)}{\pi} \sim 0.12$$

- Convergence of HQE for charm-hadron lifetimes ?

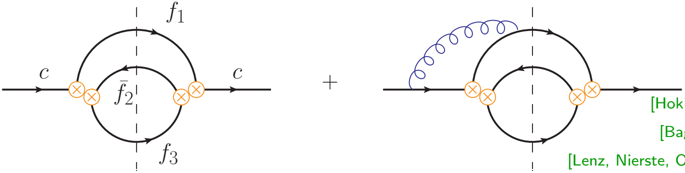
HQE: diagrams



$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right]$$

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \ldots$$

Dimension-3 contribution



[Hokim, Pham (1984)]
[Bagan et al. (1994)]
[Lenz, Nierste, Ostermaier (1997)]
[Greub, Liniger (2000, 2001)]
[Krinner, Lenz, Rauh (2013)]

- Determine dimension-3 contribution $\Gamma_3 = \Gamma_0\, c_3$

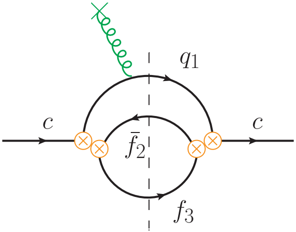
$$c_3 = c_3^{s\bar{d}u} + c_3^{s\bar{s}u} + c_3^{d\bar{d}u} + c_3^{d\bar{s}u} + c_3^{se\bar{\nu}_e} + c_3^{s\mu\bar{\nu}_\mu} \qquad \Gamma_0 = \frac{G_F^2 m_c^5}{192\pi^2}$$

- Perturbative expansion in α_s

$$c_3 = c_3^{(0)} + \frac{\alpha_s}{4\pi} c_3^{(1)} + \ldots$$

- Universal for all heavy hadrons with a c-quark
- There is no non-perturbative parameter at dimension-3

Dimension-5 two-quark contributions



[Bigi et al. (1992)]

[Blok, Shifman (1993)]

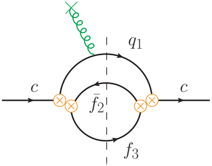
- Determine power corrections for two-quark operators Γ_5
- Non-perturbative parameters $\langle \mathcal{O}_5 \rangle$ arise:
  - ▷ Kinetic $\mu_\pi^2(D)$ and chromo-magnetic $\mu_G^2(D)$
- Now dependent on the spectator quark in D-meson
- Γ_5 for semi-leptonic modes known at NLO

  [Becher, Boos, Lunghi (2007), Alberti et al. (2013, 2014), Mannel, Pivovarov, Rosenthal (2015)]
- Γ_5 for non-leptonic modes known at LO [Blok, Shifman (1993)]

Dimension-6 Darwin operator contribution

- Darwin parameter

$$2m_H\,\rho_D^3(H) = \langle H(p_H)|\bar{Q}_v(iD_\mu)(iv\cdot D)(iD^\mu)Q_v|H(p_H)\rangle$$



- In the bottom decays: $m_b \sim m_c \gg \Lambda_{\rm QCD}$
  - ▷ Known for semi-leptonic modes upto NLO

    [Gremm, Kapustin (1996)], [Mannel, Pivovarov (2019)]
  - ▷ Recently determined for non-leptonic modes at LO

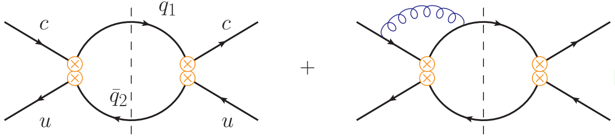
    [Lenz, Piscopo, AR (2020)], [Mannel, Moreno, Pivovarov (2020)]

- In the charm decays: $m_c \gg m_s \sim \Lambda_{\rm QCD}$
  - ▷ One cannot simply adopt results from bottom to charm decays (!)

    [see also talk by Matteo Fael on Wednesday]
  - ▷ Determined for semi-leptonic modes at LO

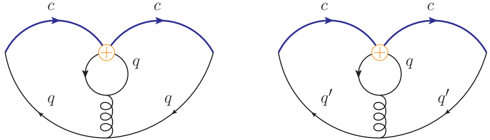
    e.g. [Gambino, Kamenik (2010)]
    [Fael, Mannel, Vos (2019)]
  - ▷ Results for non-leptonic modes at LO are new!

Four-quark contributions



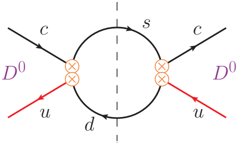
[Uraltsev (1996)]
[Neubert, Sachrajda (1996)]
[Beneke et al. (2002)]
[Franco et al. (2002)]

- Determine $\tilde{\Gamma}_6, \tilde{\Gamma}_7$ starting at one-loop
- Non-perturbative input $\langle \tilde{\mathcal{O}}_6 \rangle$, $\langle \tilde{\mathcal{O}}_7 \rangle$
  - ▷ Dim-6: 4 Bag parameters $B_{1,2}$, $\varepsilon_{1,2}$
  - ▷ Dim-7: 12 Bag parameters ρ_i, σ_i $(i = 1, \ldots 6)$ (in HQET)
- New determination of dim-6 Bag parameters from HQET SR both for D^+- and D_s^+-mesons (incl. so-called eye-contractions)

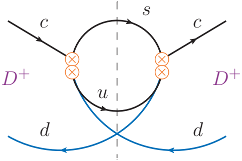


[King, Lenz, Rauh (in progress)]

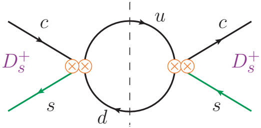
Dominant four-quark contributions



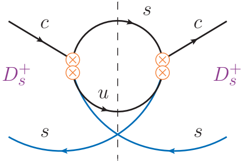
$\sim B_1 - B_2 \approx 0$ (helicity suppression)



Large! (no helicity suppression)



CKM-dominant,
helicity suppressed



CKM-suppressed,
no helicity suppression

Lifetimes and their ratios: theory status

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right]$$

$$\tau(D^+_{(s)})/\tau(D^0) = 1 + \left[\Gamma^{\rm HQE}(D^0) - \Gamma^{\rm HQE}(D^+_{(s)}) \right] \tau^{\rm exp}(D^+_{(s)})$$

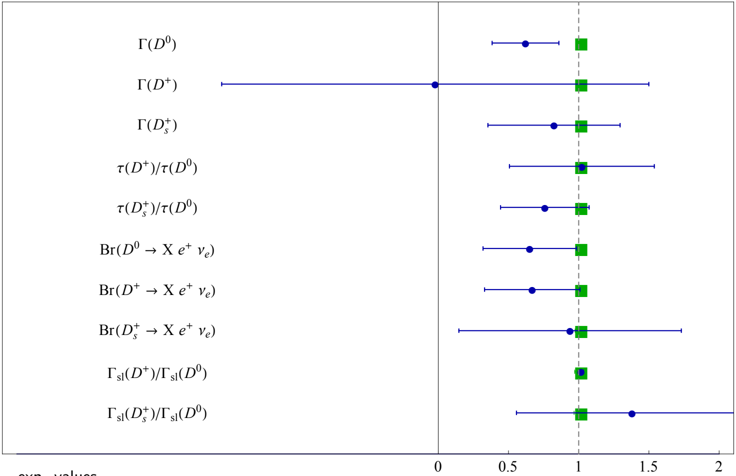
	SL	NL
Γ_3	NNNLO†	NLO
Γ_5	NLO	LO
Γ_6	LO	LO [New!]
Γ_7	LO	–
$\tilde{\Gamma}_6$	NLO	NLO*
$\tilde{\Gamma}_7$	LO	LO

	Source
$\langle \mathcal{O}_5 \rangle$	Heavy quark symmetry; Spectroscopy relations
$\langle \mathcal{O}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET Sum Rules [New!]
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA

* Penguin operator contribution is still missing

† see talk by Matteo Fael on Wednesday

Preliminary numerical results



Green - exp. values
Blue - th. predictions

Note: All quantities are normalised to the corresponding experimental values

Conclusion and Outlook

- Update of D-meson lifetimes and their ratios incl.
  - ▷ First computation of the Darwin operator contribution in NL decays
  - ▷ New dim-6 Bag parameters for both D- and D_s-mesons from HQET SR
- In agreement with data though huge theoretical uncertainties
- To improve theory predictions one needs
  - ▷ NNLO corrections at dimension-3
  - ▷ Dimension-6 Bag parameters (Lattice QCD) [M. Black, O. Witzel]
  - ▷ Dimension-7 Bag parameters (Lattice QCD) [M. Black, O. Witzel]
  - ▷ NNLO dimension-6 with four-quark operators [U. Nierste, M. Steinhauser (CRC project)]
  - ▷ NLO dimension-7 with four-quark operators [U. Nierste, M. Steinhauser (CRC project)]
  - ▷ Dimension-7 with two-quark operators
  - ▷ Dimension-8 with four-quark operators
  - ▷ NLO for dimension-5 and dimension-6 with two-quark operators
  - ▷ Experimental data on various moments in semileptonic decays to extract HQE parameters ($\mu_\pi^2(D)$, $\mu_G^2(D)$, $\rho_D^3(D)$, etc.) [BESIII]

Backup

Definition of non-perturbative parameters

- Dimension-5 two-quark

$$
\begin{aligned}
2m_D\,\mu_\pi^2(D) &= -\langle D(p_D)|\bar{c}_v(iD_\mu)(iD^\mu)c_v|D(p_D)\rangle,\\
2m_D\,\mu_G^2(D) &= \langle D(p_D)|\bar{c}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})c_v|D(p_D)\rangle,
\end{aligned}
$$

- Dimension-6 two-quark

$$
\begin{aligned}
2m_D\,\rho_D^3(D) &= \langle D(p_D)|\bar{c}_v(iD_\mu)(iv\cdot D)(iD^\mu)c_v|D(p_D)\rangle,\\
2m_D\,\rho_{LS}^3(D) &= \langle D(p_D)|\bar{c}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu})\,c_v|D(p_D)\rangle,
\end{aligned}
$$

- Dimension-6 four-quark

$$
\langle D_q|(\bar{c}_v\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma^\mu(1-\gamma_5)c_v)|D_q\rangle = f_{D_q}^2 M_{D_q}^2\,B_1(D_q)
$$

$$
\langle D_q|(\bar{c}_v(1-\gamma_5)q)(\bar{q}(1+\gamma_5)c_v)|D_q\rangle = f_{D_q}^2 M_{D_q}^2\,B_2(D_q)
$$

$$
\langle D_q|(\bar{c}_v\gamma_\mu(1-\gamma_5)t^a q)(\bar{q}\gamma^\mu(1-\gamma_5)t^a c_v)|D_q\rangle = f_{D_q}^2 M_{D_q}^2\,\varepsilon_1(D_q)
$$

$$
\langle D_q|(\bar{c}_v(1-\gamma_5)t^a q)(\bar{q}(1+\gamma_5)t^a c_v)|D_q\rangle = f_{D_q}^2 M_{D_q}^2\,\varepsilon_2(D_q)
$$