10th International Workshop on Charm Physics

Light-front Distribution Amplitudes of heavy-mesons and quarkonia

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- Light - front distribution amplitudes
- Non-perturbative continuum tools for QCD
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Light-front Distribution Amplitudes

- QCD Factorization involves matrix elements which are convolution integrals

$$\langle \pi^+\pi^- | (\bar{u}b)_{\mathrm{V-A}}(\bar{u}d)_{\mathrm{V-A}} | \bar{B}_d \rangle \to \int_0^1 d\xi \, du \, dv \, \phi_B(\xi)\phi_\pi(u)\phi_\pi(v) \, T(\xi, u, v; m_b)$$

$T(\xi, u, v; m_b)$ — **Hard - scattering kernels**

$\phi_B(u), \ \phi_\pi(v)$ — **Meson's Light-front distribution Amplitudes (LCDAS)**

$$\phi_M(x) \neq \phi^{\mathrm{asy}}(x) = 6x(1-x)$$

- **Leading-twist LCDA for pseudoscalar meson**

$$\langle 0|\bar{q}_f(y_2 n)\mathcal{W}\,[y_2 n, y_1 n]\,\gamma \cdot n\,\gamma_5\, q_g(y_1 n)|M(P)\rangle$$
$$= i f_M \, n \cdot P \int_0^1 dx \, e^{-i n\cdot P(y_1 x + y_2 \bar{x})} \phi_M(x, \mu)$$

f_M = weak decay constant

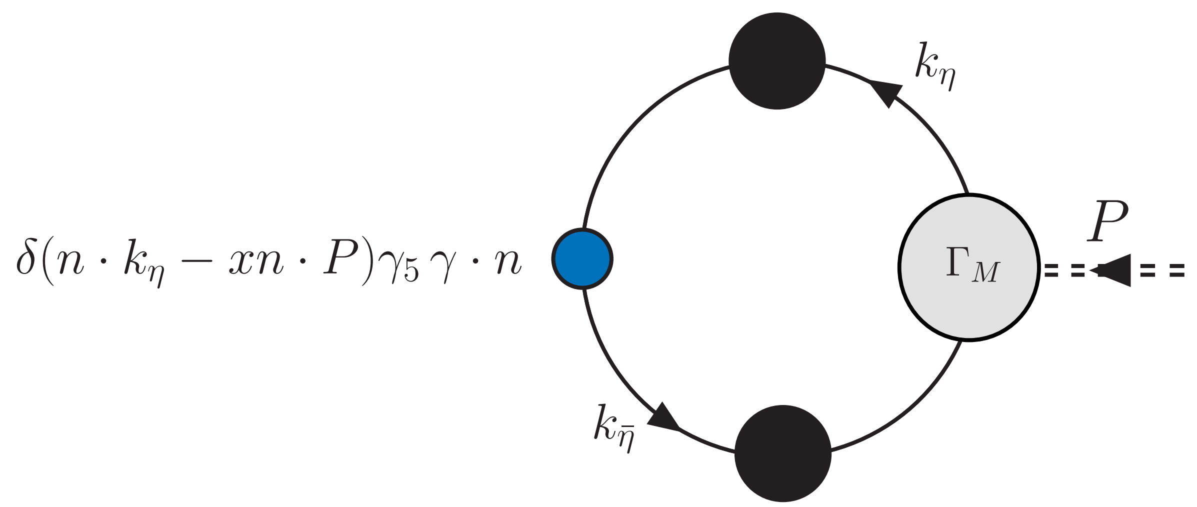
$n^2 = 0$

$n \cdot P = -m_M$

Light-front Distribution Amplitudes

- **Light-front projection of the Bethe- Salpeter wave function**

$$f_M \phi_M(x,\mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x n\cdot P)\gamma_5\,\gamma\cdot n\;\chi_M(k_\eta, k_{\bar\eta})$$



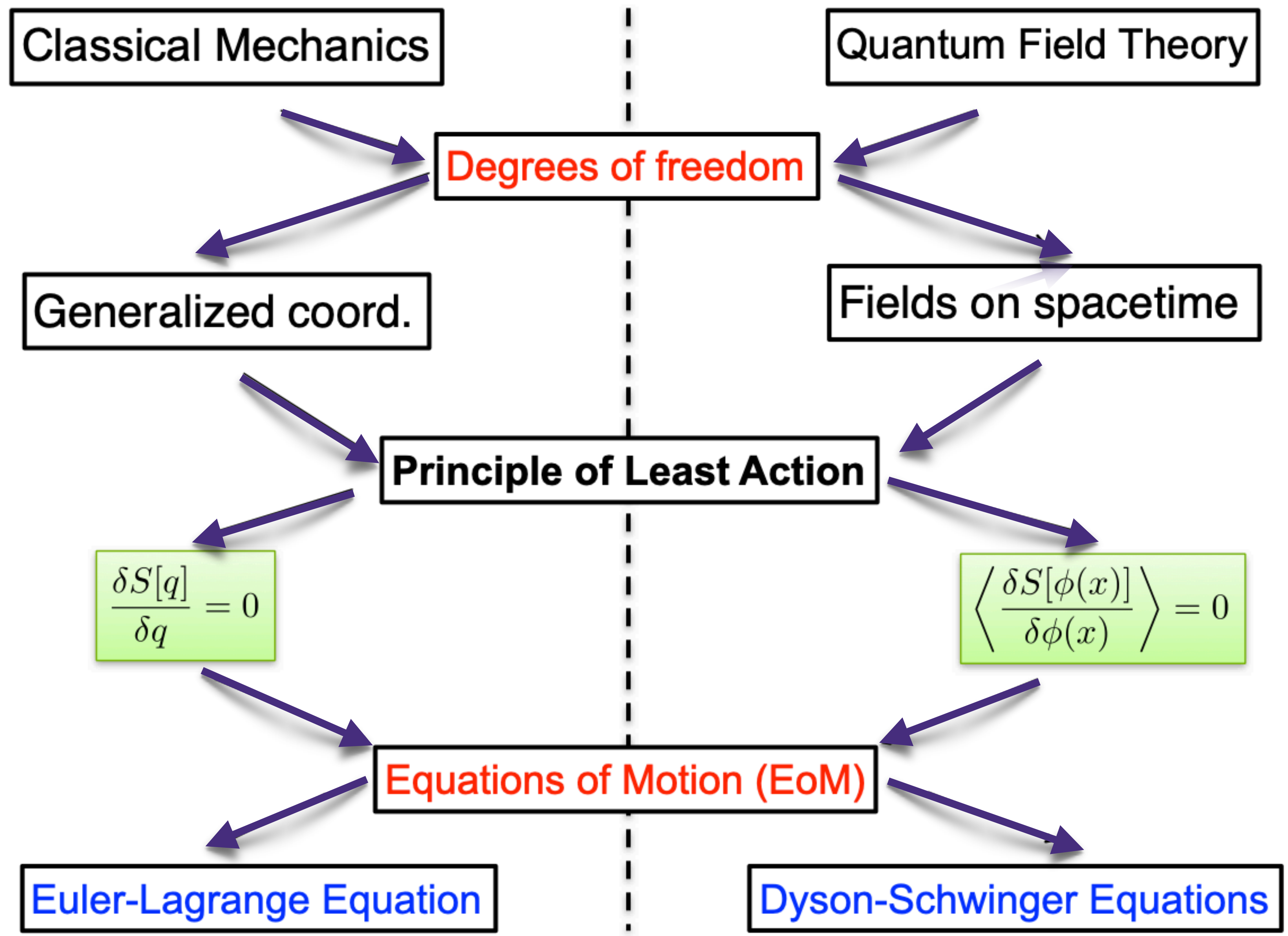
$$\chi_M(k_\eta, k_{\bar\eta}) := S(k_\eta)\Gamma_M(k,P)S(k_{\bar\eta})$$
- **Meson's Bethe-Salpeter wave function**

$$\Gamma_M(k,P)$$
- **Meson's Bethe-Salpeter Amplitude**

$$S(k_\eta)$$
- **Quark propagator**

L. Chang, I.C. Cloët, J.J. Cobos-Martínez, C.D. Roberts, S.M.Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)
J. Segovia, L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, Phys. Lett. B 731 (2014)

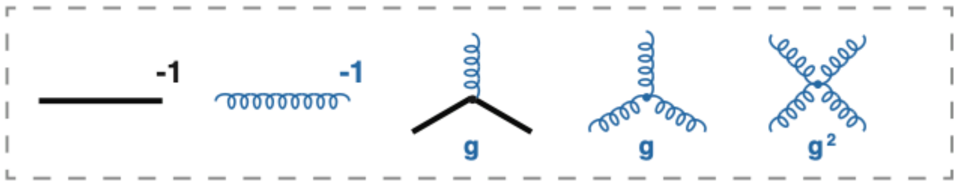
Non-perturbative continuum tools for QCD



QCD's Dyson-Schwinger Equations (DSEs)

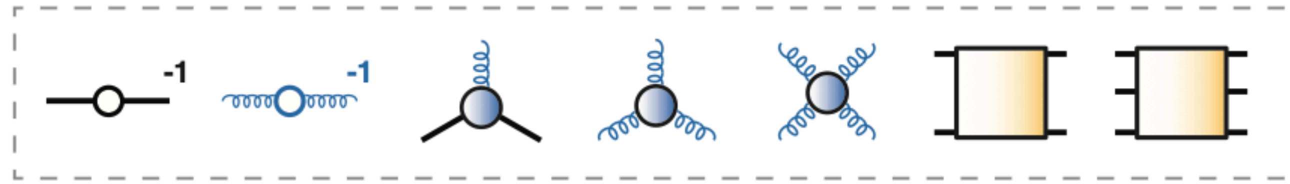
QCD's classical action:

$$S = \int d^4x \left[\bar{\psi}\,(\not{\partial} + ig\not{A} + m)\,\psi + \tfrac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} \right]$$

$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} = e^{-\Gamma}$$



DSE = Quantum equation of motion: obtained from path integral, relate n-point functions

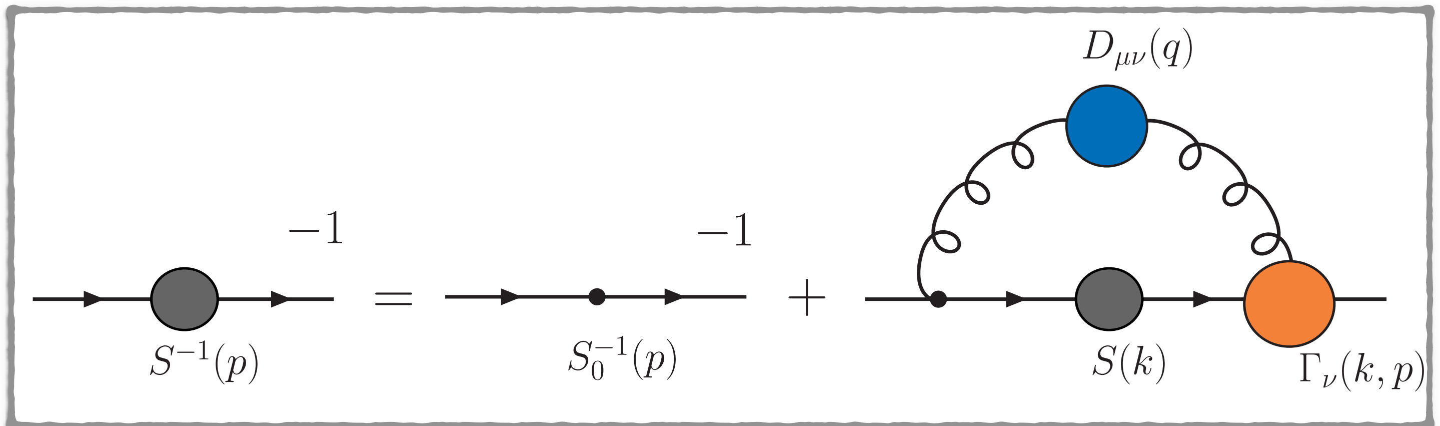
- Infinitely many coupled equations
- Continuum methods: Reproduce perturbation theory, but non-perturbative
- Systematic truncations: neglect higher n-point functions to obtain **closed system**

Capture two emergent phenomena:

- Dynamical chiral symmetry breaking
- Confinement

QCD's Dyson-Schwinger Equations (DSEs)

Quark Dyson-Schwinger Equation



$$M_f(p^2,\mu^2) = \frac{B_f(p^2,\mu^2)}{A_f(p^2,\mu^2)}$$

$$S_f^{-1}(p) = Z_2^f\,(i\gamma\cdot p + m_f^{\text{bm}}) + Z_1^f g^2 \int^\Lambda \frac{d^4k}{(2\pi)^4} D_{\mu\nu}^{ab}(q)\frac{\lambda^a}{2}\gamma_\mu S_f(k)\Gamma_{\nu,f}^b(k,p)$$

Quark propagator

$$S_f^{-1}(p) = i\gamma\cdot pA_f(p^2,\mu) \;+ A_f(p^2,\mu)$$

$$= A_f(p^2,\mu)[i\gamma\cdot p + M_f(p^2)]$$

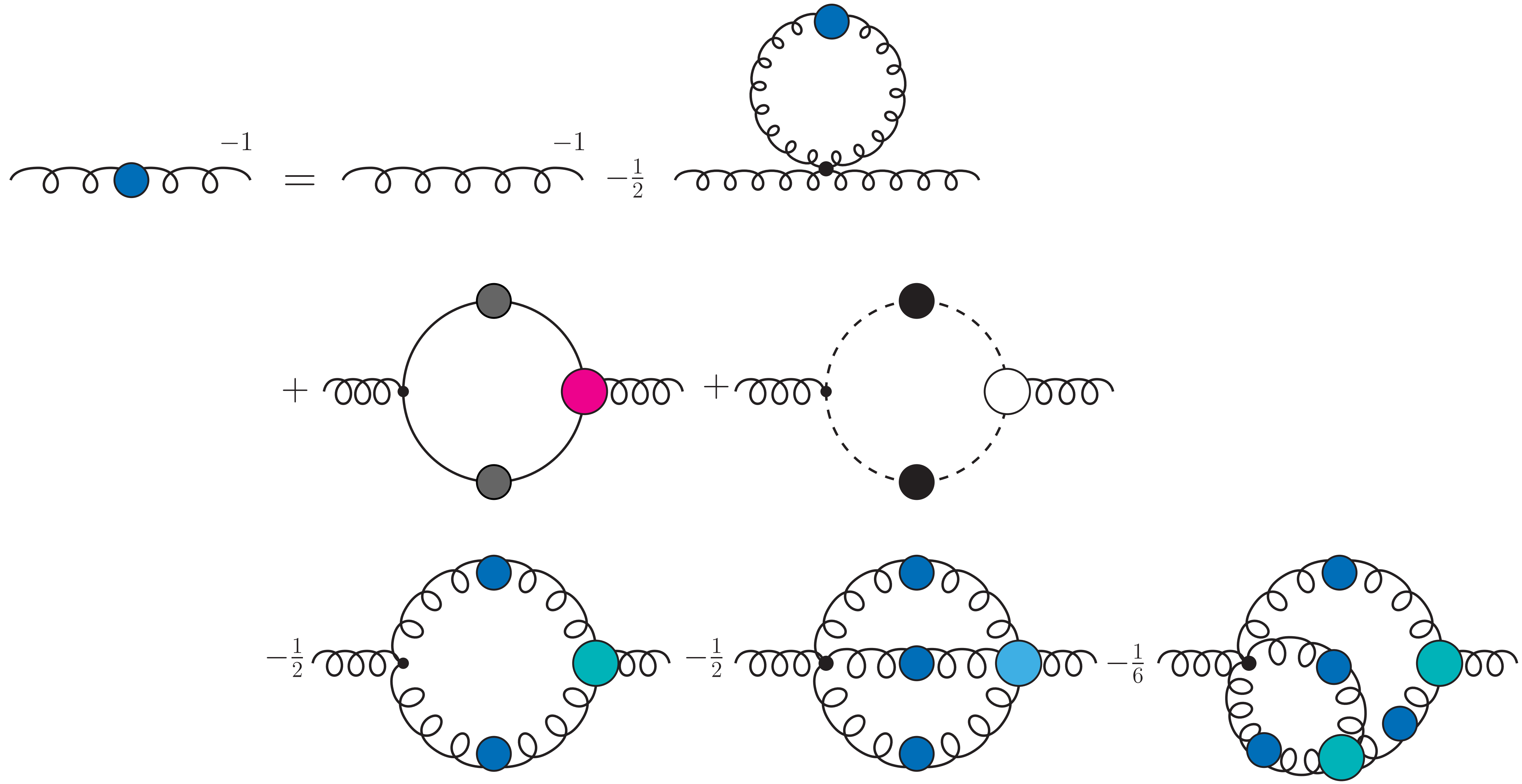
- $D_{\mu\nu}^{ab}(q)$ = Dressed gluon propagator
- $\Gamma_\nu^b(k,p)$ = Dressed quark-gluon vertex
- Z_2 = Quark wave function renormalization constant
- Z_4 = Quark-gluon vertex renormalization constant

Renormalization condition:

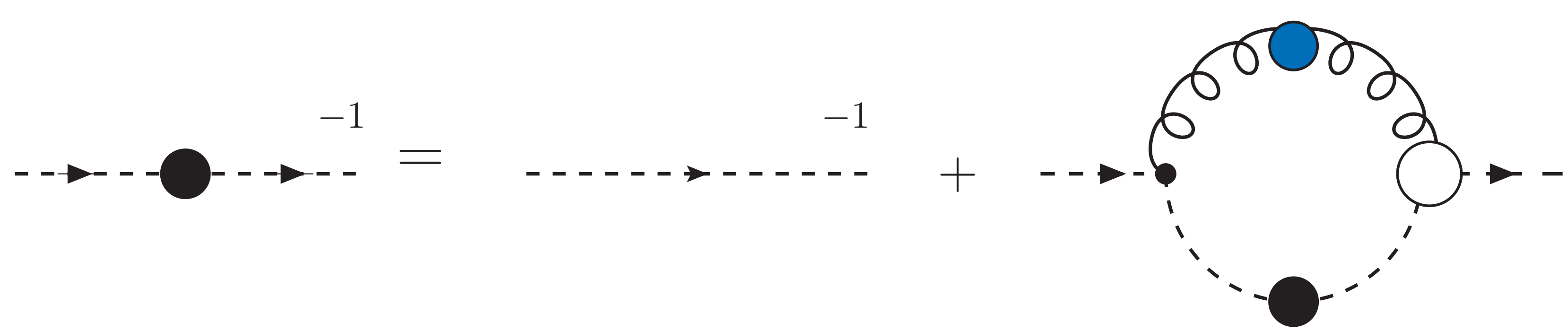
$$A_f(p^2) = A_f(p^2)\big|_{p^2=\mu^2} = 1 \qquad S_f^{-1}(p)\,\big|_{p^2=\mu^2} = i\gamma\cdot p \;+ m_f(\mu)$$

QCD’s Dyson-Schwinger Equations (DSEs)

Gluon

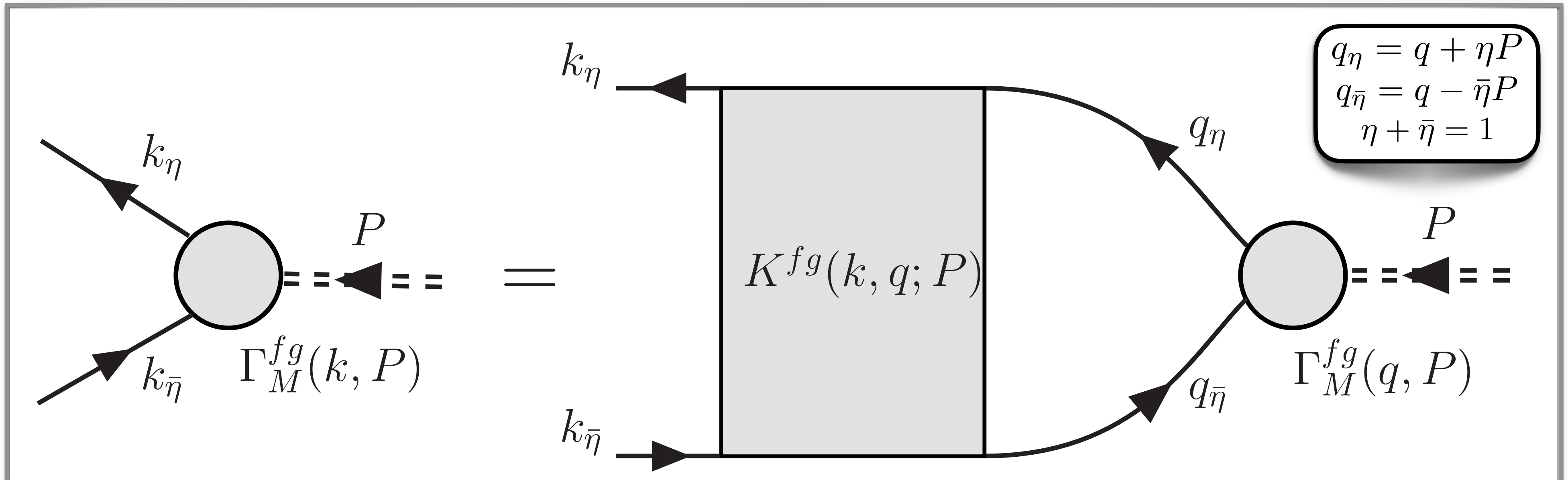


Ghost



Bethe-Salpeter Equations for QCD bound-states

BSE = Bound-state equation for meson



$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

- $K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel
- $S_f(q_\eta)$ = Dressed quark propagator
- $\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)

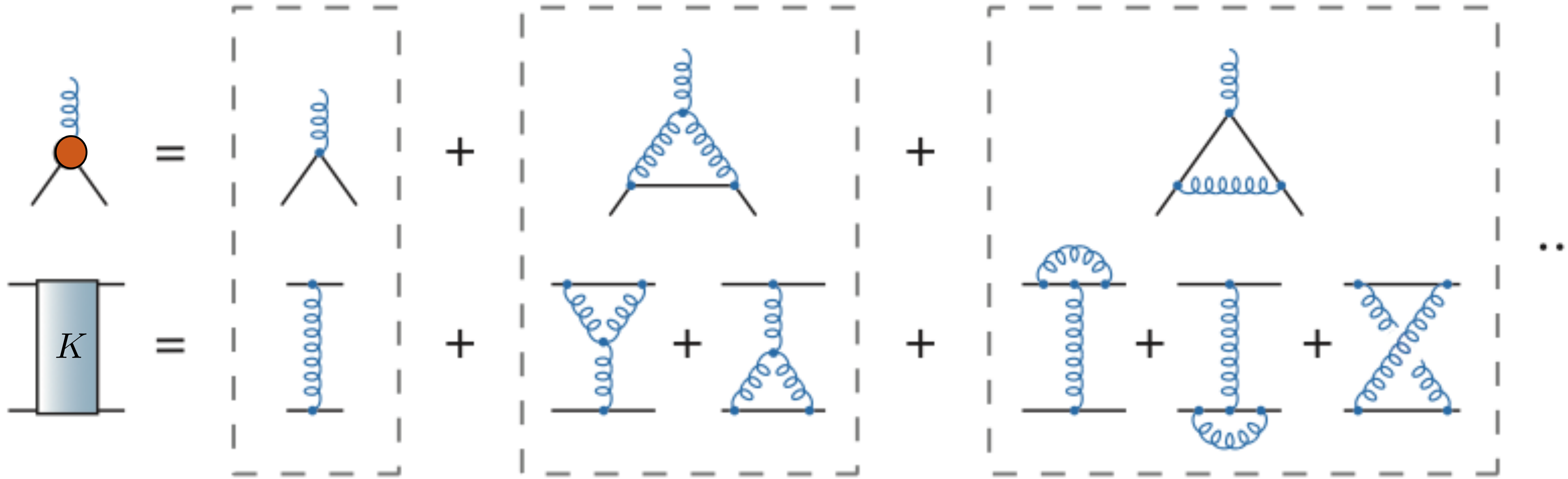
$$\Gamma_M(k,P) = \sum_{i=1}^{N} T_M^i(k,P) F_i(k,P)$$

General solution for Poincaré invariant pseudoscalar BSA →

$$\Gamma_M^{fg}(k,P) = \gamma_5 \Big[iE_M^{fg}(k,P) + \gamma\cdot P\, F_M^{fg}(k,P) + \gamma\cdot k\; k\cdot P\, G_M^{fg}(k,P) + \sigma_{\mu\nu} k_\mu P_\nu\, H_M^{fg}(k,P) \Big]$$

Truncation schemes and symmetries

- **DSE/BSE:** Kernel can be derived in accordance with chiral symmetry



Truncation must preserve **AV-WTI**, which ensures that we will have massless pions in the chiral limit.

AV-WTI: $$P_\mu \Gamma^{fg}_{5\mu}(k;P) = S_f^{-1}(k_\eta)\, i\gamma_5 + i\gamma_5 S_g^{-1}(k_{\bar{\eta}})$$

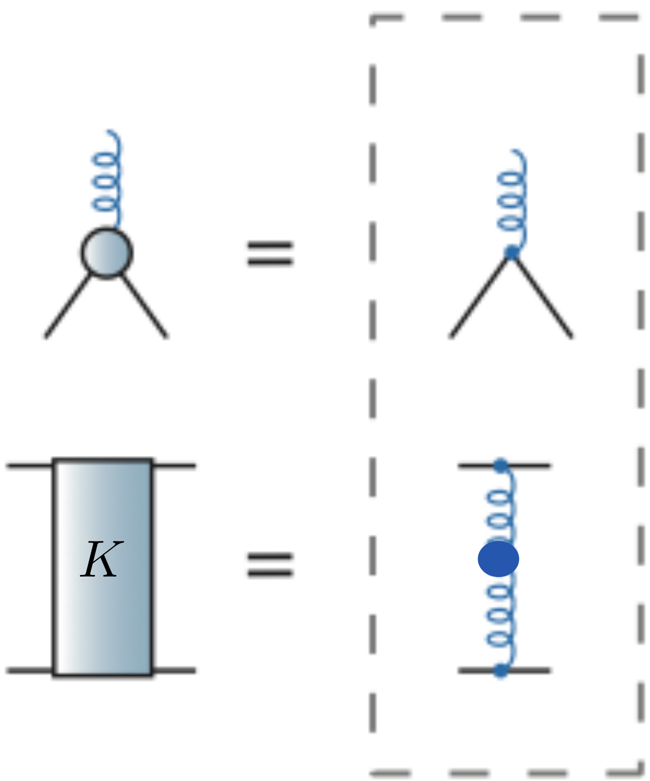
Axial vector-vertex:

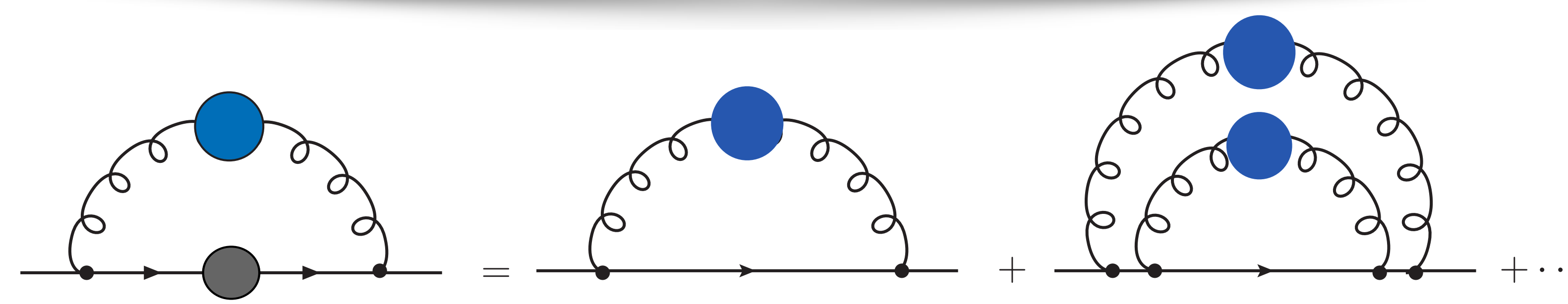
$$\Gamma^{fg}_{5\mu}(k;P) = Z_2^f \gamma_5\gamma_\mu + \int^\Lambda \frac{d^4q}{(2\pi)^4} K_{fg}(q,k;P)\, S_f(q_\eta)\Gamma^{fg}_{5\mu}(q;P) S_g(q_{\bar{\eta}})$$

Rainbow-Ladder truncation

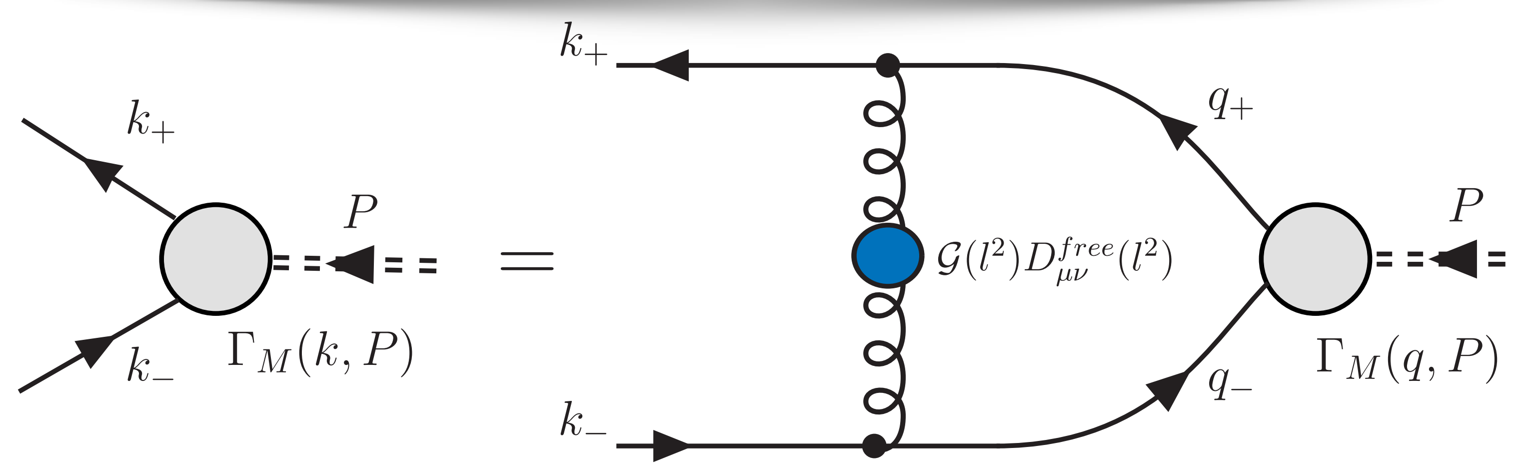
Leading truncation

$$Z_1^f g^2 D_{\mu\nu}(q)\Gamma_{\nu,f}(k,p) = \left(Z_2^f\right)^2 \mathcal{G}(q^2) D_{\mu\nu}^{\text{free}}(q)\frac{\lambda^a}{2}\gamma_\nu$$





$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right) D_{\mu\nu}^{\text{free}}(l)\,\gamma_\mu\frac{\lambda^a}{2}\gamma_\nu\frac{\lambda^a}{2}$$



$\mathcal{G}(q)$: **Effective gluon interaction**

Gluon interaction model

An interaction ansatz for $\mathscr{G}(q^2)$ that has proven its merits in meson and baryon phenomenology can be decomposed as

$$\boxed{\frac{\mathcal{G}_f(q^2)}{q^2} = \mathcal{G}_f^{\rm IR}(q^2) + 4\pi\tilde{\alpha}_{\rm PT}(q^2)}$$

- Infrared part: Two-Models

MT-Model: $\mathcal{G}_f^{\rm IR}(q^2) = \dfrac{4\pi^2}{\omega_f^6} q^2 D_f\, e^{-q^2/\omega_f^2}$

QC-Model: $\mathcal{G}_f^{\rm IR}(q^2) = \dfrac{8\pi^2}{\omega_f^4} D_f\, e^{-q^2/\omega_f^2}$

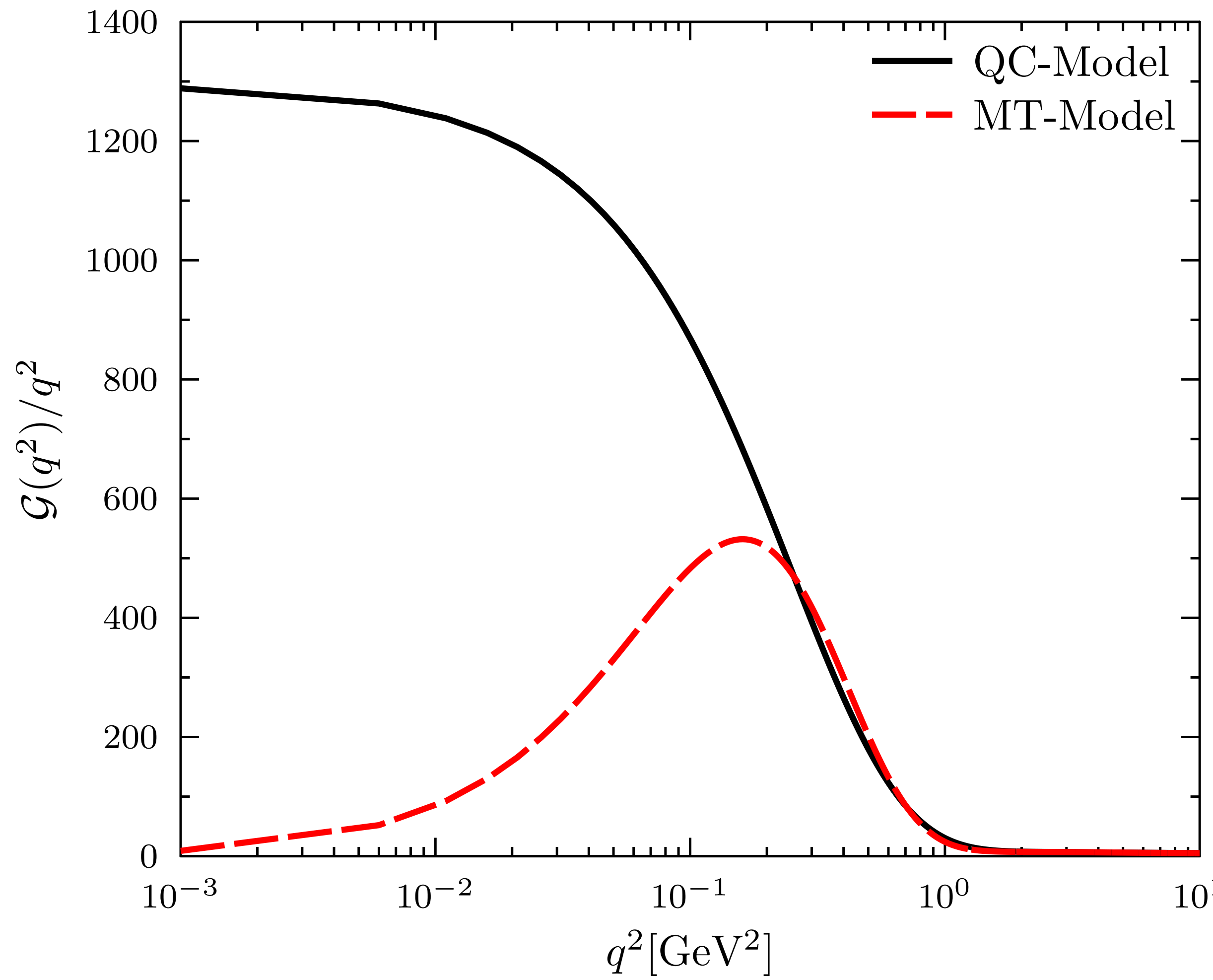
- Ultraviolet part: Model

$$4\pi\tilde{\alpha}_{\rm PT}(q^2) = \frac{8\pi^2\gamma_m \mathcal{F}(q^2)}{\ln\left[\tau + \left(1 + q^2/\Lambda_{\rm QCD}^2\right)^2\right]}$$

$$\gamma_m = 12/(33 - 2N_f),\ \ N_f = 4,\ \ \Lambda_{\rm QCD} = 0.234\,{\rm GeV}$$

$$\mathcal{F}(q^2) = [1 - \exp(-q^2/4m_t^2)]/q^2,\ \ m_t = 0.5\ {\rm GeV}$$

$$\tau = e^2 - 1$$



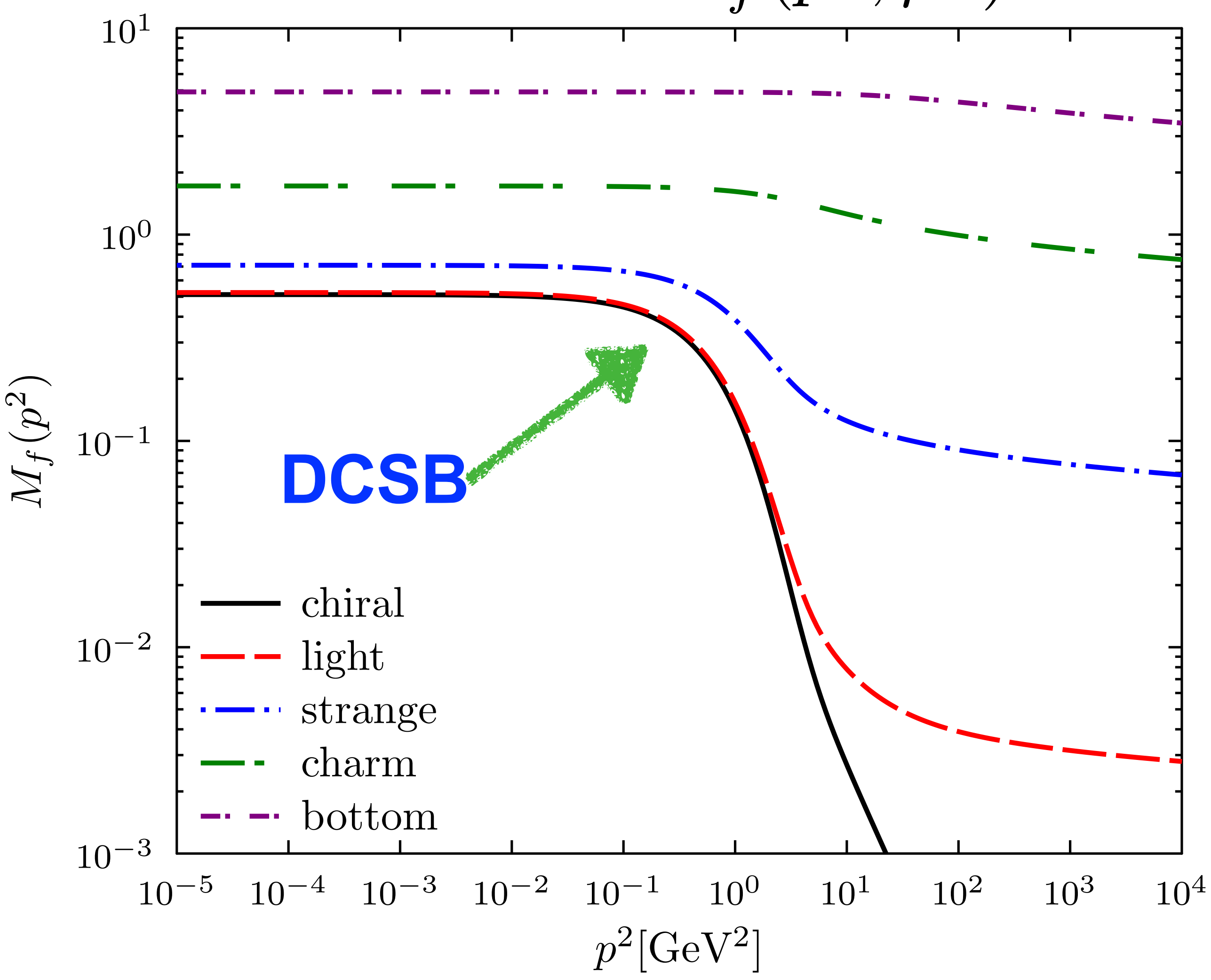
P. Maris and P. C. Tandy, Phys. Rev. C 60, 055214 (1999)
S. X. Qin, L. Chang, Y. X. Liu, C. D. Roberts and D. J. Wilson, Phys. Rev. C 84, 042202 (2011)

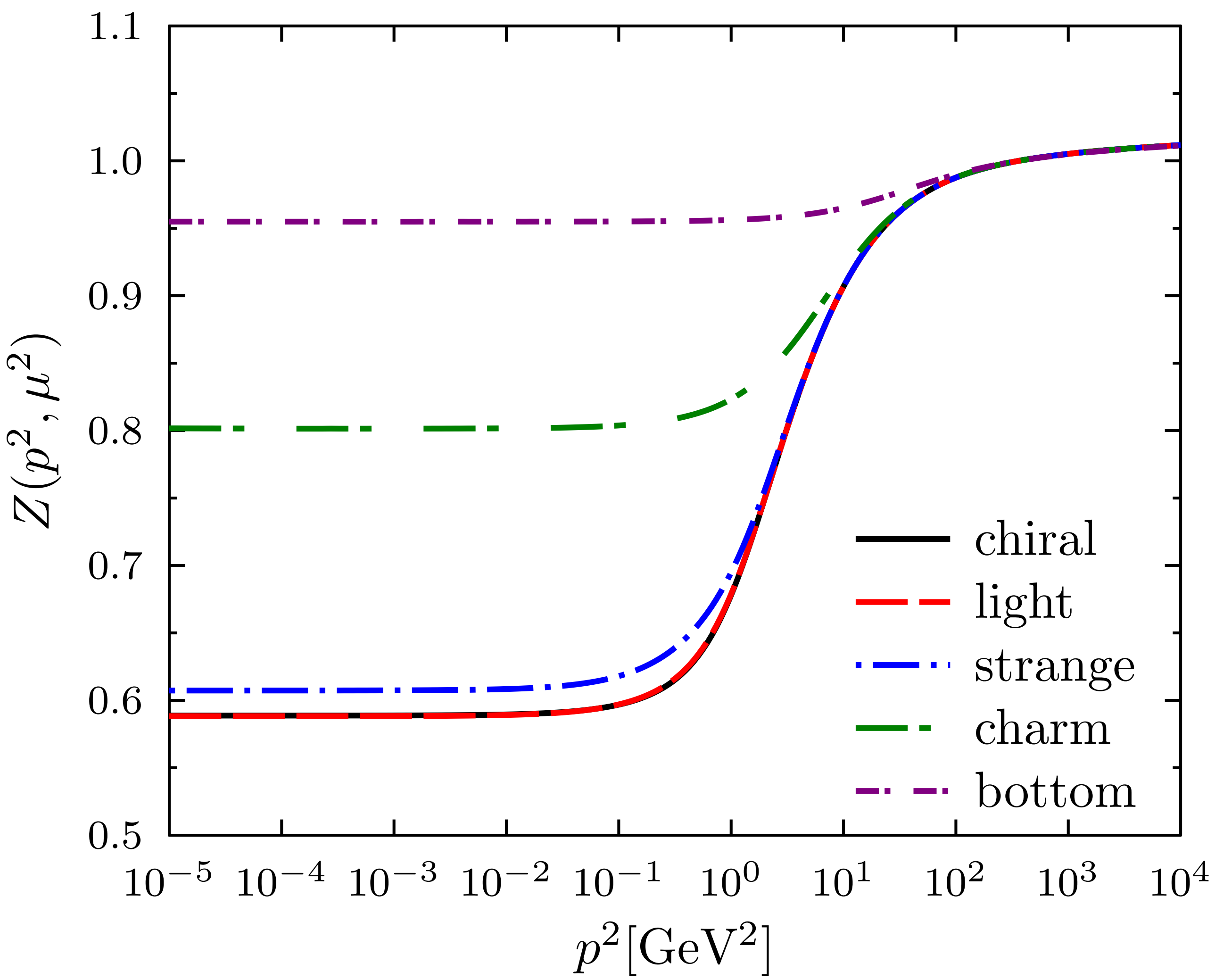
Numerical solutions: Quark DSE

REAL AXIS SOLUTION

$$M_f(p^2,\mu^2) = \frac{B_f(p^2,\mu^2)}{A_f(p^2,\mu^2)}$$

$$Z_f(p^2,\mu^2) = 1/A_f(p^2,\mu^2)$$





Quark Propagator: Dynamical chiral symmetry breaking generates 'constituent-quark masses'.

Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

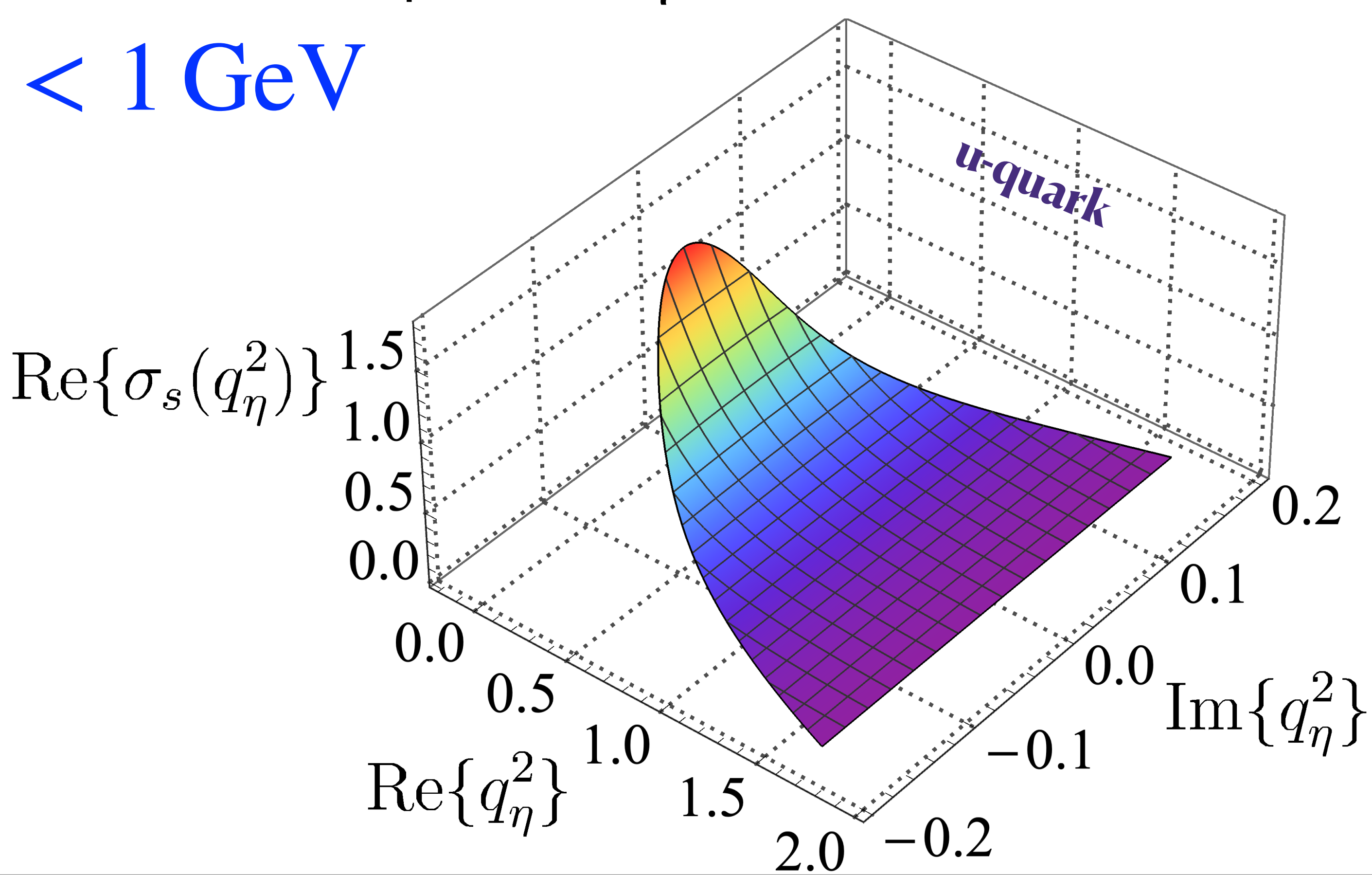
COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \, \sigma_{\rm v}(q_\eta^2) + \sigma_{\rm s}(q_\eta^2)$$
$$q_\eta^2 = q^2 - \eta^2 m_M^2 + 2i\eta \, m_M |q| z_q$$

$$\sigma_{\rm v}^f(q_\eta^2) = \frac{A_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$
$$\sigma_{\rm s}^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

- **Light meson:** For instant, the pion with $\eta = 1/2$.

$$m_M = m_\pi < 1\,\mathrm{GeV}$$



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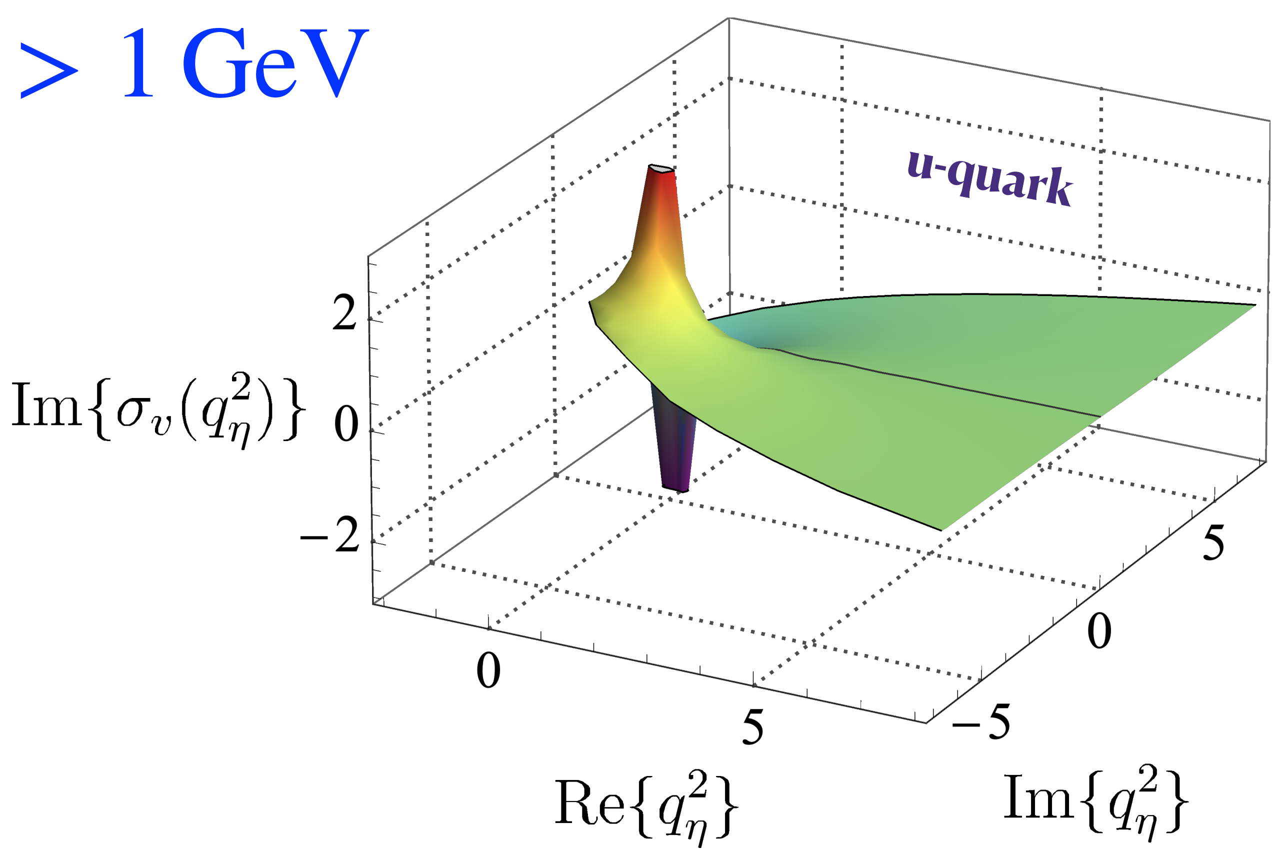
- **Heavy meson:** For instant, the D meson with $\eta = 0.6$

$$m_M = m_D > 1\,\mathrm{GeV}$$



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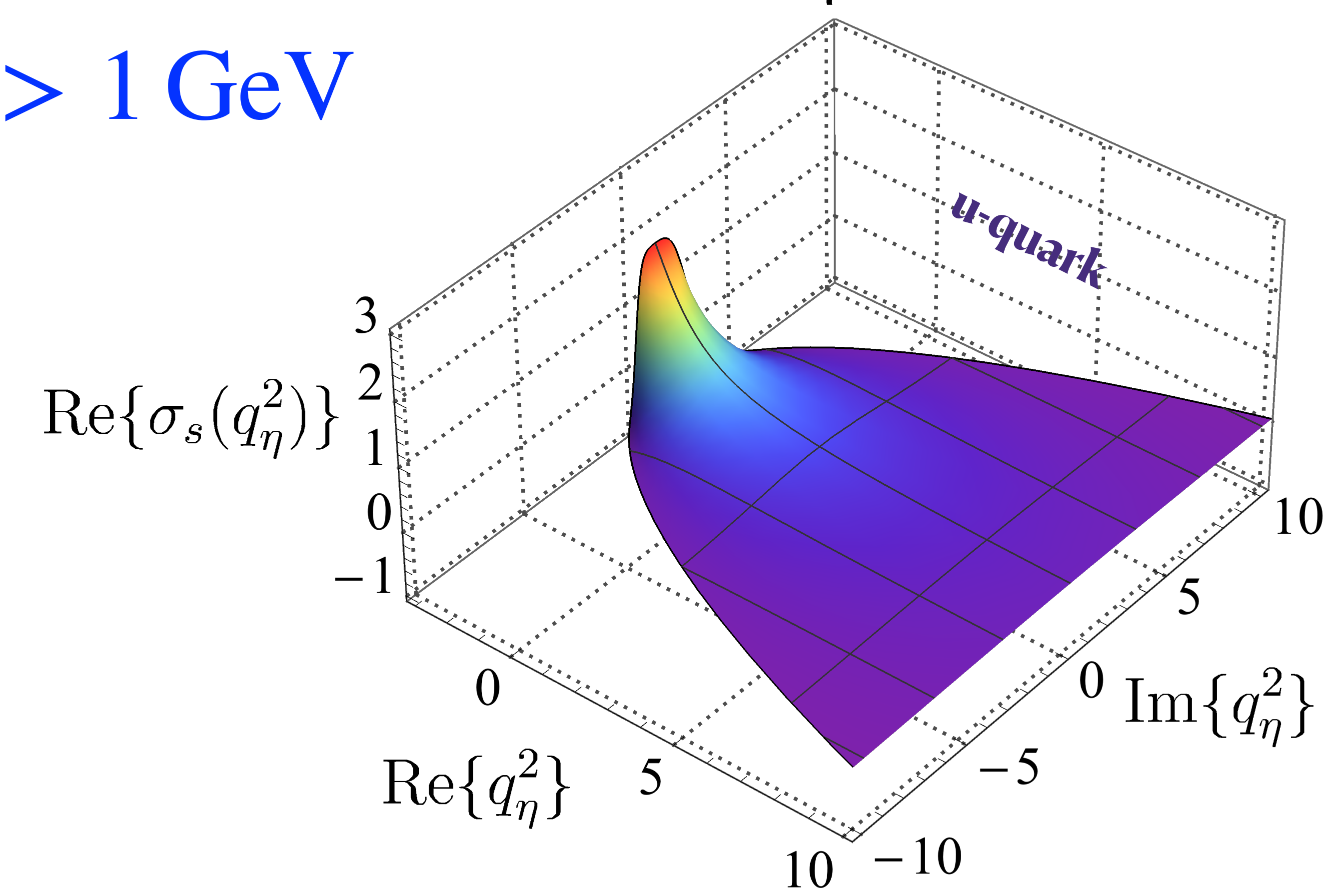


Numerical solutions: BSE

For the bound state masses we solve an artificial eigenvalue problem:

$$\lambda_n(P^2)\Gamma_M^{fg}(k,P) = \int^\Lambda \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta)\Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

$$\Gamma_M(k,P) = \sum_{i=1}^N T_M^i(k,P) F_i(k^2, z_k, P^2), \qquad z_k = k\cdot P/|k||P$$

Chebyschev polynomials

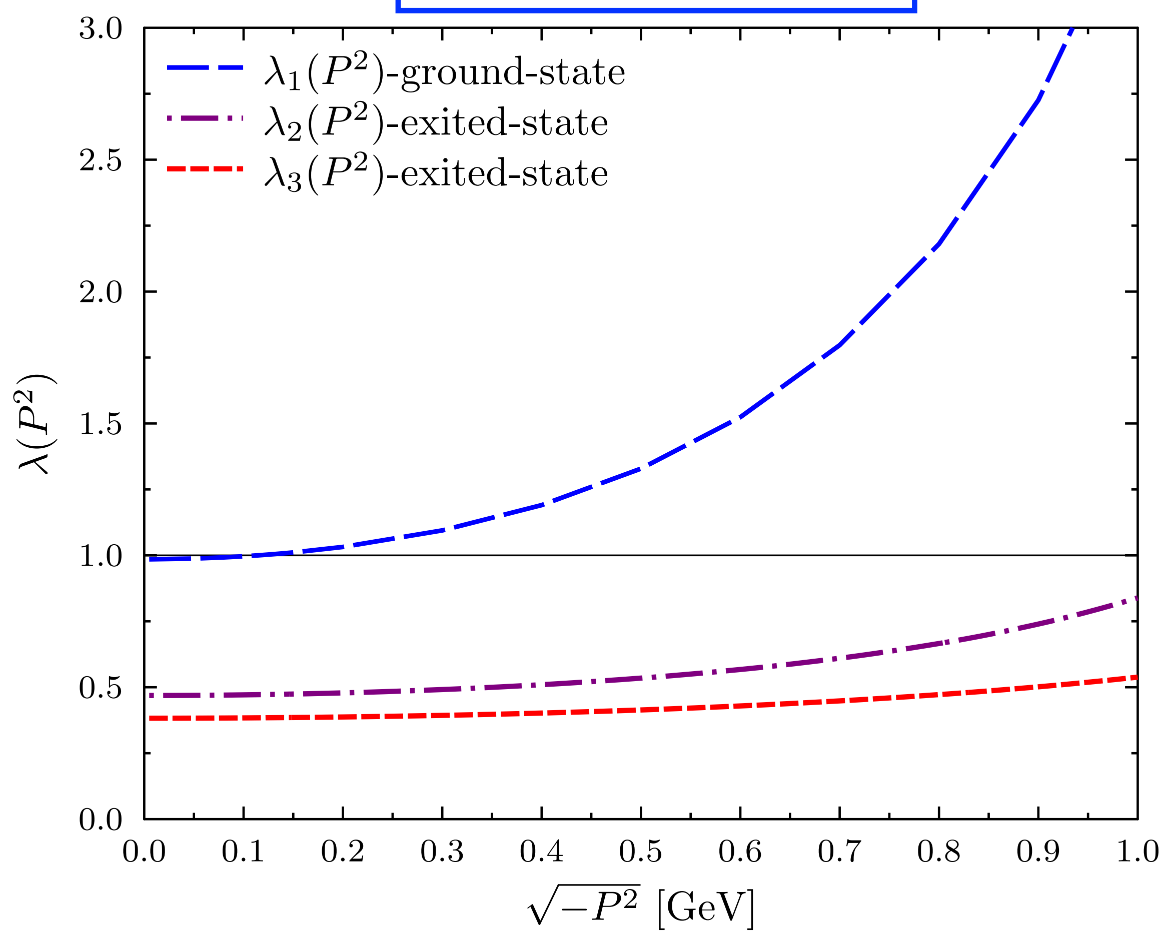
$$\mathcal{F}_i(k,P) = \sum_{m=0}^{\infty} {}^m\mathcal{F}_i(k^2,P^2) U_m(z_k)$$

Chebyschev`s Moments

$$\frac{2}{\pi}\int_0^1 \sqrt{1-x^2}\, U_m(x) U_n(x) = \delta_{mn}$$

$$\boxed{\lambda(P^2)\, {}^m\mathcal{F}^{\alpha i}(P) = (\mathbf{K}_\beta^\alpha)_{n\,j}^{mi}(P)\, {}^n\mathcal{F}^{\beta j}(P)}$$

Eigenvalue trajectory



Basically to obtain the meson mass we have find the root of $\lambda(P^2) - 1 = 0$.

Light mesons & quarkonia

Pseudoscalar channel

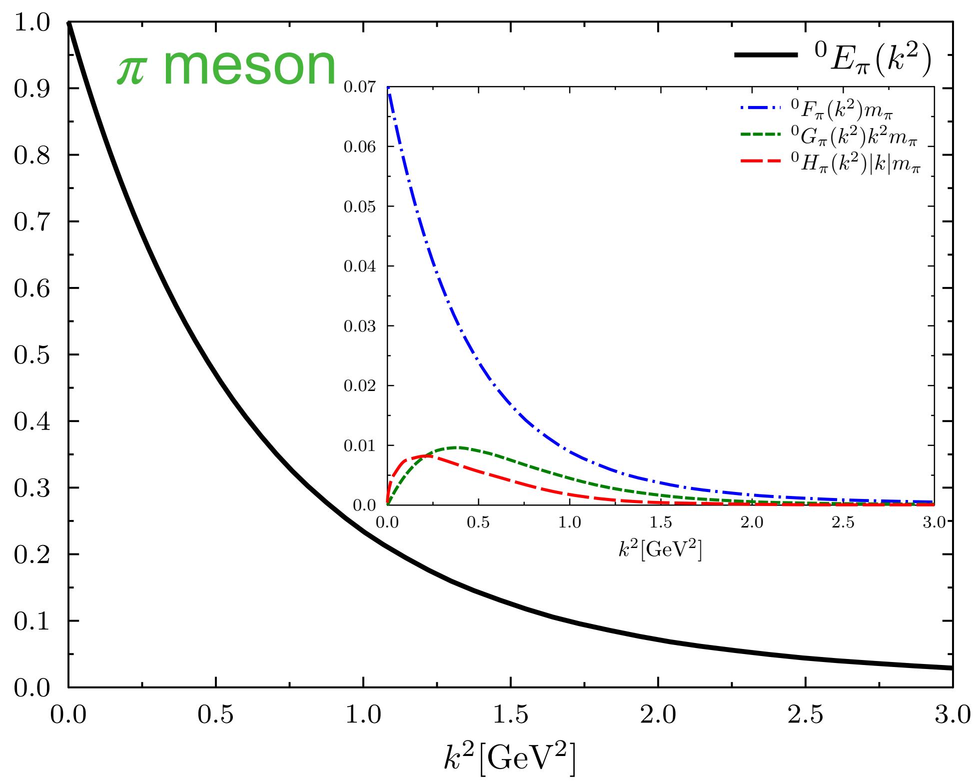
Mesons/Properties	$m_{\rm ps}$	$m_{\rm ps}^{\rm exp}$	$\epsilon_r^m\,\%$	$f_{\rm ps}$	$f_{\rm ps}^{\rm exp/lQCD}$	$\epsilon_r^f\,\%$
$\pi(u\bar{d})$	0.140	0.138	1.449	0.094	0.092	2.174
$K(s\bar{u})$	0.494	0.494	0	0.110	0.110	0
$\eta_c(c\bar{c})$	3.012	2.984	0.938	0.270	0.279	3.226
$\eta_b(b\bar{b})$	9.392	9.398	0.064	0.491	0.472	4.025

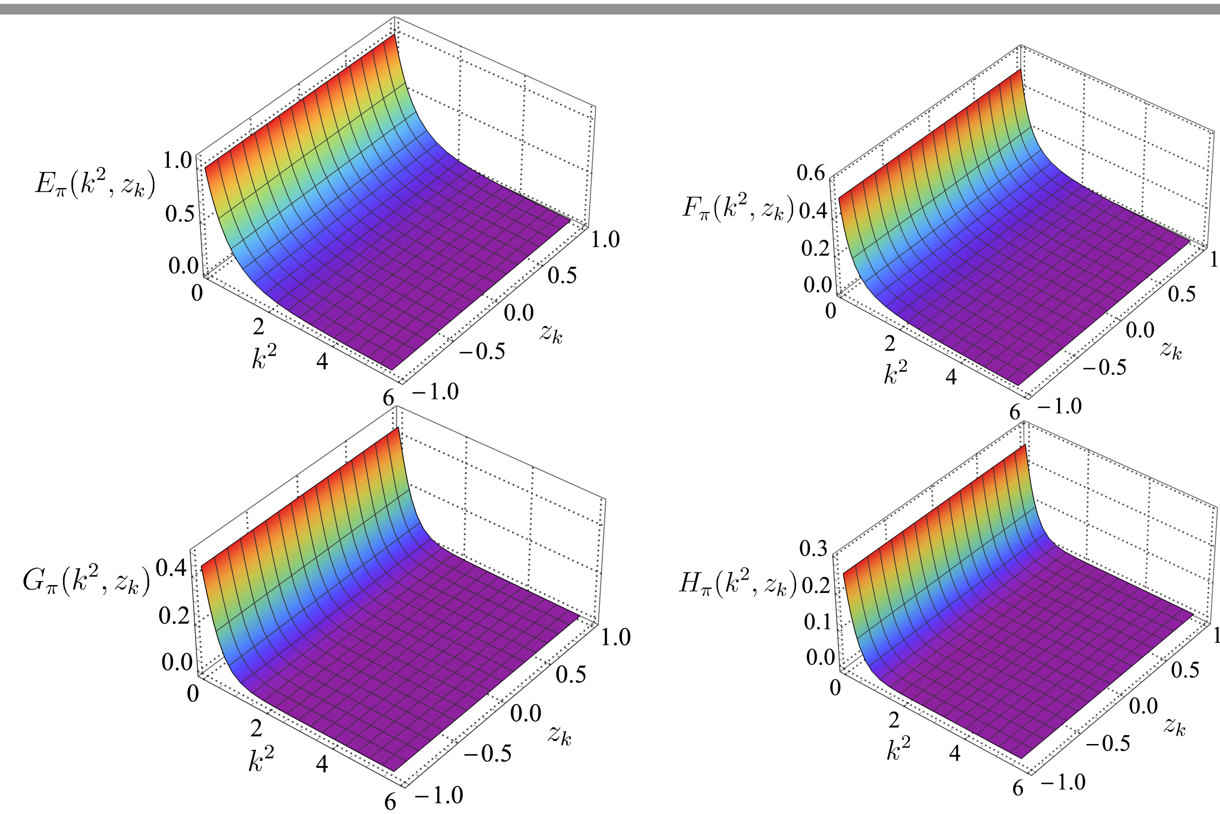
F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020).

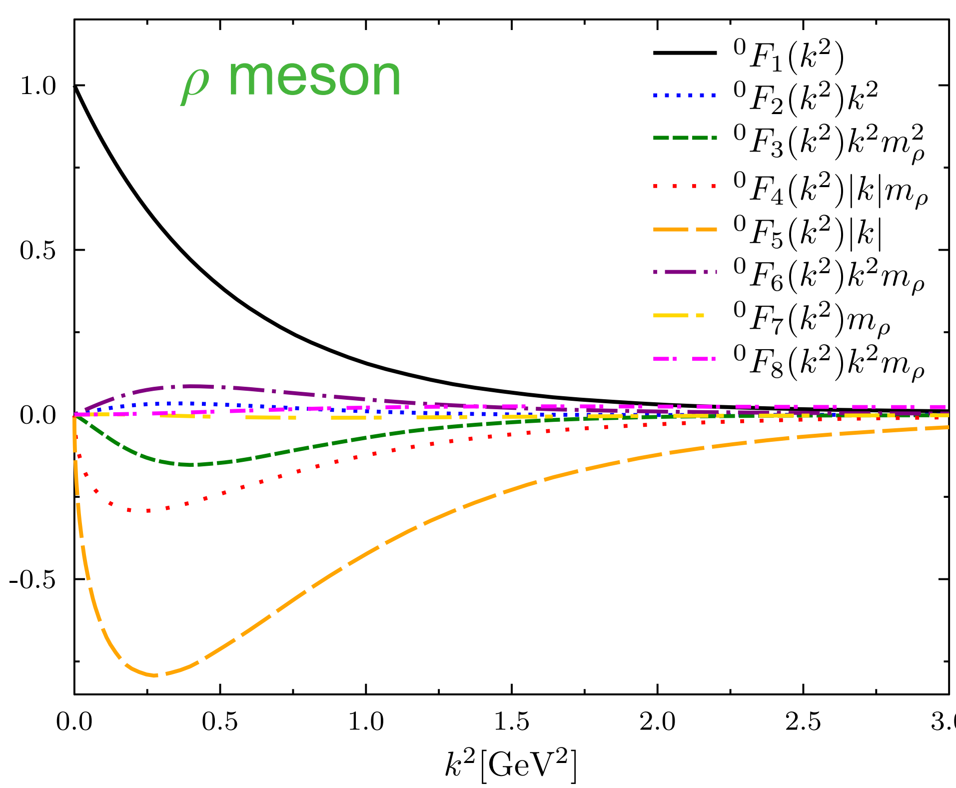
Vector channel

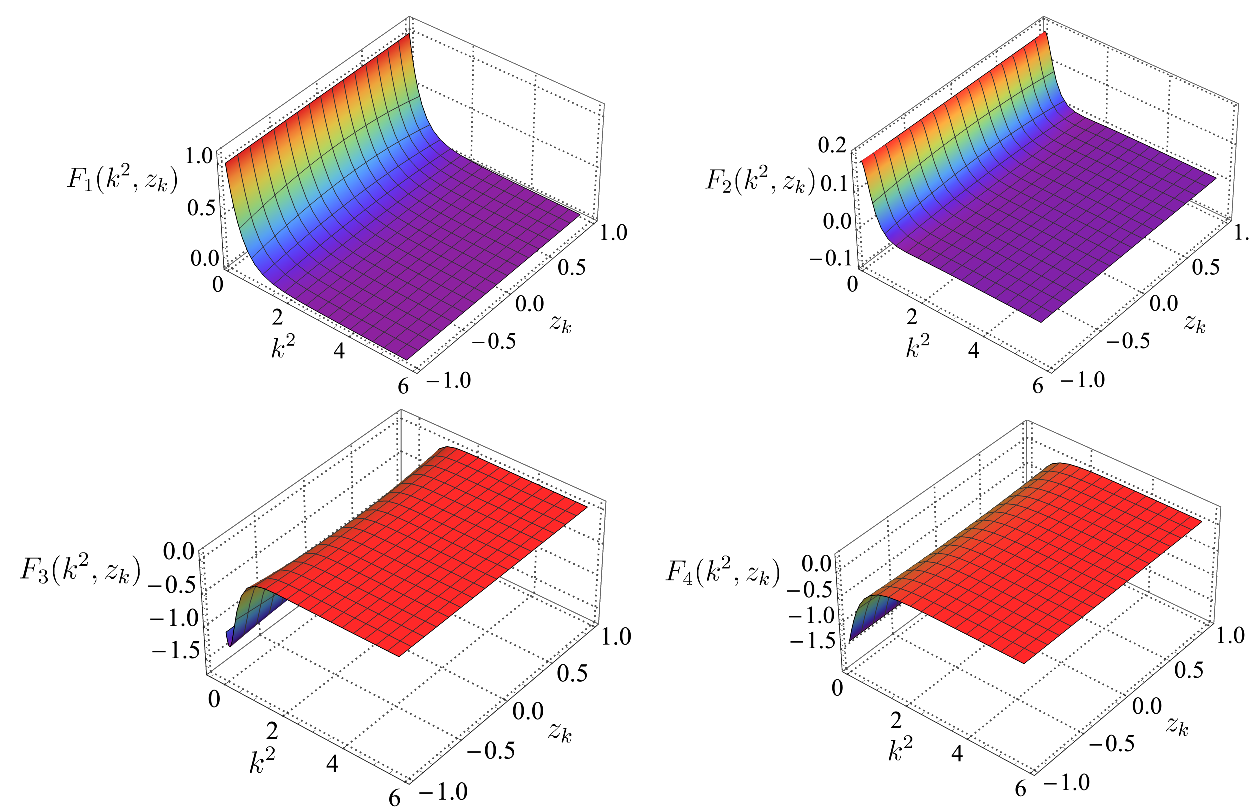
Mesons/Properties	m_M	$m_M^{\rm exp}$	$\epsilon_r^m\,\%$	f_M	$f_M^{exp/\rm lQCD}$	$\epsilon_r^f\,\%$
$\rho(u\bar{d})$	0.730	0.775	5.810	0.145	0.153	5.229
$\phi(s\bar{s})$	1.070	1.019	5.197	0.187	0.168	11.309
$K^*(s\bar{u})$	0.942	0.896	5.134	0.177	0.159	11.321
$J/\Psi(c\bar{c})$	3.124	3.097	0.872	0.277	0.294	5.782
$\Upsilon(b\bar{b})$	9.411	9.460	0.518	0.594	0.505	17.624

Light mesons & quarkonia









Heavy-light mesons

We have introduced a different flavor dependence of the dressing functions in the heavy-quark sector.

Pseudoscalar channel

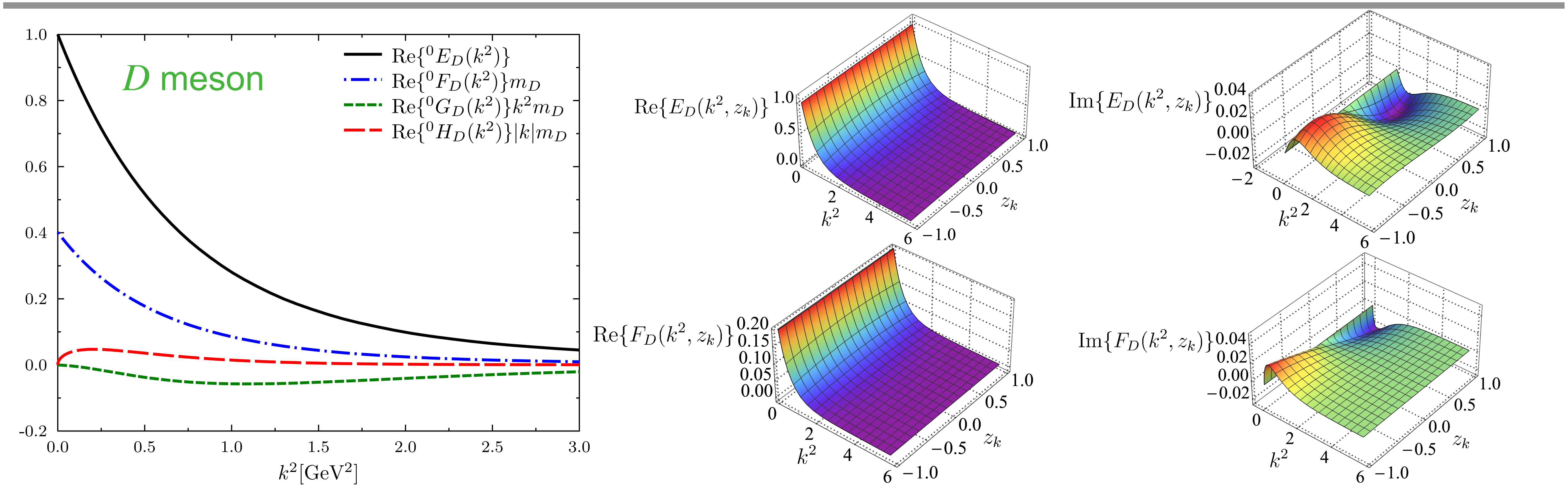
Mesons/Properties	m_{ps}	$m_{\text{ps}}^{\text{exp}}$	ϵ_r^m %	f_{ps}	$f_{\text{ps}}^{\text{exp/lQCD}}$	ϵ_r^f %
$D_u(c\bar{u})$	1.867	1.869	0.107	0.144	0.150	4.000
$D_s(c\bar{s})$	2.015	1.968	2.388	0.179	0.177	1.130
$B_u(b\bar{u})$	5.277	5.279	0.037	0.132	0.138	4.348
$B_s(b\bar{s})$	5.383	5.367	0.171	0.128	0.161	20.497
$B_c(b\bar{c})$	6.282	6.274	0.112	0.280	0.346	19.075

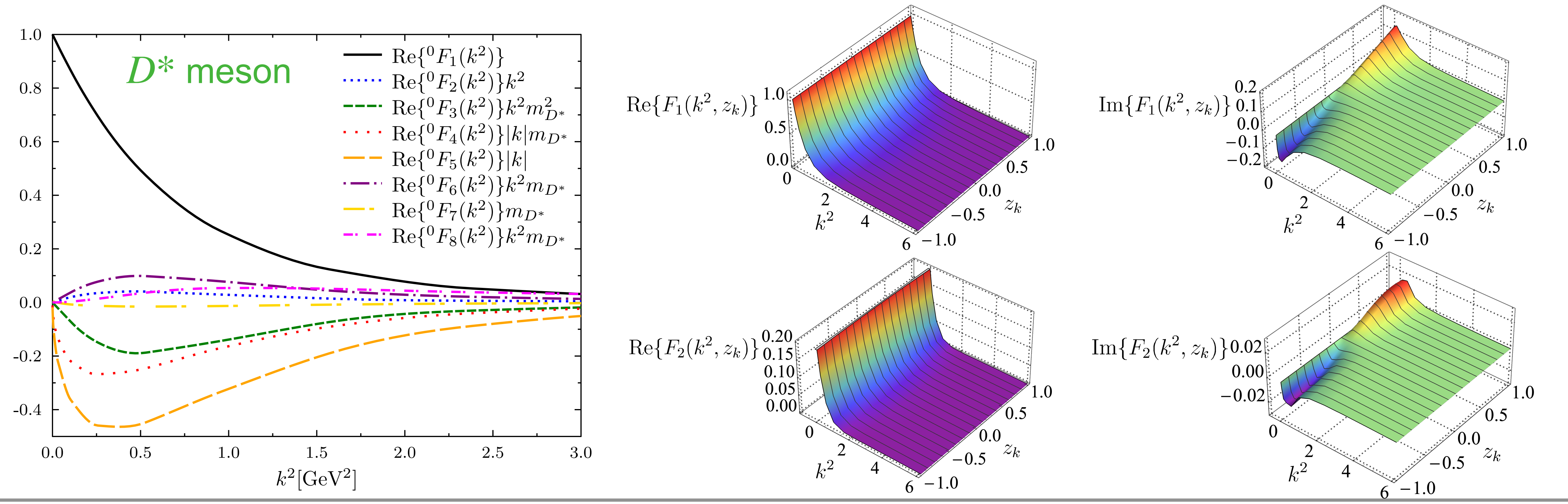
F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Vector channel

Mesons/Properties	m_M	m_M^{exp}	ϵ_r^m %	f_M	$f_M^{exp/\text{lQCD}}$	ϵ_r^f %
$D_u^*(c\bar{u})$	2.021	2.009	0.597	0.165	0.158	4.430
$D_s^*(c\bar{s})$	2.169	2.112	2.699	0.205	0.190	7.895

Heavy-light mesons





Extracting LCDAs

- Projection of BS wave-function onto the light front

$$f_M \phi_M(x,\mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \, \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \, \delta(n\cdot k_\eta - x n \cdot P) \gamma_5 \, \gamma \cdot n \, \chi_M(k_\eta, k_{\bar\eta})$$

$$\chi_M(k_\eta, k_{\bar\eta}) := S(k_\eta)\Gamma_M(k,P)S(k_{\bar\eta})$$

- Interpolation of quark propagator and Bethe-Salpeter amplitudes

$$S_f(q) = \sum_{k=1}^{N}\left[\frac{z_k^f}{i\gamma\cdot q + m_k^f} + \frac{\left(z_k^f\right)^*}{i\gamma\cdot q + \left(m_k^f\right)^*}\right] \quad \mathcal{F}_i(k,P) = \sum_{j=0}^{2}\int_{-1}^{1} dz\, \rho_{\nu_j}(z) \frac{\mathcal{U}_j \Lambda_{\mathcal{F}_i}^{2n_j}}{(k^2 + z\,k\cdot P + \Lambda_{\mathcal{F}_i}^2)^{n_j}}$$

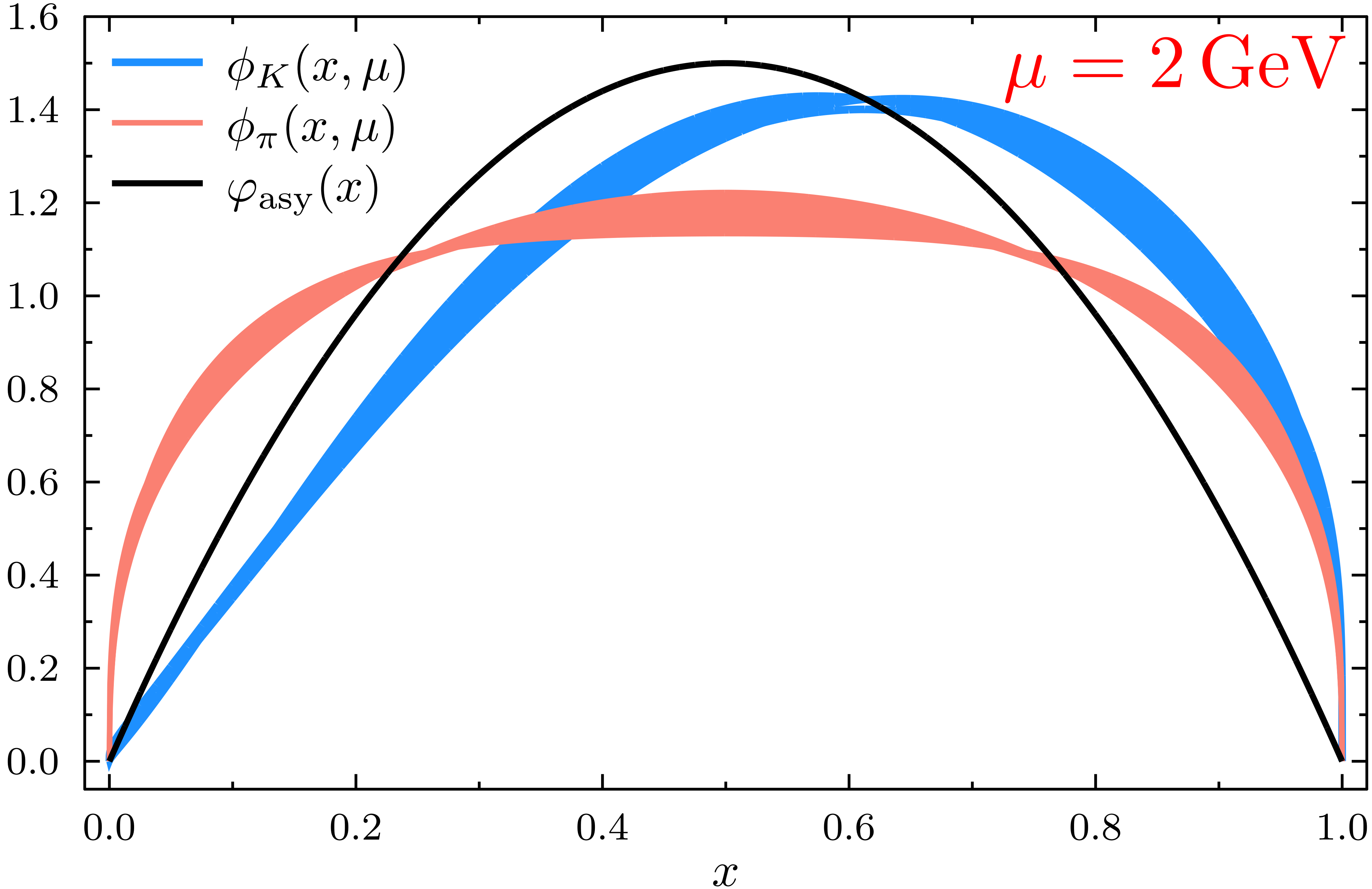
- Computing Mellin moments

$$\langle x^m \rangle = \int_0^1 dx\, x^m \phi_M(x,\mu) \qquad \langle x^0 \rangle = \int_0^1 dx \phi_M(x,\mu) = 1$$

- Reconstructing LCDAs

$$\boxed{\begin{gathered}\phi_\pi^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha)\,[x\bar{x}]^{\alpha-1/2}\,[1 + a_2 C_2^{\alpha}(2x-1)] \\ \phi_H^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha,\beta)\, 4x\bar{x}\, e^{4\,\alpha x\bar{x} + \beta(x-\bar{x})}\end{gathered}}$$

Extracting LCDAs



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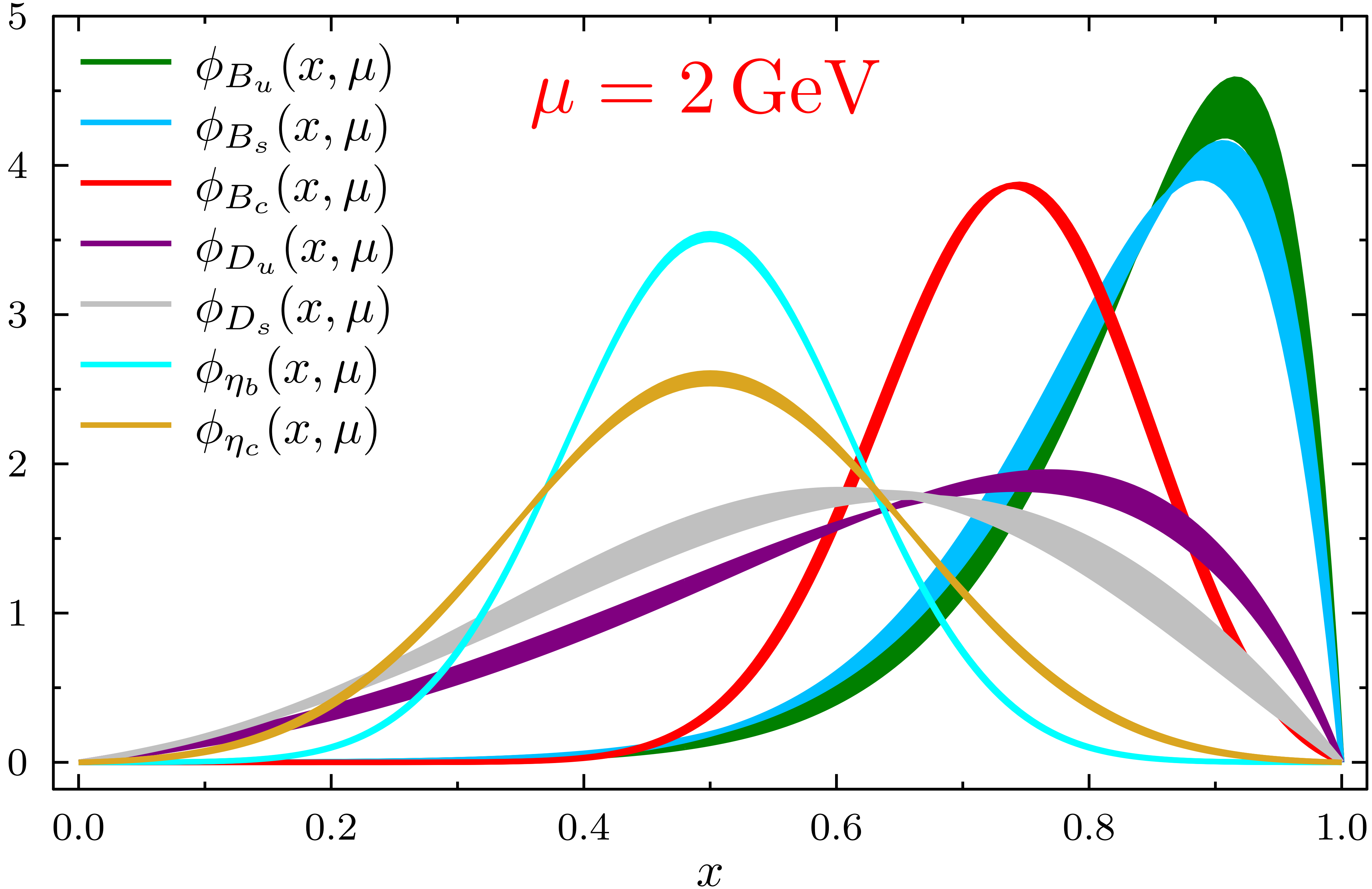
Extracting LCDAs



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Conclusions

- We have employed the combined approach of the DSE for the quark and BSE with a flavor-dependent, slightly modified interaction to first compute the mass spectrum and weak decay constants of the pseudoscalar π, K, D, D_s, B, B_s and B_c mesons and η_c and η_b quarkonia.

- We also have obtained the corresponding LCDA at a physical mass with appropriate projections of the Bethe-Salpeter amplitudes on the light front.

  - We Mesons make up of two quarks of the same flavor, each carrying the same amount of momentum fraction of the bound state on the light front.
  - Mesons make up of two different flavor, the heaviest valence quark inside the meson carries a greater amount of the meson momentum.

Thank you