Exclusive hadronic tau decays as probes of non-SM interactions

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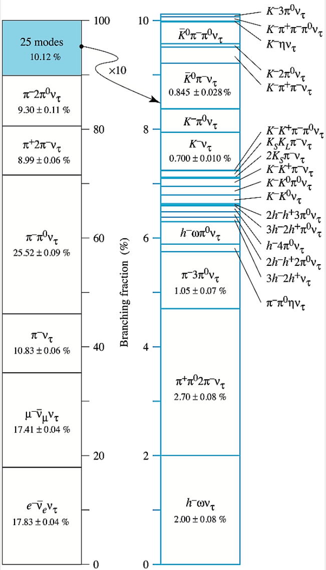
Based on:

S. Gonzàlez-Solís, A. Miranda, J. Rendón, P. Roig; Phys.Lett.B 804 (2020) 135371

S. Gonzàlez-Solís and P. Roig; Eur. Phys. J. C79 (2019) 436

R. Escribano, S. Gonzàlez-Solís, M. Jamin and P. Roig; JHEP 1409 (2014) 042

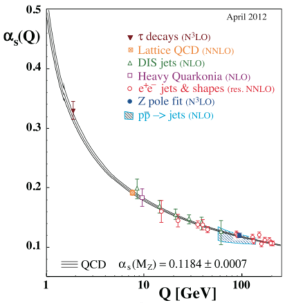
- Tau properties:
  - Mass: $m_\tau = 1.77686(12)$ GeV
  - Lifetime: $\tau_\tau = 2.903(15) \times 10^{-13}$ s
- The only lepton heavy enough to decay into hadrons:
  - Very rich phenomenology
  - Test of QCD and EW interactions
- For the test:
  - Precise measurements needed
  - Hadronic uncertainties under control
- Tau decays: tool to search for New Physics



- **Inclusive** decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

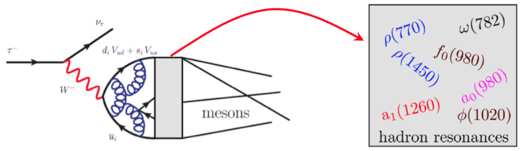
  Fundamental SM parameters: $\alpha_s(m_\tau), m_s, |V_{us}|$

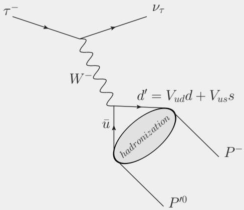


- **Exclusive** decays: $\tau^- \to (PP, PPP, \ldots)\nu_\tau$

  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)





- Invariant mass distribution ($\tau^- \to P^- P^0 \nu_\tau$)

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\mathrm{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

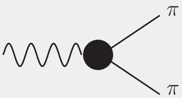
$$\times \left\{ \left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^-P^0}^{3/2}(s) |F_V^{P^-P^0}(s)|^2 + 3\frac{\Delta_{P^-P^0}^2}{s^2} \lambda_{P^-P^0}^{1/2}(s) |F_S^{P^-P^0}(s)|^2 \right\},$$

Decay channel	Standard Model	Resonances
$\tau^- \to \pi^-\pi^0\nu_\tau$	Pion form factor, $(g-2)_\mu$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K^-K_S\nu_\tau$	Kaon form factor, $(g-2)_\mu$	$\rho(770), \rho(1450), \rho(1700)$
$\tau^- \to K_S\pi^-\nu_\tau$	$K\pi$ form factor, $K_{\ell 3}$, $\|V_{us}\|$	$K^*(892), K^*(1410)$
$\tau^- \to K^-\eta^{(\prime)}\nu_\tau$	$K\pi$ form factor, $K_{\ell 3}$, $\|V_{us}\|$	$K^*(1410)$
$\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$	isospin violation, 2nd class currents	$a_0(980)$

- **Important** experimental activity: BaBar, Belle, Belle-II.

The Pion vector form factor $F_V^\pi(s)$

- Classic object of low-energy QCD

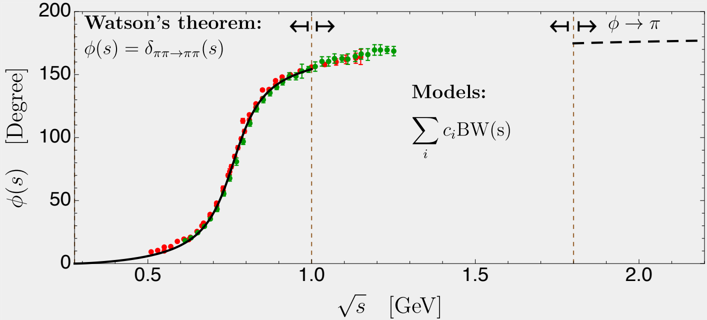


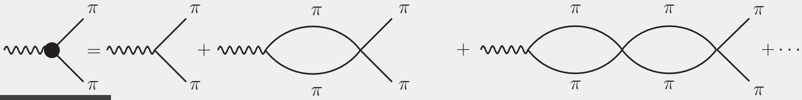
- How to determine $F_V^\pi(s)$ experimentally?
  - $e^+e^- \to \pi^+\pi^-$
  - $\tau^- \to \pi^-\pi^0\nu_\tau$
- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
  - For the intermediate energy region: models

- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right],$$

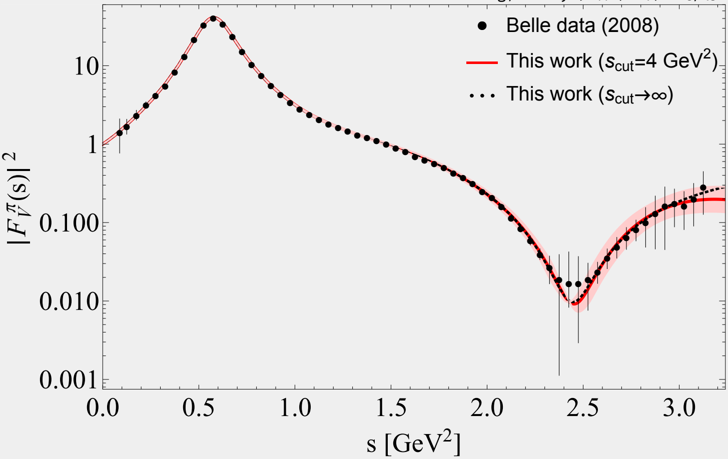
- Form Factor phase $\phi(s)$:





Form factor modulus squared

González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436



Resonance	**Model parameter** (M, Γ) [MeV]	**Pole position** (M, Γ) [MeV]
$\rho(1450)$	1376(6), 603(22)	1289(8), 540(16)
$\rho(1700)$	1718(4), 465(9)	1673(4), 445(8)

Form factor phase

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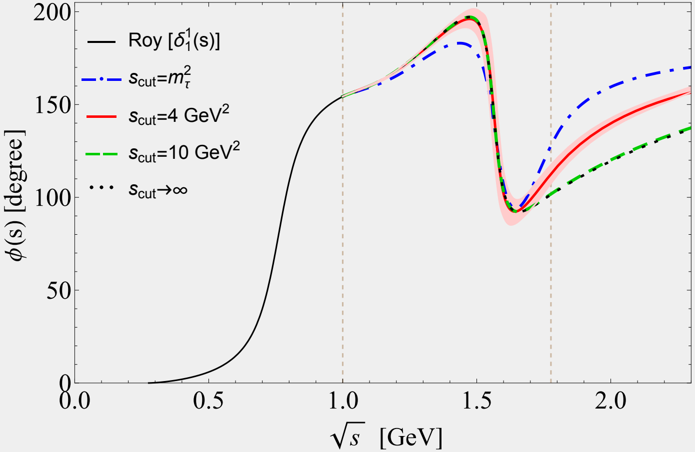

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- $\langle r^2 \rangle_V^\pi = 6\alpha_1$

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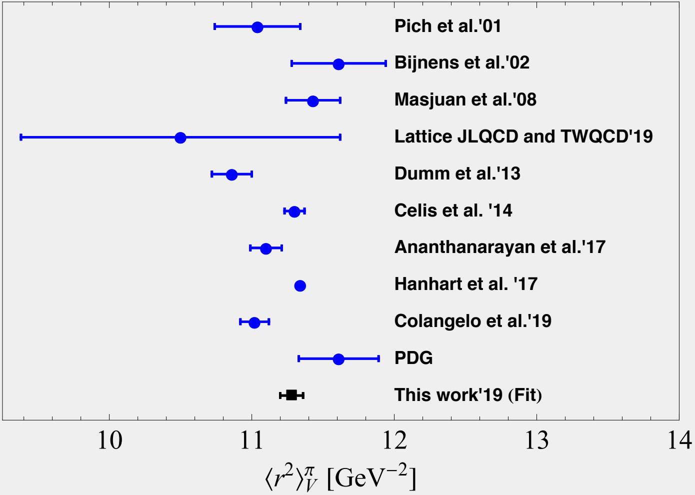


$$\frac{d\Gamma(\tau^- \to K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2}{768\pi^3} M_\tau^3 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \sigma_K^3(s) |F_V^K(s)|^2 \,,$$

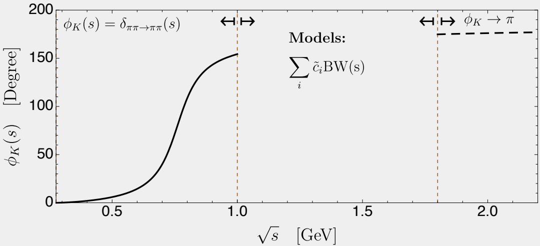
- Chiral Perturbation Theory $\mathcal{O}(p^4)$

$$F_V^K(s)|_{\rm ChPT} = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right] = F_V^\pi(s)|_{\rm ChPT} \,,$$

- Phase dispersive representation with $\alpha_{1,2}$ from $F_V^\pi(s)$

$$F_V^K(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi_K(s')}{(s')^3(s' - s - i\varepsilon)}\right] \,,$$

- Form Factor phase $\phi_K(s)$:



Fit results to BaBar $\tau^- \to K^- K_S \nu_\tau$ data

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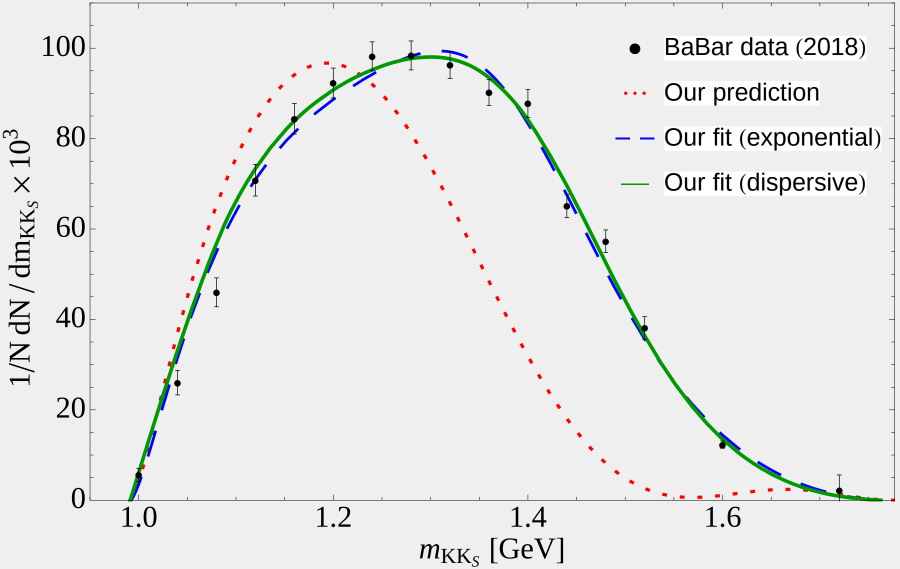


- RχT with two resonances: $K^*(892)$ and $K^*(1410)$:

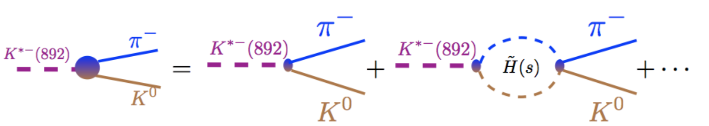


figure courtesy of D. Boito

$$\widetilde{F}_V^{K\pi}(s) = \frac{m_{K^\star}^2 - \kappa_{K^\star}\widetilde{H}_{K\pi}(0) + \gamma s}{D(m_{K^\star},\, \gamma_{K^\star})} - \frac{\gamma s}{D(m_{K^{\star\prime}},\, \gamma_{K^{\star\prime}})},$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \mathrm{Re}\left[H_{K\pi}(s)\right] - i m_n \Gamma_n(s),$$

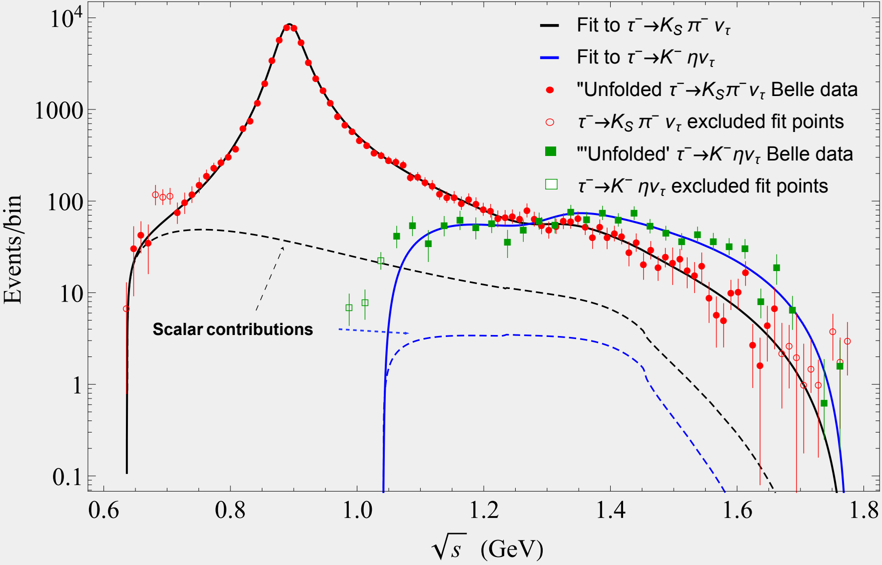
$$\kappa_n = \frac{192\pi F_K F_\pi}{\sigma_{K\pi}(m_{K^*}^2)} \frac{\gamma_{K^*}}{m_{K^*}}, \quad \Gamma_n(s) = \Gamma_n \frac{s}{m_n^2} \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_n^2)},$$

- We have a phase with two resonances:

$$\delta^{K\pi}(s) = \tan^{-1}\left[\frac{\mathrm{Im} F_V^{K\pi}(s)}{\mathrm{Re} F_V^{K\pi}(s)}\right],$$

Combined fit to $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$

Escribano, Gonzàlez-Solís, Jamin and Roig, JHEP 1409 (2014) 042



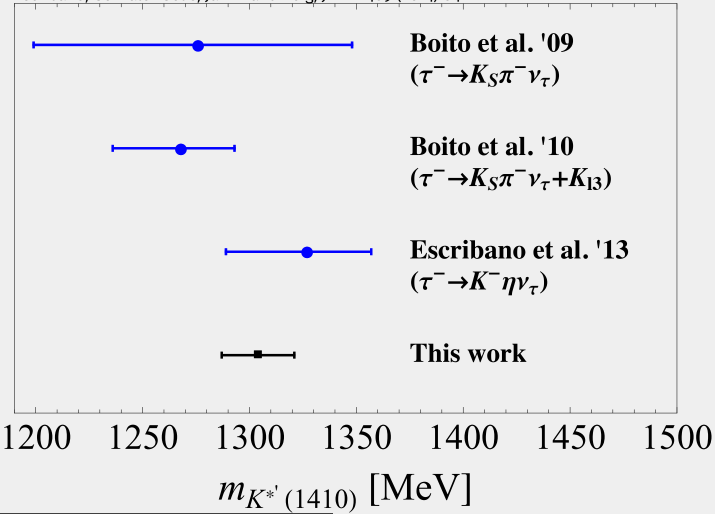
Fit results

- Different choices regarding linear slopes and resonance mixing parameters ($s_{cut} = 4$ GeV^2)

Fitted value	Reference Fit	Fit A	Fit B	Fit C
$\bar{B}_{K\pi}(\%)$	0.404 ± 0.012	0.400 ± 0.012	0.404 ± 0.012	0.397 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.402)	(0.394)	(0.400)	(0.394)
M_{K^*}	892.03 ± 0.19	892.04 ± 0.19	892.03 ± 0.19	892.07 ± 0.19
Γ_{K^*}	46.18 ± 0.42	46.11 ± 0.42	46.15 ± 0.42	46.13 ± 0.42
$M_{K^{*\prime}}$	1305^{+15}_{-18}	1308^{+16}_{-19}	1305^{+15}_{-18}	1310^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	168^{+52}_{-44}	212^{+66}_{-54}	174^{+58}_{-47}	184^{+56}_{-46}
$\gamma_{K\pi} \times 10^2$	$= \gamma_{K\eta}$	$-3.6^{+1.1}_{-1.5}$	$-3.3^{+1.0}_{-1.3}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.9 ± 0.7	23.6 ± 0.7	23.8 ± 0.7	23.6 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2	11.6 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.58 ± 0.10	1.62 ± 0.10	1.57 ± 0.10	1.66 ± 0.09
$(B^{th}_{K\eta}) \times 10^4$	(1.45)	(1.51)	(1.44)	(1.58)
$\gamma_{K\eta} \times 10^2$	$-3.4^{+1.0}_{-1.3}$	$-5.4^{+1.8}_{-2.6}$	$-3.9^{+1.4}_{-2.1}$	$-3.7^{+1.0}_{-1.4}$
$\lambda'_{K\eta} \times 10^3$	20.9 ± 1.5	$= \lambda'_{K\pi}$	21.2 ± 1.7	$= \lambda'_{K\pi}$
$\lambda''_{K\eta} \times 10^4$	11.1 ± 0.4	11.7 ± 0.2	11.1 ± 0.4	11.8 ± 0.2
χ^2/n.d.f.	$108.1/105 \sim 1.03$	$109.9/105 \sim 1.05$	$107.8/104 \sim 1.04$	$111.9/106 \sim 1.06$

- Most precise determination of the $K^*(1410)$ parameters

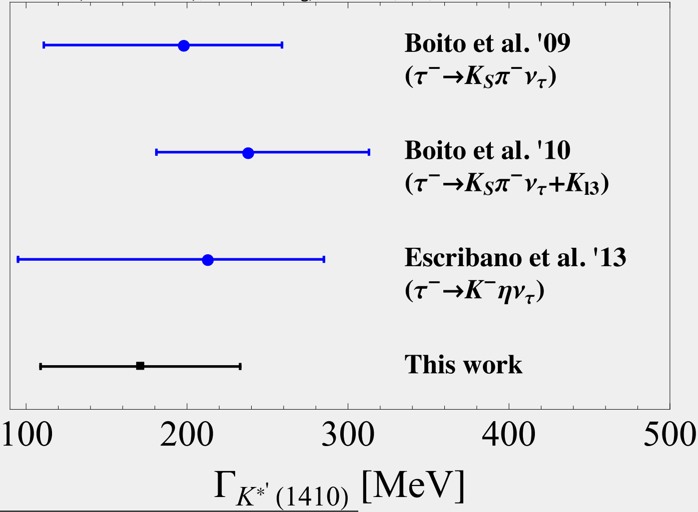
Escribano, Gonzàlez-Solís, Jamin and Roig, JHEP 1409 (2014) 042



$K^*(1410)$ PHYSICAL PARAMETERS

- Most precise determination of the $K^*(1410)$ parameters

Escribano, Gonzàlez-Solís, Jamin and Roig, JHEP 1409 (2014) 042



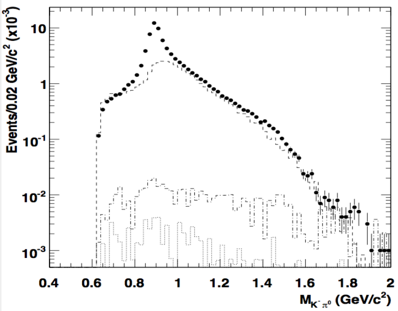
Low-energy parameters

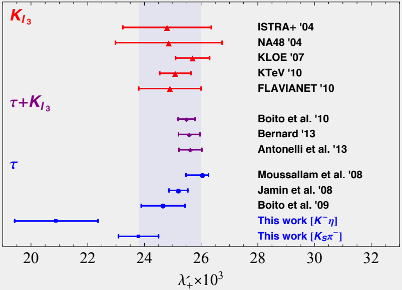
- Low-energy parameters

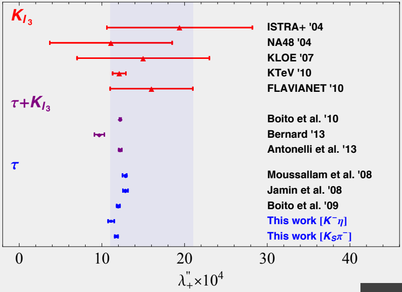
$$\left.\begin{aligned}\lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3}\\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3}\end{aligned}\right\}\text{isospin violation?}$$

$$\left.\begin{aligned}\lambda''_{K\pi} &= (11.8 \pm 0.2)\cdot 10^{-4}\\ \lambda''_{K\eta} &= (11.1 \pm 0.5)\cdot 10^{-4}\end{aligned}\right\}\text{isospin violation?}$$

$\tau^- \to K^-\pi^0\nu_\tau$ (PRD 76 (2007) 051104)







Tau lepton: SM vs non-SM

- $2.6\sigma(2.4\sigma)$ LFU deviation from $|g_\tau/g_\mu|(|g_\tau/g_e|)$ in $W^- \to \tau^-\bar{\nu}_\tau$
- 2.8σ deviation CP asymmetry in $\tau^- \to K_S\pi^-\nu_\tau$:

  $A_{CP} = -3.6(2.3)(1.1) \times 10^{-3}$ (exp) vs $A_{CP} = 3.6(1) \times 10^{-3}$ (th),
- $\tau^- \to \nu_\tau \bar{u} D$ $(D =, d, s)$ as probes on non-SM interactions

$$\mathcal{L}_{CC} = -\frac{G_F V_{uD}}{\sqrt{2}}\Big[(1+\epsilon_L^\tau)\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma^5)D$$
$$+\epsilon_R^\tau\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1+\gamma^5)D + \bar{\tau}(1-\gamma^5)\nu_\tau \cdot \bar{u}(\epsilon_S^\tau - \epsilon_P^\tau\gamma^5)D$$
$$+\epsilon_T^\tau\bar{\tau}\sigma_{\mu\nu}(1-\gamma^5)\nu_\tau\, \bar{u}\sigma^{\mu\nu}(1-\gamma^5)D\Big] + h.c.,$$

  - **Garcés** et.al. [JHEP 1712, 027 (2017)]; **Miranda** et.al. [JHEP 1811, 038 (2018)]; **Cirigliano** et.al. [Phys.Rev.Lett. 122 (2019) no.22, 221801]; **Rendón** et.al. [Phys.Rev. D 99, no. 9, 093005 (2019)]; **Gonzàlez-Solís** et.al. [Phys.Lett.B 804 (2020) 135371]

Strangeness-conserving transitions ($\Delta S = 0$)

- **One meson** decay $\tau^- \to \pi^- \nu_\tau$ ($G_F \tilde{V}^e_{uD} = G_F \left(1 + \epsilon^e_L + \epsilon^e_R\right) V_{uD}$)

$$\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}^e_{ud}|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \times \left(1 + \delta^{\tau\pi}_{\rm em} + 2\Delta^{\tau\pi} + \mathcal{O}(\epsilon^\tau_i)^2 + \mathcal{O}(\delta^{\tau\pi}_{\rm em}\epsilon^\tau_i)\right),$$

  - Inputs: $f_\pi = 130.2(8)$ MeV (FLAG 1902.08191); $\delta^{\tau\pi}_{\rm em} = 1.92(24)\%$; $|\tilde{V}^e_{ud}| = 0.97420(21)$ (β decays, PDG).

- **Constraint** for the NP effective couplings:

$$\Delta^{\tau\pi} \equiv \epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon^\tau_P = (-0.12 \pm 0.68)\times 10^{-2},$$

  - Errors (hierarchy): $f_\pi, BR, \delta^{\tau\pi}_{\rm em}$

Strangeness-conserving transitions ($\Delta S = 0$)

- Partial decay width for **two-meson** decays

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384\pi^3 s} \left(1 - \frac{s}{m_\tau^2}\right)^2 \lambda^{1/2}(s, m_P^2, m_{P'}^2)$$

$$\times \left[(1 + 2(\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e)) X_{VA} + \epsilon_S^\tau X_S + \epsilon_T^\tau X_T + (\epsilon_S^\tau)^2 X_{S^2} + (\epsilon_T^\tau)^2 X_{T^2} \right],$$

$$X_{VA} = \frac{1}{2s^2} \left\{ 3 \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \Delta_{PP'}^2 + \left(C_{PP'}^V\right)^2 |F_+^{PP'}(s)|^2 \left(1 + \frac{2s}{m_\tau^2}\right) \lambda(s, m_P^2, m_{P'}^2) \right\},$$

$$X_S = \frac{3}{s\, m_\tau} \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{m_d - m_u},$$

$$X_T = \frac{6}{s\, m_\tau} C_{PP'}^V \,\mathrm{Re}\left[F_T^{PP'}(s) \left(F_+^{PP'}(s)\right)^*\right] \lambda(s, m_P^2, m_{P'}^2),$$

$$X_{S^2} = \frac{3}{2\, m_\tau^2} \left(C_{PP'}^S\right)^2 |F_0^{PP'}(s)|^2 \frac{\Delta_{PP'}^2}{(m_d - m_u)^2},$$

$$X_{T^2} = \frac{4}{s} |F_T^{PP'}(s)|^2 \left(1 + \frac{s}{2\, m_\tau^2}\right) \lambda(s, m_P^2, m_{P'}^2),$$

- No experimental data
- Theoretical assumptions only

$$\mathrm{Im}F_T^{PP'}(s) = \sigma_{PP'}(s) t_+^*(s) F_T^{PP'}(s)\,,$$

$$F_T^{PP'}(s) = F_T^{PP'}(0)\exp\left[\frac{s}{\pi}\int_{s_{\rm th}}^{s_{\rm cut}}\frac{ds'}{s'}\frac{\delta_T^{PP'}(s')}{(s'-s-i0)}\right]\,,$$

- $s_{\rm th} = (m_P + m_{P'})^2$: two-meson production threshold
- In the elastic region: $\delta_T^{PP'}(s) = \delta_+^{PP'}(s)$
- We guide the phase to $\pi \Rightarrow$ asymptotic $1/s$ dictated by pQCD
- $F_T^{PP'}(0)$: ChPT with tensor fields+lattice

Strangeness-conserving transitions ($\Delta S = 0$)

- **Global fit** to one and two meson decays

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\mathrm{th}} - \bar{N}_k^{\mathrm{exp}}}{\sigma_{\bar{N}_k^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{\pi\pi}^{\mathrm{th}} - BR_{\pi\pi}^{\mathrm{exp}}}{\sigma_{BR_{\pi\pi}^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{KK}^{\mathrm{th}} - BR_{KK}^{\mathrm{exp}}}{\sigma_{BR_{KK}^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{\tau\pi}^{\mathrm{th}} - BR_{\tau\pi}^{\mathrm{exp}}}{\sigma_{BR_{\tau\pi}^{\mathrm{exp}}}} \right)^2$$

- $\bar{N}_k^{\mathrm{th}}$: normalized distribution for $\tau^- \to \pi^-\pi^0\nu_\tau$

$$\bar{N}^{\mathrm{th}} \equiv \frac{1}{N_{\mathrm{events}}} \frac{dN_{\mathrm{events}}}{ds} = \frac{1}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)} \frac{d\Gamma(s, \epsilon_i^\tau, \epsilon_j^e)}{ds} \Delta^{\mathrm{bin}}$$

- Data: unfolded distribution measured by Belle (0805.3773)
- Constraints:
  - $BR(\tau^- \to \pi^-\pi^0\nu_\tau)^{\mathrm{exp}} = 25.49(9)\%$
  - $BR(\tau^- \to K^-K^0\nu_\tau)^{\mathrm{exp}} = 1.486(34) \times 10^{-3}$
  - $BR(\tau^- \to \pi^-\nu_\tau)^{\mathrm{exp}} = 10.82(5)\%$

Strangeness-conserving transitions ($\Delta S = 0$)

- **Bounds** for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8}\,{}^{+0.2}_{-0.1} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-0.9}\,{}^{+0.1}_{-0.0} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5\,{}^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4}\,{}^{+0.0}_{-0.1} \pm 0.2 \end{pmatrix} \times 10^{-2},$$

- Errors:
  - *i*) Statistic (1st)
  - *ii*) Systematic: pion vector form factor (2nd), quark masses (3rd) and tensor form factor (4th)

$$\rho_{ij} = \begin{pmatrix} 1 & 0.684 & -0.493 & -0.545 \\ & 1 & -0.337 & -0.372 \\ & & 1 & 0.463 \\ & & & 1 \end{pmatrix},$$

- One meson decay $\tau^- \to K^- \nu_\tau$

$$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2 |\tilde{V}_{us}^e|^2 f_K^2 m_\tau^3}{16\pi} \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 \times \left(1 + \delta_{\rm em}^{\tau K} + 2\Delta^{\tau K} + \mathcal{O}(\epsilon_i^\tau)^2 + \mathcal{O}(\delta_{\rm em}^{\tau K}\epsilon_i^\tau)\right),$$

  - Inputs: $f_K = 155.7(7)$ MeV (FLAG 1902.08191); $\delta_{\rm em}^{\tau\pi} = 1.98(31)\%$; $|\tilde{V}_{us}^e| = 0.2231(7)$ (PDG).

- **Constraint** for the NP effective couplings:

$$\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon_P^\tau = (-0.41 \pm 0.93) \times 10^{-2},$$

  - Errors (hierarchy): $f_K, |V_{us}|, BR, \delta_{\rm em}^{\tau K}$

Strangeness-changing transitions ($|\Delta S| = 1$)

- Global fit to one and two meson decays

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\pi}^{\rm th} - BR_{K\pi}^{\rm exp}}{\sigma_{BR_{K\pi}^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left(\frac{BR_{\tau K}^{\rm th} - BR_{\tau K}^{\rm exp}}{\sigma_{BR_{\tau K}^{\rm exp}}} \right)^2$$

- $\bar{N}_k^{\rm th}$: distribution for $\tau^- \to K_S \pi^- \nu_\tau$

$$\bar{N}^{\rm th} \equiv \frac{dN_{\rm events}}{ds} = \frac{N_{\rm events}}{\Gamma(\epsilon_i^\tau, \epsilon_j^e)} \frac{d\Gamma(s, \epsilon_i^\tau, \epsilon_j^e)}{ds} \Delta^{\rm bin}$$

- Data: unfolded distribution measured by Belle (0706.2231)
- Constraints:
  - $BR(\tau^- \to K_S \pi^- \nu_\tau)^{\rm exp} = 0.404(2)\%$ (Belle)
  - $BR(\tau^- \to K^- \eta \nu_\tau)^{\rm exp} = 1.55(8) \times 10^{-4}$ (PDG)
  - $BR(\tau^- \to K^- \nu_\tau)^{\rm exp} = 6.96(10) \times 10^{-3}$ (PDG)

- **Bounds** for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \pm 0.3 \\ 0.4 \pm 0.9 \pm 0.2 \\ 0.8^{+0.8}_{-0.9} \pm 0.3 \\ 0.9 \pm 0.7 \pm 0.3 \end{pmatrix} \times 10^{-2},$$

- Errors: Statistic (fit)+systematic (tensor form factor).

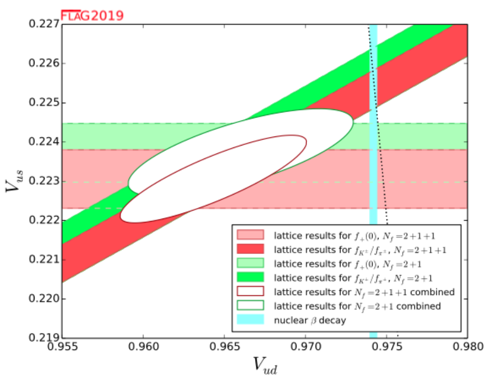
$$\rho_{ij} = \begin{pmatrix} 1 & 0.854 & -0.147 & 0.437 \\ & 1 & -0.125 & 0.373 \\ & & 1 & -0.055 \\ & & & 1 \end{pmatrix},$$

Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Precision experimental data on kaon decays (FLAG'19, 1902.08191):

$$|V_{us}| f_+^{K\pi}(0) = 0.2165(4)\,, \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} = 0.2760(4)\,,$$

- Correlation between $|V_{us}|$ and $|V_{ud}|$



Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Combination to **one and two meson decays**

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2},$$

- Errors: Statistic$\pm V_{CKM} \pm \delta_{\rm em}^{\tau\pi(K)}$$\pm$tensor form factor$\pm$quark masses

$$\mathcal{A} = \begin{pmatrix} 1 & 0.055 & 0.000 & -0.279 & -0.394 \\ & 1 & -0.997 & -0.015 & -0.022 \\ & & 1 & 0.000 & 0.000 \\ & & & 1 & 0.243 \\ & & & & 1 \end{pmatrix},$$

Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Combination to **one and two meson decays**

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm0.6 & \pm0.6 & \pm0.0 & \pm0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm0.0 & \pm0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm0.2 & \pm0.0 & \pm0.0 & \pm0.6 & \pm0.1 \end{pmatrix} \times 10^{-2},$$

- Comparison with other bounds (assuming LFU):
  - Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$ [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]
  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

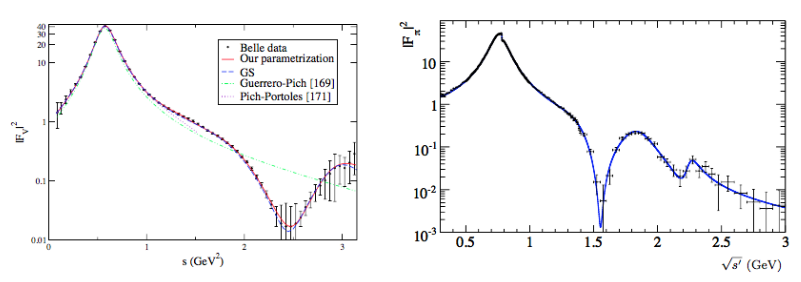
$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_S^\tau \\ \epsilon_P^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2},$$

- Hadronic τ decays as a privileged tool for the **investigation of QCD**...
- ...but also as a laboratory of **New Physics**
- Hadronic Tau decays as **golden modes** at Belle-II
- SM input: Form Factors from dispersion relations
  - (Competitive) **Bounds** on the NP effective couplings
- A lot of **interesting physics** to be done in the tau sector

THE PION VECTOR FORM FACTOR $F_V^{\pi}(s)$

- How to determine $F_V^{\pi}(s)$ experimentally?

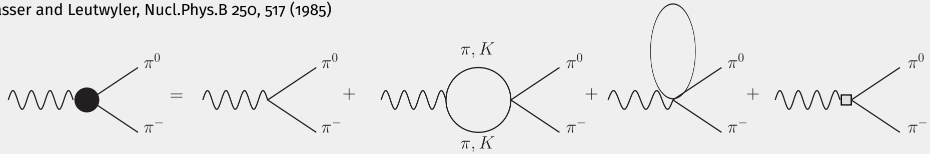
  $\tau^- \to \pi^-\pi^0\nu_\tau$ (Belle PRD 78 (2008) 072006) and $e^+e^- \to \pi^+\pi^-$ (BaBar PRD 86 (2012) 032013)



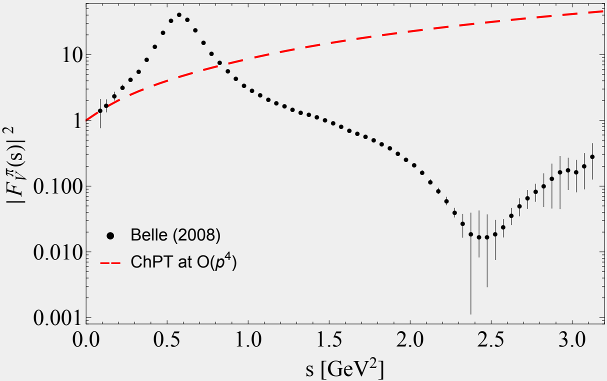
- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
  - For the intermediate energy region: models

Pion vector form factor: ChPT $\mathcal{O}(p^4)$

Gasser and Leutwyler, Nucl.Phys.B 250, 517 (1985)

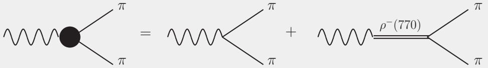


$$F_V^\pi(s)|_{\mathrm{ChPT}} = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right),$$



Pion vector Form Factor: ChPT with resonances

- Resonace Chiral Theory:



$$F_V^\pi(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_\rho^2 - s} \stackrel{F_V G_V = F_\pi^2}{\Longrightarrow} \frac{M_\rho^2}{M_\rho^2 - s}\,,$$

- Expansion in s and comparing ChPT and RχT:

$$F_V^\pi(s) \;=\; 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)\,,$$

$$F_V^\pi(s) \;=\; 1 + \left(\frac{s}{M_\rho^2}\right) + \left(\frac{s}{M_\rho^2}\right)^2 + \cdots$$

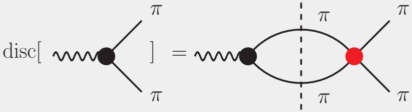
- Chiral coupling estimate: $L_9^r(M_\rho) = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} \simeq 7.2 \times 10^{-3}$

- Combining ChPT and RχT:

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\,,$$

Dispersive representation

- Unitarity:

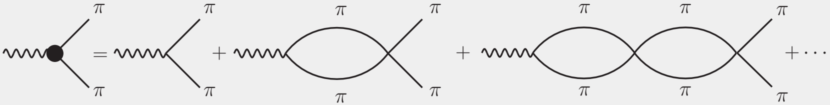


$$\mathrm{disc}F_V(s) = 2i\sigma_\pi(s)F_V(s)T_1^{1*}(s) = 2iF_V(s)\sin\delta_1^1(s)e^{-i\delta_1^1(s)}\,,$$

$$F_V(s) = \frac{1}{2i\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\mathrm{disc}F_V(s')}{s'-s-i\varepsilon}\,,$$

- Analytic solution (Omnès equation):

$$F_V(s) = P(s)\Omega(s)\,, \quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds'\,\frac{\delta_1^1(s')}{s'(s'-s-i\varepsilon)}\right\}\,,$$



RχT + Omnès: Exponential representation

- Resummation of final-state interactions to all orders (Omnès)

$$F_V^\pi(s) = P_n(s) \exp\left\{ \frac{s^n}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{(s')^n} \frac{\delta_1^1(s')}{s' - s - i\varepsilon} \right\},$$

- Get a model for the phase from $\pi\pi \to \pi\pi$ scattering at $\mathcal{O}(p^2)$

$$T(s) = \frac{s - m_\pi^2}{F_\pi^2} \to T_1^1(s) = \frac{s\sigma_\pi^2(s)}{96\pi F_\pi^2} \to \delta_1^1(s) = \sigma_\pi(s) T_1^1(s) = \frac{s\sigma_\pi^3(s)}{96\pi F_\pi^2},$$

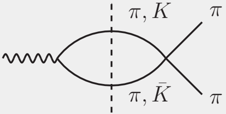
- Omnès exponentiation of the full loop function

$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp\left\{ -\frac{s}{96\pi^2 F_\pi^2} A_\pi(s, \mu^2) \right\}.$$

RχT + Omnès: Exponential representation

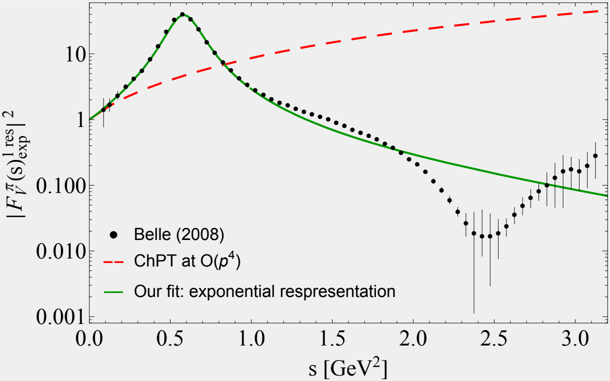
Guerrero and Pich, PLB 412, 382 (1997)

- Incorporation of the (off-shell) ρ width:



$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_\pi(s)^3\theta(s-4m_\pi^2)+\sigma_K(s)^3\theta(s-4m_K^2)\right] ,$$

$$F_V^\pi(s)\big|_{\rm exp}^{1\,\rm res} = \frac{M_\rho^2}{M_\rho^2-s-iM_\rho\Gamma_\rho(s)}\exp\left\{-\frac{s}{96\pi^2F_\pi^2}\mathrm{Re}\left[A_\pi(s,\mu^2)+\frac{1}{2}A_K(s,\mu^2)\right]\right\} .$$



RχT + Omnès: Exponential representation

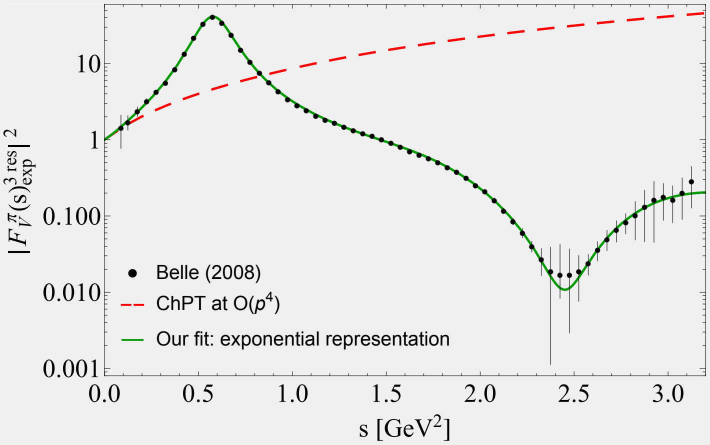
- Incorporation of the $\rho'(1450), \rho''(1700)$

$$
\begin{aligned}
F_V^\pi(s)\Big|_{\rm expo}^{\rm 3\,res} \;=\;& \frac{M_\rho^2 + s\left(\gamma e^{i\phi_1} + \delta e^{i\phi_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\} \\
& -\gamma \frac{s\, e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{ -\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \mathrm{Re}A_\pi(s)\right\} \\
& -\delta \frac{s\, e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{ -\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \mathrm{Re}A_\pi(s)\right\},
\end{aligned}
$$

where

$$
\Gamma_{\rho',\rho''}(s) \;=\; \Gamma_{\rho',\rho''} \frac{M_{\rho',\rho''}}{\sqrt{s}} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}\,.
$$

RχT + Omnès: Exponential representation

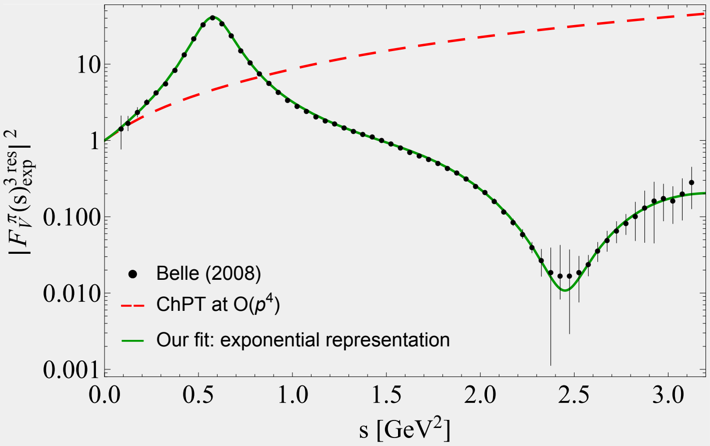


$$M_\rho = 775.2(4)\,\mathrm{MeV}\,, \quad \gamma = 0.15(4)\,, \quad \phi_1 = -0.36(24)\,,$$

$$M_{\rho'} = 1438(39)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 535(63)\,\mathrm{MeV}\,, \quad \delta = -0.12(4)\,, \phi_2\,, = -0.02(45)\,,$$

$$M_{\rho''} = 1754(91)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 412(102)\,\mathrm{MeV}\,, \quad \chi^2_{\mathrm{dof}} = 0.92$$

RχT + Omnès: Exponential representation



Resonance	Model parameter (M, Γ) [MeV]	Pole position (M, Γ) [MeV]
$\rho(770)$	775.2(4)	762.0(3), 143.0(2)
$\rho(1450)$	1438(39), 535(63)	1366(38), 488(48)
$\rho(1700)$	1754(91), 412(102)	1718(82), 397(88)

Dispersive representation

- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right] ,$$

  - Low-energy observables:

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots ,$$

$$\langle r^2\rangle_V^\pi\Big|_{\rm ChPT}^{\mathcal{O}(p^4)} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2F_\pi^2}\left[2\log\left(\frac{M_\pi^2}{\mu^2}\right) + \log\left(\frac{M_K^2}{\mu^2}\right) + 3\right] ,$$

$$\langle r^2\rangle_V^\pi = 6\alpha_1 , \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right) , \quad \alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{s'^{k+1}} .$$

  - $s_{\rm cut}$: cut-off to check stability

Central results

- Fit results (central value ± stat fit error ± syst th. error)

$$\alpha_1 = 1.88(1)(1)\ \mathrm{GeV}^{-2},\ \alpha_2 = 4.34(1)(3)\ \mathrm{GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3\ \mathrm{MeV}\,,$$

$$M_{\rho'} = 1376 \pm 6^{+18}_{-73}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141}\ \mathrm{MeV}\,,$$

$$M_{\rho''} = 1718 \pm 4^{+57}_{-94}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53}\ \mathrm{MeV}\,,$$

$$\gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}\,, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99}\,,$$

$$\delta = -0.13 \pm 0.01^{+0.00}_{-0.05}\,, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90}\,,$$

- Physical pole mass and width

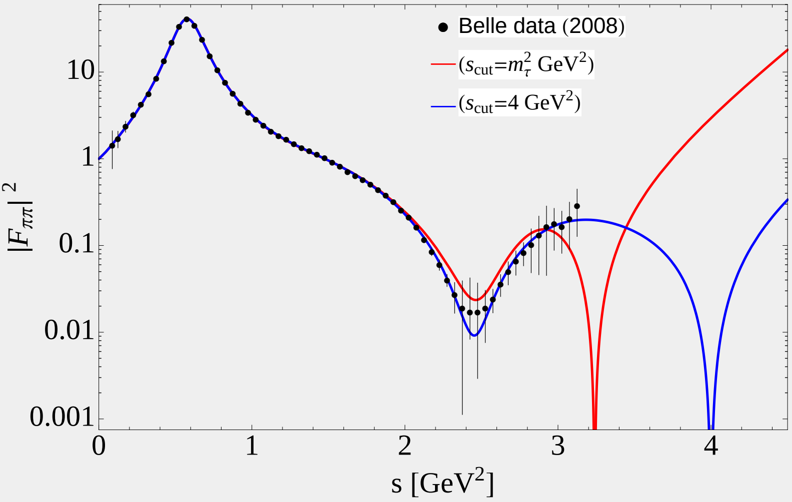
$$M_\rho^{\mathrm{pole}} = 760.6 \pm 0.8\ \mathrm{MeV}\,, \quad \Gamma_\rho^{\mathrm{pole}} = 142.0 \pm 0.4\ \mathrm{MeV}\,,$$

$$M_{\rho'}^{\mathrm{pole}} = 1289 \pm 8^{+52}_{-143}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'}^{\mathrm{pole}} = 540 \pm 16^{+151}_{-111}\ \mathrm{MeV}\,,$$

$$M_{\rho''}^{\mathrm{pole}} = 1673 \pm 4^{+68}_{-125}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''}^{\mathrm{pole}} = 445 \pm 8^{+117}_{-49}\ \mathrm{MeV}\,,$$

DISP. REPRESENTATION: SINGULARITIES AT $s = s_{\rm cut}$

- Modulus squared of the pion form factor $s_{\rm cut} = m_\tau, 4$ GeV²



Dispersive Fits to the Pion Vector Form Factor

- Fits for different values of $s_{\rm cut}$ and matching at 1 GeV

Fits	Parameter	$s_{\rm cut}$ [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1	α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.89(1)	1.89(1)
	α_2 [GeV^{-4}]	4.40(1)	4.34(1)	4.32(1)	4.32(1)
	m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
	M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$	$=m_\rho$
	$M_{\rho'}$ [MeV]	1365(15)	1376(6)	1313(15)	1311(5)
	$\Gamma_{\rho'}$ [MeV]	562(55)	603(22)	700(6)	701(28)
	$M_{\rho''}$[MeV]	1727(12)	1718(4)	1660(9)	1658(1)
	$\Gamma_{\rho''}$ [MeV]	278(1)	465(9)	601(39)	602(3)
	γ	0.12(2)	0.15(1)	0.16(1)	0.16(1)
	ϕ_1	$-0.69(1)$	$-0.66(1)$	$-1.36(10)$	$-1.39(1)$
	δ	$-0.09(1)$	$-0.13(1)$	$-0.16(1)$	$-0.17(1)$
	ϕ_2	$-0.17(5)$	$-0.44(3)$	$-1.01(5)$	$-1.03(2)$
	χ^2/d.o.f	1.47	0.70	0.64	0.64

Variant (I)

- Fits for different matching point and with $s_{\mathrm{cut}} = 4$ GeV

Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.88(1)	1.88(1)
	α_2 [GeV^{-4}]	4.35(1)	4.35(1)	4.34(1)	4.34(1)
	m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1394(6)	1374(8)	1351(5)	1376(6)
	$\Gamma_{\rho'}$ [MeV]	592(19)	583(27)	592(2)	603(22)
	$M_{\rho''}$[MeV]	1733(9)	1715(1)	1697(3)	1718(4)
	$\Gamma_{\rho''}$ [MeV]	562(3)	541(45)	486(7)	465(9)
	γ	0.12(1)	0.12(1)	0.13(1)	0.15(1)
	ϕ_1	$-0.44(3)$	$-0.60(1)$	$-0.80(1)$	$-0.66(1)$
	δ	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$
	ϕ_2	$-0.38(3)$	$-0.51(2)$	$-0.62(1)$	$-0.44(3)$
	χ^2/d.o.f	0.75	0.74	0.68	0.70

Variant (II): intermediate states other than $\pi\pi$

- Fit A: $\rho' \to K\bar{K}$ and $\rho'' \to K\bar{K}$
- Fit B: $\rho' \to K\bar{K}$ + $\rho' \to \omega\pi$

Parameter	$s_{\rm cut} = 4$ GeV²		
	Fit A	Fit B	reference fit
α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.88(1)
α_2 [GeV^{-4}]	4.37(1)	4.35(1)	4.34(1)
m_ρ [MeV]	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	1373(5)	1441(3)	1376(6)
$\Gamma_{\rho'}$ [MeV]	462(14)	576(33)	603(22)
$M_{\rho''}$[MeV]	1775(1)	1733(9)	1718(4)
$\Gamma_{\rho''}$ [MeV]	412(27)	349(52)	465(9)
γ	0.13(1)	0.15(3)	0.15(1)
ϕ_1	$-$0.80(1)	$-$0.53(5)	$-$0.66(1)
δ	$-$0.14(1)	$-$0.14(1)	$-$0.13(1)
ϕ_2	$-$0.44(2)	$-$0.46(3)	$-$0.44(3)
χ^2/d.o.f	0.93	0.70	0.70

Variant (III)

- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)} + \frac{s}{\pi}\int_{s_{\rm cut}}^{\infty} ds' \frac{\delta_{\rm eff}(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s)$$

- Properties for $\delta_{\rm eff}(s)$
  - $\delta_{\rm eff}(s_{\rm cut}) = \delta_1^1(s_{\rm cut})$ and $\delta_{\rm eff}(s) \to \pi$ for large s to recover $1/s$

$$\delta_{\rm eff}(s) = \pi + \left(\delta_1^1(s_{\rm cut}) - \pi\right)\frac{s_{\rm cut}}{s}$$

  - Integrating the piece with $\delta_{\rm eff}(s)$

$$\begin{aligned} F_V^\pi(s) &= e^{1-\frac{\delta_1^1(s_{\rm cut})}{\pi}}\left(1-\frac{s}{s_{\rm cut}}\right)^{\left(1-\frac{\delta_1^1(s_{\rm cut})}{\pi}\right)\frac{s_{\rm cut}}{s}}\left(1-\frac{s}{s_{\rm cut}}\right)^{-1} \\ &\times \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s) \\ \Sigma(s) &= \sum_{i=0}^{\infty} a_i\omega^i(s)\,, \quad \omega(s) = \frac{\sqrt{s_{\rm cut}}-\sqrt{s_{\rm cut}-s}}{\sqrt{s_{\rm cut}}+\sqrt{s_{\rm cut}-s}} \end{aligned}$$

Variant (III)

The resulting fit parameters are found to be

$$a_1 = 2.99(12)\,,$$
$$M_{\rho'} = 1261(7)\,\mathrm{MeV}\,,\quad \Gamma_{\rho'} = 855(15)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1600(1)\,\mathrm{MeV}\,,\quad \Gamma_{\rho''} = 486(26)\,\mathrm{MeV}\,,$$
$$\gamma = 0.25(2)\,,\quad \phi_1 = -1.90(6)\,,$$
$$\delta = -0.15(1)\,,\quad \phi_2 = -1.60(4)\,,$$

with a $\chi^2/\mathrm{d.o.f} = 32.3/53 \sim 0.61$ for the one-parameter fit, and

$$a_1 = 3.03(20)\,,\quad a_2 = 1.04(2.10)\,,$$
$$M_{\rho'} = 1303(19)\,\mathrm{MeV}\,,\quad \Gamma_{\rho'} = 839(102)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1624(1)\,\mathrm{MeV}\,,\quad \Gamma_{\rho''} = 570(99)\,\mathrm{MeV}$$
$$\gamma = 0.22(10)\,,\quad \phi_1 = -1.65(4)\,,$$
$$\delta = -0.18(1)\,,\quad \phi_2 = -1.34(14)\,,$$

with a $\chi^2/\mathrm{d.o.f} = 35.6/52 \sim 0.63$ for the two-parameter fit.

Variant (IV)

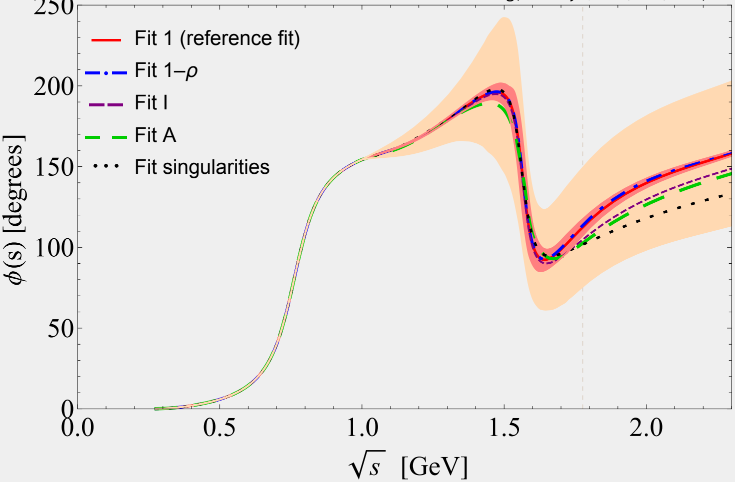
- Fits for different s_{cut} and allowing the ρ-mass to float

Fits	Parameter	s_{cut} [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1-ρ	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.89(1)	1.88(1)
	α_2 [GeV^{-4}]	4.37(3)	4.34(1)	4.31(3)	4.34(1)
	m_ρ [MeV]	773.9(3)	773.8(3)	773.9(3)	773.9(3)
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1382(71)	1375(11)	1316(9)	1312(8)
	$\Gamma_{\rho'}$ [MeV]	516(165)	608(35)	728(92)	726(26)
	$M_{\rho''}$[MeV]	1723(1)	1715(22)	1655(1)	1656(8)
	$\Gamma_{\rho''}$ [MeV]	315(271)	455(16)	569(160)	571(13)
	γ	0.12(13)	0.16(1)	0.18(2)	0.17(1)
	ϕ_1	−0.56(35)	−0.69(1)	−1.40(19)	−1.41(8)
	δ	−0.09(3)	−0.13(1)	−0.17(4)	−0.17(3)
	ϕ_2	−0.19(69)	−0.45(12)	−1.06(10)	−1.05(11)
	χ^2/d.o.f	1.09	0.70	0.63	0.66

Systematic theoretical errors

- Including $\rho'^{(\prime)} \to K\bar{K}$, $\rho' \to \omega\pi$ into the $\rho'^{(\prime)}$ width
- Different matching points with the (elastic) $\pi\pi$ phase shift

González-Solís and Roig, Eur.Phys.J C79 (2019) no.5, 436



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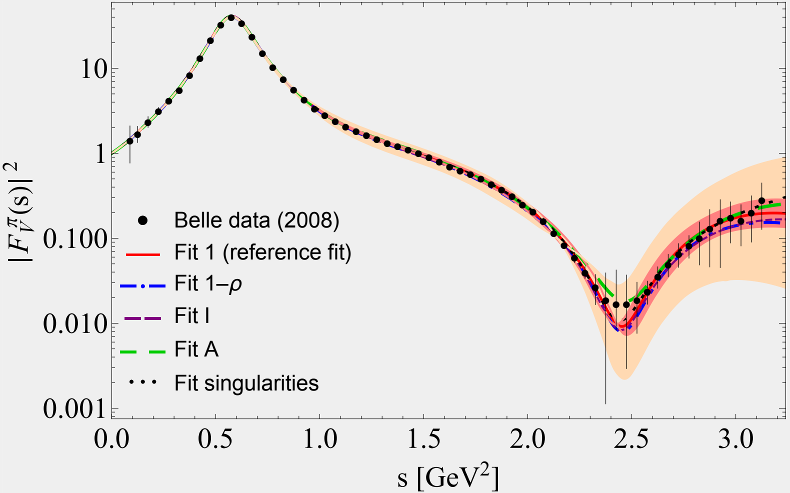
González-Solís and Roig , Eur.Phys.J C79 (2019) no.5, 436



Low-energy observables

References	$\langle r^2\rangle_V^\pi$ (GeV^{-2})	c_V^π (GeV^{-4})
Colangelo et al. [55]	11.07 ± 0.66	3.2 ± 1.03
Bijnens et al. [32]	11.22 ± 0.41	3.85 ± 0.60
Pich et al. [6]	11.04 ± 0.30	3.79 ± 0.04
Bijnens et al. [33]	11.61 ± 0.33	4.49 ± 0.28
de Troconiz et al. [56]	11.10 ± 0.03	3.84 ± 0.02
Masjuan et al. [57]	11.43 ± 0.19	3.30 ± 0.33
Guo et al. [58]	–	4.00 ± 0.50
Lattice [59]	10.50 ± 1.12	3.22 ± 0.40
Ananthanarayan et al. [60]	11.17 ± 0.53	[3.75, 3.98]
Ananthanarayan et al. [61]	[10.79, 11.3]	[3.79, 4.00]
Schneider et al. [48]	10.6	3.84 ± 0.03
Dumm et al. [7]	10.86 ± 0.14	3.84 ± 0.03
Celis et al. [8]	11.30 ± 0.07	4.11 ± 0.09
Ananthanarayan et al. [62]	11.10 ± 0.11	–
Hanhart et al. [63]	$11.34 \pm 0.01 \pm 0.01$	–
Colangelo et al. [39]	11.02 ± 0.10	–
PDG [42]	11.61 ± 0.28	–
This work	11.28 ± 0.08	3.94 ± 0.04

Sum rule	s_{cut} (GeV^2)			Fit Eq. (42)
	4	10	∞	
α_1	1.52	1.66	1.75	$1.88 \pm 0.01 \pm 0.01$
α_2	4.26	4.30	4.31	$4.34 \pm 0.01 \pm 0.03$

$\rho(1450)$ AND $\rho(1700)$ RESONANCE PARAMETERS

Reference	Model parameter $(M_{\rho'}, \Gamma_{\rho'})$ [MeV]	Pole position $(M_{\rho'}, \Gamma_{\rho'})$ [MeV]	Data
ALEPH	$1328 \pm 15, 468 \pm 41$	$1268 \pm 19, 429 \pm 31$	τ
ALEPH	$1409 \pm 12, 501 \pm 37$	$1345 \pm 15, 459 \pm 28$	$\tau + e^+e^-$
Belle	$1428(15)(26), 413(12)(57)$	$1384(16)(29), 390(10)(48)$	τ
Dumm et. al'13	—	$1440 \pm 80, 320 \pm 80$	τ
Celis et. al.'14	$1497 \pm 7, 785 \pm 51$	$1278 \pm 18, 525 \pm 16$	τ
Bartos et. al.	—	$1342 \pm 47, 492 \pm 138$	e^+e^-
Bartos et. al.	—	$1374 \pm 11, 341 \pm 24$	τ
This work	$1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$	$1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$	τ
Reference	**Model parameter** $(M_{\rho''}, \Gamma_{\rho''})$ [MeV]	**Pole position** $(M_{\rho''}, \Gamma_{\rho''})$ [MeV]	**Data**
ALEPH	$= 1713, = 235$	$1700, 232$	τ
ALEPH	$1740 \pm 20, = 235$	$1728 \pm 20, 232$	$\tau + e^+e^-$
Belle	$1694 \pm 41, 135 \pm 36^{+50}_{-26}$	$1690 \pm 94, 134 \pm 36^{+49}_{-28}$	τ
Dumm et. al.'13	—	$1720 \pm 90, 180 \pm 90$	τ
Celis et. al.'14	$1685 \pm 30, 800 \pm 31$	$1494 \pm 37, 600 \pm 17$	τ
Bartos et. al.	—	$1719 \pm 65, 490 \pm 17$	e^+e^-
Bartos et. al.	—	$1767 \pm 52, 415 \pm 120$	τ
This work	$1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$	$1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$	τ

Kaon form factor: exponential representation

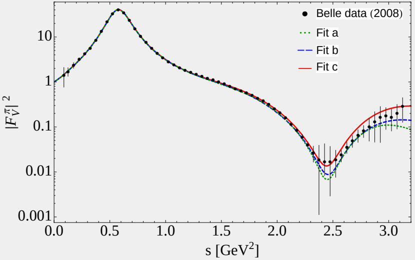
- Different resonance mixing contribution than $F_V^\pi(s)$:

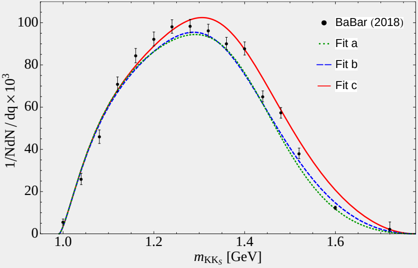
$$
\begin{aligned}
F_V^K(s) &= \frac{M_\rho^2 + s\left(\tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2}\right)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\} \\
&\quad -\tilde{\gamma}\frac{s\, e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\} \\
&\quad -\tilde{\delta}\frac{s\, e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\}, \\
\Gamma_{\rho',\rho''}(s) &= \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}\theta(s - 4m_\pi^2)\,.
\end{aligned}
$$

- Extract the phase $\tan\phi_{KK}(s) = \mathrm{Im}F_V^K(s)/\mathrm{Re}F_V^K(s)$
- Use a three-times subtracted dispersion relation

Combined analysis of $F_V^\pi(s)$ and $\tau^- \to K^- K_S \nu_\tau$

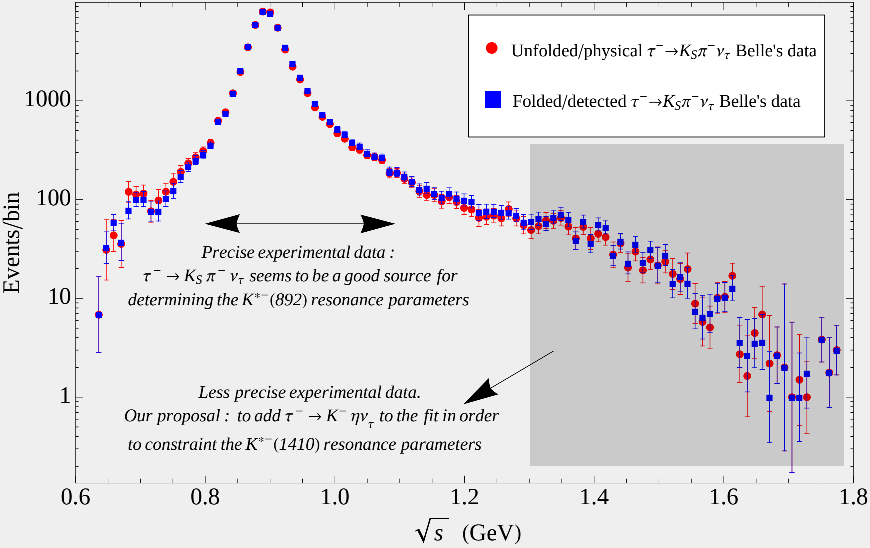
Parameter	$s_{\rm cut} = 4$ [GeV^2]		
	Fit *a*	Fit *b*	Fit *c*
α_1	1.88(1)	1.89(1)	1.87(1)
α_2	4.34(2)	4.31(2)	4.38(3)
$\tilde{\alpha}_1$	$=\alpha_1$	$=\alpha_1$	1.88(24)
$\tilde{\alpha}_2$	$=\alpha_2$	$=\alpha_2$	4.38(29)
m_ρ [MeV]	$=773.6(9)$	$=773.6(9)$	$=773.6(9)$
M_ρ [MeV]	$=m_\rho$	$=m_\rho$	$=m_\rho$
$M_{\rho'}$ [MeV]	1396(19)	1453(19)	1406(61)
$\Gamma_{\rho'}$ [MeV]	507(31)	499(51)	524(149)
$M_{\rho''}$[MeV]	1724(41)	1712(32)	1746(1)
$\Gamma_{\rho''}$ [MeV]	399(126)	284(72)	413(362)
γ	0.12(3)	0.15(3)	0.11(11)
$\tilde{\gamma}$	0.11(2)	$=\gamma$	0.11(5)
ϕ_1	$-0.23(26)$	0.29(21)	$-0.27(42)$
$\tilde{\phi}_1$	$-1.83(14)$	$-1.48(13)$	$-1.90(67)$
δ	$-0.09(2)$	$-0.07(2)$	$-0.10(5)$
$\tilde{\delta}$	$=0$	$=0$	$-0.01(4)$
ϕ_2	$-0.20(31)$	0.27(29)	$-1.15(71)$
$\tilde{\phi}_2$	$=0$	$=0$	0.40(3)
χ^2/d.o.f	1.52	1.19	1.25





Belle $\tau^- \to K_S\pi^-\nu_\tau$ measurement

$\tau^- \to K_S\pi^-\nu_\tau$ Belle's data Phys. Lett. **B 654 (2007) 65** [arXiv:0706.2231]



$K\pi$ Form Factor: Dispersive representation

- Dispersion relation with subtractions:

$$F_V^{K\pi}(s) \;=\; P(s)\exp\left[\alpha_1 \frac{s}{m_{\pi^-}^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta^{K\pi}(s')}{(s')^3(s'-s-i0)}\right] ,$$

- $\alpha_1 = \lambda_+^{'}$ and $\alpha_1^2 + \alpha_2 = \lambda_+^{''}$ are low energies parameters:

$$F_V^{K\pi}(t) = 1 + \frac{\lambda_+^{'}}{M_{\pi^-}^2} t + \frac{1}{2}\frac{\lambda_+^{''}}{M_{\pi^-}^4} t^2 ,$$

- $s_{\rm cut}$: cut-off to check stability
- Parameters to Fit: $\lambda_+^{\prime}$, $\lambda_+^{\prime\prime}$, m_{K^*} , γ_{K^*} , $m_{K^{*\prime}}$, $\gamma_{K^{*\prime}}$

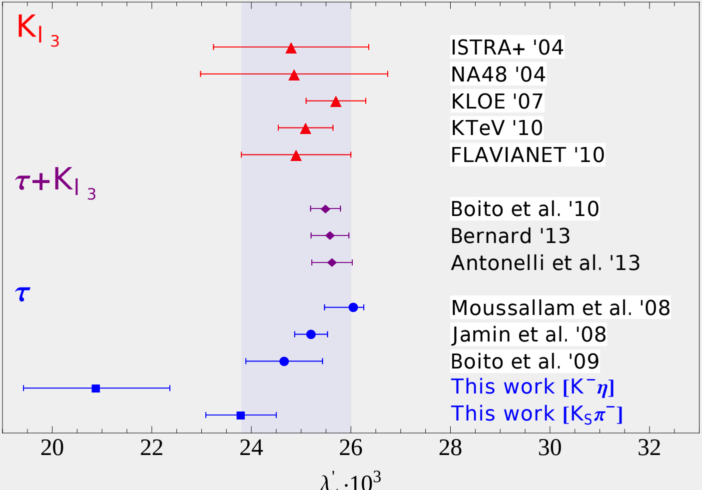
Fit results

- Reference fit results obtained for different values of s_{cut}

Parameter	3.24	4	9	∞
$\bar{B}_{K\pi}(\%)$	0.402 ± 0.013	0.404 ± 0.012	0.405 ± 0.012	0.405 ± 0.012
$(B^{th}_{K\pi})(\%)$	(0.399)	(0.402)	(0.403)	(0.403)
M_{K^*}	892.01 ± 0.19	892.03 ± 0.19	892.05 ± 0.19	892.05 ± 0.19
Γ_{K^*}	46.04 ± 0.43	46.18 ± 0.42	46.27 ± 0.42	46.27 ± 0.41
$M_{K^{*\prime}}$	1301^{+17}_{-22}	1305^{+15}_{-18}	1306^{+14}_{-17}	1306^{+14}_{-17}
$\Gamma_{K^{*\prime}}$	207^{+73}_{-58}	168^{+52}_{-44}	155^{+48}_{-41}	155^{+47}_{-40}
$\gamma_{K\pi}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$	$= \gamma_{K\eta}$
$\lambda'_{K\pi} \times 10^3$	23.3 ± 0.8	23.9 ± 0.7	24.3 ± 0.7	24.3 ± 0.7
$\lambda''_{K\pi} \times 10^4$	11.8 ± 0.2	11.8 ± 0.2	11.7 ± 0.2	11.7 ± 0.2
$\bar{B}_{K\eta} \times 10^4$	1.57 ± 0.10	1.58 ± 0.10	1.58 ± 0.10	1.58 ± 0.10
$(B^{th}_{K\eta}) \times 10^4$	(1.43)	(1.45)	(1.46)	(1.46)
$\gamma_{K\eta} \times 10^2$	$-4.0^{+1.3}_{-1.9}$	$-3.4^{+1.0}_{-1.3}$	$-3.2^{+0.9}_{-1.1}$	$-3.2^{+0.9}_{-1.1}$
$\lambda'_{K\eta} \times 10^3$	18.6 ± 1.7	20.9 ± 1.5	22.1 ± 1.4	22.1 ± 1.4
$\lambda''_{K\eta} \times 10^4$	10.8 ± 0.3	11.1 ± 0.4	11.2 ± 0.4	11.2 ± 0.4
$\chi^2/\text{n.d.f.}$	$105.8/105$	$108.1/105$	$111.0/105$	$111.1/105$

Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

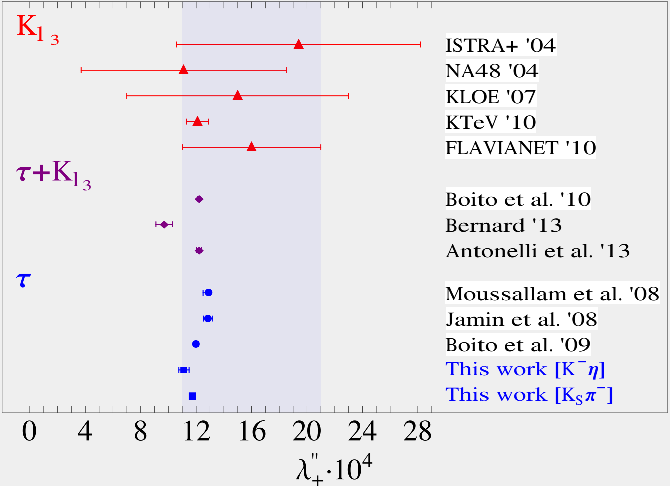
$$\left.\begin{aligned} \lambda'_{K\pi} &= (23.9 \pm 0.9)\cdot 10^{-3} \\ \lambda'_{K\eta} &= (20.9 \pm 2.7)\cdot 10^{-3} \end{aligned}\right\}\text{isospin violation?}$$



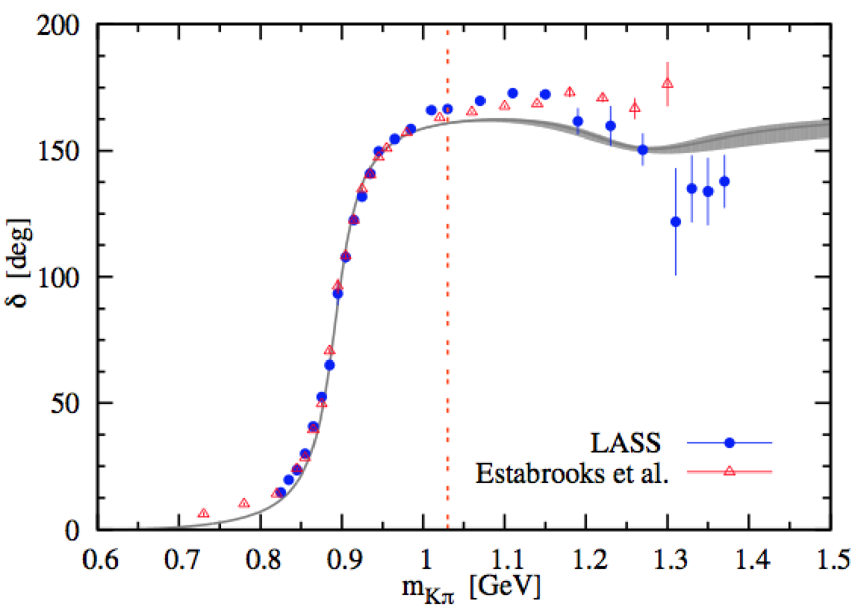
Results of the combined $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ analysis

$\lambda''_{K\pi} = (11.8 \pm 0.2) \cdot 10^{-4}$

$\lambda''_{K\eta} = (11.1 \pm 0.5) \cdot 10^{-4}$



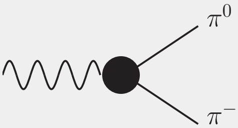
$K\pi$ PHASE

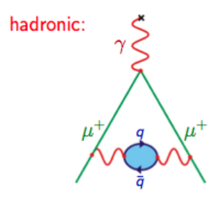


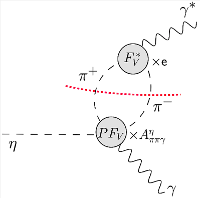
200
150
100
50
0
δ [deg]
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
m$_{K\pi}$ [GeV]
LASS
Estabrooks et al.

The pion vector form factor: Motivation

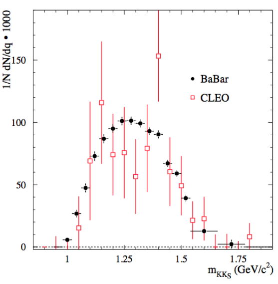
- Enters the description of many physical processes







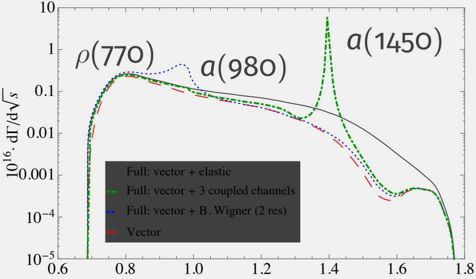
- BaBar measurement of $\tau^- \to K^- K_S \nu_\tau$ (PRD 98 (2018) no.3, 032010)

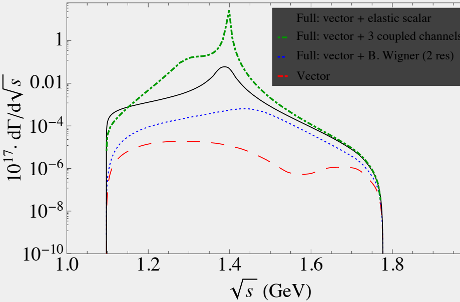


- good quality data
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

Research achievements: Isospin-violating $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$ decays

Phys. Rev. D 94 (2016) no.3, 034008





- $\tau^- \to \pi^-\eta\nu_\tau$ $\sqrt{s}$ (GeV)
  - Theory predictions: $BR \sim 1 \times 10^{-5}$ (Escribano'16, Moussallam'14)
  - BaBar: $BR < 9.9 \cdot 10^{-5}$ 95% CL , Belle: $BR < 7.3 \cdot 10^{-5}$ 90% CL
- $\tau^- \to \pi^-\eta'\nu_\tau$
  - Theory predictions: $BR \sim [10^{-7}, 10^{-6}]$ (Escribano'16)
  - BaBar: $BR < 4 \cdot 10^{-6}$ 90% CL

Challenging for Belle I

SMEFT WITH DIMENSION 6 OPERATORS

- $\tau^- \to \nu_\tau \bar{u} D$ $(D = d, s)$

$$\begin{aligned}\mathcal{L}_{CC} &= -\frac{G_F V_{uD}}{\sqrt{2}} \Big[(1+\epsilon_L^\tau)\bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma^5)D \\ &\quad +\epsilon_R^\tau \bar{\tau}\gamma_\mu(1-\gamma^5)\nu_\tau \cdot \bar{u}\gamma^\mu(1+\gamma^5)D + \bar{\tau}(1-\gamma^5)\nu_\tau \cdot \bar{u}(\epsilon_S^\tau - \epsilon_P^\tau\gamma^5)D \\ &\quad +\epsilon_T^\tau \bar{\tau}\sigma_{\mu\nu}(1-\gamma^5)\nu_\tau \, \bar{u}\sigma^{\mu\nu}(1-\gamma^5)D \Big] + h.c.,\end{aligned}$$

- ϵ_i $(i = L, R, S, P, T)$ are effective couplings characterizing NP
  - Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$ [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]
  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_S^\tau \\ \epsilon_P^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2},$$

Strangeness-conserving transitions ($\Delta S = 0$)

- **One meson** decay $\tau^- \to \pi^- \nu_\tau$

$$
\begin{aligned}
\Gamma(\tau^- \to \pi^- \nu_\tau) &= \frac{G_F^2 |\tilde{V}_{ud}^e|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \\
&\times \left(1 + \delta_{\rm em}^{\tau\pi} + 2\Delta^{\tau\pi} + \mathcal{O}(\epsilon_i^\tau)^2 + \mathcal{O}(\delta_{\rm em}^{\tau\pi}\epsilon_i^\tau)\right),
\end{aligned}
$$

- Constraint for the NP effective couplings (this work):

$$
\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon_P^\tau = (-0.12 \pm 0.68) \times 10^{-2},
$$

  - Errors (hierarchy): $f_\pi, BR, \delta_{\rm em}^{\tau\pi}$

- Cirigliano et.al. (PRL 122 (2019) no.22 221801)

$$
\epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon_P^\tau = (-0.15 \pm 0.67) \times 10^{-2},
$$

Strangeness-conserving transitions ($\Delta S = 0$)

- **One meson** decay $\tau^- \to \pi^- \nu_\tau$ ($G_F \tilde{V}^e_{uD} = G_F \left(1 + \epsilon^e_L + \epsilon^e_R\right) V_{uD}$)

$$\Gamma(\tau^- \to \pi^- \nu_\tau) = \frac{G_F^2 |\tilde{V}^e_{ud}|^2 f_\pi^2 m_\tau^3}{16\pi} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \left(1 + \delta^{\tau\pi}_{\rm em} + 2\Delta^{\tau\pi} + \mathcal{O}(\epsilon_i^\tau)^2 + \mathcal{O}(\delta^{\tau\pi}_{\rm em}\epsilon_i^\tau)\right),$$

- Inputs: $f_\pi = 130.2(8)$ MeV (FLAG 1902.08191); $\delta^{\tau\pi}_{\rm em} = 1.92(24)\%$; $|\tilde{V}^e_{ud}| = 0.97420(21)$ (β decays, PDG);
- Constraint for the NP effective couplings (this work):

$$\Delta^{\tau\pi} \equiv \epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon^\tau_P = (-0.12 \pm 0.68) \times 10^{-2},$$

Errors (hierarchy): $f_\pi, BR, \delta^{\tau\pi}_{\rm em}$

- $\Gamma(\tau \to \pi\nu)/\Gamma(\pi \to \mu\nu)$: tighter constraints (not used in this work)

$$\epsilon^\tau_L - \epsilon^e_L - \epsilon^\tau_R - \epsilon^e_R - \frac{m_\pi^2}{m_\tau(m_u + m_d)}\epsilon^\tau_P + \frac{m_\pi^2}{m_\mu(m_u + m_d)}\epsilon^\mu_P = (-0.38 \pm 0.27) \times 10^{-2},$$

Strangeness-conserving transitions ($\Delta S = 0$)

- Bounds for the non-SM effective couplings

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 0.6^{+2.3}_{-1.8}\,^{+0.2}_{-0.1} \pm 0.4 \\ 0.3 \pm 0.5^{+1.1}_{-0.9}\,^{+0.1}_{-0.0} \pm 0.2 \\ 9.7^{+0.5}_{-0.6} \pm 21.5\,^{+0.0}_{-0.1} \pm 0.2 \\ -0.1 \pm 0.2^{+1.1}_{-1.4}\,^{+0.0}_{-0.1} \pm 0.2 \end{pmatrix} \times 10^{-2},$$

- Comparison with other bounds (assuming LFU):
  - Semileptonic kaon decays: $\epsilon_S^\mu = -0.039(49) \cdot 10^{-2}$, $\epsilon_T^\mu = 0.05(52) \cdot 10^{-2}$ [González-Alonso, Martin Camalich JHEP 1612 (2016) 052]
  - (Excl. and incl.) Tau decays [Cirigliano et al. PRL 122 (2019) no.22, 221801]:

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_S^\tau \\ \epsilon_P^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2},$$

Global fit to $\Delta S = 0$ and $|\Delta S| = 1$ transitions

- Combination to **one and two meson decays**

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2} \,,$$

- Our limits on ϵ_i can be translated into bounds on the NP scale Λ through

$$\Lambda \sim v \left(V_{uD} \epsilon_i \right)^{-1/2} \,,$$

where $v = (\sqrt{2} G_F)^{-1/2} \sim 246$ GeV. Our bounds range $\Lambda \sim 10$ TeV, which are quite restricted compared to the energy scale probed in semileptonic kaon decays $\mathcal{O}(500)$ TeV.

Prospects for tau physics at Belle-II

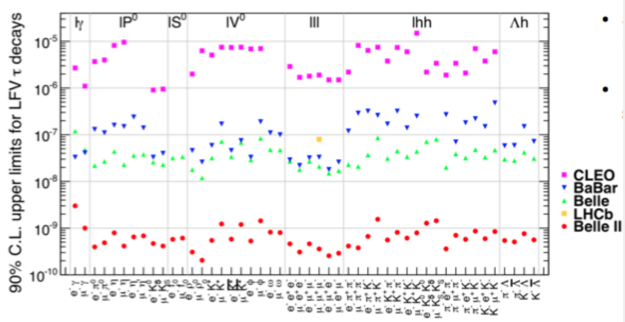
- Huge amount of **data** to be delivered

Experiment	Number of τ pairs
LEP	$\sim 3 \times 10^5$
CLEO	$\sim 1 \times 10^7$
BaBar	$\sim 5 \times 10^8$
Belle	$\sim 9 \times 10^8$
Belle II	$\sim 10^{12}$

- Broad **program** of tau lepton physics:
  - Searches for Lepton Flavor Violation (LFV)
  - CP violation
  - Second Class Currents
  - and much more (Michel parameters, precision m_τ, EDM, ...)
- See "The Belle II Physics Book" (1808.10567)

Searches for charged LFV

- Tau as a tool to probe **non-SM** interactions:
  - radiative: $\tau^- \to \ell^- \gamma$
  - leptonic: $\tau^- \to \ell^- \ell^+ \ell^-$
  - semi-leptonic: $\tau^- \to \ell^- h(h)$ $(h = P, S, V \ldots)$

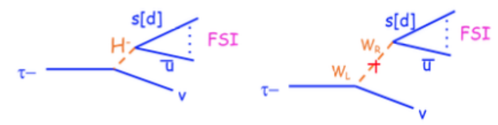


- Belle-II will push the current bound forward by at least **one order of magnitude!**
- Observation of charged **LFV** would be a clear signal of **New Physics**

CP VIOLATION IN $\tau \to K_S \pi^\pm \nu_\tau$

$$A_\tau = \frac{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S \nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S \nu_\tau)},$$

- SM prediction: $A_\tau \approx 2\mathrm{Re}(\epsilon) \approx (3.6 \pm 0.1) \times 10^{-3}$ (Bigi, Sanda'05, Grossman, Nir'11)
- Exp. measurement: $(-3.6 \pm 2.3 \pm 1.1) \times 10^{-3}$ (BaBar 2011) 2.8σ **from the SM**
- **New physics?** Very difficult to explain:
  - Charged Higgs, $W_L - W_R$ mixings (Devi, Dhargyal, Sinha' 2014)



  - Tensor interactions (Rendón, Roig, Toledo 2019)
- An improved A_τ measurement is a **priority for Belle II**

Second class currents (SCC) in $\tau \to \pi\eta\nu_\tau$

- SCC: $J^{PG} = 0^{+-}, 0^{-+}, 1^{++}, 1^{--}$ **not yet observed!**
- In the SM, $\tau \to \pi\eta\nu_\tau$ decays proceed via SCC with **tiny BRs** $\leq \mathcal{O}(10^{-5})$ (Escribano, SG-S and Roig, Phys.Rev.D 94 (2016) no.3 034008, Moussallam et.al. '14)
- **Searched** for at last-generation B-factories
  - $BR < 7.3 \times 10^{-5}$ (Belle), $BR < 9.9 \times 10^{-5}$ (BaBar)
- The observation of SCC via $\tau \to \pi\eta\nu_\tau$ is a **priority at Belle-II**
- Clear signal could suggest **New Physics**