Lepton pair production in di-pion τ lepton decays

$$\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$$

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Outline

Motivation

Semileptonic decays

- Are ideal to study the dynamics of the strong interactions in the resonance region.
- Test the fundamental parameters of the SM (α_s, V_{us}, ...).
- To probe non-standard interactions.

$\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$

- Pollute searches of LFV or LNV.
  $\tau^- \to \ell^{(\prime)-} \ell^+ \ell^-$ $(\sim 10^{-54})$[1].
  $\tau^- \to \nu_\tau \mu^- \mu^- \pi^+$ $(=0)$ [2]
- Verify radiative corrections of the HVP entering a_μ using hadronic tau decays data [3]. .
- To include in the MC Generator TAUOLA [4].

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[1] G.H.T., G. L.C. and P. Roig, Eur. Phys. J. C 79, 84 (2019).
[2] G.L.C. and N. Quintero, Phys. Rev. D85, 076006 (2012).
[3] JHEP 08, 002.; PRD 74, 071301; PRD 102, 114017 (2020).
[4] O. Shekhovtsova et al., Phys. Rev. D 86, 113008 (2012).

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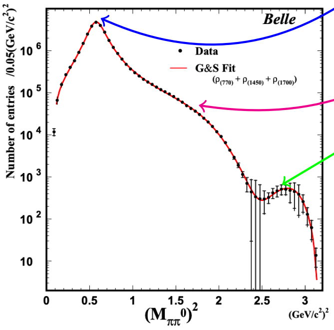
Introduction

- $\tau^- \to \nu_\tau \pi^- \pi^0 \to$ dominant channel (τ). Extract pion FF, determine ρ resonance parameters, test CVC.
- $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma \to$ hadronization of the weak current, for the di-pion observables at the few percent level.
- $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ $(\ell = e, \mu) \to$ hadronization of the weak current in an extended kinematical domain. Sources of background, CLFV decays of τ leptons.

The $\tau^- \to \nu_\tau \pi^- \pi^0$ decay

$$\langle \pi^-(p_-)\pi^0(p_0)|\bar{d}\gamma_\mu u|0\rangle = \sqrt{2}F_\pi(t)(p_- - p_0)_\mu \qquad t = (p_- + p_0)^2$$

$$F_\pi(t) = \frac{1}{1+\beta e^{i\phi}+\gamma}\left[\mathrm{BW}_\rho(t) + \beta e^{i\phi}\mathrm{BW}_{\rho'}(t) + \gamma \mathrm{BW}_{\rho''}(t)\right],$$



[5]

$\mathcal{BR}(\tau^- \to \nu_\tau \pi^- \pi^0) =$

$(25.49 \pm 0.09)\%$ via τ data[6]

$(24.84 \pm 0.14 \pm 0.22)\%$ e^+e^- via CVC[7]

$a_\mu^{had,LO} =$

$(693.9 \pm 3.9 \pm 0.7) \times 10^{-10}$ e^+e^- based[6]

$(703.0 \pm 4.4) \times 10^{-10}$ τ based[8]

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[5] M. Fujikawa et al. (Belle Collab.), PRD78, 072006 (2008).

[6] P.A. Zyla et al. (PDG), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[7] M. Davier et al., Eur. Phys. J. C66, 127 (2010).

[8] M. Davier et al., Eur. Phys. J. C74, 2803 (2014).

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The radiative $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decay

The matrix element has the general structure[9]

$$\mathcal{M}(\gamma) = eG_F V_{ud}^* \epsilon^{*\mu}(k) \left[\frac{(p_0 - p_-)_\nu f_+(t)}{2P \cdot k} \overline{u}(q)\gamma^\nu(1-\gamma_5)\left(\not{P} - \not{k} + M_\tau\right)\gamma_\mu u_\tau(P) \right.$$
$$\left. + \quad (V_{\mu\nu} - A_{\mu\nu})\,\overline{u}(q)\gamma^\nu(1-\gamma_5)u_\tau(P) \right]. \qquad (1)$$

Momentum convention $\tau^-(P) \to \nu_\tau(q)\pi^-(p_-)\pi^0(p_0)\gamma(k)$.

- First term $\rightarrow$photon emission off the τ lepton.
- Second term $\rightarrow$ structure-dependent, Vector and Axial-vector components, hadronization of the weak current $W^- \to \pi^-\pi^0\gamma$.

[9] V. Cirigliano, G. Ecker, and H. Neufeld, JHEP 08, 002.

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The radiative $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decay

The vector form factors for $V_{\mu\nu} = V_{\mu\nu}^{LO} + \hat{V}_{\mu\nu}$

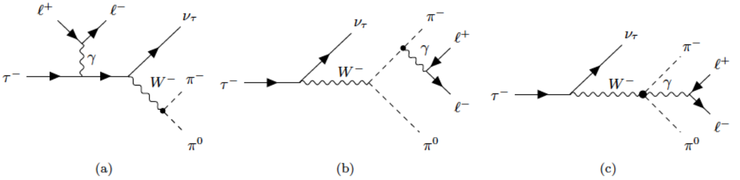
$$\begin{aligned} V_{\mu\nu}^{LO} &= f_+(t')\frac{p_{-\mu}}{p_- \cdot k}(p_- + k - p_0)_\nu - f_+(t')g_{\mu\nu} \\ &+ \frac{f_+(t') - f_+(t)}{(p_- + p_0)\cdot k}(p_- + p_0)_\mu (p_0 - p_-)_\nu \, . \end{aligned} \tag{2}$$

here $t' = (p_- + p_0 + k)^2$

$$\begin{aligned} \hat{V}^{\mu\nu} &= v_1 \left(p_- \cdot k g^{\mu\nu} - p_-^\mu k^\nu\right) + v_2 \left(p_0 \cdot k g^{\mu\nu} - p_0^\mu k^\nu\right) \\ &+ v_3 \left(p_0 \cdot k p_-^\mu - (p_- \cdot k p_0^\mu\right) p_-^\nu + v_4 \left(p_0 \cdot k p_-^\mu - p_- \cdot k p_0^\mu\right)(p_- + p_0 + k)^\nu \, , \end{aligned} \tag{3}$$

$$\begin{aligned} A^{\mu\nu} &= i a_1 \varepsilon^{\mu\nu\rho\sigma}(p_0 - p_-)_\rho k_\sigma + i a_2 W^\nu \epsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma + i a_3 \varepsilon^{\mu\nu\rho\sigma} k_\rho W_\sigma \\ &+ i a_4 (p_0 + k)^\nu \varepsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma \, . \end{aligned} \tag{4}$$

The $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decay



Similar structure to the radiative case:

$$\mathcal{M}(\ell\ell) = \frac{e^2}{k^2} G_F V_{ud}^* l^\mu \left\{ \frac{(p_- - p_0)_\nu}{k^2 - 2P \cdot k} f_+(t) \overline{u}_\nu(q) \gamma^\nu (1 - \gamma_5) \left(\not{P} - \not{k} + M_\tau \right) \gamma_\mu u_\tau(P) \right.$$
$$\left. + \quad (V_{\mu\nu} - A_{\mu\nu}) L^\nu \right\}, \quad (5)$$

Momentum convention: $\tau^-(P) \to \nu_\tau(q) \pi^-(p_-) \pi^0(p_0) \ell^+(p_{\ell+}) \ell^-(p_{\ell-})$

$$L^\nu = \overline{u}_\nu(q) \gamma^\nu (1 - \gamma_5) u_\tau(P)$$
$$l^\mu = \overline{u}(p_{\ell-}) \gamma^\mu v(p_{\ell+})$$
$$k = p_{\ell+} + p_{\ell-}$$

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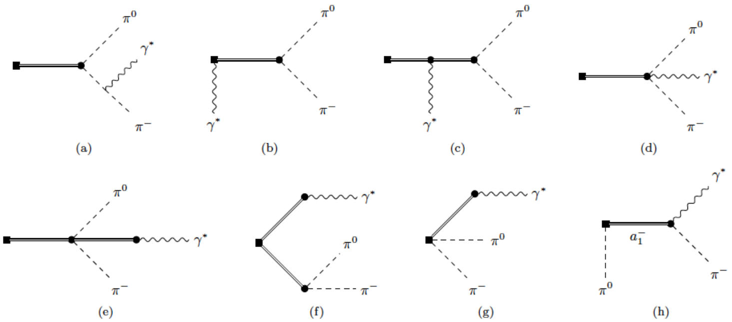
The $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ decay

- $V_{\mu\nu} = V^{LO}_{\mu\nu} + \hat{V}_{\mu\nu}$ as in the radiative case, but depending on (t, t', k^2).

$$\hat{V}^{\mu\nu} = \ldots + v_5 \left(k^2 g^{\mu\nu} - k^\mu k^\nu\right) + v_6 \left(k^2 p_-^\mu - p_- \cdot k k^\mu\right) p_0^\nu + v_7 \left(p_0 \cdot k k^\mu - k^2 p_0^\mu\right) p_-^\nu .$$

- Vector contributions to $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$



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Structure-dependent pieces

$$
\begin{aligned}
v_1 &= \frac{F_V G_V}{f^2}\left[2D_\rho^{-1}\left(k^2\right)+2D_\rho^{-1}\left(t'\right)+tD_\rho^{-1}(t)D_\rho^{-1}\left(k^2\right)+tD_\rho^{-1}(t)D_\rho^{-1}\left(t'\right)\right] \\
&+ \frac{{F_V}^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right)-D_\rho^{-1}\left(t'\right)+\left(t'-k^2\right)D_\rho^{-1}\left(t'\right)D_\rho^{-1}\left(k^2\right)\right] \\
&+ \frac{{F_A}^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2-m_\pi^2+\frac{t}{2}\right)D_{a_1}^{-1}\left[\left(p_-+k\right)^2\right], \\
v_2 &= \frac{F_V G_V t}{f^2}\left[-D_\rho^{-1}(t)D_\rho^{-1}\left(k^2\right)-D_\rho^{-1}(t)D_\rho^{-1}\left(t'\right)\right] \\
&+ \frac{{F_V}^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right)-D_\rho^{-1}\left(t'\right)-\left(t'-k^2\right)D_\rho^{-1}\left(t'\right)D_\rho^{-1}\left(k^2\right)\right] \\
&+ \frac{{F_A}^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2-m_\pi^2-k\cdot p_-\right)D_{a_1}^{-1}\left[\left(p_-+k\right)^2\right], \\
v_3 &= \frac{{F_A}^2}{f^2 m_{a_1}^2}D_{a_1}^{-1}\left[\left(p_-+k\right)^2\right], \\
v_4 &= \frac{{F_V}^2}{f^2}D_\rho^{-1}\left(t'\right)D_\rho^{-1}\left(k^2\right)-\frac{2F_V G_V}{f^2}D_\rho^{-1}(t)D_\rho^{-1}\left(t'\right),
\end{aligned}
\tag{6}
$$

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$$v_5 = \frac{F_V{}^2}{2f^2}\left[-D_\rho^{-1}\left(k^2\right) - D_\rho^{-1}\left(t'\right) - k\cdot\left(p_0 - p_-\right) D_\rho^{-1}\left(t'\right) D_\rho^{-1}\left(k^2\right)\right]$$
$$+ \frac{F_A{}^2}{f^2 m_{a_1}^2}\left(m_{a_1}^2 - m_\pi^2 + \frac{t}{2}\right) D_{a_1}^{-1}\left[\left(p_- + k\right)^2\right],$$
$$v_6 = \frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}\left(k^2\right) + \frac{F_V{}^2}{f^2} D_\rho^{-1}\left(t'\right) D_\rho^{-1}\left(k^2\right),$$
$$v_7 = \frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}\left(k^2\right) + \frac{F_V{}^2}{f^2} D_\rho^{-1}\left(t'\right) D_\rho^{-1}\left(k^2\right) + \frac{F_A{}^2}{f^2 m_{a_1}^2} D_{a_1}^{-1}\left[\left(p_- + k\right)^2\right]. \quad (7)$$

$$D_R(s) = m_R^2 - s - i m_R \Gamma_R(s),$$

$$t' = W = \left(p_- + p_0 + k\right)^2$$

- To asses our model dependent error we evaluate with the two sets:

$$F_V = \sqrt{2} f, \quad G_V = \frac{f}{\sqrt{2}}, \quad F_A = f, \quad (8)$$

$$F_V = \sqrt{3} f, \quad G_V = \frac{f}{\sqrt{3}}, \quad F_A = \sqrt{2} f. \quad (9)$$

Axial-Vector contributions to $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$

- Same expression as in the real photon case ($W = P - q = p_- + p_0 + k$):

$$\begin{aligned} A^{\mu\nu} &= i a_1 \varepsilon^{\mu\nu\rho\sigma} \left(p_0 - p_-\right)_\rho k_\sigma + i a_2 W^\nu \epsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma \\ &+ i a_3 \varepsilon^{\mu\nu\rho\sigma} k_\rho W_\sigma + i a_4 \left(p_0 + k\right)^\nu \varepsilon^{\mu\lambda\rho\sigma} p_{-\lambda} p_{0\rho} k_\sigma , \end{aligned} \tag{10}$$

- At $O(p^4)$ only a_1 and a_2, from the Wess-Zumino-Witten functional [10], contribute and are the same as in the real photon case

$$\begin{aligned} a_1 &= \left[8\pi^2 f^2\right]^{-1} , \\ a_2 &= -\left[4\pi^2 f^2 \left(t' - m_\pi^2\right)\right]^{-1} . \end{aligned} \tag{11}$$

[10] Phys. Lett. B 37, 95 (1971) and Nucl. Phys. B 223, 422 (1983).

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Branching Ratio and Lepton-Pair Spectrum

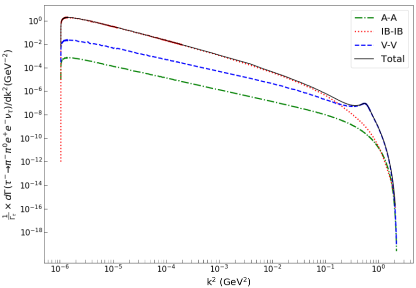
Results

Contribution	$\ell^+\ell^- = e^+e^-$	$\ell^+\ell^- = e^+e^-$ (9) for F_V, F_A & G_V	$\ell^+\ell^- = \mu^+\mu^-$	$\ell^+\ell^- = \mu^+\mu^-$ (9) for F_V, F_A & G_V
IB	$2.213(11) \times 10^{-5}$		$5.961(3) \times 10^{-8}$	
VV	$6.745(36) \times 10^{-7}$	$9.571(48) \times 10^{-7}$	$5.462(4) \times 10^{-8}$	$9.429(7) \times 10^{-8}$
AA	$1.91(1) \times 10^{-8}$		$1.663(1) \times 10^{-9}$	
IB-V	$-3.83(18) \times 10^{-7}$	$-1.02(18) \times 10^{-7}$	$1.337(4) \times 10^{-8}$	$2.126(5) \times 10^{-8}$
IB-A	$9.1(4.5) \times 10^{-9}$		$2.85(3) \times 10^{-9}$	
V-A	$5.2(2.1) \times 10^{-9}$	$4.5(2.6) \times 10^{-9}$	$-1.73(3) \times 10^{-10}$	$-1.65(5) \times 10^{-10}$
Total	$2.245(13) \times 10^{-5}$	$2.302(13) \times 10^{-5}$	$1.319(2) \times 10^{-7}$	$1.795(2) \times 10^{-7}$

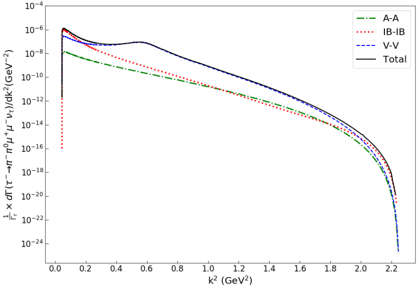
BR & lepton-pair distributions

$$\begin{aligned} \mathrm{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 e^+ e^-\right) &= (2.27 \pm 0.03) \times 10^{-5} \\ \mathrm{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 \mu^+ \mu^-\right) &= (1.55 \pm 0.25) \times 10^{-7} \end{aligned}$$

$\ell^+\ell^- = e^+e^-$



$\ell^+\ell^- = \mu^+\mu^-$



Conclusions

- We computed the BR and the lepton-pair distributions of the 5-body decays $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ $(\ell = e, \mu)$. They are of $\mathcal{O}(\alpha^2)$.
- SD contributions ($W^- \to \pi^- \pi^0 \gamma^*$), calculated using RChT framework.
- We get:
  $\mathrm{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 e^+ e^-\right) = (2.27 \pm 0.03) \times 10^{-5}$, which could be discovered already with BaBar or Belle data, and
  $\mathrm{BR}\left(\tau^- \to \nu_\tau \pi^- \pi^0 \mu^+ \mu^-\right) = (1.55 \pm 0.25) \times 10^{-7}$, that will challenge the capabilities of Belle-II.
- Improving background rejection for LFV or LNV searches by adding the matrix elements derived here to the MC generator TAUOLA.

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THANKS!

Backup

The radiative $\tau^- \to \nu_\tau \pi^- \pi^0 \gamma$ decay

In particular, the form factors entering the $\hat{V}^{\mu\nu}$ tensor (SD) are calculated using RChT framework [11] (when $k^2 \to 0$):

$$\begin{aligned}
v_1 &= \frac{F_V G_V}{f^2 m_\rho^2}\left[2 + 2m_\rho^2 D_\rho^{-1}(t') + tD_\rho^{-1}(t) + tm_\rho^2 D_\rho^{-1}(t) D_\rho^{-1}(t')\right] \\
&+ \frac{F_V^2}{2f^2 m_\rho^2}\left[-1 - m_\rho^2 D_\rho^{-1}(t') + t' D_\rho^{-1}(t')\right] + \frac{F_A^2}{f^2 m_{a_1}^2}\left[m_{a_1}^2 - m_\pi^2 + \frac{t}{2}\right] D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_2 &= \frac{F_V G_V t}{f^2 m_\rho^2}\left[-D_\rho^{-1}(t) - m_\rho^2 D_\rho^{-1}(t) D_\rho^{-1}(t')\right] + \frac{F_V^2}{2f^2 m_\rho^2}\left[-1 - m_\rho^2 D_\rho^{-1}(t') - t' D_\rho^{-1}(t')\right] \\
&+ \frac{F_A^2}{f^2 m_{a_1}^2}\left[m_{a_1}^2 - m_\pi^2 - p_- \cdot k\right] D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_3 &= \frac{F_A^2}{f^2 m_{a_1}^2} D_{a_1}^{-1}\left[(p_- + k)^2\right], \\
v_4 &= -\frac{2F_V G_V}{f^2} D_\rho^{-1}(t) D_\rho^{-1}(t') + \frac{F_V^2}{f^2 m_\rho^2} D_\rho^{-1}(t'),
\end{aligned} \tag{12}$$

[11] Nucl. Phys. B 321, 311 (1989); Phys. Lett. B 223, 425 (1989).

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Kinematics

- The unpolarized squared amplitude of a five-body decay as $\tau^- \to \nu_\tau \pi^- \pi^0 \ell^+ \ell^-$ depends on eight independent variables. In general 3n-7 [12].
- Depending upon the observable we are interested in, integrate some or all of the variables. VEGAS for Python.
- We will compute the invariant mass distribution of the lepton pair, i.e., k^2 distribution.
- The kinematical domain being $\left[4m_\ell^2, (M_\tau - 2m_\pi)^2\right]$.

[12] R. Kumar, Phys. Rev. 185, 1865 (1969).

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Resonance Chiral Theory

- Owning the resonances transform as nonets under $U(3)_V$, i.e., $R \xrightarrow{G} hRh^\dagger$,
- with $R = \frac{1}{\sqrt{2}}\lambda^a R^a$ and R stands for V or A.
- Then the resonances will come represented as 2-rank antisymmetric tensors [13].

[13] Nucl. Phys. B 321, 311 (1989)

The lagrangian density for the resonances is given by

$$\mathcal{L}_R = \sum_{R=V,A,S,P} \left[\mathcal{L}_{Kin}(R) + \mathcal{L}_{int}(R)\right],$$

with its corresponding terms:

$$\mathcal{L}_{Kin} = -\frac{1}{2}\langle \nabla^\alpha R_{\alpha\mu} \nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\mu\nu} R^{\mu\nu}\rangle,$$

and the interaction terms at order $D = 2$

$$\mathcal{L}_2^V = \frac{1}{2\sqrt{2}}\langle V_{\mu\nu}\left(F_V f_+^{\mu\nu} + iG_V[u^\mu, u^\nu]\right)\rangle,$$

$$\mathcal{L}_2^A = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle, \qquad \mathcal{L}_2^P = id_m\langle P_\chi\rangle,$$

and $f_\pm = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$, $u^\mu = iu^\dagger (D^\mu U)\, u^\dagger$.

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