Based on the work
Eur. Phys. J. C **80,** 506 (2020).

3 June 2021 – 10th International Workshop on Charm Physics

$\tau \to \mu\mu\mu$ at a rate of one out of 10^{14} tau decays?

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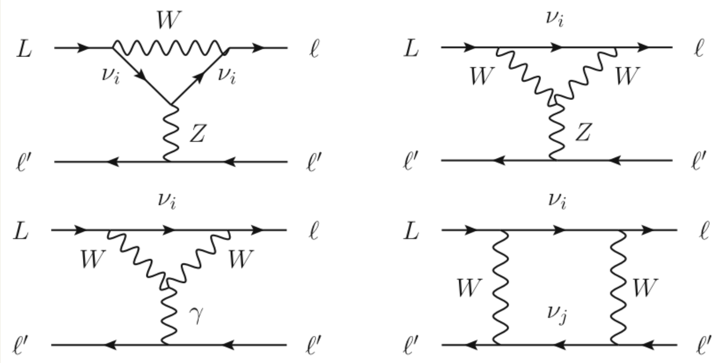
CLFV from mixing



$$\mathcal{L}_{\text{Weak}} = -\frac{g}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \bar{\ell}\,\gamma_\alpha P_L \nu_\ell\, W^{\alpha\dagger} + \text{h.c.}$$

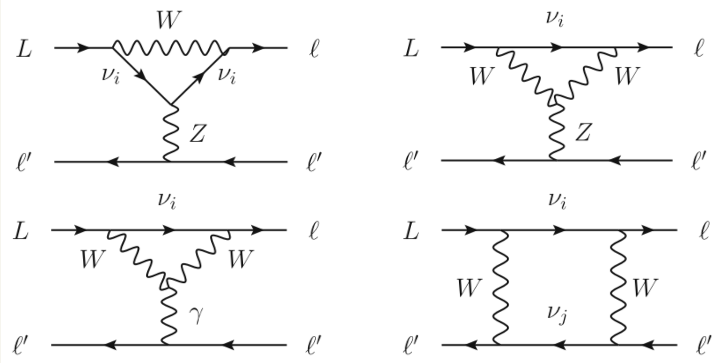
$$\nu_\ell = \sum_{i=1,2,3} U_{\ell i}\nu_i \quad \rightarrow \quad \mathcal{L}_{\text{Weak}} = -\frac{g}{2\sqrt{2}} \sum_{\substack{\ell=e,\mu,\tau \\ i=1,2,3}} U_{\ell i}\,\bar{\ell}\,\gamma_\alpha P_L \nu_i\, W^{\alpha\dagger} + \text{h.c.}$$

$$\sum_i U_{\ell i} U^*_{\ell' i} = \delta_{\ell\ell'}$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{\text{CP}}} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\text{CP}}} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\text{CP}}} & c_{13}\,s_{23} \\ s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\text{CP}}} & -c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\text{CP}}} & c_{13}\,c_{23} \end{pmatrix}$$

CLFV from mixing



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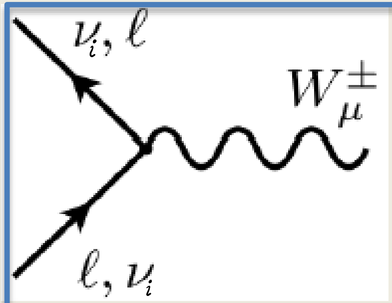
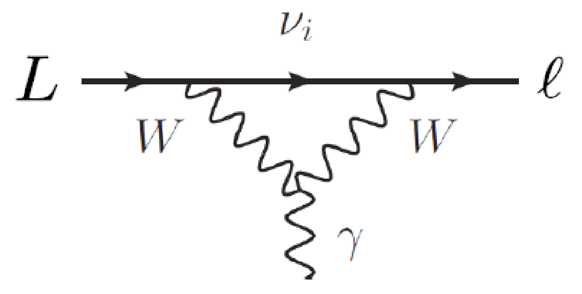
$$\nu_\ell = \sum_{i=1,2,3} U_{\ell i}\nu_i \;\rightarrow\; \mathcal{L}_{\text{Weak}} = -\frac{g}{2\sqrt{2}} \sum_{\substack{\ell=e,\mu,\tau \\ i=1,2,3}} U_{\ell i}\,\bar{\ell}\,\gamma_\alpha P_L \nu_i \, W^{\alpha\dagger} + \text{h. c.}$$

$$\sum_i U_{\ell i} U^*_{\ell' i} = \delta_{\ell\ell'}$$

Unitarity leads to GIM suppression (Glashow, Iliopoulos, Maiani, 1970)



$$= -\frac{ig}{\sqrt{2}} U^{(*)}_{\ell i} \gamma_\mu P_L$$



Suppression in $L \to \ell\gamma$

Much simpler calculation:

$$\frac{\Gamma(L^- \to \ell^- \gamma)}{\Gamma(L^- \to e^- \bar{\nu}_e \nu_L)} = \frac{3\alpha}{32\pi} \left| \sum_{i=1}^{3} U_{Li}^* U_{\ell i} \frac{m_i^2}{M_W^2} \right|^2 =$$

Parameters from NUFIT4.1 normal ordering, PDG 2020

2×10^{-53}	$\tau \to \mu\gamma$
8×10^{-55}	$\tau \to e\gamma$
4×10^{-55}	$\mu \to e\gamma$

Insensitive to m_1!

Petkov, Sov. J. Nucl. Phys. 25 (1977) 340
Cheng & Li *Phys.Rev.Lett.* 45 (1980) 1908

Current Bound: $\mathrm{Br}(\mu \to e\,\gamma) < 4.2 \times 10^{-13}$

MEG Collaboration
EPJ C76 434

$\sum m_i < 0.170$ eV (95% CL)

Plank + BAO, Astron. Astrophys. 594 (2016) A13

Other GIM-breaking forms

For normal ordering and $m_1 = 0.170$ eV,

$$\left|\sum_{i=1}^{3} U_{Li}^* U_{\ell i} \frac{m_i^2}{M_W^2}\right|^2 \sim 10^{-50}$$

$$\left|\sum_{i=1}^{3} U_{Li}^* U_{\ell i} \frac{m_i^2}{M_W^2} \log\left(\frac{m_i^2}{M_W^2}\right)\right|^2 \sim 10^{-46}$$

Neutrino-mass dependence in sum breaks GIM mechanism, type of dependence drastically changes order of amplitude.

$$\left|\sum_{i=1}^{3} U_{Li}^* U_{\ell i} \log\left(\frac{m_i^2}{M_W^2}\right)\right|^2 \sim 10^{-3}$$ (?!)

N.B. This expression is independent of M_W.

That's peculiar...

Eur. Phys. J. C 8, 513–516 (1999)
DOI 10.1007/s100529901088

Lepton flavor changing in neutrinoless τ decays

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Received: 29 October 1998 / Published online: 11 March 1999

Abstract. Neutrino oscillations, as recently reported by the Super-Kamiokande collaboration, imply that lepton numbers could be violated, and $\tau^\pm \to \mu^\pm + \ell^+ + \ell^-$, $\tau^\pm \to \mu^\pm + \rho^0$ are some typical examples. We point out that in these neutrinoless modes, the GIM cancelation is much milder with only a logarithmic behavior $\log(m_j/m_k)$ where $m_{j,k}$ are the neutrino masses. This is in sharp contrast with the vanishingly small amplitude $\tau^\pm \to \mu^\pm + \gamma$ strongly suppressed by the quadratic power $(m_j^2 - m_k^2)/M_W^2$. In comparison with the hopelessly small branching ratio $B(\tau^\pm \to \mu^\pm + \gamma) \approx 10^{-40}$, the $B(\tau^\pm \to \mu^\pm + \ell^+ + \ell^-)$ could be larger than 10^{-14}. The latter mode, if measurable, could give one more constraint to the lepton mixing angle $\sin 2\theta_{jk}$ and the neutrino mass ratio m_j/m_k, and therefore is complementary to neutrino oscillation experiments.

The first calculation

S. T. Petkov (1977) performed this calculation **ignoring external momentum scales**.

$$\Gamma_{\mathrm{ZML}}(L \to \ell\ell\ell) = \frac{\alpha^2 G_F^2 m_L^5}{(4\pi)^5} \left| \sum_{i=2}^{3} U_{Li}^* U_{\ell i} \frac{\Delta m_{i1}^2}{M_W^2} \log \frac{\Delta m_{i1}^2}{M_W^2} \right|^2$$

with $\Delta m_{ij}^2 = m_i^2 - m_j^2$, $m_1 < m_2 < m_3$ and $m_1 \to 0$.

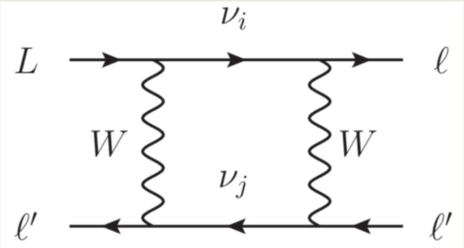
Petkov, Sov. J. Nucl. Phys. 25 (1977) 340

$$\log \frac{M_W^2}{\Delta m_{ij}^2} \sim 60$$

Process	BR
$\tau \to \mu\mu\mu$	2.0×10^{-53}
$\tau \to eee$	1.1×10^{-54}
$\mu \to eee$	4.1×10^{-54}

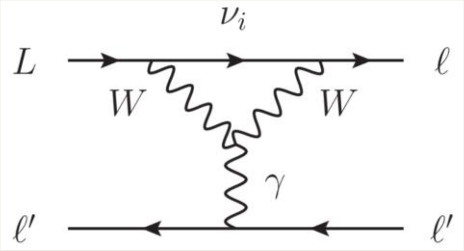
PDG ‘18 values

Boxes



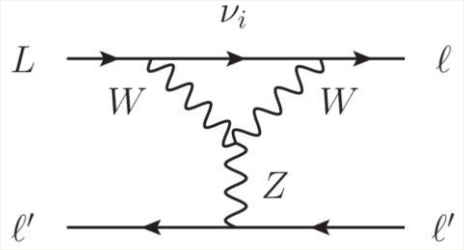
$$= x(1 + \log x)$$

γ Penguins



$$= -4x$$

Z Penguins



$$= x(3 + \log x)$$

Zero-momentum limit (ZML):

$$\mathcal{P} \ll m_i \ll M_W$$

Allow for substantial simplification of loop diagrams, later codified as **Inami-Lim loop functions.**

These are leading $x \to 0$ behavior in ZML.

$$x \equiv \frac{m_\nu^2}{M_W^2}$$

$\mathcal{P}$ = external mass & momentum scales of the problem, $m_L, m_\ell, s, t, \ldots$

Prog. of Theo. Physics, Volume 65, Issue 1, January 1981, Pages 297–314

Physical Limit (PL):

$$m_\nu \ll \mathcal{P} \ll M_W$$

10^{-14} claim:

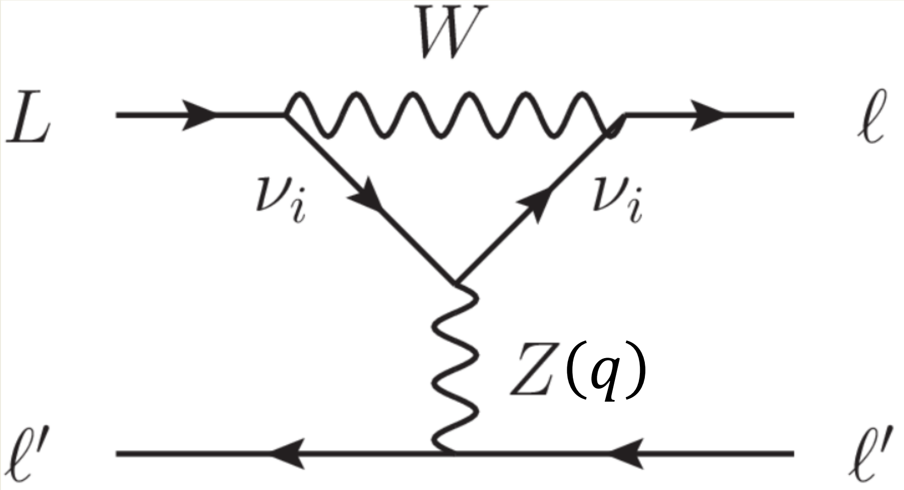
Moving to PL generates a $\log m_i$ divergence in the Z penguin. This involves an expansion about $q^2 = 0$:

$$f_0(x_i) + (q^2/M_W^2) f_1(x_i) + \cdots$$

$$f_0(x_i) \sim x_i \log x_i$$

$$f_1(x_i) \sim \boldsymbol{\log x_i}$$

Pham, EPJC 8 (1999) 513



$$Br \propto \left| \sum_{i=2}^{3} U_{Li}^* U_{\ell i} \log\left(m_i^2/m_1^2\right) \right|^2$$

$$Br(\tau \to \mu \ell^+ \ell^-) \geq 10^{-14}$$

Physical Limit (PL):

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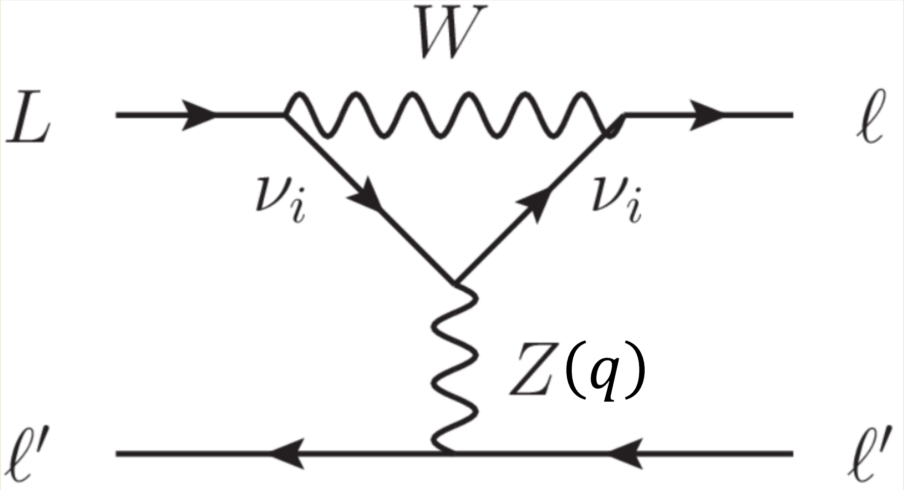


Buchalla, Buras, Harlander
NPB 349 (1991) 1

Concerns:

1. Nontrivial gauge-dependence cancellation
2. q^2 is physically limited by $q^2 > 4m_{\ell'}^2$, so the expansion **cannot give correct $m_i \to 0$ behavior**
3. We desire the $m_i \to 0$ limit to recover the SM without fine-tuning of ratios m_i/m_j

If m_1 is very small, Pham's predictions start to look dramatic:

Process	Br (Pham) $m_1 = 10^{-2}$ eV	Br (Pham) $m_1 = 10^{-8}$ eV
$\tau \to \mu\mu\mu$	10^{-16}	10^{-13}
$\tau \to eee$	10^{-16}	10^{-14}
$\mu \to eee$	10^{-22}	10^{-17}

Current limits:

$$\text{Br}_{\text{expt}}(\tau \to \ell\ell'\ell') \lesssim 10^{-8}$$

HFLAV Coll., EPJ C (2021) **81**: 226

$$\text{Br}_{\text{expt}}(\mu \to eee) < 1.0 \times 10^{-12}$$
$(90\%\ CL)$

SINDRUM Coll., NPB 299 (1988) 1

Expected limits:

- Mu3e
  $\mu \to eee \sim 10^{-15} - 10^{-16}$
- HL-LHC ($3000\ \text{fb}^{-1}$)
  $\tau \to \mu\mu\mu \sim 10^{-9}$
- Belle II ($50\ \text{ab}^{-1}$)
  $\tau \to \mu\mu\mu \sim 10^{-10}$

Cerri et al, hep-ph/1812.07638.
Belle II Physics Book, 1808.10567.

Belle prospects (from Belle II physics book)

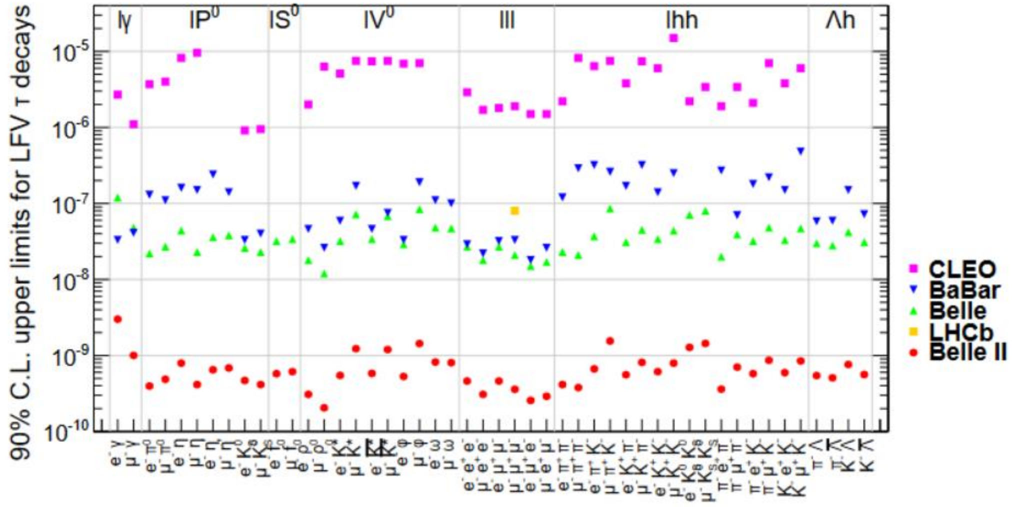
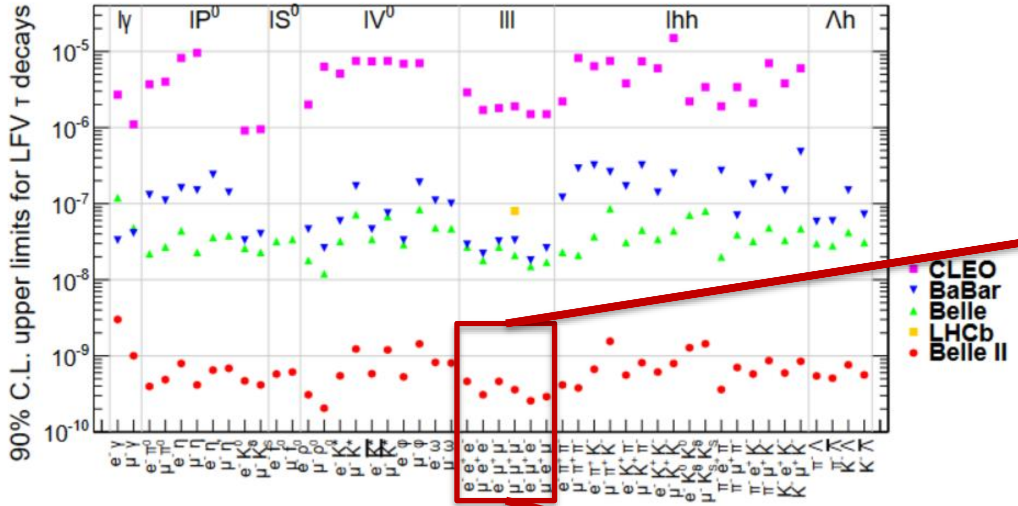


Fig. 189: Current 90% C.L. upper limits for the branching fraction of τ LFV decays obtained in the CLEO, BaBar, and Belle experiments. Purple boxes, blue inverted triangles, green triangles and yellow boxes show CLEO, BaBar, Belle and LHCb results, respectively, while red circles express the Belle II future prospects, where they are extrapolated from Belle results assuming the integrated luminosity of 50 ab^{-1}.

Belle prospects (from Belle II physics book)



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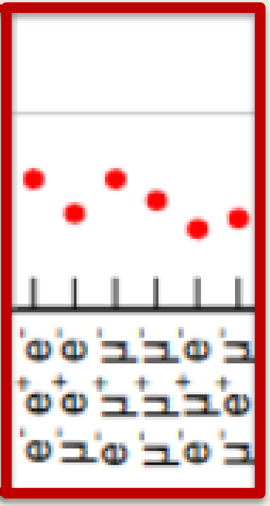


2018: A numerical look

Hernández-Tomé, López Castro & Roig performed a numerical analysis in PL.

Including Z-penguins and box diagrams, there is still a question of gauge dependence but, critically, no logarithmic divergence was found.

Their results agree in magnitude with ZML expression.

Hernández-Tomé et al., EPJ C (2020) 80: 438

Decay channel	~~Our~~ result
$\mu^- \to e^- e^+ e^-$	7.4×10^{-55}
$\tau^- \to e^- e^+ e^-$	3.2×10^{-56}
$\tau^- \to \mu^- \mu^+ \mu^-$	6.4×10^{-55}
$\tau^- \to e^- \mu^+ \mu^-$	2.1×10^{-56}
$\tau^- \to \mu^- e^+ e^-$	5.2×10^{-55}

The Goal

We want to determine the analytic form of

$$\Gamma(L \to \ell\ell'\ell') = \frac{\alpha^2 G_F^2 m_L^5}{(4\pi)^5} \left| \sum_{i=2}^{3} U_{Li}^* U_{\ell i} \textbf{(???)} \right|^2$$

in the PL $m_\nu \ll \mathcal{P} \ll M_W$

The Task

60 diagrams including boxes, Z and γ penguins, Goldstone diagrams, and wavefunction renormalization.

`FeynArts`: generate diagrams & CTs in Feynman gauge

`Form` (w/ `FormCalc` package): reduce amplitudes to one-loop tensor integrals, retaining all $m_{\nu i}$, $\mathcal{P}$, and M_W dependence

`Mathematica`, `FeynCalc`, and `Package-X`: independent analysis of master integrals and exhaustive numerical sanity checks

The Task

FeynArts: T. Hahn, Comput. Phys. Commun. 140, 418 (2001). (arXiv:hep-ph/0012260)

Form: J. Kuipers, T. Ueda, J.A.M.Vermaseren, J. Vollinga, Comput. Phys. Commun. 184, 1453 (2013). (arXiv:1203.6543)

FormCalc: T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. 118, 153 (1999). (arXiv:hep-ph/9807565)

Mathematica: Wolfram Research, Inc., Mathematica, Version 12.3, Champaign, IL (2021)

FeynCalc: V. Shtabovenko, R. Mertig, F. Orellana, Comput. Phys. Commun. 207, 432 (2016). (arXiv:1601.01167)

Package-X: H.H. Patel, Comput. Phys. Commun. 197, 276 (2015). (arXiv:1503.01469)

Beneke, Smirnov, NPB 522 (1998) 321
Smirnov, Springer Tracts Mod. Phys. 177

Method of Regions

Example expansion: (p_1, p_2, p_3 are $\ell^{(\prime)}$ momenta, P is L momentum)

$$\int \frac{d^d k}{(2\pi)^d}\left[(k^2 - M_W^2)((k+p_1)^2 - m_\nu^2)\left((k+(P-p_3))^2 - M_W^2\right)\right]^{-1}$$

Large region ($k \sim M_W$):

$$-\frac{1}{M_W^2} - \frac{m_\nu^2}{M_W^4}\left(\frac{1}{\varepsilon} + \log\left(\frac{\mu^2}{M_W^2}\right)\right) - \frac{1}{M_W^4}\left(\frac{m_\ell^2}{2} + m_\nu^2 + \frac{s}{12}\right) + \mathcal{O}\left(\frac{\mathcal{P}^4}{M_W^6}\right)$$

Small region ($k \sim \mathcal{P}, m_\nu$)

$$+\frac{m_\nu^2}{M_W^2}\left(\frac{1}{\varepsilon} + \log\left(\frac{\mu^2}{m_\nu^2}\right)\right) + \frac{m_\nu^2}{M_W^4} + \mathcal{O}\left(\frac{\mathcal{P}^2 m_\nu^2}{M_W^6}\right)$$

Beneke, Smirnov, NPB 522 (1998) 321
Smirnov, Springer Tracts Mod. Phys. 177

Method of Regions

Example expansion: (p_1, p_2, p_3 are $\ell^{(\prime)}$ momenta, P is L momentum)

$$\int \frac{d^d k}{(2\pi)^d}\left[(k^2 - M_W^2)\big((k+p_1)^2 - m_\nu^2\big)\left((k+(P-p_3))^2 - M_W^2\right)\right]^{-1}$$

Combine to give expansion of integral:

$$-\frac{1}{M_W^2} + \frac{1}{M_W^4}\left(-\frac{m_\mu^2}{2} - m_\nu^2 - \frac{s}{12} + m_\nu^2 \log\left(\frac{M_W^2}{m_\nu^2}\right)\right) + \mathcal{O}\left(\frac{\mathcal{P}^4}{M_W^6}\right)$$

Series of series

Master integrals are expanded analytically using MOR + `Package-X`.

- First, separate $M_W \gg \{\mathcal{P}, m_\nu\}$

$$\mathcal{A} = \sum_n f_n(\mathcal{P}^2, m_\nu^2) \left(\frac{1}{M_W^2}\right)^n$$

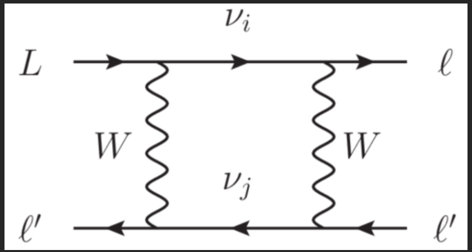
- Next, carefully account for all small parameters $m_\nu \ll \mathcal{P} \ll M_W$ and regions

$$\mathcal{A} = \sum_{nmij} c_{nm,ij} \left(\frac{\mathcal{P}^2}{M_W^2}\right)^n \left(\frac{m_\nu^2}{\mathcal{P}^2}\right)^m \log^i\left(\frac{\mathcal{P}^2}{M_W^2}\right) \log^j\left(\frac{m_\nu^2}{\mathcal{P}^2}\right)$$

- Use GIM mechanism and keep leading term in m_ν to obtain correct expansion

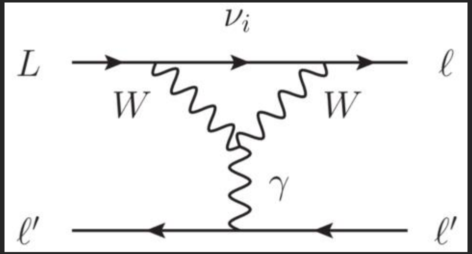
$$\mathcal{A} \to \frac{\mathcal{P}^4}{M_W^4} \frac{m_\nu^2}{\mathcal{P}^2} \log^i\left(\frac{\mathcal{P}^2}{M_W^2}\right) \log^j\left(\frac{m_\nu^2}{\mathcal{P}^2}\right)$$

Boxes



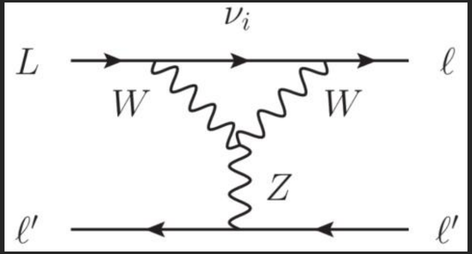
$$\sim \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}\log\left(\frac{m_L^2}{M_W^2}\right)$$

γ Penguins



$$\sim \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}$$

Z Penguins



$$\sim \frac{\mathcal{P}^4}{M_W^4}\frac{m_\nu^2}{\mathcal{P}^2}\log\left(\frac{m_L^2}{M_W^2}\right)$$

$$L = \{\mu, \tau\}$$

Logarithmic enhancement is replaced by a mild $\log\left(\frac{M_W}{m_\tau}\right) = 3.8$ or $\log\left(\frac{M_W}{m_\mu}\right) = 6.6$

Results

One final MOR application in phase space integration to get leading order in m_ℓ/m_L, we find

$$\Gamma(L \to \ell\ell\ell) = \frac{G_F^2 \alpha^2 m_L^5}{(4\pi)^5} \left| \sum_{i=2}^{3} U_{Li}^* U_{\ell i} \frac{\Delta m_{i1}^2}{M_W^2} \right|^2$$

$$\times \left[\log^2 x_L + 2 \log x_L - \frac{1}{6} \log x_\ell + \frac{19}{18} + \frac{17}{18}\pi^2 - \frac{1}{\sin^2\theta_W}\left(\log x_L + \frac{11}{12}\right) + \frac{3}{8\sin^4\theta_W} \right]$$

Results

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Note: Final expression is independent of m_1 due to absence of neutrino mass logarithms!

Results

One final MOR application in phase space integration to get leading order in m_ℓ/m_L, we find

$$\Gamma(L \to \ell\ell'\ell') = \frac{G_F^2\alpha^2 m_L^5}{(4\pi)^5}\left|\sum_{i=2}^{3} U_{Li}^* U_{\ell i}\frac{\Delta m_{i1}^2}{M_W^2}\right|^2$$

$$\times\left[\frac{2}{3}\log^2 x_L + \frac{25}{18}\log x_L - \frac{1}{6}\log x_{\ell'} + \frac{55}{108} + \frac{2}{3}\pi^2 - \frac{1}{\sin^2\theta_W}\left(\frac{\log x_L}{2} + \frac{11}{24} + \frac{\pi^2}{18}\right) + \frac{1}{\sin^4\theta_W}\left(\frac{3}{16} + \frac{\pi^2}{36}\right)\right]$$

$\ell \neq \ell'$

Results

Eur. Phys. J. C **80,** 506 (2020)

Table 1 Branching ratio for the CLFV decays $L \to \ell\ell'\ell'$ in the ZML and the PL for normal ordering (NO) and inverted ordering (IO) of neutrino masses. The ratio between the two is also reported. In the ZML we assume $m_1 = 0$ (NO) or $m_3 = 0$ (IO)

	Branching ratio (NO)			Branching ratio (IO)		
	ZML	PL	ZML/PL	ZML	PL	ZML/PL
$\mu \to eee$	4.1×10^{-54}	2.9×10^{-55}	14	6.1×10^{-54}	4.6×10^{-55}	14
$\tau \to \mu\mu\mu$	2.0×10^{-53}	5.8×10^{-55}	34	2.0×10^{-53}	5.8×10^{-55}	34
$\tau \to \mu ee$	1.3×10^{-53}	3.8×10^{-55}	35	1.3×10^{-53}	3.8×10^{-55}	35
$\tau \to eee$	1.1×10^{-54}	3.3×10^{-56}	34	6.1×10^{-55}	1.9×10^{-56}	32
$\tau \to e\mu\mu$	7.6×10^{-55}	2.1×10^{-56}	36	4.1×10^{-55}	1.2×10^{-56}	34

ZML = Expression from Petkov with updated parameters
PL = Result using our expression

Notice that universally $PL < ZML$ because of the replacement $\log x_i \rightsquigarrow \log x_L$

A remark on the missing logarithm

$$O_6^{L\ell'} = (\bar{\nu}_i \gamma^\mu P_L L)(\bar{\ell}' \gamma_\mu P_L \nu_j),$$
$$O_6^{\ell\ell'} = (\bar{\ell} \gamma^\mu P_L \nu_i)(\bar{\nu}_j \gamma_\mu P_L \ell'),$$
$$O_8 = m_{\nu i}^2 (\bar{\ell} \gamma^\mu P_L L)(\bar{\ell}' \gamma_\mu P_L \ell')$$

$$C_6^{L\ell'} = \frac{4G_F}{\sqrt{2}} U_{\ell' j} U_{Li}^*, \qquad C_6^{\ell\ell'} = \frac{4G_F}{\sqrt{2}} U_{\ell' j}^* U_{\ell i},$$
$$C_8(\mu) = \frac{G_F^2}{2\pi^2} \log\left(\frac{M_W^2}{\mu^2}\right) U_{\ell i} U_{Li}^*.$$

Our (unconfirmed) suspicion:

The evolution of C_8 cannot proceed to below the scale of the process, $\mathcal{P}$, so no $\log M_W/m_\nu$ terms arise.

In ZML, $\mathcal{P} \to 0$ allows the full evolution to the neutrino mass, generating these logarithms.

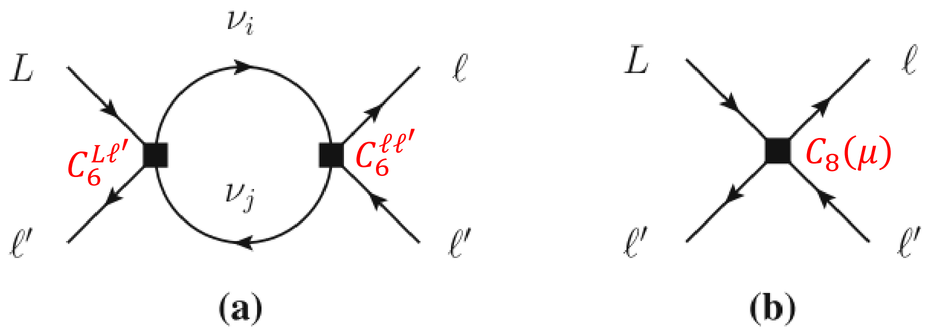

Fig. 2 Example of two diagrams mediating $L \to \ell\ell'\ell'$ in a low energy effective field theory description

Conclusions

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- Can $Br(L \to 3\ell)$ be as large as 10^{-14} in the Standard Model with PMNS neutrino mixing? Unfortunately, no.
- There are no unregulated $\log m_i$ divergences in the neutrino mass.
- If near- or distant-future experiments (Mu3e, Belle II, HL-LHC) see these processes, it will certainly evidence the existence of New Physics beyond $\mathrm{SM} + m_\nu \neq 0$ (and hopefully betray its nature).

Backup

The Method of Regions (MOR)

Beneke, Smirnov, NPB 522 (1998) 321
Smirnov, Springer Tracts Mod. Phys. 177

To perform expansions in small parameters $m_\nu^2/\mathcal{P}^2$ and $\mathcal{P}^2/M_W^2$, we use MOR rather than contend with full analytic form of basis integrals.

1. Identify disparate scales involved in integral,
2. introduce cut-off which partitions the regions,
3. perform the expansion in each integrand relevant to that region,
4. expand each regions' domain to the full domain and sequester cut-off scale dependence into "compensating terms"
5. the compensating terms cancel order-by-order and what remains is the desired expansion.

MOR: 1D Example

Beneke, Smirnov, NPB 522 (1998) 321
Smirnov, Springer Tracts Mod. Phys. 177

With the hierarchy $0 < m \ll q$, we want to calculate

$$F(q,m,\varepsilon) = \int_0^\infty \frac{k^{-\varepsilon} dk}{(k+m)(k+q)} = -\frac{\pi}{\sin \pi\varepsilon} \frac{q^{-\varepsilon} - m^{-\varepsilon}}{q-m}$$

Large Region: $k \sim q$

$$F_{\text{large}} \sim \int_0^\infty \frac{k^{-\varepsilon-1} dk}{k+q} \left[1 - \left(\frac{m}{k}\right) + \left(\frac{m}{k}\right)^2 - \cdots\right] = \frac{\pi}{q^{1+\varepsilon} \sin \pi\varepsilon} \sum_{n=0}^{\infty} \left(\frac{m}{q}\right)^n$$

Small Region: $k \sim m$

$$F_{\text{small}} = \frac{1}{q} \int_0^\infty \frac{k^{-\varepsilon} dk}{k+m} \left[1 - \left(\frac{k}{q}\right) + \left(\frac{k}{q}\right)^2 - \cdots\right] = -\frac{\pi}{q\, m^{\varepsilon} \sin \pi\varepsilon} \sum_{n=0}^{\infty} \left(\frac{m}{q}\right)^n$$

MOR: 1D Example

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What about compensating terms?

Large region: 1st order compensating term

$$\int_0^\Lambda \frac{k^{-\varepsilon-1}dk}{k+q} = -\frac{1}{\varepsilon q\,\Lambda^\varepsilon} + \mathcal{O}\left(\frac{1}{q^2}\right)$$

Small region: 1st order compensating term

$$\frac{1}{q}\int_\Lambda^\infty \frac{k^{-\varepsilon}dk}{k+m} = +\frac{1}{\varepsilon q\,\Lambda^\varepsilon} + \mathcal{O}\left(\frac{1}{q^2}\right)$$