QCD-based estimates of direct *CP* asymmetry in charm decays

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□ Direct CP asymmetry in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ decays

AK, A.Petrov Phys. Lett. B **774** (2017), 235 [arXiv:1706.07780 [hep-ph]].

- Asymmetry is nonvanishing if the decay amplitude $A(D^0 \to f)$ consists of two parts,

$$A(D^0 \to f) = A_f^{(1)} e^{i\delta_1} e^{i\phi_1} + A_f^{(2)} e^{i\delta_2} e^{i\phi_2},$$

with different weak $\phi_1 \neq \phi_2$ and strong $\delta_1 \neq \delta_2$ phases:

$$A(\bar{D}^0 \to \bar{f}) = A_f^{(1)} e^{i\delta_1} e^{-i\phi_1} + A_f^{(2)} e^{i\delta_2} e^{-i\phi_2},$$

- the CP-violating asymmetry: $f = \pi^+\pi^-,\ K^+K^-$

$$a_{CP}^{dir}(f) = \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to \bar{f})}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to \bar{f})}$$

$$= \frac{|A(D^0 \to f)|^2 - |A(\bar{D}^0 \to \bar{f})|^2}{|A(D^0 \to f)|^2 + |A(\bar{D}^0 \to \bar{f})|^2} \sim \frac{A_f^{(1)}}{A_f^{(2)}} \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2).$$

- is a calculable estimate and/or upper bound for $a_{\rm CP}^{\rm dir}(f)$ in SM ?

□ Single Cabibbo-suppressed (SCS) decays

- effective Hamiltonian: $O_{i\geq 3}$ with $c_i \ll c_{1,2}$ are neglected

$$H_{eff} = \frac{G_F}{\sqrt{2}}\Big\{\lambda_d\big(c_1\ O_1^d + c_2\ O_2^d\big) + \lambda_s\big(c_1\ O_1^s + c_2\ O_2^s\big)\Big\}\,,$$

$$O_1^d = (\bar{u}\Gamma_\mu d)\,\big(\bar{d}\Gamma^\mu c\big)\ ,\ O_2^d = \big(\bar{d}\Gamma_\mu d\big)\,(\bar{u}\Gamma^\mu c) \underset{d\to s}{\Longrightarrow}\ O_1^s\,,\ O_2^s$$

- hereafter using a compact notation:

$$\mathcal{O}^D \equiv \frac{G_F}{\sqrt{2}}\sum_{i=1,2} c_i O_i^D\,,\quad (D = d, s)\,.$$

$$\lambda_D = V_{uD}V_{cD}^*,\ \ (D = d, s, b)\,,$$

- CKM unitarity in SM:

$$\sum_{D=d,s,b}\lambda_D = 0\,,\quad \text{or}\quad \lambda_d = -(\lambda_s + \lambda_b)\,,$$

$$\lambda_b \ll \lambda_{s,d},\quad \mathrm{Im}(\lambda_b) \neq 0$$

□ Decomposition of decay amplitudes

- separating the contributions of $\mathcal{O}^d_{1,2}$ and $\mathcal{O}^s_{1,2}$ operators

$$A(D^0 \to \pi^+\pi^-) = \lambda_d \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle ,$$

$$A(D^0 \to K^+K^-) = \lambda_s \langle K^+K^- | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^+K^- | \mathcal{O}^d | D^0 \rangle ,$$

- replacing $\lambda_d = -(\lambda_s + \lambda_b)$
- it is sufficient to calculate the "penguin" type amplitudes

$$\mathcal{P}^s_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle , \quad \mathcal{P}^d_{KK} = \langle K^+K^- | \mathcal{O}^d | D^0 \rangle ,$$

a rather general definition of "penguin": the operator contains a $\bar{q}q$ pair where q does not belong to the valence content of the final state,

□ Decomposition of decay amplitudes

- separating the $O(\lambda_b)$ contribution with CP-phase

$$A(D^0 \to \pi^+\pi^-) = -\lambda_s \mathcal{A}_{\pi\pi}\left\{1 + \frac{\lambda_b}{\lambda_s}\left(1 + r_\pi \exp(i\delta_\pi)\right)\right\},$$

$$A(D^0 \to K^+K^-) = \lambda_s \mathcal{A}_{KK}\left\{1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right\},$$

with the notation:

$$\mathcal{A}_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle - \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle ,$$
$$\mathcal{A}_{KK} = \langle K^+K^- | \mathcal{O}^s | D^0 \rangle - \langle K^+K^- | \mathcal{O}^d | D^0 \rangle ,$$

$$r_\pi = \left| \frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}} \right| , \quad r_K = \left| \frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}} \right| , \; \delta_{\pi(K)} = arg[\mathcal{P}^{s(d)}_{\pi\pi(KK)}] - arg[\mathcal{A}_{\pi\pi(KK)}]$$

- to a good approximation

$$-\lambda_s \mathcal{A}_{\pi\pi} \simeq A(D^0 \to \pi^+\pi^-) , \quad \lambda_s \mathcal{A}_{KK} \simeq A(D^0 \to K^+K^-)$$

□ Direct CP-asymmetry

- In terms of the parameters entering the decomposition:

$$a_{CP}^{dir}(K^+K^-) = \frac{-2 r_b r_K \sin\delta_K \sin\gamma}{1 - 2 r_b r_K \cos\gamma\cos\delta_K + r_b^2 r_K^2},$$

$$a_{CP}^{dir}(\pi^+\pi^-) = \frac{2 r_b r_\pi \sin\delta_\pi \sin\gamma}{1 + 2 r_b \cos\gamma(1 + r_\pi \cos\delta_\pi) + r_b^2(1 + 2 r_\pi \cos\delta_\pi + r_\pi^2)}$$

- the CKM elements involved:

$$\frac{\lambda_b}{\lambda_s} \equiv r_b e^{-i\gamma}, \quad r_b = \left|\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right|.$$

- a "clean" observable (after time-integration)

$$\begin{aligned}\Delta a_{CP}^{dir} &= a_{CP}^{dir}(K^+K^-) - a_{CP}^{dir}(\pi^+\pi^-)\\ &= -2 r_b \sin\gamma(r_K \sin\delta_K + r_\pi \sin\delta_\pi) + O(r_b^2).\end{aligned}$$

- a QCD-based calculation of $\mathcal{P}_{\pi\pi}^s$ and $\mathcal{P}_{KK}^d$
- combined with $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ extracted from experiment
  $\Rightarrow$ an estimate of r_π and r_K

Calculation of the "penguin" hadronic matrix element

- The method employing: QCD Light-Cone Sum Rules (LCSRs) with pion or kaon distribuiton amplitudes (DAs) and quark-hadron duality
- formulated and used earlier for the $B \to \pi\pi$ decays
  - nonfactorizable soft gluon emission

    AK, Nucl. Phys. B **605** (2001) 558 [hep-ph/0012271];
  - charming penguins (to be used here)

    AK, T. Mannel and B. Melic, Phys. Lett. B **571** (2003) 75 [hep-ph/0304179];
  - annihilation contribution

    AK, T. Mannel, M. Melcher and B. Melic, Phys. Rev. D **72** (2005) 094012 [hep-ph/0509049].

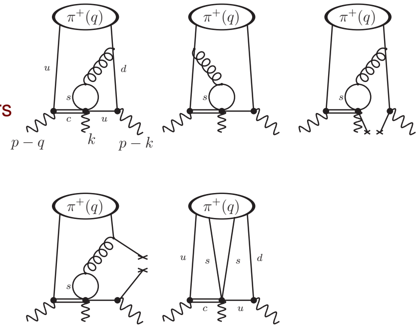
□ Calculation of the "penguin" matrix elements

- correlation function for $D \to \pi^+\pi^-$ ($\pi \to K$, $s \to d$ for $D \to K^+K^-$)

$$F_\alpha(p,q,k) = i^2 \int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{-i(p-k)y} \langle 0|T\{j^{(\pi)}_{\alpha 5}(y) O^s_{1,2}(0) j^{(D)}_5(x)\}|\pi^+(q)\rangle$$

$$c_1 O^s_1 + c_2 O^s_2 = 2c_1 \widetilde{O}^s_2 + \left(\frac{c_1}{3} + c_2\right) O^s_2\,, \qquad \widetilde{O}^s_2 = \left(\bar{s}\Gamma_\mu \frac{\lambda^a}{2} s\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} c\right)$$

- we actually calculate: $\langle \pi^+\pi^- | \widetilde{O}^s_2 | D^0 \rangle$
- OPE diagrams for $D \to \pi^+\pi^-$ in terms of pion LCDAs:

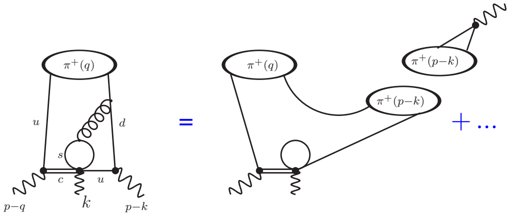


- some details:
  - LCSR's for $D \to \pi, K$ form factors
  - finite quark masses m_c, m_s
  - $SU(3)$ not used, only isospin
  - tw 2,3 accuracy, fact. tw 5,6
  - selection of diagrams

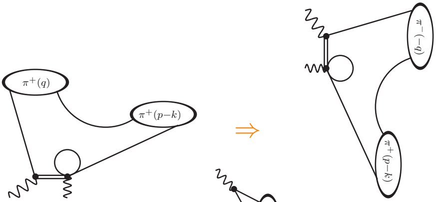
(see eariler $B \to \pi\pi$ papers)

□ Obtaining LCSR

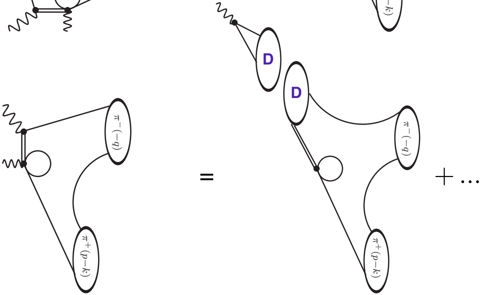
- **step 1:**
  Dispersion relation in the pion channel ⊕ duality



- **step 2:**
  Analytic continuation
  $P^2 = (p - q - k)^2 < 0 \Rightarrow \quad P^2 = m_D^2$



- **step 3:**
  Dispersion relation in the D channel ⊕ duality



Numerical estimates

- LCSR input: quark masses, pion, kaon DAs, Borel scales, effective thresholds from the LCSR calculation of $D \to \pi$, $D \to K$ and pion form factor
- hadronic matrix elements calculated from the sum rules

$$\langle \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle = (9.50 \pm 1.13) \times 10^{-3} \exp[i(-97.5^o \pm 11.6)]\,\mathrm{GeV}^3\,,$$
$$\langle K^+K^-|\widetilde{\mathcal{Q}}_2^d|D^0\rangle = (13.9 \pm 2.70) \times 10^{-3} \exp[i(-71.6^o \pm 29.5)]\,\mathrm{GeV}^3\,,$$

- converting into an estimate of the penguin amplitude:

$$\mathcal{P}^s_{\pi\pi} \simeq 2c_1 \frac{G_F}{\sqrt{2}} \langle \pi^+\pi^-|\widetilde{O}_2^s|D^0\rangle, \quad \mathcal{P}^d_{KK} \simeq 2c_1 \frac{G_F}{\sqrt{2}} \langle K^+K^-|\widetilde{O}_2^d|D^0\rangle$$

$$|\mathcal{P}^s_{\pi\pi}| = (1.96 \pm 0.23) \times 10^{-7}\mathrm{GeV}\,,$$
$$|\mathcal{P}^d_{KK}| = (2.86 \pm 0.56) \times 10^{-7}\mathrm{GeV}\,,$$

the quoted uncertainties are only parametrical !

- using experimentally measured branching fractions [PDG]:

$$r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015\,.$$

□ Results for direct CP asymmetry

- CKM averages yield $r_b\, sin\gamma = 0.64 \times 10^{-3}$,
- the difference of asymmetries:

$$\Delta a_{CP}^{dir} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)$$

- the resulting upper limits: (independent of strong phases)

$$\left|a_{CP}^{dir}(\pi^-\pi^+)\right| < 0.012 \pm 0.001\%, \quad \left|a_{CP}^{dir}(K^-K^+)\right| < 0.009 \pm 0.002\%$$
$$\left|\Delta a_{CP}^{dir}\right| < 0.020 \pm 0.003\%\,.$$

- Assuming that δ_π and δ_K are given by the calculated phases of $\mathcal{P}^s_{\pi\pi}$ and $\mathcal{P}^d_{KK}$:

$$a_{CP}^{dir}(\pi^-\pi^+) = -0.011 \pm 0.001\%, \; a_{CP}^{dir}(K^-K^+) = 0.009 \pm 0.002\%,$$
$$\Delta a_{CP}^{dir} = 0.020 \pm 0.003\%\,.$$

- The most recent LHCb collaboration result:

$\Delta a_{CP}^{dir} = (-0.154 \pm 0.029)\%$

R. Aaij *et al.* [LHCb Collaboration], arXiv:1903.08726 [hep-ex] (2019)

□ Accuracy of the LCSR prediction , other decay modes?

- the same method reproduces measured BR and $|A_{CP}|$ of the $(\bar{c}c)$ penguin-dominated $B \to K\pi$ modes

  AK, talk at Mainz Workshop on NL decays, January 2019 https://indico.mitp.uni-mainz.de/event/177,

  work in progress

- parametric accuracy of LCSR:
  twist-4, $O(\alpha_s^2)$; neglected terms of $O(s_0^{\pi,K}/m_D^2)$;
  – improvable, needs dedicated calculation,

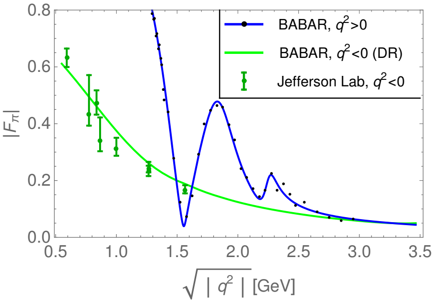
- systematic errors from semilocal duality in π and D channels
  – controlled by the pion e.m. and $D \to \pi, K$ form factor LCSRs

- we do not calculate the total amplitude of $D \to \pi^+\pi^-, K^+K^-$ in which several “topologies” contribute,

  – demands including multiloop diagrams in LCSRs
  – equally difficult is $D^0 \to K^0\bar{K}^0$

□ How accurate is the local duality approximation

- the spacelike $\to$ timelike transition for the pion e.m. form factor:



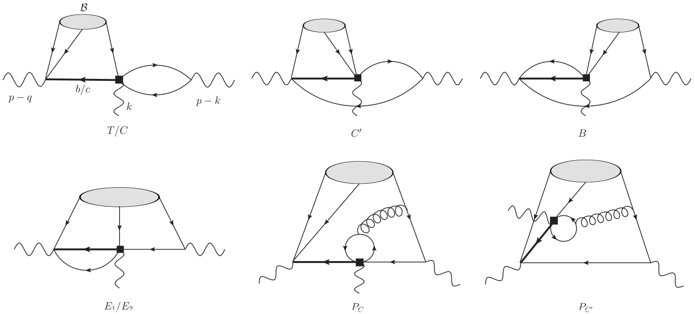
[AK, S.Cheng, A.Rusov, 2007.05550]

- a possibility to match spacelike with timelike via dispersion relation,
  – an enhancement due to $f_0(J^P = 0^+)$ resonances ?, additional model-depenedence

□ Heavy baryon two-body decays

[Hua-Yu Jiang, AK, Fu-Sheng Yu and Shan Cheng, work in progress]

- Extending the LCSR $\oplus$ duality method to nonleptonic heavy baryon decays, strating from $\Lambda_b \to p\pi^-, pK^-$
- using LCDA of proton(nucleon): a possibility to include all topologies in LO,



- $\Lambda_c \to p\pi^0, \Sigma K$ SCS amplitudes as a byproduct

Summary

- using QCD-based tools (QCD light-cone sum rules, quark-hadron duality) it is possible to estimate hadronic matrix elements for nonleptonic charm decays
- the magnitude of direct CP-violation in $D \to \pi^+\pi^-$ and $D \to K^+K^-$ can be predicted; the result is significantly smaller than the latest LHCb result
- future perspective: other $D_{(s)}$ and charmed baryon modes, improved parametrical accuracy, model of duality violation

Backup

The light-cone sum rule

$$\langle \pi^+\pi^- | \widetilde{O}_2^s | D^0 \rangle = -i \frac{\alpha_s C_F m_c^2}{8\pi^3 m_D^2 f_D} \Bigg[\int_0^{s_0^\pi} ds e^{-s/M_1^2} \int_{u_0^D}^{1} \frac{du}{u} e^{\left(m_D^2 - \frac{m_c^2}{u}\right)/M_2^2}$$

$$\times \Bigg\{ P^2 \int_0^1 dz I(zuP^2, m_s^2) \Bigg(z(1-z)\varphi_\pi(u)$$

$$i + (1-z)\frac{\mu_\pi}{2m_c} \Bigg[\left(2z + \frac{m_c^2}{uP^2}\right) u\varphi_p(u) + \frac{1}{3}\left(2z - \frac{m_c^2}{uP^2}\right)\left(\varphi_\sigma(u) - \frac{u\varphi_\sigma'(u)}{2}\right) \Bigg] \Bigg)$$

$$- \frac{\mu_\pi m_c}{4} \int_0^1 dz I(-z\bar{u}m_c^2/u, m_s^2) \frac{\bar{u}^2}{u} \Bigg[\left(1 + \frac{3m_c^2}{uP^2}\right)\varphi_p(1) + \left(1 - \frac{5m_c^2}{uP^2}\right)\frac{\varphi_\sigma'(1)}{6} \Bigg] \Bigg\}$$

$$+ \frac{2\pi^2}{3} m_c(-\langle \bar{q}q \rangle) \int_{u_0^D}^{1} \frac{du}{u^2} e^{\left(m_D^2 - \frac{m_c^2}{u}\right)/M_2^2} \Bigg\{ I(uP^2, m_s^2) \Bigg(2\varphi_\pi(u) + \frac{\mu_\pi}{m_c} \Bigg[3u\varphi_p(u)$$

$$+ \frac{\varphi_\sigma(u)}{3} - \frac{u\varphi_\sigma'(u)}{6} \Bigg] \Bigg) \Bigg\} \Bigg]_{P^2 \to m_D^2},$$

the loop integral: $I(\ell^2, m_q^2) = \frac{1}{6} + \int_0^1 dx x(1-x) \ln\left[\frac{m_q^2 - x(1-x)\ell^2}{\mu^2}\right]$.