

10th International Workshop on Charm Physics (CHARM 2020)

Hadronic Charm Meson Decays @ **BES**II

Yu-Lan Fan Wuhan University

On behalf of the BESIII Collaboration

CHARM 2020, May 31st — Jun 5th, 2021, Mexico, virtual conference

Outline

- Introduction
- Strong Phase Parameters
- Amplitude Analyses
- Absolute Branching Fractions

1

• Summary

Beijing Electron Positron Collider II(BEPCII)

Double storage ring ~240 m

Linac ~200 m

BESIII detector

2004: Started upgrade BEPCII/BESIII $\sim \sqrt{s} = 2.0 \sim 4.9 \text{ GeV}$ $\sim \mathcal{L} = 1 \times 10^{33} \text{ cm}^{-2} s^{-1}$ (April 2016) 2008: Test run 2009-now: τ -charm physics runs

Beijing Electron Positron Collider II(BEPCII)

Double storage ring ~240 m



BESIII detector

Charm Data and Analysis Method



Data production

Data samples	√s (GeV)	Int. $\mathcal{L}(fb^{-1})$	×CLEO-c
<i>D⁰D¯⁰/D⁻D⁺</i>	3.773	2.93	3.6×
$D_s D_s^*$	4.178	3.19	5.3×
$D_s D_{s^*}$	4.189-4.226	3.13	-

- Pair production @ threshold
- Quantum-correlated neutral *D*-meson pairs created in the decay of the $\Psi(3770)$ resonance
- Fully reconstructed event
- Clean background



Hadronic Parameters

Strong Phase Difference









HFLAV

PDG 2021 $\rightarrow D_x^- K^+ \pi^+$

 $\rightarrow D^{*0}K$

68.3%

95.5%

200

γ [°]

 $B^+ \rightarrow D^0 K^+$





 $\frac{\langle B^{-} \to \overline{D}^{\circ} K^{-} \rangle}{\langle B^{-} \to D^{\circ} K^{-} \rangle} = r_{B} e^{i(\delta_{B} - \gamma)} \qquad \frac{\langle D \to f \rangle}{\langle D \to f \rangle} = r_{D} e^{i\delta_{D}}$

The strong-phase difference $\delta_{\rm D}$ can be measured directly in the decays of quantum-correlated neutral **D**-meson pairs created in the decay of the $\Psi(3770)$ resonance, which ensures a binned modelindependent measurement of the CKM angle $\gamma(\phi_3)$ with **B** decays.

Works of hadronic parameters in BESIII

- Strong-phase parameters in $D \rightarrow K_{S/I} \pi^+ \pi^-, D \rightarrow K_{S/I} K^+ K^-$ PRD 101, 112002(2020), PRL 124, 241802(2020) PRD 102, 052008(2020)
- Strong-phase difference in $D \rightarrow K^{-}\pi^{+}\pi^{-}$ and $D \rightarrow K^{-}\pi^{+}\pi^{0}$, JHEP 05, 164(2021)

Hadronic Parameters

Strong-Phase Parameters in $D^0 \rightarrow K^0_{S/L} \pi^+ \pi^-$



[Phys. Rev. Lett. 124, 241802 (2020)] [Phys. Rev. D 101, 112002 (2020)]

- Self-conjugate multi-body decays (GGSZ approach)
- Three-binning schemes [PRD 82, 112006(2010)]
- The phase space is partitioned into **eight pairs** of irregularly shaped bins(*i*)



- > 2-D fits are performed on all DT events.
- > The comparison of the hadronic parameters c_i, s_i



 m_+^2, m_-^2 invariant mass square of $K_s \pi^+, K_s \pi^-$

 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

 C_i , S_i is the amplitudeweighted average of $\cos 4\delta_D$, $\sin \Delta \delta_D$ in the *i*th region of the Dalitz plot (D_i)

The most precise measurements to date of the strong-phase parameters c_i , s_i

Strong-Phase Difference in $D \rightarrow K^0_{g,J} K^+ K^-$



 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

Phys.Rev.D 102 (2020) 5, 052008

Self-conjugate multi-body decays (**GGSZ** approach)

Hadronic Parameters

this work

BaBar model

- Equal-Δδ_p binning scheme(**N=2,3,4**) [PRD 78, 034023 (2008)]
- The phase space is partitioned into 2/3/4 pairs of irregularly shaped bins(i)



- 2-D fits are performed on all DT events.
- The comparison of the hadronic parameters c_i , s_i



This is the most precise measurement to date of the strong-phase difference in these decays.

Hadronic Parameters

Strong-Phase Difference in

 $D \rightarrow K^{-}\pi^{+}\pi^{+}\pi^{-}$ and $D \rightarrow K^{-}\pi^{+}\pi^{0}$



JHEP 05, 164(2021)

- > Using multi-body $D \rightarrow K n\pi$ decays
- 2-D fits are performed on all DT events.
- **Global analysis and Equal-** $\Delta\delta_D$ **binning scheme(N=4)(Back-up)** ([Phys.Lett.B 802 (2020) 135188])

Scans of $\Delta \chi^2$ in global 2-D parameter space 350F 350 300 BESIII $\Delta \chi^2 = 2.30$ $\Delta \chi^2 = 6.18$ 250 200 () 200 م 150 م орания 200 () 2 CLEO-c $\Delta \chi^2 = 2.30$ $\Delta \chi^2 = 6.18$ 100 50 0 01 02 03 04 05 06 07 08 09 01 02 03 04 05 06 07 08 09 R_{K3π} $R_{\kappa_{\pi\pi^0}}$ $R_{K3\pi} = 0.52^{+0.12}_{-0.10}$ $R_{K\pi\pi^0} = 0.78 + 0.04$ $\delta_D^{K3\pi} = (167^{+31}_{-19})^{\circ} \qquad \qquad \delta_D^{K\pi\pi^0} = (196^{+14}_{-15})^{\circ}$ (Back-up) $r_D^{K3\pi} = (5.46 \pm 0.09) \times 10^{-2}$ $r_D^{K\pi\pi^0} = (4.40 + 0.11) \times 10^{-2}$

The region of $(R_{K3\pi}, \delta_D^{K3\pi})$ parameter space encompassed by the 2σ and 3σ confidence intervals is significantly more constrained.

Impact of the results on the $\boldsymbol{\gamma}$



600 suppressed B decays $\sim (71^{+7}_{.9})^{\circ}$ 60,000 suppressed B decays $\sim (71^{+5}_{.7})^{\circ}$



Amplitude Analyses of D_(s) Decays



 $D_{s}^{+} \rightarrow \pi^{+}\pi^{0}\eta$ $D_{s}^{+} \rightarrow K_{s}^{0}\pi^{+}\pi^{0}$ $D^{+} \rightarrow K_{s}^{0}K^{+}\pi^{0}$ $D_{s}^{+} \rightarrow K^{+}K^{-}\pi^{+}$ $D_{s}^{+} \rightarrow K^{-}K^{+}\pi^{+}\pi^{0}$

Amplitude Analysis

Dalitz Plot Analysis of $D_{s}^{+} \rightarrow \pi^{+}\pi^{0}\eta$

PRL123, 112001 (2019)



W-annihilation dominant \triangleright



- 1239 DT candidate events with 97.7% purity
- **Unbinned ML fit** ($\chi^2/n_{dof} = 82.8/77$) \geq

Amplitude	ϕ_n (rad)	FF_n	
$D_s^+ \rightarrow \rho^+ \eta$	0.0 (fixed)	$0.783 \pm 0.050 \pm 0.021$	(20.0 σ)
$D_s^+ \rightarrow (\pi^+ \pi^0)_V \eta$	$0.612 \pm 0.172 \pm 0.342$	$0.054 \pm 0.021 \pm 0.025$	(5.7 σ)
$D_s^+ \rightarrow a_0(980)\pi$	$2.794 \!\pm\! 0.087 \!\pm\! 0.044$	$0.232 {\pm} 0.023 {\pm} 0.033$	(16.2 σ)

- Improved precision $Br(D_s^+ \rightarrow \pi^+ \pi^0 \eta) = (9.50 \pm 0.28 \pm 0.41)\%$
- $Br(D_s^+ \to \pi^+ \pi^0 \eta) = (9.50 \pm 0.28 \pm 0.41)\%$ $Br(D_s^+ \to a_0 (980)^{+(0)} \pi^0, \ a_0 (980)^{+(0)} \to \pi^{+(0)} \eta) = (1.46 \pm 0.15 \pm 0.23)\%$ **First observation** >
- Large *W*-annihilation rate in *SP* mode (estimated to be 0.84 ± 0.23)

$\mathcal{L} = 3.19 \text{ fb}^{-1}$ (a) $\sqrt{s} = 4.178 \text{ GeV}$

Dalitz plot and fit projections



Amplitude Analysis **Dalitz Plot Analysis of** $D^+ \rightarrow K^0_S K^+ \pi^0$



 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

arXiv:2104.09131

> Topological diagrams contributing to the SCS decay of $D^+ \rightarrow K^*(892)^+ K_S^0$



Nominal fitting results of Dalitz plot analysis

Dominant Amplitude	Magnitude	Phase ϕ (°)	FF (%)	Significance
$D^+ \to K^*(892)^+ K_S^0$	1.0 (fixed)	0.0 (fixed)	57.1 ± 2.6	29.6σ
$D^+ \to \bar{K}^* (892)^0 K^+$	0.41 ± 0.04	162 ± 10	10.2 ± 1.5	11.6σ
$D^+ \to (K^+ \pi^0)_{\mathcal{S}-\text{wave}} K_S^0$	2.02 ± 0.37	140 ± 14	3.9 ± 1.5	5.2σ
$D^+ \to (K^0_S \pi^0)_{\mathcal{S}-\text{wave}} K^+$	3.14 ± 0.46	-173.7 ± 9.7	9.7 ± 2.6	7.4σ

BF	This work	PDG
$\frac{\mathcal{B}(D^+ \to K^*(892)^+ (K^+ \pi^0) K_S^0)}{\mathcal{B}(D^+ \to K^+ K_S^0 \pi^0)}$	$(57.1 \pm 2.6_{\rm stat.} \pm 4.2_{\rm syst.})\%$	
$\frac{\mathcal{B}(D^+ \to \bar{K}^* (892)^0 (\ddot{K}^0_S \pi^0) K^+)}{\mathcal{B}(D^+ \to K^+ K^0_S \pi^0)}$	$(10.2 \pm 1.5_{\rm stat.} \pm 2.2_{\rm syst.})\%$	
$\mathcal{B}(D^+ \to K^*(892)^+ K_S^0)$	$(8.69 \pm 0.40_{\rm stat.} \pm 0.64_{\rm syst.} \pm 0.51_{\rm Br.}) \times 10^{-3}$	$(17 \pm 8) \times 10^{-3}$
$\mathcal{B}(D^+ \to \bar{K}^*(892)^0 K^+)$	$(3.10 \pm 0.46_{\rm stat.} \pm 0.68_{\rm syst.} \pm 0.18_{\rm Br.}) \times 10^{-3}$	$(3.74^{+0.12}_{-0.20}) \times 10^{-3}$

- > 692 DT candidate events with 97.4% purity
- First amplitide analysis



- a factor of 4.6 improved!
- differs from the theoretical predictions by about 4σ
- agree with PDG value and theoretical calculation

Amplitude Analysis

Amplitude Analysis of $D_s^+ \rightarrow K_S^0 \pi^+ \pi^0$



- Amplitude analysis of D_s⁺ decay to a 3-body pseudoscalar meson is a powerful tool for studying the VP channels of the SCS D_s⁺ decays $\mathcal{L} = 6.32 \text{ fb}^{-1} \text{ (a) } \sqrt{s} = 4.178 4.226 \text{ GeV}$
- > 609 DT events with purity of 83.1%; Unbinned ML fit

Dalitz plot and projections



Amplitude analysis results

Amplitude	Magnitude (ρ_n)	Phase (ϕ_n)	FF (%)	Significance (σ)
$D_s^+ \to K_S^0 \rho^+$	1.0(fixed)	0.0(fixed)	$50.2 \pm 7.2 \pm 3.9$	>10
$D_s^+ \to K_S^0 \rho(1450)^+$	2.7 ± 0.5	$2.2\pm0.2\pm0.1$	$20.4\pm4.3\pm4.4$	>10
$D_s^+ \to K^*(892)^0 \pi^+$	0.4 ± 0.1	$3.2\pm0.2\pm0.1$	$8.4\pm2.2\pm0.9$	5.0
$D_s^+ \to K^*(892)^+ \pi^0$	0.3 ± 0.1	$0.2\pm0.2\pm0.2$	$4.6\pm1.4\pm0.4$	4.0
$D_s^+ \to K^* (1410)^0 \pi^+$	0.8 ± 0.2	$0.2\pm0.3\pm0.1$	$3.3\pm1.6\pm0.5$	3.7

$$F \text{ measurement of } D_s^+ \to K_s^0 \pi^+ \pi^0$$

$$Br(D_s^+ \to K_s^0 \pi^+ \pi^0) = (5.43 \pm 0.30_{stat.} \pm 0.15_{syst.}) \times 10^{-3}$$

$$A_{CP} = \frac{Br(D_s^+) - Br(D_s^-)}{Br(D_s^+) + Br(D_s^-)} = (2.7 \pm 5.5_{stat.} \pm 0.9_{syst.}) \times 10^{-3}$$

Precision improved by a factor of 3 compared to CLEO-c result *[PRD 80, 112004(2009)].* No evidence for CPV

BFs of intermediate processes

Intermediate process	BF (10^{-3})
$D_s^+ \to K_S^0 \rho^+$	$2.73 \pm 0.42 \pm 0.22$
$D_s^+ \to K_S^0 \rho(1450)^+$	$1.11 \pm 0.24 \pm 0.24$
$D_s^+ \to K^* (892)^0 \pi^+$	$0.45 \pm 0.12 \pm 0.05$
$D_s^+ \to K^*(892)^+ \pi^0$	$0.25 \pm 0.08 \pm 0.02$
$D_s^+ \to K^* (1410)^0 \pi^+$	$0.18 \pm 0.09 \pm 0.03$

5σ off the theoretical prediction *PRD 100, 093002(2019)*

Consistent with the theoretical prediction [PRD 84, 074019 (2011)]

Amplitude Analysis of $D_s^+ \rightarrow K^+ K^- \pi^+$



arXiv:2011.08041

 $\mathcal{L} = 3.19 \text{ fb}^{-1} @ \sqrt{s} = 4.178 \text{ GeV}$

- Obvious Differences between FFs of BaBar and CLEO-c
- 4399 DT candidate events with 99.6% purity(BABAR(96307±369, 95%); CLEO-c(14400, 85%))
- > Unbinned ML fit , $\chi^2/N_{dof} = 290/280$
- Model- independent PWA in low K+K- mass region (<u>Back-up</u>)

Amplitude	Magnitude (ρ)	Phase (ϕ)	FFs (%)	Significance (σ)
$D_s^+ \to \bar{K}^* (892)^0 K^+$	1.0 (fixed)	0.0 (fixed)	$48.3 \pm 0.9 \pm 0.6$	> 20
$D_s^+ \to \phi(1020)\pi^+$	$1.09 \pm 0.02 \pm 0.01$	$6.22 {\pm} 0.07 {\pm} 0.04$	$40.5 \pm 0.7 \pm 0.9$	> 20
$D_s^+ \to S(980)\pi^+$	$2.88 \pm 0.14 \pm 0.17$	$4.77 {\pm} 0.07 {\pm} 0.07$	$19.3 \pm 1.7 \pm 2.0$	> 20
$D_s^+ \to \bar{K}_0^* (1430)^0 K^+$	$1.26 \pm 0.14 \pm 0.16$	$2.91 {\pm} 0.20 {\pm} 0.23$	$3.0 {\pm} 0.6 {\pm} 0.5$	8.6
$D_s^+ \to f_0(1710)\pi^+$	$0.79 \pm 0.08 \pm 0.14$	$1.02 \pm 0.12 \pm 0.06$	$1.9{\pm}0.4{\pm}0.6$	9.2
$D_s^+ \to f_0(1370)\pi^+$	$0.58 \pm 0.08 \pm 0.08$	$0.59 {\pm} 0.17 {\pm} 0.46$	$1.2 \pm 0.4 \pm 0.2$	6.4

> Branching fraction measurement $D_s^+ \rightarrow K^+ K^- \pi^+$

$\mathcal{B} \ (D_s^+ \to K^+ K^- \pi^+)(\%)$	Collaboration
$5.55 \pm 0.14_{\rm stat} \pm 0.13_{\rm sys}$	CLEO [23]
$5.06 \pm 0.15_{\rm stat} \pm 0.21_{\rm sys}$	Belle [24]
$5.78 \pm 0.20_{\rm stat} \pm 0.30_{\rm sys}$	BaBar [25]
$5.47 \pm 0.08_{\rm stat} \pm 0.13_{\rm sys}$	BESIII(this analysis)

$$B(D_{s}^{+} \to \overline{K}^{*}(892)^{0}K^{+}) = (3.94 \pm 0.12)\%$$

$$B(D_{s}^{+} \to \phi(1020)\pi^{+}) = (4.60 \pm 0.17)\%$$

Consistent with theoretical prediction. PRD 93, 114010 (2016)



Amplitude Analysis of $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$



arXiv:2103.02482

 $\mathcal{L} = 6.32 \text{ fb}^{-1} @ \sqrt{s} = 4.178 - 4.226 \text{ GeV}$

- Large BF of this CF decay with a large systematic uncertainty
- > Study K_1 and f_1 mesons by $D_s^+ \rightarrow AP$; Study the 2-body $D_s^+ \rightarrow VV(\phi \rho^+, \overline{K}^{*0}K^{*+})$



Inconsistent between different experiments

 $R_{K_{1}(1270)} = \frac{Br(K_{1}(1270) \to K^{*}\pi)}{Br(K_{1}(1270) \to K\rho)}$ $R_{K_1(1270)}$ Process Experiment $1.18 \pm 0.43 \ D^0 \rightarrow K^- K_1^+(1270) \ \text{CLEO} \ [19]$ $0.11 \pm 0.06 \ D^0 \rightarrow K^+ K_1^-(1270) \ \text{CLEO} \ [19]$ [19] PRD85, 122002 (2012) $0.19 \pm 0.10 \ D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ BESIII [20] [20] PRD95, 072010 (2017) $0.24 \pm 0.04 \ D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ LHCb [21] [21] EPJC78. 443 (2018) $0.45 \pm 0.05 \ B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$ Belle [22] (Fit 1) [22] PRD83. 032005 (2011) $0.30 \pm 0.04 \ B^+ \rightarrow J/\psi K^+ \pi^+ \pi^-$ Belle [22] (Fit 2) [23] NPB187, 1(1981) $0.38 \pm 0.13 \ K^- p \to K^- \pi^- \pi^+ p$ ACCMOR [23] [24] JHEP05. 143(2017) $0.45 \pm 0.14 \ D^0 \rightarrow K^- K_1^+(1270) \ \text{CLEO} \ [24]$

- 3088 DT candidate events with a purity of 97.5%
- First amplitude analysis ; Unbinned ML fit ($\chi^2/N_{dof} = 288.6/273$)





Amplitude Analysis Amplitude Analysis of $D_s^+ \rightarrow K^- K^+ \pi^+ \pi^0$

arXiv:2103.02482



Fit results

Label	Amplitude	Phase (ϕ_n)	FF (%)	SS (σ)
I	$D_s^+[S] \to \phi \rho^+$	0.0 (fixed)	$42.64 \pm 1.30 \pm 0.77$	>20
II	$D_s^+[P] \to \phi \rho^+$	$1.64 \pm 0.05 \pm 0.02$	$8.58 \pm 0.69 \pm 0.37$	15.2
III	$D_s^+[D] \to \phi \rho^+$	$1.58 \pm 0.06 \pm 0.02$	$4.89 \pm 0.79 \pm 0.47$	8.4
	$D_s^+ o \phi ho^+$		$56.17 \pm 1.05 \pm 1.24$	
IV	$D_s^+[S] \to \bar{K}^{*0} K^{*+}$	$1.13 \pm 0.06 \pm 0.03$	$15.49 \pm 0.81 \pm 0.36$	>20
V	$D_s^+[P] ightarrow ar{K}^{*0} K^{*+}$	$2.82 \pm 0.07 \pm 0.03$	$6.13 \pm 0.50 \pm 0.19$	16.2
VI	$D_s^+[D] \rightarrow \bar{K}^{*0}K^{*+}$	$1.76 \pm 0.07 \pm 0.03$	$4.00 \pm 0.47 \pm 0.34$	12.5
	$D_s^+ o \bar{K}^{*0} K^{*+}$		$22.44 \pm 0.81 \pm 0.32$	
VII	$D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \to K^- \rho^+$	$5.36 \pm 0.06 \pm 0.10$	$9.81 \pm 0.80 \pm 0.46$	>20
	$D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[S] \to \bar{K}^{*0}\pi^0$		$0.69 \pm 0.13 \pm 0.12$	
	$D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[S] \to K^{*-}\pi^+$		$1.27 \pm 0.27 \pm 0.25$	
VIII	$D_s^+ \to \bar{K}_1^0(1270)K^+, \ \bar{K}_1^0(1270)[S] \to K^*\pi$	$0.09 \pm 0.14 \pm 0.12$	$1.87 \pm 0.39 \pm 0.36$	7.2
	$D_s^+ \to \bar{K}_1^0(1270)K^+, \ \bar{K}_1^0(1270)[D] \to \bar{K}^{*0}\pi^0$		$0.22 \pm 0.05 \pm 0.03$	
	$D^+_s \to \bar{K}^0_1(1270) K^+, \bar{K}^0_1(1270) [D] \to K^{*-} \pi^+$	•••	$0.41 \pm 0.10 \pm 0.05$	
IX	$D_s^+ \to \bar{K}_1^0(1270)K^+, \ \bar{K}_1^0(1270)[D] \to K^*\pi$	$1.62 \pm 0.15 \pm 0.12$	$0.64 \pm 0.16 \pm 0.08$	5.5
	$D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \to K^*\pi$		$2.57 \pm 0.42 \pm 0.42$	
	$D_s^+ \to \bar{K}_1^0(1400)K^+, \bar{K}_1^0(1400)[S] \to \bar{K}^{*0}\pi^0$		$2.67 \pm 0.36 \pm 0.17$	
	$D_s^+ \to \bar{K}_1^0(1400) K^+, \ \bar{K}_1^0(1400)[S] \to K^{*-} \pi^+$		$4.90 \pm 0.65 \pm 0.29$	
X	$D_s^+ \to \bar{K}_1^0(1400)K^+, \ \bar{K}_1^0(1400)[S] \to K^*\pi$	$5.66 \pm 0.08 \pm 0.05$	$7.23 \pm 0.95 \pm 0.41$	12.0
XI	$D_s^+ ightarrow a_0^0(980) ho^+$	$2.33 \pm 0.10 \pm 0.09$	$1.61 \pm 0.29 \pm 0.21$	6.0
	$D_s^+ \to f_1(1420)\pi^+, f_1(1420) \to K^{*-}K^+$		$0.87 \pm 0.17 \pm 0.07$	• • •
	$D_s^+ \to f_1(1420)\pi^+, f_1(1420) \to K^{*+}K^-$	272	$0.87 \pm 0.17 \pm 0.07$	· · ·
XII	$D_s^+ \to f_1(1420)\pi^+, f_1(1420) \to K^{*\mp}K^{\pm}$	$5.14 \pm 0.10 \pm 0.05$	$1.35 \pm 0.28 \pm 0.11$	6.5
XIII	$D_s^+ \to f_1(1420)\pi^+, f_1(1420) \to a_0^0(980)\pi^0$	$5.77 \pm 0.14 \pm 0.07$	$0.65 \pm 0.24 \pm 0.12$	3.6
XIV	$D_s^+ \to \eta(1475)\pi^+, \eta(1475) \to a_0^0(980)\pi^0$	$0.98 \pm 0.08 \pm 0.06$	$3.28 \pm 0.38 \pm 0.25$	9.7

 $\frac{Br(D_s^+ \to \overline{K}_1^0(1270)K^+, \overline{K}_1^0(1270) \to K^{*-}\pi^+)}{Br(D_s^+ \to \overline{K}_1^0(1270)K^+, \overline{K}_1^0(1270) \to K^-\rho^+)} = 0.17 \pm 0.04$

BF measurements

 $Br(D_s^+ \to K^- K^+ \pi^+ \pi^0) = (5.42 \pm 0.10_{stat.} \pm 0.17_{syst.})\%$

Precisions are significantly improved!

BF for the intermediate processes

Dominant

Obtaining a much better precision than PDG value

Consistent with the theory prediction *PRD 49, 269(1994)*

Process	BF (%)	
$D_s^+[S] \to \phi \rho^+$	$2.31 \pm 0.08 \pm 0.08$	
$D_s^+[P] \to \phi \rho^+$	$0.45 \pm 0.04 \pm 0.02$	
$D^+_*[D] \to \phi \rho^+$	$0.26 \pm 0.04 \pm 0.03$	
$D_s^+ \to \phi \rho^+$	$3.06 \pm 0.08 \pm 0.12$)
$D_s^+[S] \to K^{*0}K^{*+}$	$0.84 \pm 0.05 \pm 0.03$	
$D_s^+[P] \to \bar{K}^{*0}K^{*+}$	$0.33 \pm 0.03 \pm 0.01$	
$D_s^+[D] \to \bar{K}^{*0} K^{*+}$	$0.21 \pm 0.03 \pm 0.02$	
$D_s^+ o ar{K}^{*0} K^{*+}$	$1.21 \pm 0.05 \pm 0.04$	J
$D_s^+ \to \bar{K}_1^0(1270)K^+, \ \bar{K}_1^0(1270) \to K^- \rho^+$	$0.50 \pm 0.04 \pm 0.03$	
${}^+_s \to \bar{K}^0_1(1270)K^+, \bar{K}^0_1(1270)[S] \to K^*\pi$	$0.10 \pm 0.02 \pm 0.02$	
$K_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270)[D] \to K^*\pi$	$0.04 \pm 0.01 \pm 0.01$	
$D_s^+ \to \bar{K}_1^0(1270)K^+, \bar{K}_1^0(1270) \to K^*\pi$	$0.14 \pm 0.02 \pm 0.02$	
$D_s^+ \to \bar{K}_1^0(1400)K^+, \bar{K}_1^0(1400) \to K^*\pi$	$0.39 \pm 0.05 \pm 0.03$	
$D_s^+ \to a_0^0(980) \rho^+$	$0.07 \pm 0.02 \pm 0.01$	
$D_s^+ \to f_1(1420)\pi^+, f_1(1420) \to K^{*\mp}K^{\pm}$	$0.07 \pm 0.02 \pm 0.01$	
$f_s^+ \to f_1(1420)\pi^+, f_1(1420) \to a_0^0(980)\pi^0$	$0.04 \pm 0.01 \pm 0.01$	
$D_s^+ \to \eta(1475)\pi^+, \eta(1475) \to a_0^0(980)\pi^0$	$0.17 \pm 0.02 \pm 0.01$	

$$R_{K_{1}(1270)} \equiv \frac{Br(K_{1}(1270) \to K^{*}\pi)}{Br(K_{1}(1270) \to K\rho)} = 0.51 \pm 0.12_{stat.} \pm 0.09_{syst}$$

This *R* value is consistent with the results using CLEO data [JHEP 05, 143(2017)] and Belle data (Fit 1) [PRD 83, 032005(2011)] within uncertainties.

Amplitude Amplitude Analysis of other $D_{(s)}$ Decays

Analysis



Amplitude analysis of $D^0 \rightarrow K^0_{s} K^+ K^$ arXiv:2006.02800 Amplitude analysis of $D^+ \to K_s \pi^+ \pi^+ \pi^-$ PRD 100, 072008 (2019) Amplitude analysis of $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$ PRD 99, 092008 (2019) Amplitude analysis of $D_s^+ \rightarrow K_s^0 K^- \pi^+ \pi^+$ PRD 103, 092006 (2021) Amplitude analysis of $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$ **BESIIIPreliminaryary**

Absolute Branching Fractions of $D_{(s)}$ Decays **Absolute BFs**

Fourteen exclusive hadronic $D^{0(+)}$ decays to η PRL 124, 241803 (2020)

Two body
$$D^{0(+)} \to \phi P \ (P = K^{+}, \pi^{+(0)}, \eta)$$
 PLB 798, 135017 (2019)

 $D^+ \rightarrow K_{S/I} K^+(\pi^0)$ PRD 99, 032002 (2019)

Inclusive decays $D^{\theta(+)} \rightarrow \phi X$ PRD 100, 072006 (2019)

Singly Cabiboo-suppressed decays $D \rightarrow \omega \pi \pi$ PRD 102, 052003 (2020)

- $D^{\theta(+)} \rightarrow \eta \pi \pi$ PRD 101, 052009 (2020) $D^{\theta(+)} \rightarrow KK\pi\pi$ PRD 102, 052006 (2020)
- $D_{s} \rightarrow K_{S/I} K^{+}$ PRD 99, 112005 (2019)
- $D_s \rightarrow PP$ **JHEP 08, 146 (2020)** > Some hadronic $D_{(s)}$ covered in "Radiative and Rare Charm Decays at BESIII"

Absolute BFs

Exclusive hadronic *D* **Decays to** η

PRL 124,241803(2020)

 \rightarrow

- Key potential backgrounds in some LFU tests
- > Known D^{0}/D^{+} exclusive decays to η only account for 44%/16%
- Crucial to address the tensions found in LFU tests with semileptonic B decays
- First measurements, DT method, 2D unbinned ML fit

Decay	$\Delta E_{\rm sig}~({\rm MeV})$	N _{DT}	$\epsilon_{ m sig}$ (%)	$\mathcal{B}_{sig}~(\times 10^{-4})$
$D^0 \to K^- \pi^+ \eta$	(-37, 36)	6116.2 ± 81.8	14.22	185.3(25)(31)
$D^0 \to K^0_S \pi^0 \eta$	(-57, 45)	1092.7 ± 35.2	4.66	100.6(34)(30)
$D^0 \rightarrow K^+ K^- \eta$ 5.5 σ	(-27, 27)	13.1 ± 4.0	9.53	0.59(18)(05)
$D^0 \rightarrow K^0_S K^0_S \eta$ 2.8 σ	(-29, 28)	7.3 ± 3.2	2.36	1.33(59)(18)
$D^0 \rightarrow K^- \pi^+ \pi^0 \eta$	(-44, 36)	576.5 ± 28.8	5.53	44.9(22)(15)
$D^0 \rightarrow K^0_S \pi^+ \pi^- \eta$	(-33, 32)	248.2 ± 18.0	3.80	28.0(19)(10)
$D^0 \rightarrow K^{0}_{S} \pi^0 \pi^0 \eta$	(-56, 41)	64.7 ± 9.2	1.58	17.6(23)(13)
$D^0 \rightarrow \pi^+ \pi^- \pi^0 \eta$	(-57, 45)	508.6 ± 26.0	6.76	32.3(17)(14)
$D^+ \rightarrow K^0_{\rm S} \pi^+ \eta$	(-36, 36)	1328.2 ± 37.8	6.51	130.9(37)(31)
$D^+ \rightarrow K_S^0 K^+ \eta$ 5.70	(-27, 27)	13.6 ± 3.9	4.72	1.85(52)(08)
$D^+ \to K^- \pi^+ \pi^+ \eta$	(-33, 33)	188.0 ± 15.3	8.94	13.5(11)(04)
$D^+ \rightarrow K_S^0 \pi^+ \pi^0 \eta$ 8 4 σ	(-49, 41)	48.7 ± 9.7	2.57	12.2(24)(06)
$D^+ \rightarrow \pi^+ \pi^+ \pi^- \eta$	(-40, 38)	514.6 ± 25.7	9.67	34.1(17)(10)
$\underbrace{D^+ \to \pi^+ \pi^0 \pi^0 \eta}_{}$	(-70, 49)	192.5 ± 17.1	3.86	32.0(28)(17)

Absolute Branching Fractions

	Agree with Belle results
Ì	[PRD98,030001(2018)][PRD102, 12002(2020)] with 1.3σ,
	with precision improved 2-fold!
	Greater than CLEO's results
	[PRD98,030001(2018)] [PRL93,111801(2004)] by 3.7σ
	·**

Charge-conjugated BFs and asymmetries

Decay	\mathcal{B}_{sig}^+ (×10 ⁻⁴)	$\mathcal{B}^{-}_{\overline{\mathrm{sig}}}$ (×10 ⁻⁴)	$\mathcal{A}_{CP}^{\mathrm{sig}}$ (%)
$D^0 \to K^- \pi^+ \eta$	182.1 ± 3.5	189.1 ± 3.6	$-1.9 \pm 1.3 \pm 1.0$
$D^0 \to K^0_S \pi^0 \eta$	98.4 ± 4.8	106.3 ± 5.1	$-3.9\pm3.2\pm0.8$
$D^0 \rightarrow K^- \pi^+ \pi^0 \eta$	41.7 ± 2.7	48.8 ± 3.2	$-7.9\pm4.8\pm2.5$
$D^0 o \pi^+ \pi^- \pi^0 \eta$	29.8 ± 2.2	33.3 ± 2.5	$-5.5 \pm 5.2 \pm 2.4$
$D^+ \to K^0_S \pi^+ \eta$	129.9 ± 5.3	132.3 ± 5.4	$-0.9\pm2.9\pm1.0$
$D^+ \to \pi^+ \pi^+ \pi^- \eta$	35.4 ± 2.4	33.7 ± 2.4	$+2.5 \pm 5.0 \pm 1.6$

No evidence for *CP* violation is found





 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

Summary

- \succ Large $D_{(s)}$ samples near charm threshold
- > Our results of hadronic parameters provide key inputs for the modelindependent determination of the CKM angle γ of *B* decays.
- > Amplitude analyses on 3-and 4-body hadronic decays, especially those with neutral mesons K_s^{θ} , η , π^{θ} in the final states.
- Precise measurements of absolute branching fractions of hadronic $D_{(s)}$ decays can calibrate QCD calculations.
- Planning to take more data of D_(s): 20 fb⁻¹ at 3.773 GeV; 6 fb⁻¹ at 4.178 GeV [Chinese Physics C Vol. 44, No. 4 (2020)]

BACK UP

Strong-Phase Difference

Strong-Phase Difference





equal- $\Delta \delta_D$:

according to regions of similar strong-phase difference $\varDelta \delta_{\rm D}$ optimal :

maximum sensitivity to γ in the presence of negligible background (modified) optimal :

maximum sensitivity to $\boldsymbol{\gamma}$ in the presence of significant background

[PRD 82, 112006(2010)]

Hadronic Parameters c_i, s_i

 c_i , s_i is the amplitude-weighted average of $\cos \Delta \delta_D$, $\sin \Delta \delta_D$ in the *i*th region of the Dalitz plot (D_i)

 $c_{i} = \frac{1}{\sqrt{F_{i}F_{-i}}} \int_{i} |f_{D}(m_{+}^{2}, m_{-}^{2})| |f_{D}(m_{-}^{2}, m_{+}^{2})|$ $\times \cos[\Delta \delta_{D}(m_{+}^{2}, m_{-}^{2})] dm_{+}^{2} dm_{-}^{2}$

$$\begin{split} s_i &= \frac{1}{\sqrt{F_i F_{-i}}} \int_i |f_D(m_+^2, m_-^2)| |f_D(m_-^2, m_+^2)| \\ &\times \sin[\Delta \delta_D(m_+^2, m_-^2)] dm_+^2 dm_-^2, \end{split}$$

 F_i is the fraction of events found in the *i*th bin of the flavor-specific decay $D^0 \rightarrow K^0_{S/L} \pi^+ \pi^-$, $D^0 \rightarrow K^0_{S/L} K^+ K^-$

Strong-Phase Difference



23

Hadronic Parameters

$$R_S e^{-i\delta_D^S} = \frac{\int \mathcal{A}_S^{\star}(\mathbf{x})\mathcal{A}_{\bar{S}}(\mathbf{x})\mathrm{d}\mathbf{x}}{A_S A_{\bar{S}}} \quad \text{and} \quad r_D^S = A_{\bar{S}}/A_S$$

 $R_{\rm S}$ ~ coherence factor

 r^{S} ~ amplitude ratio

 $\delta^{s} \sim CP$ -conserving strong-phase difference $A_S(x) \sim \text{decay amplitude of } D^0 \rightarrow S \text{ at a point in}$ multi-body phase space

Tag Mode

Flovour	Like sign	$K^-\pi^+\pi^+\pi^-,K^-\pi^+\pi^0,K^-\pi^+$
Flavour	Opposite sign	$K^{+}\pi^{-}\pi^{-}\pi^{+},K^{+}\pi^{-}\pi^{0},K^{+}\pi^{-}$
CD	Even	$K^+K^-,\pi^+\pi^-,K^0_S\pi^0\pi^0,K^0_L\pi^0,K^0_L\omega,\pi^+\pi^-\pi^0$
UF	Odd	$K^0_S \pi^0, K^0_S \eta, K^0_S \omega, K^0_S \eta', K^0_S \phi, K^0_L \pi^0 \pi^0$
Self-conju	ıgate	$K^0_S \pi^+ \pi^-$

Formalism and Measurement Strategy

$\left(\right)$	decay rate
$\Gamma(S)$	$S T) = \int \int \mathcal{A}_S(\mathbf{x}) \mathcal{A}_{ar{T}}(\mathbf{y}) - \mathcal{A}_{ar{S}}(\mathbf{x}) \mathcal{A}_T(\mathbf{y}) ^2 \mathrm{d}\mathbf{x} \mathrm{d}\mathbf{y}$
	$= \Big[A_S^2 A_{\bar{T}}^2 + A_{\bar{S}}^2 A_T^2 -$
	$2R_S R_T A_S A_{\bar{S}} A_T A_{\bar{T}} \cos\left(\delta_D^T - \delta_D^S\right)$
	$= A_{S}^{2} A_{T}^{2} \left[(r_{D}^{S})^{2} + (r_{D}^{T})^{2} - 2R_{S} R_{T} r_{D}^{S} r_{D}^{T} \cos \left(\delta_{D}^{T} - \delta_{D}^{S} \right) \right]$
	T ~ tag mode S ~signal mode
CP tag	$\Gamma(S CP) = A_S^2 A_{CP}^2 \left(1 + (r_D^S)^2 - 2\lambda R_S r_D^S \cos \delta_D^S \right)$
$\rho_{CP\pm}^S = \frac{1}{N(1-1)}$	$\frac{N(S CP) + N(\bar{S} CP)}{K^{-}\pi^{+} CP) + N(K^{+}\pi^{-} CP)} \cdot \frac{\mathcal{B}(D^{0} \to K^{-}\pi^{+}) + \mathcal{B}(D^{0} \to K^{+}\pi^{-})}{\mathcal{B}(D^{0} \to S) + \mathcal{B}(D^{0} \to \bar{S})} \cdot \rho_{CP\pm}^{K\pi}$
Flavour tag	$\Gamma(S S) = A_S^2 A_{\bar{S}}^2 \left[1 - R_S^2 \right]$
$ ho_{T,L}^S$	$_{S} = \left(1 + (r_{D}^{S}/r_{D}^{T})^{2} - 2(r_{D}^{S}/r_{D}^{T})R_{S}R_{T}\cos(\delta_{D}^{T} - \delta_{D}^{S})\right) / $
	$(1 + (r_D^S/r_D^T)^2 - R_T([y/r_D^T]\cos\delta_D^T - x/r_D^T]\sin\delta_D^T) -$
	$R_S([yr_D^S/(r_D^T)^2]\cos\delta_D^S - [xr_D^S/(r_D^T)^2]\sin\delta_D^S) + (x^2 + y^2)/(r_D^T)^2)$
Self-conjuga	ate
Y_i^S	$H = H \left(K_i + \left(r_D^S \right)^2 K_{-i} - 2r_D^S R_S \sqrt{K_i K_{-i}} \left[c_i \cos \delta_D^S - s_i \sin \delta_D^S \right] \right)$

Hadronic Parameters

Strong-Phase Difference in

 $D \rightarrow K^{-}\pi^{+}\pi^{+}\pi^{-}$ and $D \rightarrow K^{-}\pi^{+}\pi^{0}$



arXiv:2103.05988

► Binned $D \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{-}$ analysis (4-bins, *PLB802, 135188(2020)*)

Parameter	Global fit		Binne	ed fit	
		Bin 1	Bin 2	Bin 3	Bin 4
$R_{K3\pi}$	$0.52\substack{+0.12 \\ -0.10}$	$0.58\substack{+0.25 \\ -0.33}$	$0.78\substack{+0.50 \\ -0.21}$	$0.85\substack{+0.15 \\ -0.12}$	$0.45\substack{+0.33 \\ -0.37}$
$\delta_D^{K3\pi}$	$\left(167^{+31}_{-19}\right)^{\circ}$	$\left(131^{+124}_{-16}\right)^{\circ}$	$\left(150^{+37}_{-39}\right)^{\circ}$	$\left(176^{+57}_{-21}\right)^{\circ}$	$\left(274^{+19}_{-30}\right)^{\circ}$
$r_D^{K3\pi}$ (×10 ⁻²)	$5.46{\pm}0.09$	$5.44\substack{+0.45\\-0.14}$	$5.80\substack{+0.14 \\ -0.13}$	$5.75\substack{+0.41 \\ -0.14}$	$5.09\substack{+0.14 \\ -0.14}$
$R_{K\pi\pi^0}$	$0.78{\pm}0.04$		$0.80\pm$:0.04	
$\delta_D^{K\pi\pi^0}$	$\left(196^{+14}_{-15}\right)^{\circ}$		(200 ±	= 11)°	
$r_D^{K\pi\pi^0}~(imes 10^{-2})$	$4.40{\pm}0.11$		4.41±	:0.11	



Black dot with error bar denotes the prediction of model.



Expected γ/ϕ_3 precision of the LHCb[1] and Belle II[2] experiments and their timescales

Runs	Collected / Expected integrated luminosity	Year attained	γ/ϕ_3 sensitivity
LHCb Run-1 [7, 8 TeV]	3 fb^{-1}	2012	8°
LHCb Run-2 [13 TeV]	6 fb^{-1}	2018	4°
Belle II Run	50 ab^{-1}	2025	1.5°
LHCb upgrade I [14 TeV]	50 fb^{-1}	2030	< 1°
LHCb upgrade II [14 TeV]	300 fb^{-1}	(>)2035	< 0.4°

2.93fb⁻¹ @ $\sqrt{s} = 3.773 \text{ GeV } D^{\theta} \overline{D}^{\theta}$ 20 fb⁻¹ @ $\sqrt{s} = 3.773 \text{ GeV } D^{\theta} D^{\theta}$ in the future $D \rightarrow K_{S/L} \pi^{+} \pi^{-} \sim 0.7^{\circ}, 1.2^{\circ}, 0.8^{\circ} \sim 0.4^{\circ}$

Amplitude Analysis

Amplitude Analysis Dalitz Plot Analysis of $D^0 \rightarrow K^0_c K^+ K^-$



arXiv: 2006.02800

- _____
- $\mathcal{L} = 2.93 \text{ fb}^{-1} @ \sqrt{s} = 3.773 \text{ GeV}$

- > 1856±45 DT candidate events with 96.4% purity
- Model selection ~ LASSO method [J. R. Stat. Soc. Ser. B 58, 267 (1996)]
- > The Dalitz plot is well described by a set of six resonances
- Results of Dalitz plot analysis

Final state	Magnitude	Phase [rad]	Fit fraction [%]	Sign.[σ]
$a_0(980)^0 K_S^0$	1	0	$90\pm10\pm17$	>10
$a_0(980)^+K^-$	$0.64^{+0.14}_{-0.08}\pm0.09$	$2.94^{+0.19}_{-0.14}\pm0.06$	$34 \pm 7 \pm 6$	> 10
$\phi(1020)K^0_S$	$0.74^{+0.08}_{-0.04}\pm0.08$	$1.67 \pm 0.08 \pm 0.19$	$48 \pm 2 \pm 3$	>10
$a_2(1320)^+K^-$	$0.12 \pm 0.03 \pm 0.01$	$-2.92^{+0.21}_{-0.26}\pm0.31$	< 2.3 (@90% C.L.), CV = 1.4	3.9
$a_2(1320)^-K^+$	$0.09 \pm 0.03 \pm 0.02$	$-0.06 \pm 0.23 \pm 0.28$	< 1.6 (@90% C.L.), CV = 0.8	3.5 > 5.9
$a_0(1450)^-K^+$	$0.16^{+0.12}_{-0.05}\pm0.04$	$0.12 \pm 0.58 \pm 0.50$	< 13.2 (@90 % C.L.), CV = 2.2	3.5
Total			176 ± 20	

> The coupling constant of $a_0(980)$ to $K\overline{K}$:

 $g_{K\overline{K}} = (3.77 \pm 0.24 \pm 0.35)GeV$

The first measurement of absolute branching fraction:

$$Br(D^0 \to K_s^0 K^+ K^-) = (4.51 \pm 0.05_{stat.} \pm 0.16_{sys.}) \times 10^{-3}$$

Dalitz plot and projections



Amplitude Analysis of $D_s^+ \rightarrow K^+ K^- \pi^+$



arXiv:2011.08041

$\mathcal{L} = 3.19 \text{ fb}^{-1}$ (a) $\sqrt{s} = 4.178 \text{ GeV}$

- > A MVA method is used to suppress bkg.
- Fit to the signal D_s invariant mass after BDTG requirement



The area between the pink lines is the signal area of the sample for MIPWA

- Assuming that only S- and P-wave amplitudes are necessary at the low end of K⁺K⁻ mass spectrum
- Angular distribution can be written as partial wave analysis

$$\begin{aligned} \frac{dN}{d\cos\Theta} &= 2\pi \left| SY_0^0(\cos\Theta) + PY_1^0(\cos\Theta) \right|^2 \\ \left| S \right|^2 &= \sqrt{4\pi} \left\langle Y_0^0 \right\rangle - \sqrt{5\pi} \left\langle Y_2^0 \right\rangle, & S(P) \sim \text{amplitude of } S(P) \text{-wave} \\ \cos\phi_{SP} &= \frac{\left\langle Y_1^0 \right\rangle}{\sqrt{(2\langle Y_0^0 \rangle - \sqrt{5} \langle Y_2^0 \rangle)} \sqrt{5} \langle Y_2^0 \rangle} & \phi_{SP} \sim \text{phase difference} \\ \text{between } S \text{-wave and } P \text{-wave} \\ \left| P \right|^2 &= \sqrt{5\pi} \left\langle Y_2^0 \right\rangle, \end{aligned}$$

➤ Calculating $|S|^2$, ϕ_{SP} and $|P|^2$ in every mass interval of $m(K^+K^-)$ in the threshold region, the distribution of $|S|^2$, $|P|^2$, ϕ_{SP} and ϕ_S can be obtained...



- > The lineshape of *S(980)* is empirically parameterized by $A_{\underline{S}(980)} = \frac{1}{m_0^2 - m^2 - im_0\Gamma_0\rho_{KK}}$
- > Fitting the distribution of $|S|^2$ in (a) with $|A_s(980)|^2$

 $m_0 = (0.919 \pm 0.006_{\text{stat}}) \text{ GeV}/c^2$ $\Gamma_0 = (0.272 \pm 0.040_{\text{stat}}) \text{ GeV}.$

Choose the black one in (c)

Amplitude Analysis **Amplitude Analysis of** $D_s^+ \rightarrow \pi^+ \pi^- \pi^+$



 $\mathcal{L} = 3.19 \text{ fb}^{-1}$ (a) $\sqrt{s} = 4.178 \text{ GeV}$

- > $f_0(980)$ resonance still needs to be better understood; Important input for the global study of $D_s \rightarrow VP$
- > 13.8 K data events with 80% purity in the signal region for DP analysis
- > Dalitz Plot model: **unbinned ML fit** ($\chi^2/N_{dof} = 344/342$)
- > Amplitude analysis results (model- independent PWA in low $\pi^+\pi^-$ S-wave extraction)



Fit results by using **BABAR model** [PRD 79, 032003 (2009)]:

Decay mode	Decay fraction (%)	Amplitude	Phase (radians)
$f_2(1270)\pi^+$	$10.52 \pm 0.83 \pm 1.15$	1. (Fixed)	0. (Fixed)
$\rho(770)\pi^{+}$	$0.87 \pm 0.38 \pm 0.52$	$0.13 \pm 0.03 \pm 0.04$	$5.44 \pm 0.25 \pm 0.62$
$\rho(1450)\pi^+$	$1.26 \pm 0.40 \pm 0.53$	$0.91 \pm 0.16 \pm 0.22$	$1.03 \pm 0.32 \pm 0.51$
S-wave	$84.15 \pm 0.83 \pm 1.30$	see nex	t page
Total	$96.80 \pm 2.45 \pm 3.50$		

where **S-wave** is parameterized by an interpolation between the **N=29** control points also used by **BABAR**



With improved precision our results are compatible with BABAR measurements [PRD 79, 032003 (2009)]



Amplitude Analysis

Amplitude Analysis of $D_{s}^{+} \rightarrow \pi^{+}\pi^{-}\pi^{+}$



Choice of signal Dalitz-plot model

Different fit models are tested, and **Fit 4** is chosen as the nominal fit model:

Decay Mode			Decay fraction (%)		
	Fit 1	A Fit 2	Fit 3	Fit 4	Fit 5
$f_2(1270)\pi^+$	13.2 ± 0.6	25 ± 0.7	10.8 ± 0.8	10.5 ± 0.8	10.5 ± 0.7
$ ho(770)\pi^+$	—	1.1 = 0.5		0.9 ± 0.4	0.4 ± 0.2
$\rho(1450)\pi^+$		× ×	2.5 ± 0.5	1.3 ± 0.4	1.4 ± 0.3
$\omega(782)\pi^+$	—			,	0.3 ± 0.1
$(S$ -wave) π^+	87.7 ± 0.4	84.7 ± 0.0	85.7 ± 0.7	84.2 ± 0.8	84.1 ± 0.7
Total	100.9 ± 1.1	98.9 ± 2.0	99.0 ± 2.0	96.8 ± 2.4	96.8 ± 2.0
$-2\ln \mathcal{L}$	40401.2	40348.9	40321.4	40303.2	40276.7
Significance		6.9σ	8.7σ	3.9σ	4.8σ
Significance	—	$[Fit1 + \rho(770)]$	[Fit] + $\rho(1450)$]	$[Fit3 + \rho(770)]$	$[Fit4 + \omega(782)]$
χ^2/ν	$\frac{433.0}{404-58} = 1.25$	$\frac{393.6}{404-60} = 1.14$	$\frac{550.0}{404-60} = 1.02$	$\frac{344.4}{404-62} = 1.01$	$\frac{335.2}{404-64} = 0.99$

The results of Fit 5 are considered as the systematic uncertainties on w ("Alt. Fit")

Amplitude Analysis Amplitude Analysis of $D^0 \rightarrow K^- \pi^+ \pi^0 \pi^0$

RRD 99 092008(2019)



$\mathcal{L} = 2.93 \text{ fb}^{-1} @ \sqrt{s} = 3.773 \text{ GeV}$

- First amplitude analysis on this CF decay with large BF
- 5950 DT candidate events with a purity of 98.9%
- Unbinned ML fit results



Measurement the BF

 $Br(D^0 \to K^- \pi^+ \pi^0 \pi^0) = (8.86 \pm 0.13_{stat.} \pm 0.19_{syst.})\%$ Improved precision

Amplitude mode	FF [%]	Phase $[\phi]$	Significance [o
$D \rightarrow SS$			
$D \rightarrow (K^- \pi^+)_{S-\text{wave}} (\pi^0 \pi^0)_S$	$6.92 \pm 1.44 \pm 2.86$	$-0.75 \pm 0.15 \pm 0.47$	>10
$D \rightarrow (K^- \pi^0)_{S-\text{wave}} (\pi^+ \pi^0)_S$	$4.18 \pm 1.02 \pm 1.77$	$-2.90 \pm 0.19 \pm 0.47$	6.0
$D \to AP, A \to VP$			
$D \to K^- a_1(1260)^+, \rho^+ \pi^0[S]$	$28.36 \pm 2.50 \pm 3.53$	0 (fixed)	>10
$D \to K^- a_1(1260)^+, \rho^+ \pi^0[D]$	$0.68 \pm 0.29 \pm 0.30$	$-2.05 \pm 0.17 \pm 0.25$	6.1
$D \to K_1(1270)^- \pi^+, K^{*-} \pi^0[S]$	$0.15 \pm 0.09 \pm 0.15$	$1.84 \pm 0.34 \pm 0.43$	4.9
$D \to K_1(1270)^0 \pi^0, K^{*0} \pi^0[S]$	$0.39 \pm 0.18 \pm 0.30$	$-1.55 \pm 0.20 \pm 0.26$	4.8
$D \rightarrow K_1(1270)^0 \pi^0, K^{*0} \pi^0[D]$	$0.11 \pm 0.11 \pm 0.11$	$-1.35 \pm 0.43 \pm 0.48$	4.0
$D \to K_1(1270)^0 \pi^0, K^- \rho^+[S]$	$2.71 \pm 0.38 \pm 0.29$	$-2.07 \pm 0.09 \pm 0.20$	>10
$D \to (K^{*-}\pi^0)_A \pi^+, K^{*-}\pi^0[S]$	$1.85 \pm 0.62 \pm 1.11$	$1.93 \pm 0.10 \pm 0.15$	7.8
$D \to (K^{*0}\pi^0)_A \pi^0, K^{*0}\pi^0[S]$	$3.13 \pm 0.45 \pm 0.58$	$0.44 \pm 0.12 \pm 0.21$	>10
$D \to (K^{*0}\pi^0)_A \pi^0, K^{*0}\pi^0[D]$	$0.46 \pm 0.17 \pm 0.29$	$-1.84 \pm 0.26 \pm 0.42$	5.9
$D \rightarrow (\rho^+ K^-)_A \pi^0, K^- \rho^+ [D]$	$0.75 \pm 0.40 \pm 0.60$	$0.64 \pm 0.36 \pm 0.53$	5.1
$D \rightarrow AP, A \rightarrow SP$			
$D \rightarrow ((K^-\pi^+)_{S-\text{wave}}\pi^0)_A \pi^0$	$1.99 \pm 1.08 \pm 1.55$	$-0.02 \pm 0.25 \pm 0.53$	7.0
$D \rightarrow VS$			
$D \rightarrow (K^- \pi^0)_{S-wave} \rho^+$	$14.63 \pm 1.70 \pm 2.41$	$-2.39 \pm 0.11 \pm 0.35$	>10
$D \rightarrow K^{*-}(\pi^+\pi^0)_S$	$0.80 \pm 0.38 \pm 0.26$	$1.59 \pm 0.19 \pm 0.24$	4.1
$D \to K^{*0} (\pi^0 \pi^0)_S$	$0.12 \pm 0.12 \pm 0.12$	$1.45 \pm 0.48 \pm 0.51$	4.1
$D \to VP, V \to VP$			
$D \rightarrow (K^{*-}\pi^+)_V \pi^0$	$2.25 \pm 0.43 \pm 0.45$	$0.52 \pm 0.12 \pm 0.17$	>10
$D \rightarrow VV$			
$D \to K^{*-}\rho^+[S]$	$5.15 \pm 0.75 \pm 1.28$	$1.24 \pm 0.11 \pm 0.23$	>10
$D \to K^{*-} \rho^+[P]$	$3.25 \pm 0.55 \pm 0.41$	$-2.89 \pm 0.10 \pm 0.18$	>10
$D \to K^{*-} \rho^+[D]$	$10.90 \pm 1.53 \pm 2.36$	$2.41 \pm 0.08 \pm 0.16$	>10
$D \to (K^- \pi^0)_V \rho^+ [P]$	$0.36 \pm 0.19 \pm 0.27$	$-0.94 \pm 0.19 \pm 0.28$	5.7
$D \rightarrow (K^- \pi^0)_V \rho^+ [D]$	$2.13 \pm 0.56 \pm 0.92$	$-1.93 \pm 0.22 \pm 0.25$	>10
$D \rightarrow K^{*-}(\pi^+\pi^0)_V[D]$	$1.66 \pm 0.52 \pm 0.61$	$-1.17 \pm 0.20 \pm 0.39$	7.6
$D \to (K^- \pi^0)_V (\pi^+ \pi^0)_V [S]$	$5.17 \pm 1.91 \pm 1.82$	$-1.74 \pm 0.20 \pm 0.31$	7.6
$D \rightarrow TS$			
$D \rightarrow (K^- \pi^+)_{S-\text{wave}} (\pi^0 \pi^0)_T$	$0.30 \pm 0.21 \pm 0.30$	$-2.93 \pm 0.31 \pm 0.82$	5.8
$D \rightarrow (K^- \pi^0)_{S-\text{wave}} (\pi^+ \pi^0)_T$	$0.14 \pm 0.12 \pm 0.10$	$2.23 \pm 0.38 \pm 0.65$	4.0
TOTAL	98.54		

Amplitude Analysis of $D^+ \rightarrow K_s \pi^+ \pi^+ \pi^-$ **Amplitude Analysis**



 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

 $m(\pi^{+}\pi^{+}\pi^{-})$ (GeV/c²)

> Fit projections

PRD 100. 072008(2019)

- Measuring $D \rightarrow AP$ decay via this amplitude analysis
- 4559 DT candidate events with a purity of 97.5%
- **Unbinned ML fit** \triangleright

 $D^+ \to K_S^0 a_1(1260)^+ (\rho^0 \pi^+)$

 $D^+ \rightarrow \bar{K}_1(1400)^0 (K^{*-}\pi^+)\pi^+$

 $D^+ \to \bar{K}_1(1270)^0 (K^0_{\rm S} \rho^0) \pi^+$

 $D^+ \rightarrow \bar{K}(1460)^0 (K^{*-}\pi^+)\pi^+$

 $D^+ \to \bar{K}_1(1650)^0 (K^{*-}\pi^+)\pi^+$

 $D^+ \to \bar{K}(1460)^0 (K^0_{\rm S} \rho^0) \pi^+$

 $D^+ \rightarrow K_{\rm s}^0 \pi^+ \rho^0$

 $D^+ \rightarrow K^0_{\rm S} \pi^+ \pi^+ \pi^-$

 $D^+ \rightarrow K_S^0 a_1(1260)^+ (f_0(500)\pi^+)$

dominant

Component

Likelihood scan to determine the parameters $a_1(1260)^+$, $K(1460)^0$ and ω in the ρ - ω mixing (next page)

Branching fraction (%)

 $1.197 \pm 0.062 \pm 0.120 \pm 0.044$

 $0.163 \pm 0.021 \pm 0.053 \pm 0.006$

 $0.642 \pm 0.036 \pm 0.033 \pm 0.024$

 $0.202 \pm 0.018 \pm 0.031 \pm 0.007$

 $0.024 \pm 0.006 \pm 0.015 \pm 0.009$

 $0.048 \pm 0.012 \pm 0.042 \pm 0.002$

 $0.190 \pm 0.021 \pm 0.103 \pm 0.007$

 $0.241 \pm 0.018 \pm 0.026 \pm 0.009$

Extracting the BFs by the PDG value $Br(D^+ \to K_s \pi^+ \pi^- \pi^-) = (2.97 \pm 0.1)\%$



- Consistent with the previous measurements
- **Precisions improved!**
- $D \rightarrow Ka_1(1260)$ dominated in $D^{0(+)}$ decays(Comparing the $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$, PRD95 072010(2017), EPJC78, 443(2018)) $D \rightarrow K_1(1400)\pi$, the related BF in D^+ is found to be larger than that in D^0 by 1 order of magnitude.

Amplitude Analysis **Amplitude Analysis of** $D^+ \rightarrow K_S \pi^+ \pi^+ \pi^-$



PRD 100, 072008(2019)

\succ Likelihood scan to determine the parameters of $a_1(1260)^+$, $K(1460)^0$ and ω in the ρ - ω mixing



Amplitude Analysis Amplitude Analysis of $D_s^+ \rightarrow K_S^0 K^- \pi^+ \pi^+$

 $\mathcal{L} = 6.32 \text{ fb}^{-1}$ (a) $\sqrt{s} = 4.178 - 4.226 \text{ GeV}$

 $Br(D_{s}^{+} \rightarrow K_{s}^{0}K^{-}\pi^{+}\pi^{+}) = (1.46 \pm 0.05_{stat} \pm 0.05_{syst})\%$

Using $Br(K_{s}^{0} \rightarrow \pi^{+}\pi^{+}) = (69.20 \pm 0.05)\%$

BF measurements

PRD 103,092006(2010)

- Measuring $D \rightarrow V V$ decay via this first amplitude analysis
- 1308 DT candidate events with a purity around 94.9%
- Unbinned ML fit

Fit projections

Absolute Branching Fractions

Absolute BFs

Absolute BFs of other *D* **Decays I**

DT method

$\mathcal{L} = 2.93 \text{ fb}^{-1}$ @ $\sqrt{s} = 3.773 \text{ GeV}$ DT method

$\mathcal{L} = 2.93 \text{ fb}^{-1} @ \sqrt{s} = 3.773 \text{ GeV}$

SCS Decay of $D \rightarrow \omega \pi \pi$ [*PRD102,052003(2020*)]

Decay mode	$N_{ m SG}^{\omega/\eta}$	<i>f</i> (%)	$N_{ m SB}^{\omega/\eta}$	N ^{BKGV} _{peak}	$N_{ m DT}^{ m sig}$	Sig.	\mathcal{B}^{int}	$\mathcal{B}^{sig}(imes 10^{-3})$	$\mathcal{B}_{\rm PDG}(\times 10^{-3})$
$D^0 \rightarrow \omega \pi^+ \pi^-$	908.0 ± 39.4	74.6 ± 1.5	610.5 ± 35.1	41.4 ± 2.5	411.2 ± 48.3	12.9 <i>o</i>	0.882	$1.33 \pm 0.16 \pm 0.12$	1.6 ± 0.5
$D^+ \rightarrow \omega \pi^+ \pi^0$	474.0 ± 42.8	73.3 ± 1.2	329.0 ± 34.3		232.9 ± 49.8	7.7σ	0.872	$3.87 \pm 0.83 \pm 0.25$	
$D^0 \to \omega \pi^0 \pi^0$	20.2 ± 10.5	75.2 ± 5.6	22.1 ± 10.0	19.0 ± 1.2	-15.4 ± 13.0	0.6σ	0.862	< 1.10	
$D^0 \rightarrow \eta \pi^+ \pi^-$	151.3 ± 14.6	42.6 ± 0.9	115.0 ± 15.3	6.1 ± 0.2	96.2 ± 16.0	8.3 <i>σ</i>	0.227	$1.06 \pm 0.18 \pm 0.07$	1.09 ± 0.16
$D^+ \rightarrow \eta \pi^+ \pi^0$	61.5 ± 14.3	41.4 ± 0.7	47.3 ± 16.4		41.9 ± 15.8	3.5σ	0.224	$2.47 \pm 0.93 \pm 0.16$	1.38 ± 0.35
$D^0 \to \eta \pi^0 \pi^0$	5.7 ± 3.8	40.6 ± 3.3	13.1 ± 4.8	2.0 ± 0.1	-1.6 ± 4.3	0.1σ	0.221	< 2.38	0.38 ± 0.13

The product of the BFs of the intermediate states ω/η and π^0 in the subsequent decays of the D meson

Precision Improved ~ 2.1

Measured for the first time

Consistent with the previous measurements

DT method

 $\mathcal{L} = 2.93 \text{ fb}^{-1}$ (a) $\sqrt{s} = 3.773 \text{ GeV}$

Inclusive decays $D^{0(+)} \rightarrow \phi X$ [PRD100, 072006(2019)]

	This work	CLEO [2]	BES [1]	
$ \begin{array}{c} D^+ \to \phi X \\ D^0 \to \phi X \end{array} $	$\begin{array}{c} 1.135 \pm 0.034 \pm 0.031 \\ 1.091 \pm 0.027 \pm 0.035 \end{array}$	$\begin{array}{c} 1.03 \pm 0.10 \pm 0.07 \\ 1.05 \pm 0.08 \pm 0.07 \end{array}$	< 1.8 (90% C.L.) $1.71^{+0.76}_{-0.71} \pm 0.17$	

Consistent with previous measurements, but with much better precision

SCS Decay of $D \rightarrow \eta \eta(\pi) \pi$ [PRD101,052009(2020)]

Decay mode	N _{DT}	$\epsilon_{ m sig}(\%)$	\mathcal{B}_{sig} (×10 ⁻³)	$\mathcal{B}_{\text{CLEO}}$ (×10 ⁻³)
$D^+ o \eta \eta \pi^+$	179 ± 15	24.96 ± 0.12	$2.96 \pm 0.24 \pm 0.10$	N/A
$D^+ o \eta \pi^+ \pi^0$ $D^0 o \eta \pi^+ \pi^-$	$381 \pm 26 \\ 450 \pm 25$	$\begin{array}{c} 28.11 \pm 0.13 \\ 39.98 \pm 0.17 \end{array}$	$2.23 \pm 0.15 \pm 0.10 \\ 1.20 \pm 0.07 \pm 0.04$	$\begin{array}{c} 1.38 \pm 0.31 \pm 0.16 \\ 1.09 \pm 0.13 \pm 0.09 \end{array}$

Measured for the first time

Consistent with CLEO's measurements with $2.2\sigma, 0.6\sigma$

DT method

 $\mathcal{L} = 2.93 \text{ fb}^{-1} @ \sqrt{s} = 3.773 \text{ GeV}$

$D^{0(+)} \rightarrow K\overline{K}\pi\pi$ [PRD102, 052006(2020)]

Signal mode	$\Delta E_{\rm sig}$	$N_{ m DT}^{ m fit}$	$N_{K_S^0}^{ m sid}$	$N_{\mathrm{DT}}^{\mathrm{net}}$	$\epsilon_{ m sig}$ (%)	\mathcal{B}_{sig} (×10 ⁻³)	$\mathcal{B}_{\rm PDG}~(\times 10^{-3})$
$D^0 \rightarrow K^+ K^- \pi^0 \pi^0$	(-59, 40)	132.1 ± 13.9		132.1 ± 13.9	8.20 ± 0.07	$0.69 \pm 0.07 \pm 0.04$	
$D^0 \rightarrow K^0_S K^0_S \pi^+ \pi^-$	(-22, 22)	82.1 ± 9.7	37.8 ± 7.5	63.2 ± 10.4	5.14 ± 0.04	$0.53 \pm 0.09 \pm 0.03$	1.22 ± 0.23
$D^0 \rightarrow K^0_S K^- \pi^+ \pi^0$	(-43, 32)	278.8 ± 18.8	166.1 ± 15.1	195.8 ± 20.3	6.38 ± 0.06	$1.32 \pm 0.14 \pm 0.07$	
$D^0 \rightarrow K_S^0 K^+ \pi^- \pi^0$	(-44, 33)	124.0 ± 12.8	$9.5^{+3.7}_{-3.1}$	119.3 ± 12.9	7.94 ± 0.06	$0.65 \pm 0.07 \pm 0.02$	
$D^+ \rightarrow K^+ K^- \pi^+ \pi^0$	(-39, 30)	1311.7 ± 40.4		1311.7 ± 40.4	12.72 ± 0.08	$6.62 \pm 0.20 \pm 0.25$	26^{+9}_{-8}
$D^+ \rightarrow K^0_S K^+ \pi^0 \pi^0$	(-61, 44)	35.9 ± 7.1	$3.8^{+2.8}_{-2.0}$	34.0 ± 7.2	3.77 ± 0.02	$0.58 \pm 0.12 \pm 0.04$	
$D^+ \rightarrow K^0_S K^- \pi^+ \pi^+$	(-22, 21)	505.0 ± 24.5	74.2 ± 10.3	467.9 ± 25.0	13.24 ± 0.08	$2.27 \pm 0.12 \pm 0.06$	2.38 ± 0.17
$D^+ \rightarrow K_S^0 K^+ \pi^+ \pi^-$	(-21, 20)	284.6 ± 18.0	$15.3^{+4.9}_{-4.2}$	277.0 ± 18.2	9.39 ± 0.06	$1.89 \pm 0.12 \pm 0.05$	1.74 ± 0.18
$D^+ \rightarrow K^0_S K^0_S \pi^+ \pi^0$	(-46, 37)	101.1 ± 11.3	42.0 ± 8.1	80.1 ± 12.0	3.84 ± 0.03	$1.34 \pm 0.20 \pm 0.06$	

First direct measurement,> 8σFirst observation!Precision Improved

Absolute BFs

Absolute BFs of other $D_{(s)}$ Decays II

List of data samples collected by BESIII/BEPCII up to 2019, and the proposed samples for the remainder of the physics program

Energy	Physics motivations	Current data	Expected final data	$T_{\rm C}$ / $T_{\rm U}$
1.8 - 2.0 GeV	R values Nucleon cross-sections	N/A	0.1 fb^{-1} (fine scan)	60/50 days
2.0 - 3.1 GeV	R values Cross-sections	Fine scan (20 energy points)	Complete scan (additional points)	250/180 days
J/ψ peak	Light hadron & Glueball J/ψ decays	3.2 fb^{-1} (10 billion)	3.2 fb^{-1} (10 billion)	N/A
$\psi(3686)$ peak	Light hadron & Glueball Charmonium decays	0.67 fb^{-1} (0.45 billion)	4.5 fb^{-1} (3.0 billion)	150/90 days
$\psi(3770)$ peak	D^0/D^{\pm} decays	2.9 fb^{-1}	20.0 fb^{-1}	610/360 days
3.8 - 4.6 GeV	<i>R</i> values <i>XYZ</i> /Open charm	Fine scan (105 energy points)	No requirement	N/A
4.180 GeV	D_s decay XYZ /Open charm	3.2 fb^{-1}	6 fb ⁻¹	140/50 days
4.0 - 4.6 GeV	XYZ/Open charm Higher charmonia cross-sections	16.0 fb ⁻¹ at different \sqrt{s}	30 fb ⁻¹ at different \sqrt{s}	770/310 days
4.6 - 4.9 GeV	Charmed baryon/XYZ cross-sections	0.56 fb^{-1} at 4.6 GeV	15 fb ⁻¹ at different \sqrt{s}	1490/600 days
4.74 GeV	$\Sigma_c^+ \bar{\Lambda}_c^-$ cross-section	N/A	1.0 fb^{-1}	100/40 days
4.91 GeV	$\Sigma_c \overline{\Sigma}_c$ cross-section	N/A	1.0fb^{-1}	120/50 days
4.95 GeV	Ξ_c decays	N/A	1.0 fb^{-1}	130/50 days

[Chinese Physics C Vol. 44, No. 4 (2020)]