Masses of Doubly Heavy Tetraquarks $(QQ\overline{q}\overline{q})$ with Error Bars

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U.S. DEPARTMENT OF ENERGY

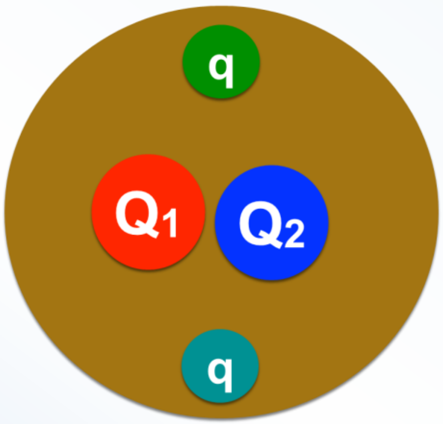
Outline

- **Introduction to doubly heavy tetraquarks ($QQ\bar{q}\bar{q}$)**
- **Singly heavy hadron masses in HQET**
- **Doubly heavy hadron masses in HQET**
- **Predictions for $QQ\bar{q}\bar{q}$ masses with error bars**
- **Conclusions/Summary**

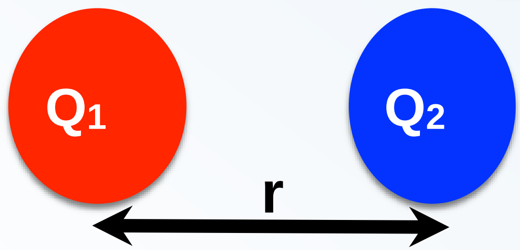
Doubly Heavy Tetraquarks ($QQ\bar{q}\bar{q}$)

- 4-quark system: 2 heavy quarks/antiquark (Q_1Q_2) and two light anti-quark/quark $\bar{q}_i\bar{q}_k$.

Doubly heavy core



spin: $1/2 \otimes 1/2 = 0 \oplus 1$



color: $3 \otimes 3 = 6 \oplus 3^*$

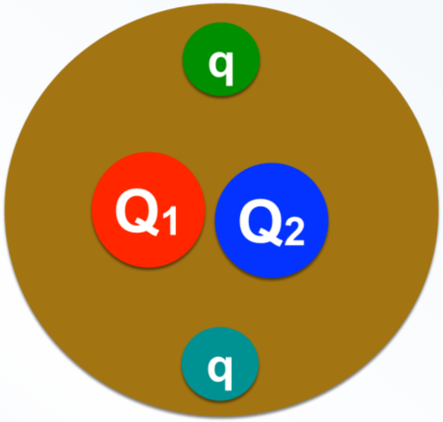
one-gluon-exchange

potential: $\alpha_s/3r$ $-2\alpha_s/3r$

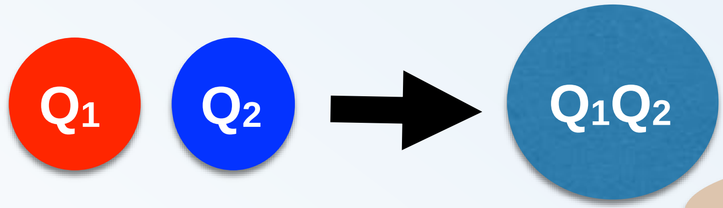
repulsive attractive

- Assuming the 2 heavy quarks forms a compact core in the attractive color Coloumb potential.

Doubly Heavy Tetraquarks ($QQ\overline{q}\overline{q}$)



Doubly heavy core



antisymmetric

symmetric

spin: $1/2 \otimes 1/2 = 0 \oplus 1$

antisymmetric

color: $3 \otimes 3 = 6 \oplus 3^*$

Overall w.f of
diquark core: Antisymmetric

flavor:

Symmetric: {Q_1Q_2}
Anti Symmetric:[Q_1Q_2]

parity:

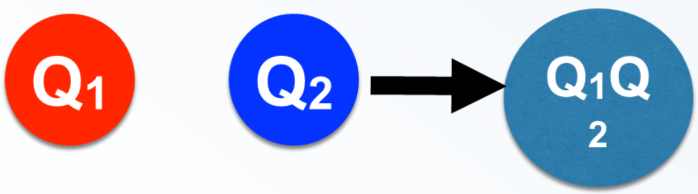
P = $(-1)^L$ = +1 (L=0)

L=0: ground state
Symmetric spatial wf

Doubly Heavy Tetraquarks ($QQ\overline{q}\overline{q}$)

doubly heavy core

spin: $1/2 \otimes 1/2 = 0 \oplus 1$



color: $3 \otimes 3 = 6 \oplus 3^*$

flavor: $\{Q_1Q_2\}$ $[Q_1Q_2]$

J^P: 1^+ 0^+

light antiquarks

q q'

$\{qq'\}, 1^+$ $[qq'], 0^+$

doubly heavy tetraquarks

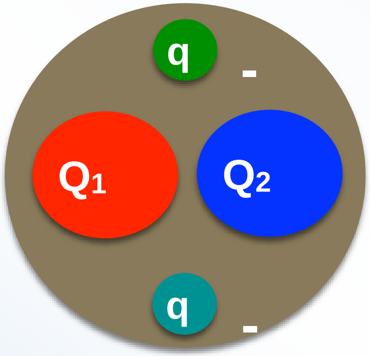
flavor	J^P
$[Q_1Q_2]$ $[qq']$	0^+
$[Q_1Q_2]$ $\{qq'\}$	1^+
$\{Q_1Q_2\}$ $[qq']$	1^+
$\{Q_1Q_2\}$ $\{qq'\}$	$(0, 1, 2)^+$

Abhishek Mohapatra-TUM

Earlier Results on $(QQ\overline{q}\overline{q})$

- Constituents can be rearranged into two color-singlet mesons. **Stable tetraquarks** below strong-decay threshold.

- Compact Core: Energy of $QQ\overline{qq}$ below the energy of two separated $Q\overline{q}$ mesons



  - **Manohar and Wise** **[Nucl.Phys. B399, 17 (1993)]**

    There must be stable $QQ\bar{q}\bar{q}$ tetraquarks in the limit $m_Q \rightarrow \infty$

  - **Karliner and Rosner** **[Phys. Rev. Lett 119, 202001(2017)]:**

    Existence of stable $bb\bar{u}\bar{d}$ tetraquark and near threshold $cc\bar{u}\bar{d}$ (1^+) and $bc\bar{u}\bar{d}$ (0^+).

  - **Eichten and Quigg** **[Phys. Rev. Lett 119, 202002 (2017)]**

    Existence of stable $bb\bar{u}\bar{d}$, $bb\bar{s}\bar{u}$, $bb\bar{s}\bar{d}$. cc and bc tetraquarks are not stable.

- **Bicudo et al.** **[Phys. Rev. D 92, 014507 (2015), Phys. Rev. D 95, 034502 (2017)]**

  Existence of stable $bb\bar{u}\bar{d}$ tetraquark.

- **Francis et al (Lattice QCD)** **[Phys. Rev. Lett 118, 142001 (2017), Phys. Rev. D. 99, 054505 (2019)]**

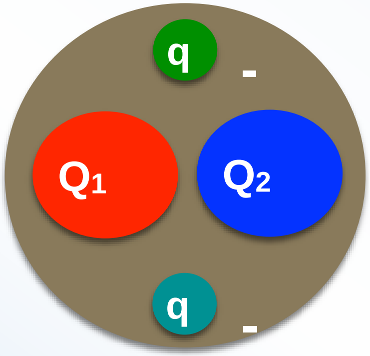
  Strong Existence of deeply bound $bb\bar{u}\bar{d}$, $bb\bar{s}\bar{u}$, $bb\bar{s}\bar{d}$. cc tetraquarks are not stable

  Also evidence of stable $bc\bar{u}\bar{d}$ (1^+) about 40 MeV below strong decay threshold.

Earlier Results on $(QQ\overline{q}\overline{q})$

- Constituents can be rearranged into two color-singlet mesons. **Stable tetraquarks** below strong-decay threshold.

- Compact Core: Energy of $QQ\overline{q}\overline{q}$ below the energy of two separated $Q\overline{q}$ mesons



- **Hadron Spectrum Colloboration [JHEP 11, 0133 (2017)]**

  Studied cc tetraquarks and found no evidence of stable bound states.

- **Junarkar et al [Phys. Rev. D 99, 034507 (2019)]**

  Strong Existence of deeply bound $bb\bar{u}\bar{d}$, $bb\bar{s}\bar{u}$, $bb\bar{s}\bar{d}$. Near threshold stable $cc\bar{u}\bar{d}$ and $cc\bar{u}\bar{s}$.

- **Leskovec et al [Phys. Rev. D 100, 014503 (2019)]**

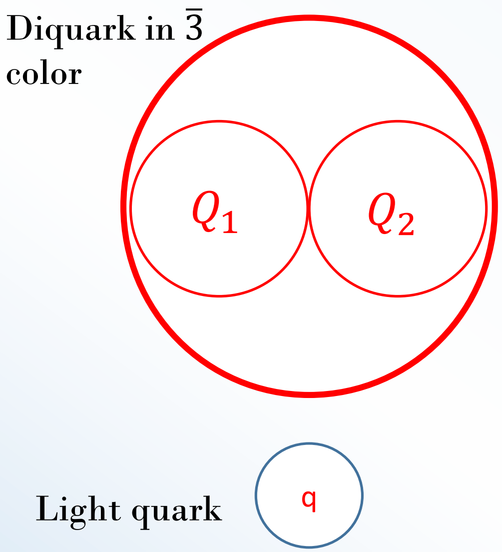
  Strong Existence of stable $bb\bar{u}\bar{d}$ (1^+) with mass around $10476 \pm 24 \pm 10$ MeV.

  All sources of major systematic errors have well quantified.

Important to have predictions of the masses with well quantified error bars

Heavy Quark-Diquark Symmetry

- Symmetry of QCD in the heavy quark limit $m_Q \to \infty$. Savage & Wise, PLB **248** (1990)
- Attractive Coulomb force: 2 heavy quarks bound together as a single diquark. ($\boldsymbol{r}_{12} \to 0; r_{1q} = r_{2q}$)



- $\boldsymbol{m_Q} \to \infty$ limit, diquark is a **point like** color source.
- For charm and bottom quark:

$$\text{diquark size} \sim \frac{1}{p_{\text{rel}}} \sim \; 1/m_Q v$$

- Energy scale of interaction of light quark with heavy quarks / diquark $\sim \Lambda_{\text{QCD}}$

- **HQDQ symmetry: $m_Q \gg m_Q v \gg \Lambda_{\text{QCD}}$**

$\Lambda_{\text{QCD}} \sim 300$ MeV $m_c v \sim 900$ MeV
$m_b v \sim 1.5$ GeV

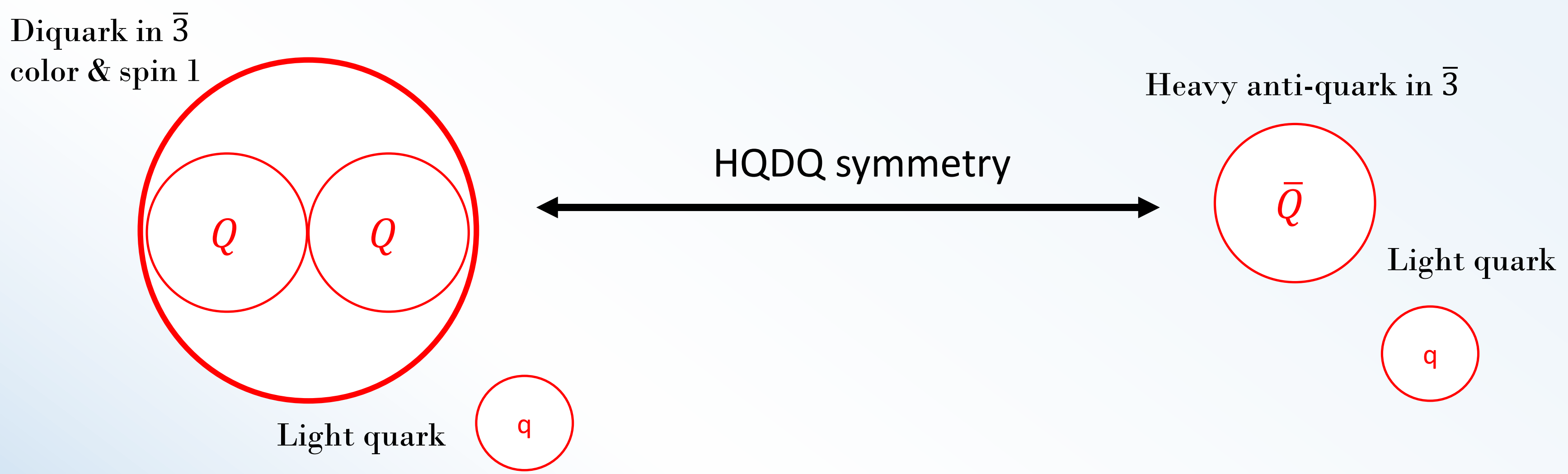
- **HQDQ symmetry:** Light quark only sees the diquark as point source $\bar{\mathbf{3}}$ color charge. Cannot resolve individual heavy quarks in the diquarks.

Heavy Quark-Diquark Symmetry

Savage & Wise, PLB **248** (1990)

- **HQDQ symmetry: $m_Q v \gg \Lambda_{\mathrm{QCD}}$**
- **HQDQ symmetry:** Light quark only sees the diquark as point source $\bar{\mathbf{3}}$ color charge. Doesn't see the individual heavy quarks in the diquarks.

Identical to situation in Heavy Mesons.



Ξ_{cc}^{++} ⟵ HQDQ ⟶ $\bar{D}^0$

Our Method

- Based on an effective Hamiltonian that depends on several coefficients $(\mathcal{E}_l, \mathcal{K}_l, \mathcal{S}_l)$ and organized as an expansion in inverse heavy quark mass.

- Estimate these coefficients in effective Hamiltonian:

  - Singly Heavy Hadrons: χ^2 fit to PDG 2020 data.

  - Doubly Heavy Baryons (DHB): χ^2 fit to lattice results.

- Determine these coefficients for $\mathbf{QQ\bar{q}\bar{q}}$ from known results for singly heavy hadrons (mesons + baryons) and DHB using HQDQ symmetry.

- Use the effective Hamiltonian to predict $\mathbf{QQ\bar{q}\bar{q}}$ masses.

- Similar to Eichten and Quigg but differs on several aspects.

Singly Heavy Hadrons

Braaten, He, AM, PRD(2021)

- Quantum numbers of the light quarks $l = \bar{q}/qq', j^p$:

Mesons	Baryons
Isospin doublet $(\bar{u}, \bar{d})$ and Isospin singlet $\bar{s}$	Antisymmetric Isospin: $[u, d]$, $([d, s], [u, s])$ Symmetric Isospin: $(dd, uu, \{u, d\})$, $(\{d, s\}, \{u, s\})$, ss
$j = 1/2, P = +, -$	$j = 0, 1, P = +, -$

- Hamiltonian up to order $1/m_Q$ from HQET:

$$H_\ell^Q = m_Q + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2m_Q} + \frac{\mathcal{S}_\ell}{2m_Q} S \cdot j_\ell$$

Spin of heavy quark: S

Hadron spin: $J = S + j_l$

- Parameters $\mathcal{E}_l, \mathcal{K}_l, \mathcal{S}_l$ depends on light QCD fields and determined from fitting to data of measured masses from PDG 2020.

Singly Heavy Hadrons

- Hamiltonian up to order $1/m_Q$ from HQET :

$$H_\ell^Q = m_Q + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2m_Q} + \frac{\mathcal{S}_\ell}{2m_Q} S \cdot j_\ell$$

$\mathcal{E}_{\ell,Q}$

Braaten, He, AM, PRD(2021)

- Isospin splittings due to $u - d$ mass difference ~5 MeV.
- Include all contributions to hadron masses > isospin splittings.
- Hyperfine splittings due to spin: ~140 MeV for charm mesons
  ~40 MeV for bottom mesons

Correction to spin coefficient $\mathcal{S}_l$ of order $1/m_Q$: contributes $\gtrsim$ 10 MeV to charm mesons

- Quantified Errors: isospin splittings + Experimental error..
- Strange quark mass $m_{s,Q}$: $\mathcal{E}_{s,Q} - \mathcal{E}_{l,Q}$ $(l = u, d)$.

Singly Heavy Hadrons

- Hamiltonian (predict masses with accuracy upto ~ 5 MeV):

$$H_\ell^Q = m_Q + \mathcal{E}_{\ell,Q} + \frac{\mathcal{S}_{\ell,Q}}{2m_Q} \boldsymbol{S} \cdot \boldsymbol{j}_\ell .$$

- Parameters determined by minimizing χ^2 fit to data (PDG 2020)

Mesons

Q	ℓ	$\mathcal{E}_{u/d,Q}$ [MeV]	$m_{s,Q}$ [MeV]	$\mathcal{S}_Q$ [GeV^2]	dof	χ^2/dof
c	$\bar{q}, \frac{1}{2}^-$	313.4 ± 2.0	102.3 ± 3.5	0.472 ± 0.012	3	0.28
b	$\bar{q}, \frac{1}{2}^-$	306.3 ± 0.2	87.6 ± 0.4	0.455 ± 0.003	2	1.23

Baryons

Q	ℓ	$\mathcal{E}_{u/d,Q}$ [MeV]	$m_{s,Q}$ [MeV]	$\mathcal{S}_Q$ [GeV^2]	dof	χ^2/dof
c	$[q\,q'], 0^+$	626.5 ± 1.1	182.9 ± 1.4		1	1.12
b	$[q\,q'], 0^+$	612.4 ± 3.0	174.8 ± 3.8		1	0.45
c	$\{q\,q'\}, 1^+$	837.7 ± 0.6	124.2 ± 0.8	0.147 ± 0.003	9	0.92
b	$\{q\,q'\}, 1^+$	819.8 ± 1.9	118.8 ± 2.2	0.137 ± 0.024	5	0.44

Braaten, He, AM, PRD(2021)

Doubly Heavy Hadrons

- Appropriate framework: **pNRQCD** effective field theory.
  [Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005)] [Mehen and Fleming, Phys. Rev. D73, 034502 (2005)]
- Effective field theory formulated in terms of heavy diquark field:
  Triplet field (T) : 3^* color state, sextet field Σ: 6 color state
- Relevant terms for Triplet field in pNRQCD Lagrangian:
  - **kinetic term:** $T^\dagger\left[\frac{\boldsymbol{D}_R^2}{2m_R}+\frac{\boldsymbol{\nabla}_r^2}{2m_r}\right]T$ $\quad m_R = m_1 + m_2 \quad m_r = \frac{m_1 m_2}{m_1 + m_2}$
  - **spin dependent term:** $T^\dagger(\boldsymbol{S}_1/m_{Q_1}+\boldsymbol{S}_2/m_{Q_2})\cdot\boldsymbol{B}\,T$
  - **orbital angular momentum term (irrelevant for L=0 diquark):**

$$(m_{Q_1}^2+m_{Q_2}^2)/[m_{Q_1}m_{Q_2}(m_{Q_1}+m_{Q_2})]\; T^\dagger\boldsymbol{L}\cdot\boldsymbol{B}\,T$$

- Hamiltonian for doubly heavy baryons up to first order in $1/m_Q$:

$$H_\ell^{Q_1Q_2} = (m_{Q_1}+m_{Q_2})+\mathcal{E}_{Q_1Q_2}+\mathcal{E}_\ell+\frac{\mathcal{K}_\ell}{2m_R}+\frac{1}{2}\left(\frac{\mathcal{S}_\ell}{2m_{Q_1}}\boldsymbol{S}_1+\frac{\mathcal{S}_\ell}{2m_{Q_2}}\boldsymbol{S}_2\right)\cdot\boldsymbol{j}_\ell.$$

Diquark spin: $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ Braaten, He, AM, PRD(2021)

- Relative diquark KE term absorbed in diquark energy $\mathcal{E}_{Q_1Q_2}$

Doubly Heavy Hadrons

- Simplify the spin-dependent term using Wigner-Eckart theorem:

$$\frac{1}{2}\left\langle\left(\frac{\mathcal{S}_\ell}{2m_{Q_1}}\boldsymbol{S}_1+\frac{\mathcal{S}_\ell}{2m_{Q_2}}\boldsymbol{S}_2\right)\cdot\boldsymbol{j}_\ell\right\rangle=\frac{\mathcal{S}_\ell}{8\mu_{Q_1Q_2}}\langle\boldsymbol{S}\cdot\boldsymbol{j}_\ell\rangle.$$

- Include $1/m_Q$ corrections to S_l for contributions > isospin splittings.

- Hamiltonian for doubly heavy hadrons (QQq, $QQ\bar{q}\bar{q}$) :

$$H_\ell^{Q_1Q_2}=(m_{Q_1}+m_{Q_2})+\mathcal{E}_{\ell,Q_1Q_2}+\frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}}\boldsymbol{S}\cdot\boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2}=\mathcal{E}_{Q_1Q_2}+\mathcal{E}_\ell+\frac{\mathcal{K}_\ell}{2\left(m_{Q_1}+m_{Q_2}\right)}$$

Braaten, He, AM, PRD(2021)

Spin splitting term different from Eichten and Quigg

[Phys. Rev. Lett 119, 202002 (2017)]

Doubly Heavy Baryons

- Hamiltonian for doubly heavy hadrons (QQq, $QQ\bar{q}\bar{q}$) :

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S} \cdot \boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\,(m_{Q_1} + m_{Q_2})}$$

- Parameters determined by minimizing χ^2 fit to lattice data for DHB.

[Briceno, Lin, Bolton, Phys. Rev. D86, 094504 (2012), Alexandrou, Drach, Jansen, Kallidonis, Kostou, , Phys. Rev. D90, 074501 (2014), Brown, Detmold, Meinel, Orignos, , Phys. Rev. D90, 094507 (2014), Mathur, Padmanath, , Phys. Rev. D99, 031501 (2019)]

Q_1Q_2	ℓ	$\mathcal{E}_{u/d,Q_1Q_2}$ [MeV]	m_{s,Q_1Q_2} [MeV]	$\mathcal{S}_{Q_1Q_2}$ [GeV^2]	dof	χ^2/dof
cc	$q,\ \frac{1}{2}^+$	319.5 ± 11.0	124.9 ± 13.4	0.363 ± 0.024	12	0.29
$[bc]$	$q,\ \frac{1}{2}^+$	275.8 ± 37.2	55.0 ± 47.0		0	
$\{bc\}$	$q,\ \frac{1}{2}^+$	309.3 ± 27.3	73.5 ± 34.3	0.181 ± 0.046	2	8×10^{-5}
bb	$q,\ \frac{1}{2}^+$	152.0 ± 25.1	130.0 ± 33.6	0.472 ± 0.075	2	2×10^{-5}

Braaten, He, AM, PRD(2021)

Doubly Heavy Tetraquarks

- Determine all the parameters in the Hamiltonian to predict $QQ\bar{q}\bar{q}$ masses

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S} \cdot \boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\,(m_{Q_1} + m_{Q_2})}$$

- Use HQDQ symmetry along with known results for $Q_1Q_2q, \bar{Q}q, and\ \bar{Q}\bar{q}\bar{q}$

cc

$$\mathcal{E}_{\ell,cc} = \mathcal{E}_{\ell',cc} + \frac{m_b - 2m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}) + \frac{m_b}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{b}} - \mathcal{E}_{\ell',\bar{b}})$$

$$m_{s,\ell,cc} = \frac{m_b - 2m_c}{2(m_b - m_c)} m_{s,\ell,\bar{c}} + \frac{m_b}{2(m_b - m_c)} m_{s,\ell,\bar{b}},$$

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}} = \mathcal{S}_{\bar{\ell},Q}.$$

Braaten, He, AM, PRD(2021)

bb

$$\mathcal{E}_{\ell,bb} = \mathcal{E}_{\ell',bb} - \frac{m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}) + \frac{2m_b - m_c}{2(m_b - m_c)}(\mathcal{E}_{\ell,\bar{b}} - \mathcal{E}_{\ell',\bar{b}})$$

$$m_{s,\ell,bb} = -\frac{m_c}{2(m_b - m_c)} m_{s,\ell,\bar{c}} + \frac{2m_b - m_c}{2(m_b - m_c)} m_{s,\ell,\bar{b}},$$

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}} = \mathcal{S}_{\bar{\ell},Q}$$

Doubly Heavy Tetraquarks

- Need to calculate all the parameters in the Hamiltonian to predict masses

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S} \cdot \boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2\,(m_{Q_1} + m_{Q_2})}$$

- Use HQDQ symmetry along with known results for $Q_1Q_2q, \bar{Q}q, and\ \bar{Q}\bar{q}\bar{q}$

bc:

$$\mathcal{E}_{\ell,bc} = \mathcal{E}_{\ell',bc} - \frac{m_c^2}{m_b^2 - m_c^2}\left(\mathcal{E}_{\ell,\bar{c}} - \mathcal{E}_{\ell',\bar{c}}\right) + \frac{m_b^2}{m_b^2 - m_c^2}\left(\mathcal{E}_{\ell,\bar{b}} - \mathcal{E}_{\ell',\bar{b}}\right).$$

$$m_{s,\ell,bc} = -\frac{m_c^2}{m_b^2 - m_c^2} m_{s,\ell,\bar{c}} + \frac{m_b^2}{m_b^2 - m_c^2} m_{s,\ell,\bar{b}}.$$

$$\mathcal{S}_{\ell,bc} = \frac{\mu_{bc}}{m_b}\mathcal{S}_{\ell,\bar{b}} + \frac{\mu_{bc}}{m_c}\mathcal{S}_{\ell,\bar{c}}.$$

Braaten, He, AM, PRD(2021)

Differs from Eichten and Quigg !! [Phys. Rev. Lett 119, 202002 (2017)]

Include corrections of order $1/m_Q$.

Doubly Heavy Tetraquarks

- Parameters for ground state doubly heavy tetraquarks:

Q_1Q_2	ℓ	$\mathcal{E}_{u/d,Q_1Q_2}$ [MeV]	m_{s,Q_1Q_2} [MeV]	$\mathcal{S}_{Q_1Q_2}$ [GeV^2]
$c\,c$	$[\bar{q}\,\bar{q}']$, 0^+	627.4 ± 11.2	176.8 ± 2.9	
$[b\,c]$	$[\bar{q}\,\bar{q}']$, 0^+	581.0 ± 37.4	173.8 ± 4.3	
$\{b\,c\}$	$[\bar{q}\,\bar{q}']$, 0^+	614.5 ± 27.5	173.8 ± 4.3	
$b\,b$	$[\bar{q}\,\bar{q}']$, 0^+	456.4 ± 25.4	172.8 ± 4.8	
$c\,c$	$\{\bar{q}\,\bar{q}'\}$, 1^+	835.7 ± 11.1	120.2 ± 1.7	0.147 ± 0.003
$[b\,c]$	$\{\bar{q}\,\bar{q}'\}$, 1^+	788.0 ± 37.3	118.1 ± 2.5	
$\{b\,c\}$	$\{\bar{q}\,\bar{q}'\}$, 1^+	821.5 ± 27.4	118.1 ± 2.5	0.145 ± 0.006
$b\,b$	$\{\bar{q}\,\bar{q}'\}$, 1^+	662.8 ± 25.2	117.5 ± 2.8	0.137 ± 0.024

Braaten, He, AM, PRD(2021)

- Errors well-quantified!!

Doubly Heavy Tetraquarks

- Results for doubly heavy tetraquarks:

Braaten, He, AM, PRD(2021)

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{S_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S}\cdot\boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2(m_{Q_1} + m_{Q_2})}$$

cc & bb tertaquarks

flavor	J^P	Eichten-Quigg	this work	threshold
$cc[\bar{u}\bar{d}]$	1^+	3978	3947 ± 11	3875
$cc[\bar{s}\bar{u}]$	1^+	4156	4124 ± 12	3975
$cc\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$4146 + (0, 21, 64)$	$4111 + (0, 22, 66) \pm 11$	$3734 + (0, 141, 0)$
$cc\{\bar{s}\bar{u}\}$	$0^+, 1^+, 2^+$		$4232 + (0, 22, 66) \pm 11$	$3833 + (0, 142, 0)$
$cc\,\bar{s}\bar{s}$	$0^+, 1^+, 2^+$		$4352 + (0, 22, 66) \pm 12$	$3937 + (0, 144, 0)$
$bb[\bar{u}\bar{d}]$	1^+	**10482**	$\mathbf{10471 \pm 25}$	10604
$bb[\bar{s}\bar{u}]$	1^+	**10643**	$\mathbf{10644 \pm 26}$	10692
$bb\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$10674 + (0, 7, 21)$	$10664 + (0, 7, 21) \pm 25$	$10559 + (0, 45, 0)$
$bb\{\bar{s}\bar{u}\}$	$0^+, 1^+, 2^+$		$10781 + (0, 7, 21) \pm 25$	$10646 + (0, 45, 0)$
$bb\,\{\bar{s}\bar{s}\}$	$0^+, 1^+, 2^+$		$10898 + (0, 7, 21) \pm 26$	$10734 + (0, 49, 0)$

Stable Tetraquarks: Masses below Strong decay threshold

Karliner and Rosner [Phys. Rev. Lett 119, 202001(2017)]

m(cc[ud]) = 3882 ± 12 MeV, m(bb[ud]) = 10398 ± 12 MeV

No stable cc tetraquarks !! Agrees with Eichten & Quigg

Doubly Heavy Tetraquarks

- Results for doubly heavy tetraquarks:

Braaten, He, AM, PRD(2021)

$$H_\ell^{Q_1Q_2} = (m_{Q_1} + m_{Q_2}) + \mathcal{E}_{\ell,Q_1Q_2} + \frac{\mathcal{S}_{\ell,Q_1Q_2}}{8\mu_{Q_1Q_2}} \boldsymbol{S}\cdot\boldsymbol{j}_\ell.$$

$$\mathcal{E}_{\ell,Q_1Q_2} = \mathcal{E}_{Q_1Q_2} + \mathcal{E}_\ell + \frac{\mathcal{K}_\ell}{2(m_{Q_1} + m_{Q_2})}$$

bc tertaquarks

flavor	J^P	Eichten-Quigg	this work	threshold
$[bc][\bar{u}\bar{d}]$	0^+	7229	7260 ± 37	7144
$[bc][\bar{s}\bar{u}]$	0^+	7406	7446 ± 37	7232
$[bc]\{\bar{u}\bar{d}\}$	1^+	7439	7473 ± 37	7190
$[bc]\{\bar{s}\bar{u}\}$	1^+		7600 ± 37	7280
$[bc]\,\bar{s}\bar{s}$	1^+		7726 ± 37	7384
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	7293 ± 28	7190
$\{bc\}[\bar{s}\bar{u}]$	1^+	7445	7480 ± 28	7280
$\{bc\}\{\bar{u}\bar{d}\}$	$0^+, 1^+, 2^+$	$7461 + (0, 11, 32)$	$7477 + (0, 14, 43) \pm 28$	$7144 + (0, 45, 0)$
$\{bc\}\{\bar{s}\bar{u}\}$	$0^+, 1^+, 2^+$		$7604 + (0, 14, 43) \pm 28$	$7232 + (0, 49, 0)$
$\{bc\}\,\bar{s}\bar{s}$	$0^+, 1^+, 2^+$		$7731 + (0, 14, 43) \pm 28$	$7355 + (0, 49, 0)$

Karliner and Rosner [Phys. Rev. Lett 119, 202001(2017)]

m([bc][ud]) = 7134 ± 13 MeV (10 MeV below threshold)

No stable bc tetraquarks !! Agrees with Eichten & Quigg

Conclusions/Summary

- Predicted masses for doubly heavy tetraquarks with well quantified error bars.

- 2 stable bb Tetraquarks:

  bb[ud] (10476 ± 25 MeV) ≈ 130 MeV below strong threshold

  bb[qs] (10655 ± 25 MeV) ≈ 50 MeV below strong threshold

  Our results agree with Eichten and Quigg within 1 error bars for most tetraquarks.

  Excellent agreement with Leskovec et al on bb[ud] = 10476 ± 24 ± 10 MeV

- No stable cc or bc tetraquarks within HQDQ symmetry.

See also **Eric Braaten's** talk on **Monday (May 31)**

Thank you !!!

Back Up Slides

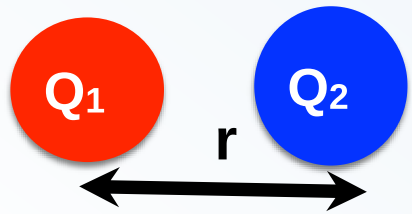
Doubly Heavy Hadrons

- Appropriate framework: **pNRQCD** effective field theory.

[Brambilla, Vairo, and Rosch, Phys. Rev. D72, 034021 (2005)]

- Effective field theory formulated in terms of heavy diquark field:
  Triplet field (T) : 3^* color state, sextet field Σ: 6 color state

- **pNRQCD Lagrangian:** Expansion in $1/m_Q$ and multipole expansion in $r = x_1 - x_2$.



color: $3 \otimes 3 = 6 \oplus 3^*$

one-gluon-exchange

potential: $\alpha_s/3r$ $-2\alpha_s/3r$

repulsive **attractive**

- Assuming attractive Coulomb potential forms a compact diquark in 3^* color state. (Focus on the triplet field T in pNRQCD)

Implications for HQDQ

- Mass hyperfine splitting relation due to HQDQ symmetry

$\Xi^*: S = 3/2$

$\Xi : S = 1/2$

$D^*: S = 1$

$D : S = 0$

$$m_{\Xi^*_{cc}} - m_{\Xi_{cc}} = \frac{3}{4}(m_{D^*} - m_D)$$

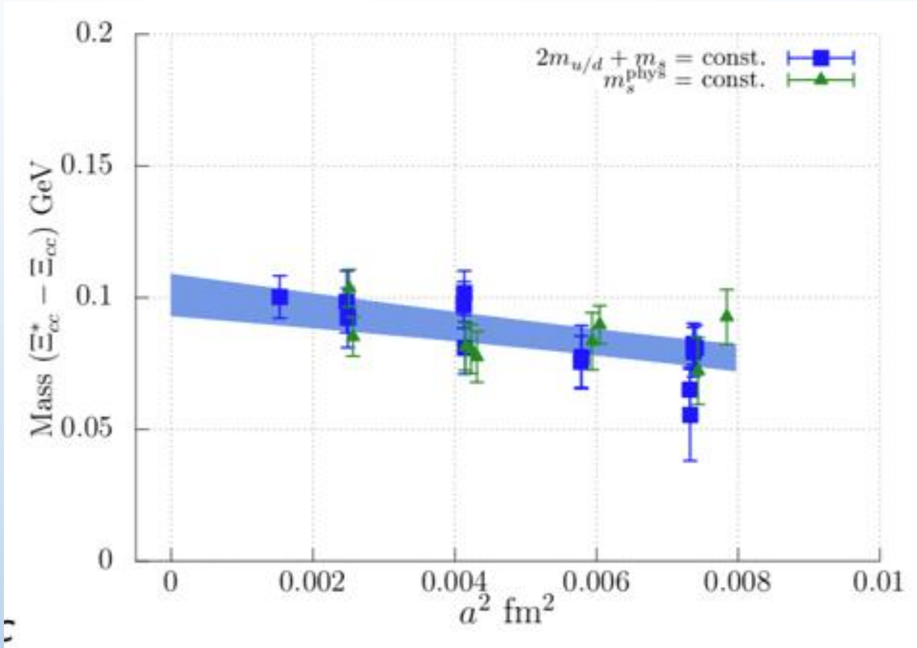
$\approx 106\ \text{MeV}$ Charm

$\approx 34\ \text{MeV}$ Bottom

(Masses of meson from PDG)

M. Savage and M. Wise, PLB **248** (1990) S. Fleming and T. Mehen, PRD **73** (2006)
N. Brambilla, A. Vairo, T. Rosch, PRD **72** (2005)

- Lattice results for Hyperfine splitting:



Brown, Detmold, Meinel, Orignos (2014)

$$\Xi^*_{bb} - \Xi_{bb} \simeq 35 \pm 5\ \text{MeV}$$

$$\Xi^*_{cc} - \Xi_{cc} \simeq 83 \pm 13\ \text{MeV}$$

RQCD QWG (2019):

$$\Xi^*_{cc} - \Xi_{cc} \simeq 100\ \text{MeV}$$

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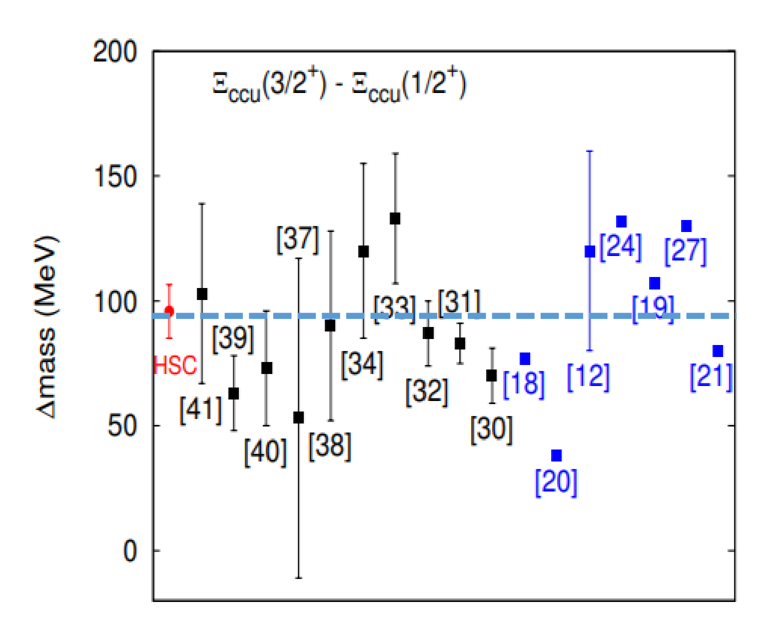

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Padmanath,, Mathur, Edwards & Peardon PRD 91.094502 (2015) & reference therein

Implications for HQDQ

- HQDQ Symmetry prediction for spin-splitting coefficient:

$$\mathcal{S}_{\ell,QQ} = \mathcal{S}_{\ell,\bar{Q}} = \mathcal{S}_{\bar{\ell},Q}.$$

$$\mathcal{S}_{\ell,bc} = \frac{\mu_{bc}}{m_b}\mathcal{S}_{\ell,\bar{b}} + \frac{\mu_{bc}}{m_c}\mathcal{S}_{\ell,\bar{c}}.$$

$$\mathcal{S}_{\ell,\bar{b}} < \mathcal{S}_{\ell,bc} < \mathcal{S}_{\ell,\bar{c}}.$$

$$\Xi^*_{bb} - \Xi_{bb} < \Xi^*_{bc} - \Xi'_{bc} < \Xi^*_{cc} - \Xi_{cc}$$

- Comparing spin-splitting coefficient S_l:

Meson

Q	ℓ	$\mathcal{S}_Q$ [GeV^2]
c	$\bar{q},\ \frac{1}{2}^-$	0.472 ± 0.012
b	$\bar{q},\ \frac{1}{2}^-$	0.455 ± 0.003

Doubly heavy baryon

Q_1Q_2	ℓ	$\mathcal{S}_{Q_1Q_2}$ [GeV^2]
cc	$q,\ \frac{1}{2}^+$	0.363 ± 0.024
$[b\,c]$	$q,\ \frac{1}{2}^+$	
$\{b\,c\}$	$q,\ \frac{1}{2}^+$	0.181 ± 0.046
$b\,b$	$q,\ \frac{1}{2}^+$	0.472 ± 0.075

- $S_{cc} < S_c$ by about 4 error bars, $S_{bb} \sim S_b$ within error bars

- HQDQ predictions works well for bottom systems !!

Implications for HQDQ

- HQDQ Symmetry prediction for spin-splitting coefficient:

$$S_{\ell,QQ} = S_{\ell,\bar{Q}} = S_{\bar{\ell},Q}. \qquad S_{\ell,bc} = \frac{\mu_{bc}}{m_b} S_{\ell,\bar{b}} + \frac{\mu_{bc}}{m_c} S_{\ell,\bar{c}}. \qquad S_{\ell,\bar{b}} < S_{\ell,bc} < S_{\ell,\bar{c}}.$$

$$\Xi^*_{bb} - \Xi_{bb} < \Xi^*_{bc} - \Xi'_{bc} < \Xi^*_{cc} - \Xi_{cc}$$

34 MeV 106 MeV

- Lattice predictions for {b,c} DHB splitting is close to b baryon splitting:

$$\Xi^*_{bc} - \Xi'_{bc} \sim 27 \pm 12 \text{ MeV}$$

$$\Xi^*_{b} - \Xi'_{b} \sim \Xi^*_{bc} - \Xi'_{bc} \sim 27 \text{ MeV}$$

Brown, Detmold, Meinel, Orignos (2014)

- {b,c} diquark: charm quark should probably be treated as light constituent