Pc(4312), Pc(4380), and Pc(4457) as double triangle cusps

arXiv:2103.06817 (to appear in PRD)

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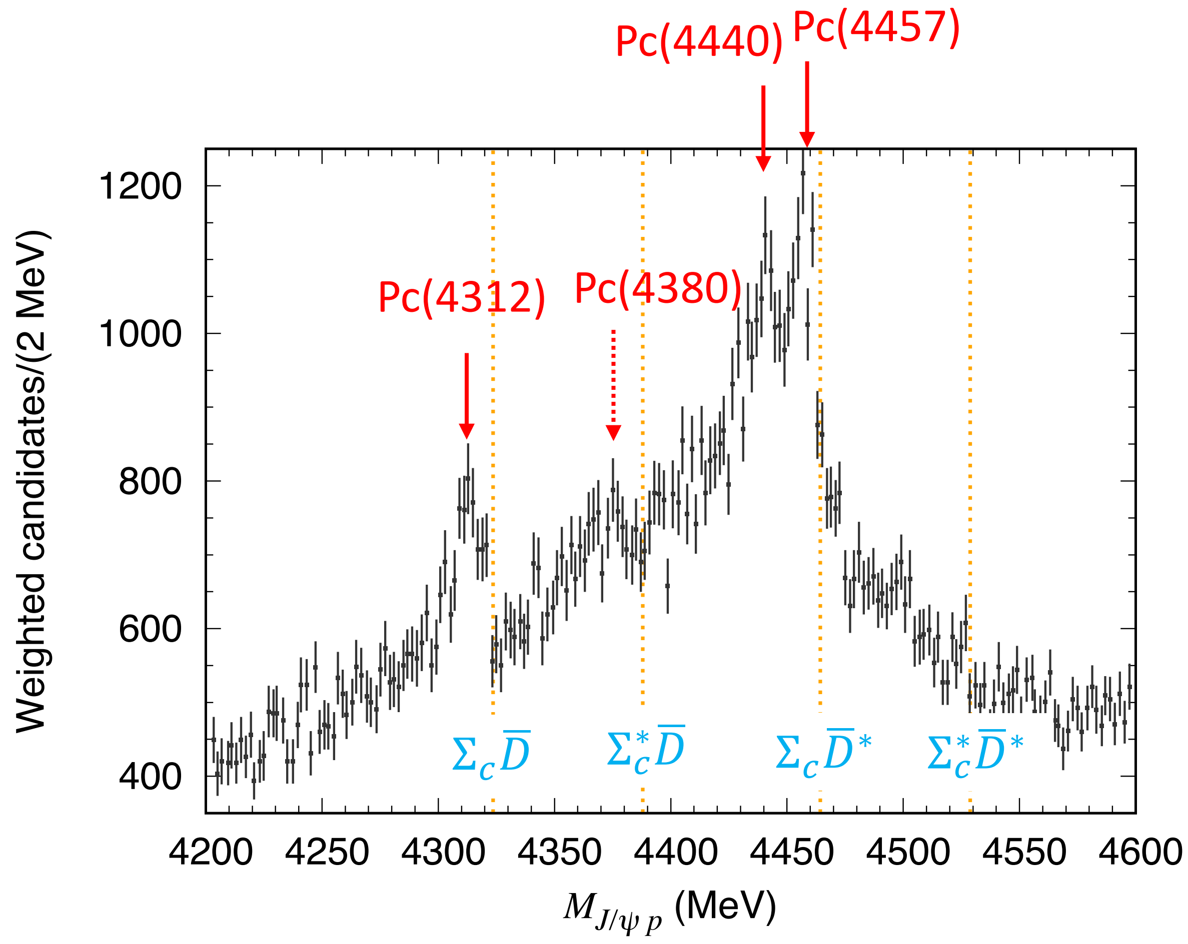
Introduction

P_c signals in $\Lambda_b^0 \to J/\psi\, p\, K^-$ data



LHCb, PRL 122, 222001 (2019)

Spectrum bumps suggest:

resonance
bound state
virtual state
(poles)

decay → $J/\psi\, p$

$c\bar{c}\ \ uud$ ← pentaquark

Peaks at slightly below $\Sigma_c^{(*)}\bar{D}^{(*)}$ thresholds

$\Sigma_c : \Sigma_c(2455)$
$\Sigma_c^* : \Sigma_c(2520)$

→ $\Sigma_c^{(*)}\bar{D}^{(*)}$ bound states (hadron molecule) ?

Other possibilities also proposed:

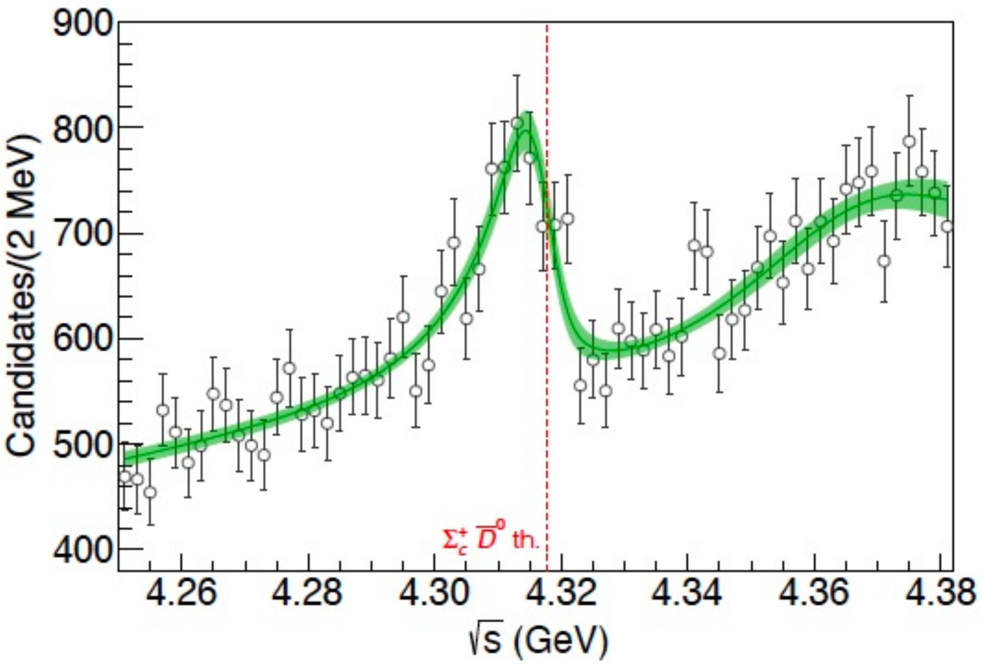
Compact constituent pentaquark, hadrocharmonium

Many papers and discussions !

Previous analysis of LHCb data ($M_{J/\psi\, p}$ distribution)

Fernandez-Ramirez et al. (JPAC), PRL 123, 092001 (2019)

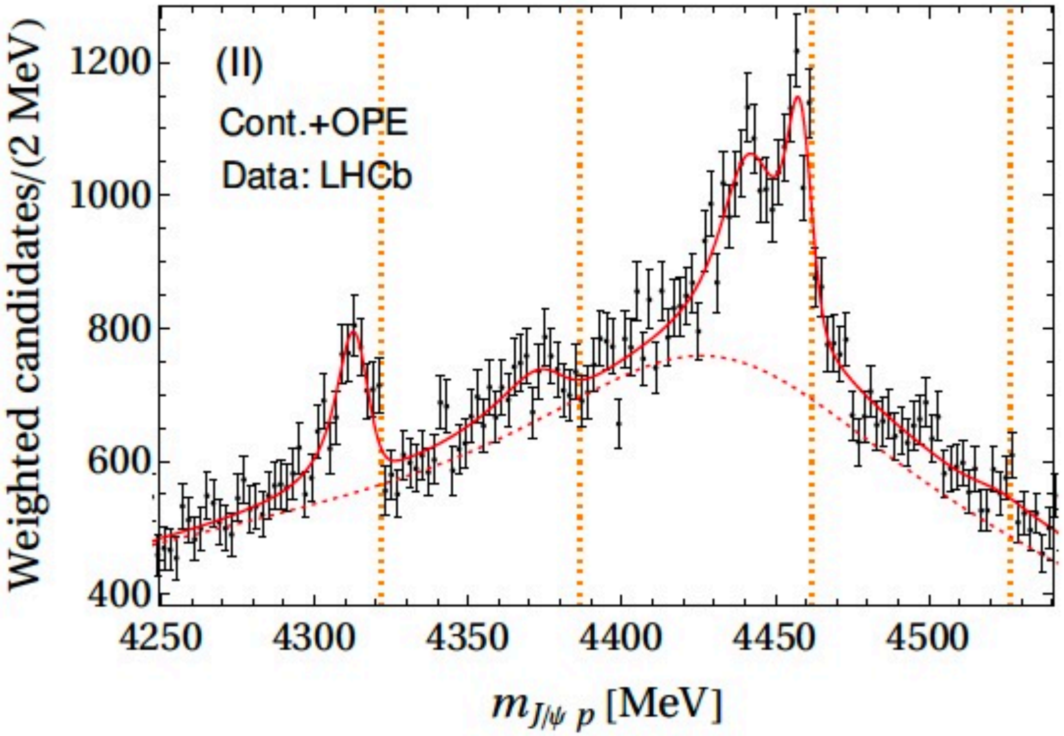
Two-channel ($\Sigma_c \bar{D}$-$J/\psi\, p$) K-matrix model for Pc(4312)



Pc(4312) is interpreted as a virtual state pole

Du et al. (Germany-China group), PRL 124, 072001 (2020)

$\Sigma_c^{(*)} \bar{D}^{(*)}$ coupled-channel model
heavy quark spin symmetry + one-pion-exchange

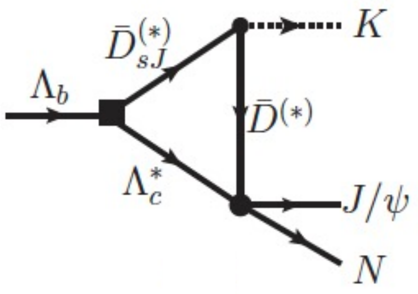


Pc(4312), Pc(4440), Pc(4380), Pc(4457) as $\Sigma_c^{(*)} \bar{D}^{(*)}$ bound states

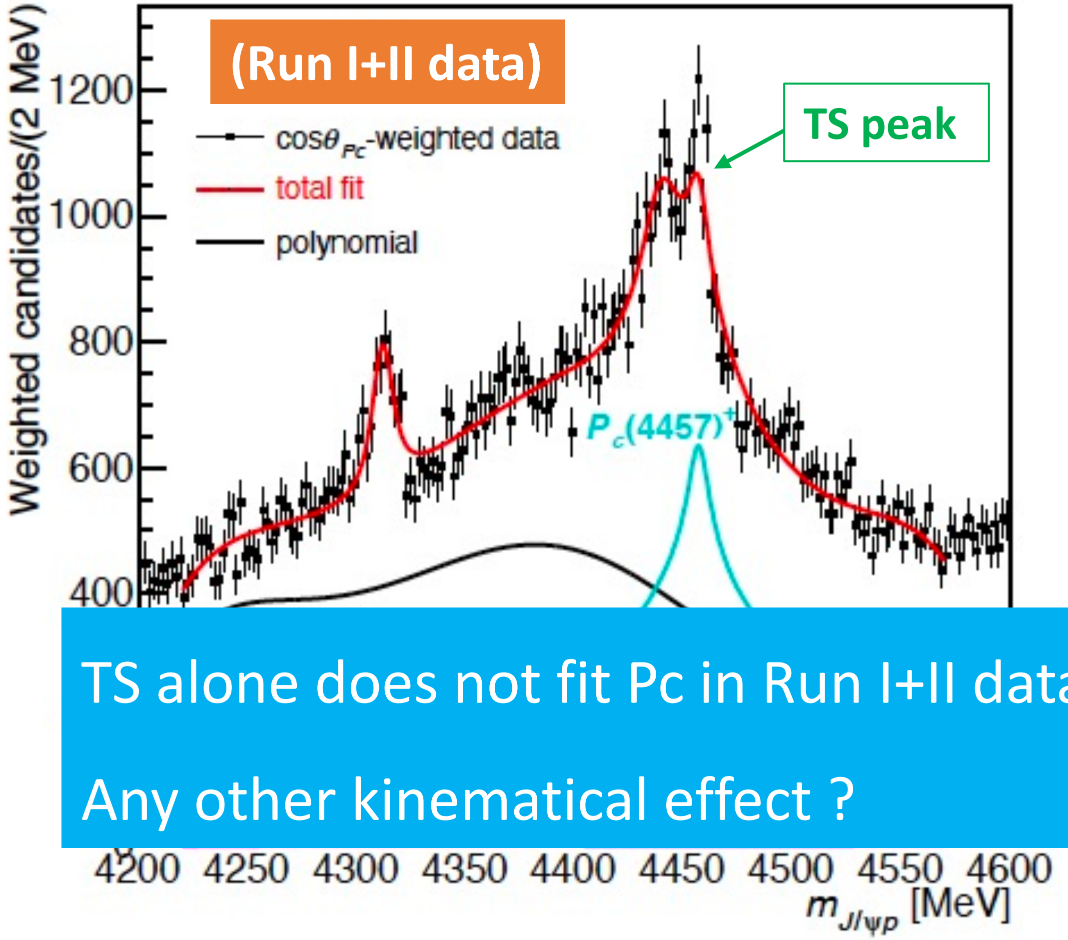
P_c as kinematical effect

Triangle singularities (TS) explored to interpret Run I data

Guo et al., PRD 92, 071502(R) (2015); Liu et al., PLB 757, 231 (2016)



LHCb, PRL 122, 222001 (2019)



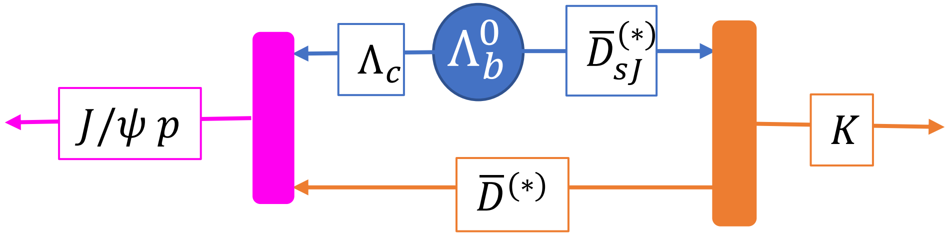
TS alone does not fit Pc in Run I+II data

Any other kinematical effect ?

TS conditions : process is kinematically allowed at classical level

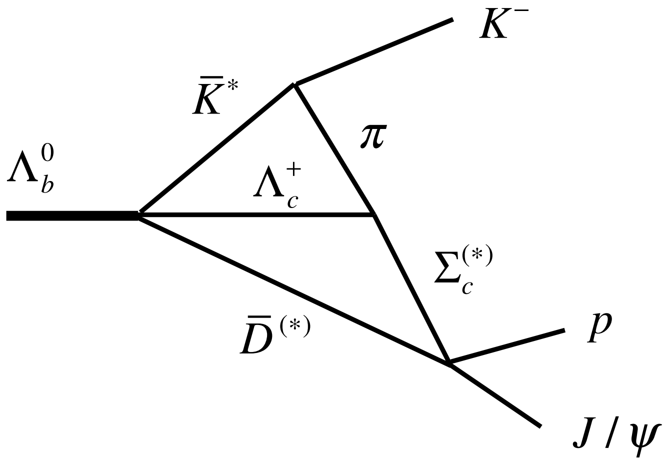
(i) on-shell intermediate states (ii) collinear internal momenta

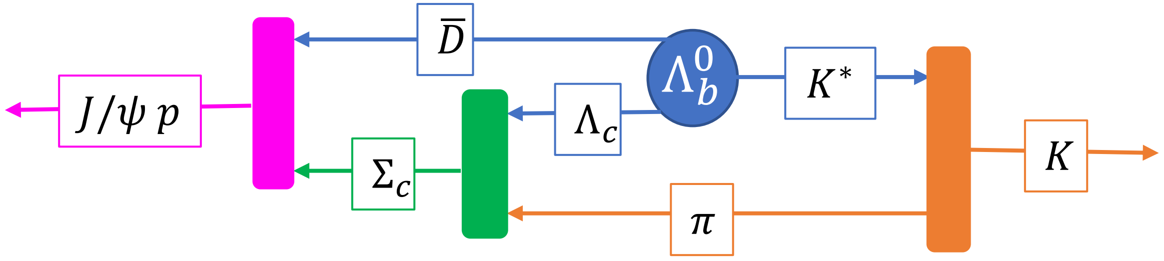
(iii) $v_{\bar{D}^{(*)}} \geq v_{\Lambda_c^*}$



Double triangle singularity (DTS)

Kinematical condition for DTS : kinematically classical process is allowed (Coleman-Norton theorem)





All intermediate states can be on-shell simultaneously (Σ_c case) → leading singularity

One (or more) state is necessarily off-shell (Σ_c^* case) → lower-order singularity

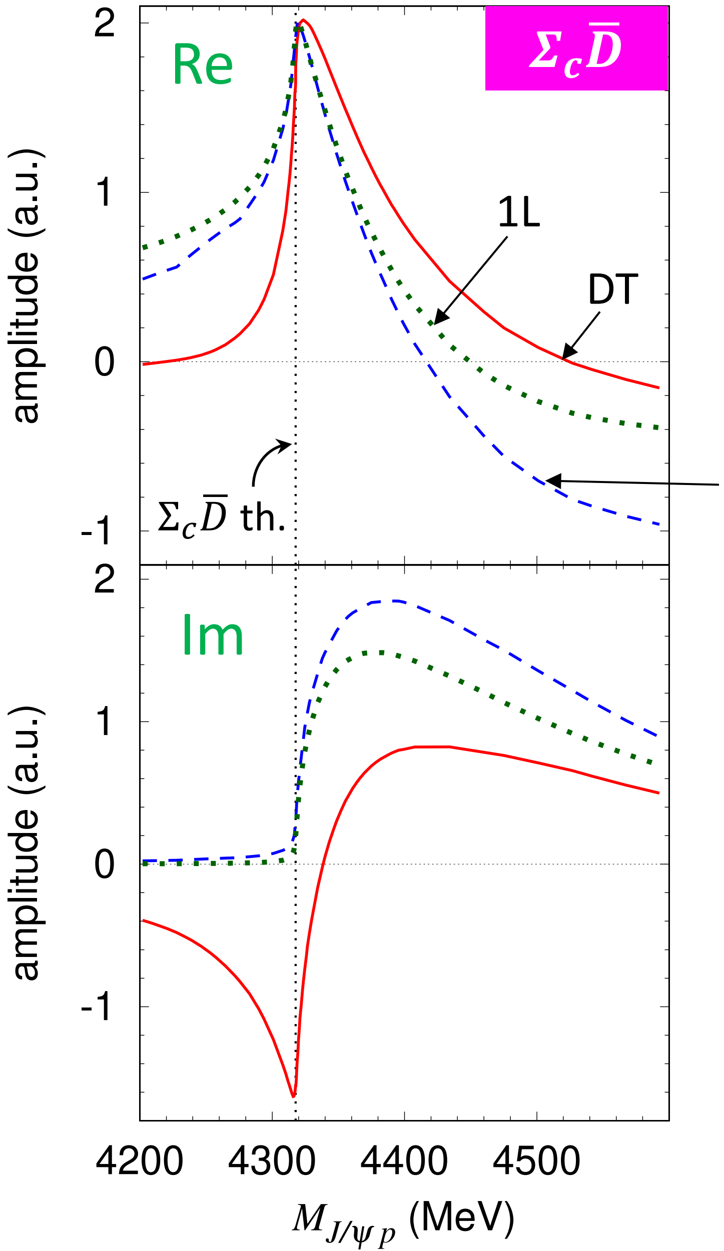
This work

- DTS causes anomalous threshold cusp significantly more singular than ordinary threshold cusp
- DT amplitudes reproduce Pc signals of LHCb data through interference with common (one-loop, tree) mechanisms
- Only Pc(4440) is required as a resonance, with width and strength significantly smaller than LHCb analysis result

New interpretation of Pc signals in LHCb data

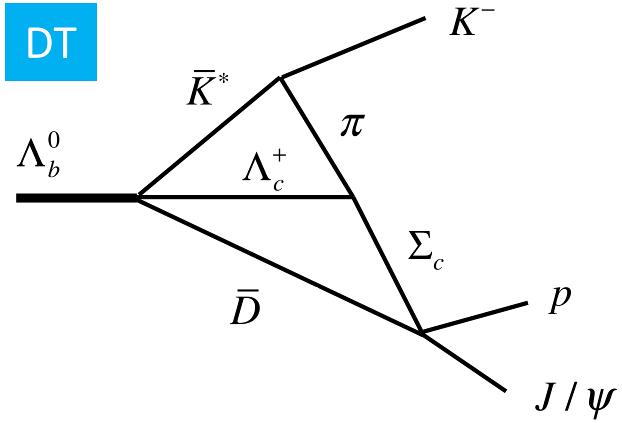
Singular behavior of double triangle amplitude

Singular behavior of double triangle amplitude

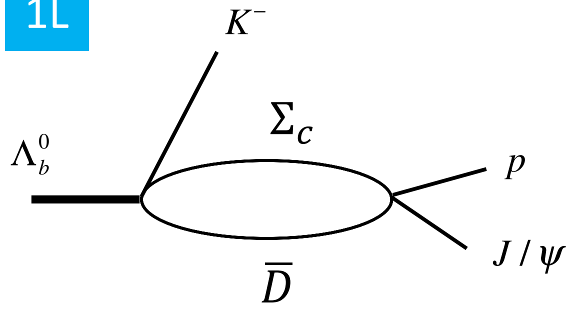


DT (m_{Λ_c} = 3 GeV)

DT

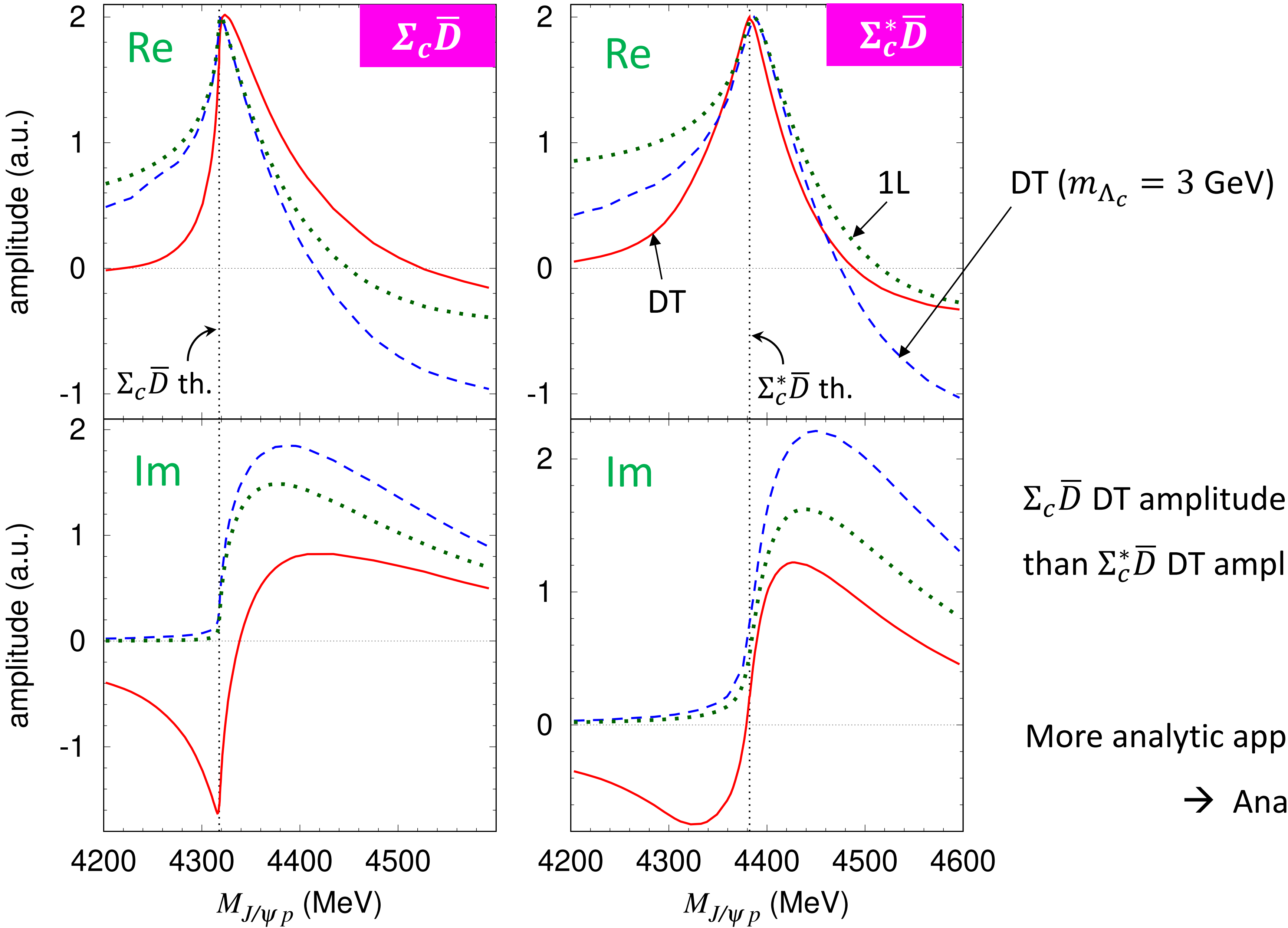


1L



- DT amplitude (leading DTS) are significantly more singular than ordinary threshold cusp (square root singularity)
- For m_{Λ_c} = 3 GeV, DT amplitude has ordinary threshold cusp
- With attractive $\Sigma_c\bar{D}$ interaction, the amplitudes become more singular (In figures, perturbative $\Sigma_c\bar{D} \rightarrow J/\psi p$ interaction is used)

Singular behavior of double triangle amplitude



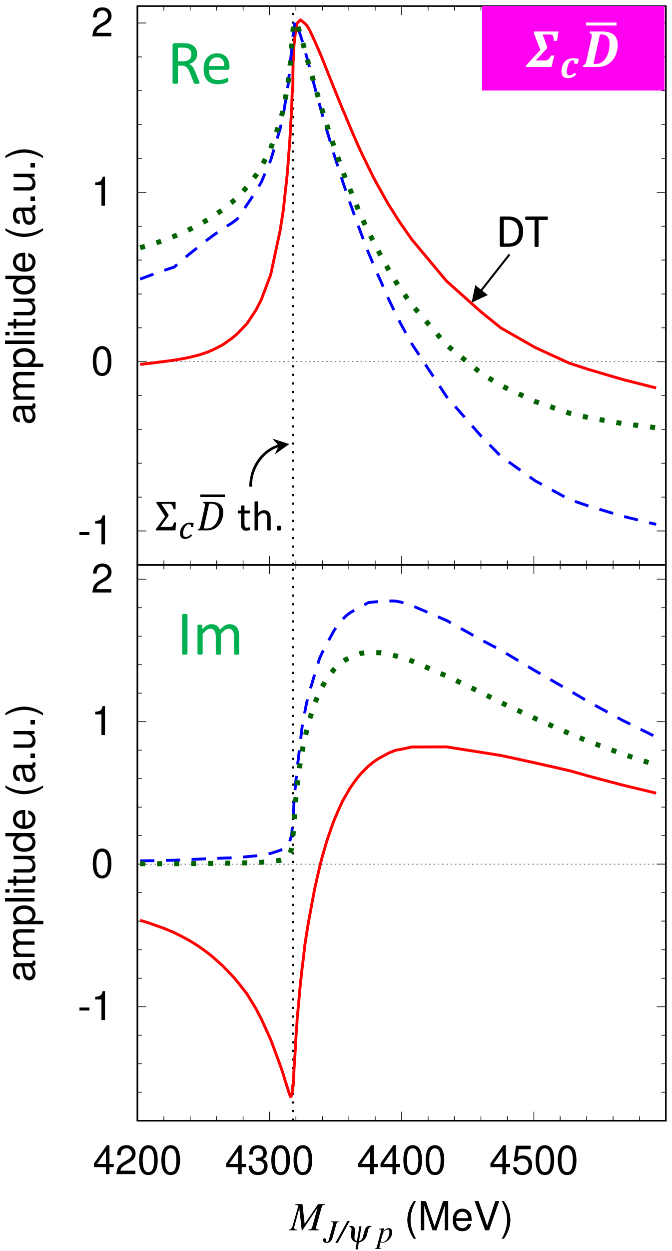
DT ($m_{\Lambda_c} = 3$ GeV)

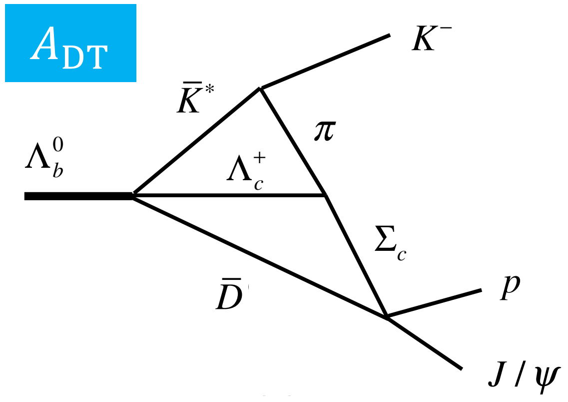
$\Sigma_c\bar{D}$ DT amplitude (leading singularity) is more singular than $\Sigma_c^*\bar{D}$ DT amplitude (lower-order singularity)

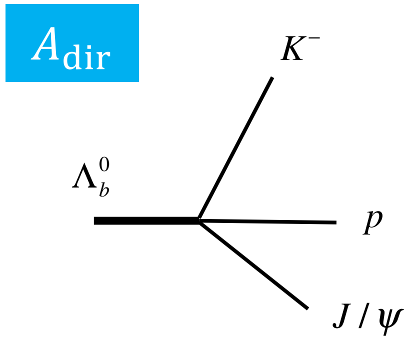
More analytic approach to examine singular behavior

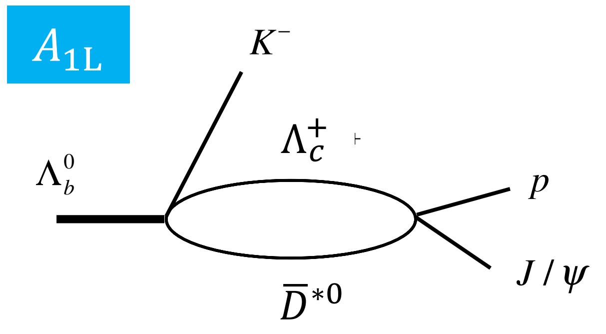
→ Analysis of Landau equation

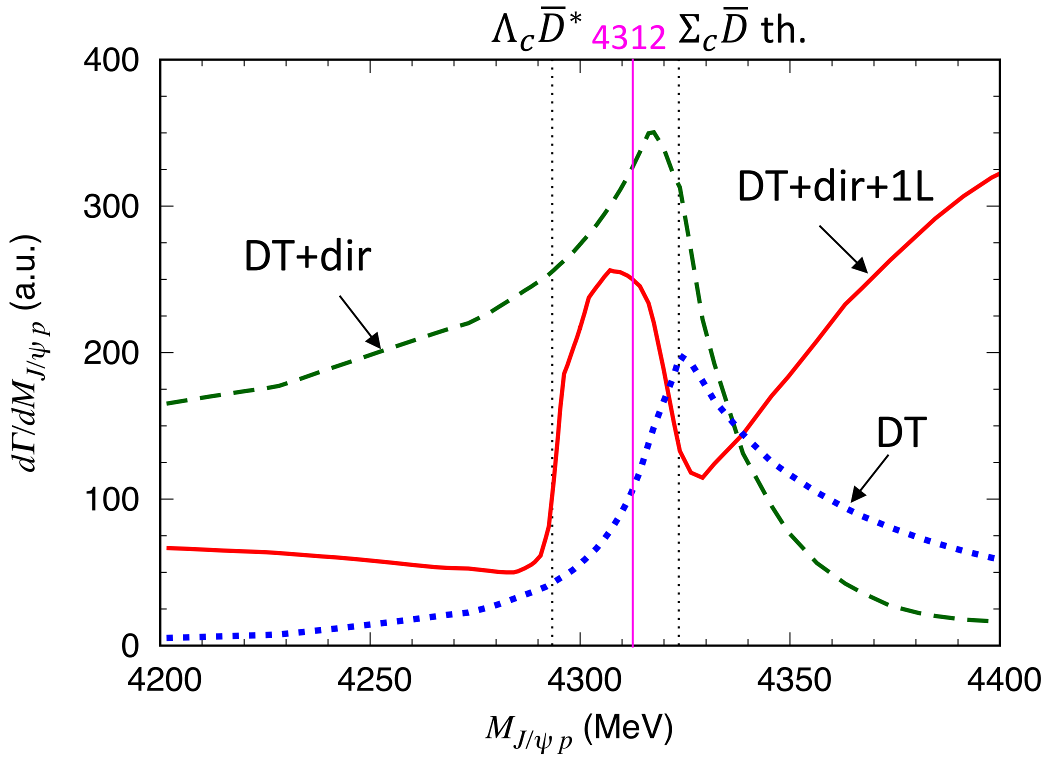
How double triangle amplitude appears as Pc ?









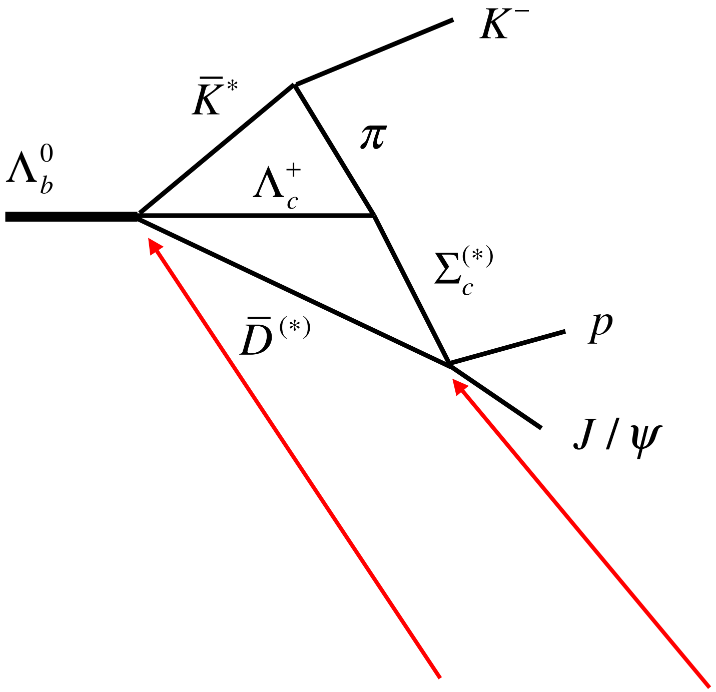


DT amplitude alone creates a peak at $\Sigma_c\bar{D}$ threshold ← not Pc

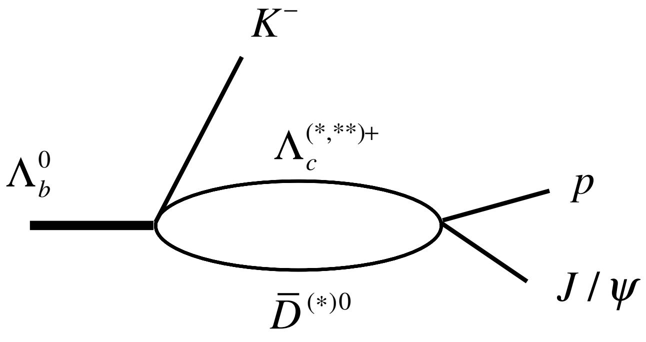
Interference among DT, one-loop, direct amplitudes play major role to create Pc peak

Analysis of LHCb data

Setup



$\Sigma_c^{(*)}\bar D^{(*)}(J^P)$
$\Sigma_c(2455)\bar D\ (1/2^-)$
$\Sigma_c(2520)\bar D\ (3/2^-)$
$\Sigma_c(2455)\bar D^*\ (1/2^-)$
$\Sigma_c(2455)\bar D^*\ (3/2^-)$
$\Sigma_c(2520)\bar D^*\ (1/2^-)$
$\Sigma_c(2520)\bar D^*\ (3/2^-)$



$\Lambda_c^{(*,**)}\bar D^{(*)}\ (J^P)$
$\Lambda_c\bar D^*\ (1/2^-)$
$\Lambda_c(2593)\bar D\ (1/2^+)$
$\Lambda_c(2625)\bar D\ (3/2^+)$

2×6 fitting parameters : $c_{\Lambda_c\bar D^{(*)}\bar K^*,\Lambda_b} \times c^{J^P}_{\psi p,\Sigma_c^{(*)}\bar D^{(*)}}$

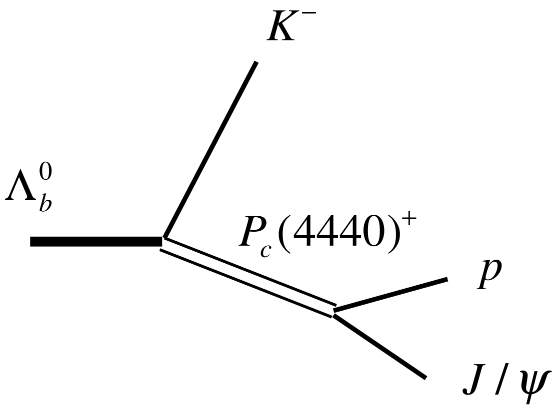
(complex couplings)

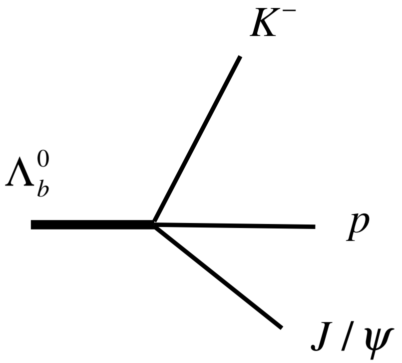
2×3 fitting parameters : $c_{\Lambda_c^{(*)}\bar D^{(*)}\bar K,\Lambda_b} \times c^{J^P}_{\psi p,\Lambda_c^{(*)}\bar D^{(*)}}$

Only color-favored weak vertices are used ↔ color-suppressed $\Lambda_b^0 \to \Sigma_c^{(*)}\bar D^{(*)}K^-$ are often used in previous models

Cannot explain Pc production rates ? Burns and Swanson, PRD 100, 114033 (2019)

Setup





$P_c(4440)$ of $J^P = 1/2^{\pm}, 3/2^{\pm}$ are examined

One direct-decay amplitude in each of

$J^P = 1/2^{\pm}, 3/2^{\pm}$ partial waves

J^P : spin-parity of $J/\psi\, p$ pair

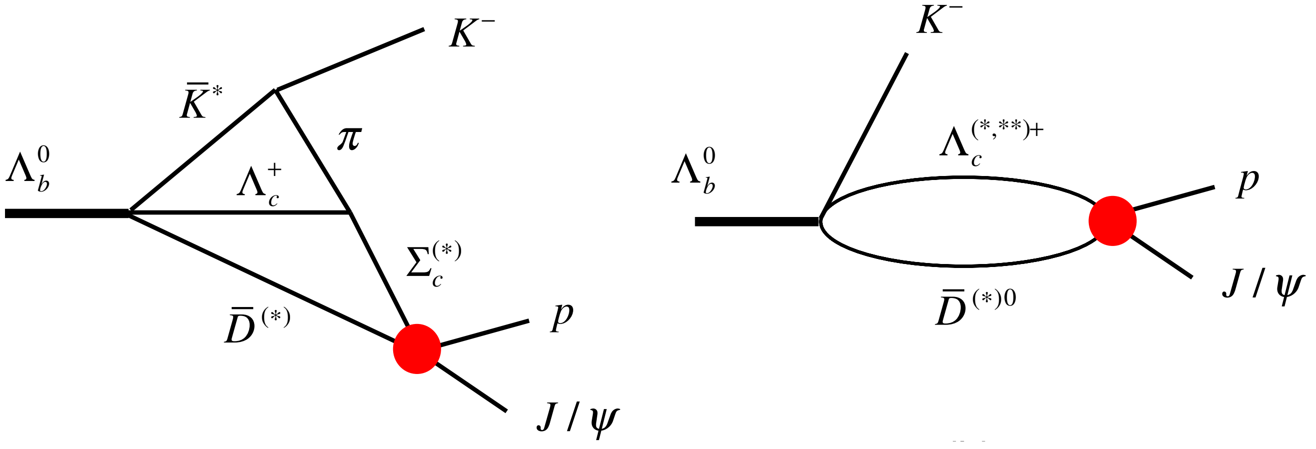
4 fitting parameters : m_{P_c} , Γ_{P_c} , $c_{P_c \bar{K},\Lambda_b} \times c^{J^P}_{\psi p,P_c}$

4 fitting parameters : $c^{J^P}_{J/\psi\, p\, \bar{K},\Lambda_b}$ (real) for each J^P

Totally 26 fitting parameters in the full model

$Y_c\bar{D}^{(*)}$ final state interactions

$Y_c = \Lambda_c^{(*,**)}, \Sigma_c^{(*)}$



Our model :

- $Y_c\bar{D}^{(*)}$ single-channel scattering (elastic unitarity)
- other possible coupled-channel effect

  → absorbed by couplings fitted to data
- Examine if fit favors attraction or repulsion

  for each channel of $Y_c\bar{D}^{(*)}$ (J^P)

Attraction : $\Sigma_c\bar{D}(1/2^-)$, $\Sigma_c^*\bar{D}(3/2^-)$, $\Sigma_c\bar{D}^*(1/2^-)$, $\Sigma_c\bar{D}^*(3/2^-)$, $\Lambda_c(2593)\bar{D}$ $(1/2^+)$, $\Lambda_c(2625)\bar{D}$ $(3/2^+)$

All interaction strengths are fixed so that $a \approx 0.5$ fm ; $p\cot\delta \sim 1/a + \mathcal{O}(p^2)$

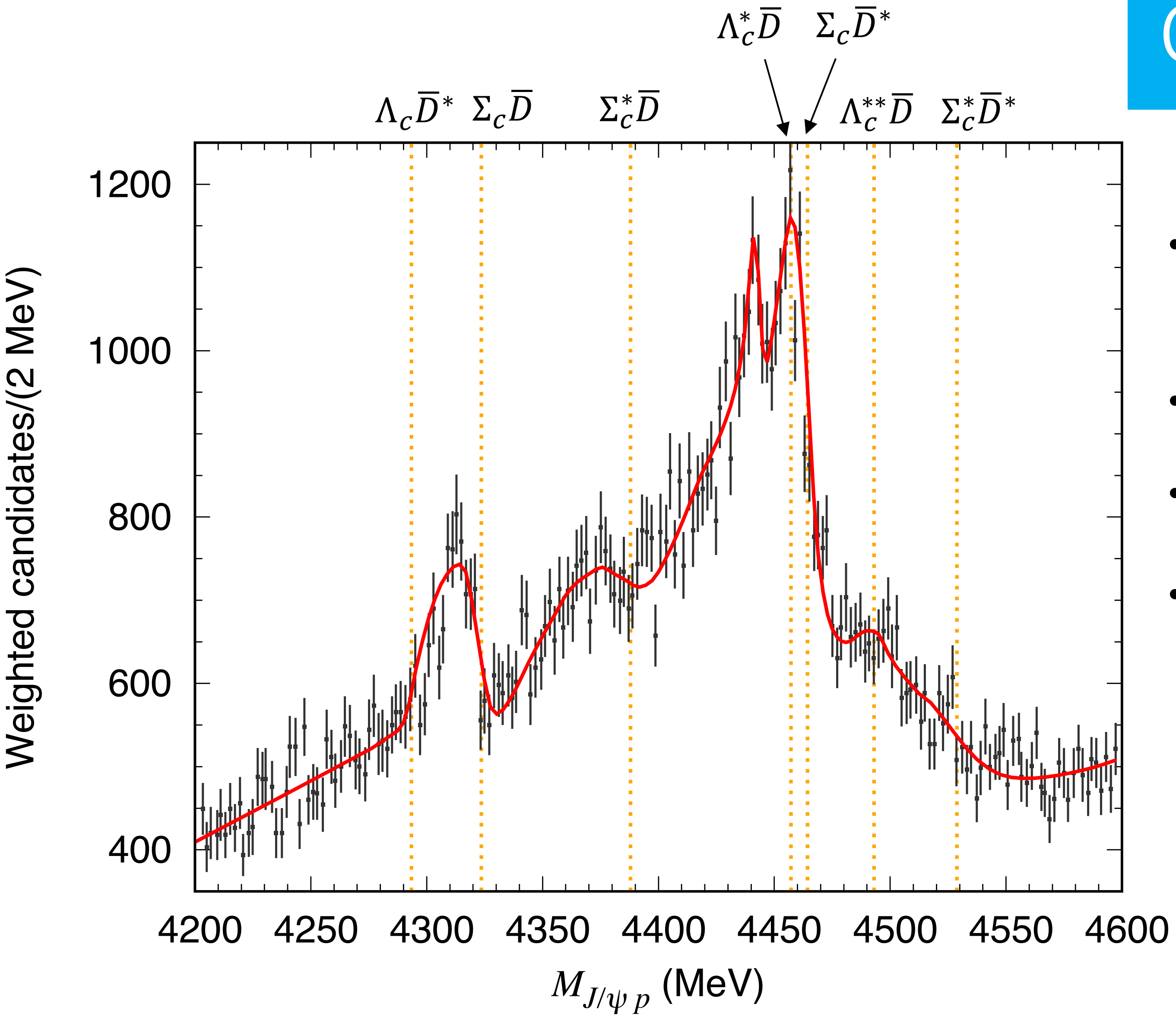
Repulsion : $\Lambda_c\bar{D}^*$ $(1/2^-)$, $\Sigma_c^*\bar{D}^*(1/2^-)$, $\Sigma_c^*\bar{D}^*(3/2^-)$ ← common interaction strength is used

$\Lambda_c\bar{D}^*$ $(1/2^-)$ interaction strength is fitted to LHCb data → $a = -0.4 \sim -0.05$ fm for $\Lambda = 0.8 \sim 2$ GeV

(Λ: cutoff in form factors)

Note: Pc-like peak positions are NOT sensitive to a values

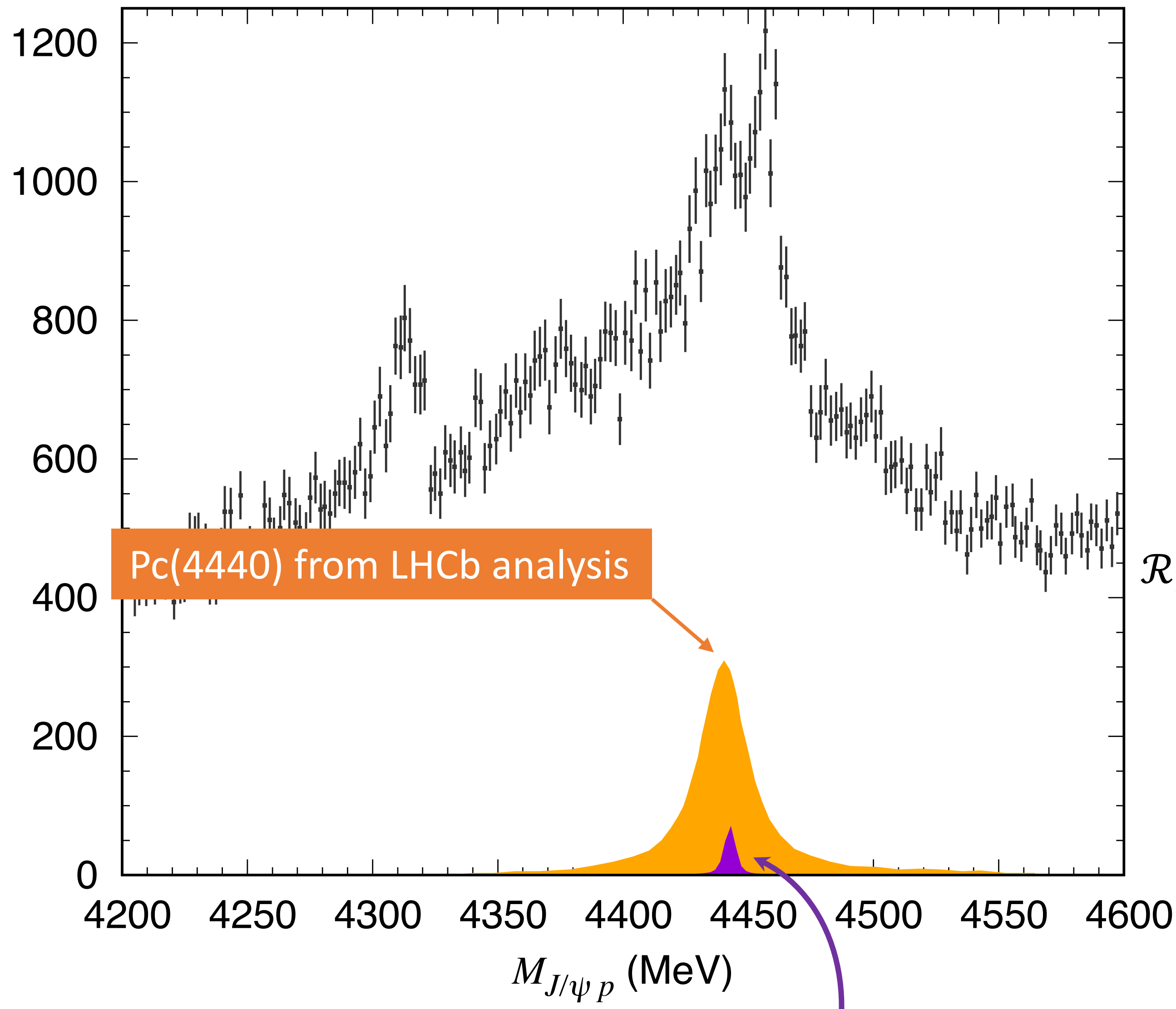
Comparison with LHCb data



— : full model (smeared by exp. resolution)

- Pc(4312), Pc(4380), Pc(4457) peaks are well described by kinematical effects; not by poles
- $\Lambda_c \bar{D}^*$ and $\Lambda_c(2625)\bar{D}$ threshold cusps fit the data
- Pc(4440) requires a resonance pole ($J^P = 3/2^-$ in figure)
- Similar fit quality when changing cutoff over $0.8 - 2$ GeV and changing $J^P = 1/2^{\pm}, 3/2^{\pm}$ for Pc(4440)

Pc(4440)



Pc(4440) from LHCb analysis

Pc(4440) from this work; interference excluded

	Mass (MeV)	Width (MeV)
This work	4443.1 ± 1.4	$\underline{2.7} \pm 2.4$
LHCb	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$\underline{20.6} \pm 4.9^{+8.7}_{-10.1}$

Pc(4440) contribution

$$\mathcal{R}_{\mathrm{LHCb}} \equiv \frac{\mathcal{B}\left(\Lambda_b^0 \to P_c^+ K^-\right) \mathcal{B}(P_c^+ \to J/\psi\, p)}{\mathcal{B}(\Lambda_b^0 \to J/\psi\, p\, K^-)} = 1.11 \pm 0.33^{+0.22}_{-0.10}\ \%$$

$$\approx \underline{22} \times \mathcal{R}_{\mathrm{This\ work}}$$

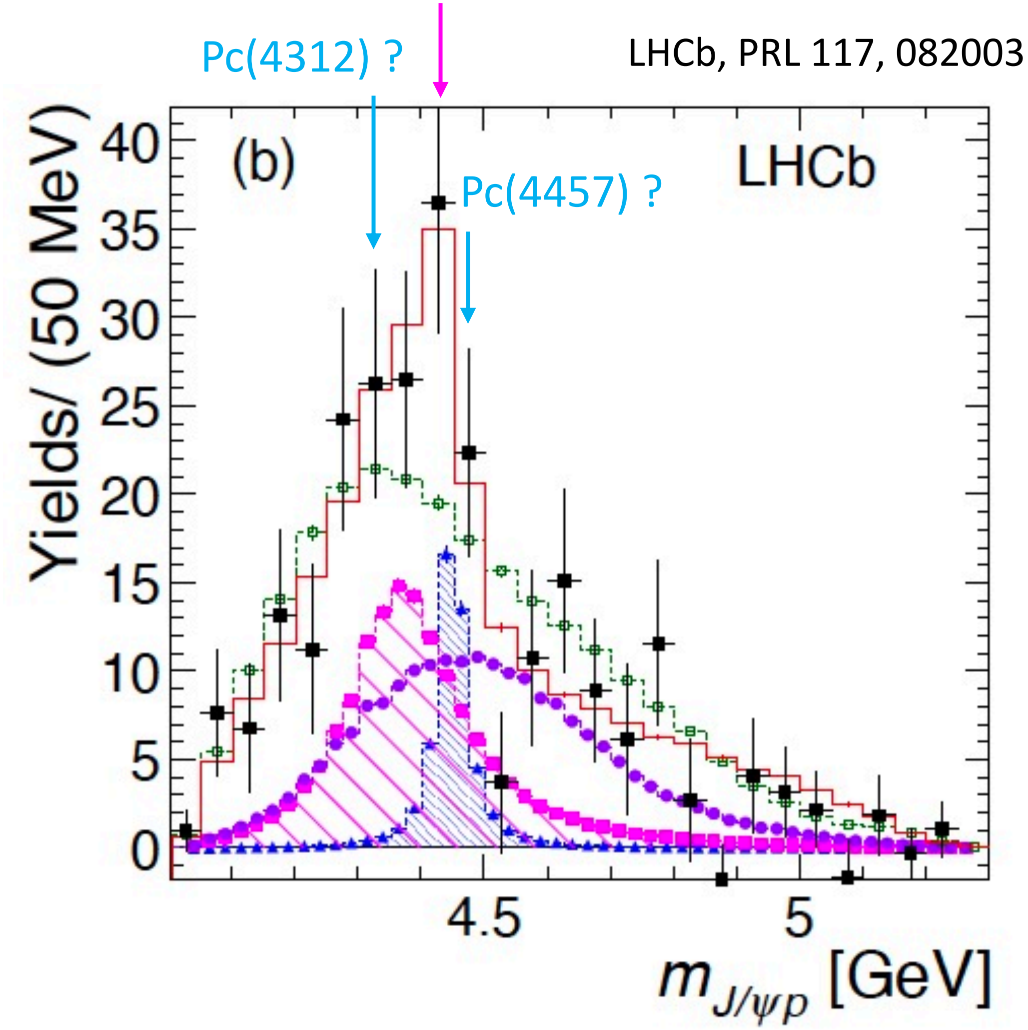
Pc(4440) from this work has significantly narrower width and weaker coupling strength than LHCb analysis

← Different strategies to fit large structure at ~ 4450 MeV

LHCb : fit with incoherent Pc(4440) and Pc(4457)

This work : mostly kinematical effect, Pc(4440) is small spike

P_c signal in $\Lambda_b^0 \to J/\psi\, p\, \pi^-$ data



Pc(4440) !

Pc(4312) ?

Pc(4457) ?

LHCb, PRL 117, 082003 (2016)

LHCb data

- $M_{J/\psi\, p}$ bin for Pc(4440) is enhanced
- No enhancement for other Pc's bins

This observation is consistent with our model because:

- $\Lambda_b^0 \to J/\psi\, p\, \pi^-$ cannot have DTS of $\Lambda_b^0 \to J/\psi\, p\, K^-$

  → no Pc(4312), Pc(4380), Pc(4457) in $\Lambda_b^0 \to J/\psi\, p\, \pi^-$
- $\Lambda_b^0 \to J/\psi\, p\, \pi^-$ can have $\Lambda_b^0 \to P_c(4440)\, \pi^-$ mechanism

  → Pc(4440) signal is possible in $\Lambda_b^0 \to J/\psi\, p\, \pi^-$

However, this data may conflict with some other Pc models

Pc signals in $\Lambda_b^0 \to J/\psi\, p\, \pi^-$ are inconclusive due to limited statistics

→ Higher statistics $\Lambda_b^0 \to J/\psi\, p\, \pi^-$ data can seriously test Pc models !

Summary

Summary

- LHCb data of $\Lambda_b^0 \to J/\psi\, p\, K^-$ with Pc structures is analyzed
- Pc(4312), Pc(4380), and Pc(4457) peaks are well described by double triangle cusps

  and their interference with common mechanisms
- Only Pc(4440) is interpreted as a resonance

  Its width and coupling strength are significantly smaller than the LHCb analysis
- The proposed interpretation of Pc structures in $\Lambda_b^0 \to J/\psi\, p\, K^-$ is completely different

  from hadron molecule and compact pentaquark models
- In future, understand other resonance-like structures near thresholds with DTS

  DTS should now be a possible option

Backup

Theoretical interpretations for Pc (many papers !)

- $\Sigma_c^{(*)}\bar{D}^{(*)}$ hadron molecule $J^P = 1/2^-$ for Pc(4312), $1/2^-$ or $3/2^-$ for Pc(4440) and Pc(4457)

  -- Coupled-channel $\Sigma_c^{(*)}\bar{D}^{(*)}$ system based on heavy quark spin symmetry (HQSS) → 7 Pc states predicted

  Liu et al. (Beihang group) , PRL 122, 242001 (2019)

  -- HQSS interactions + one-pion-exchange mechanism

  Du et al. (Germany-China group), PRL 124, 072001 (2020); Xiao et al., PRD 102, 056018 (2020)

- Constituent quark model

  -- diquark-diquark-antiquark model $J^P = 3/2^-$ for Pc(4312), $3/2^+$ for Pc(4440) , $5/2^+$ for Pc(4457)

  Ali and Parkhomenko, PLB 793, 365 (2019)

  -- pentaquark model $J^P = 1/2^-$ for Pc(4312), $3/2^-$ for Pc(4440) , $1/2^-$ for Pc(4457)

  Weng et al. (Pekin group), PRD 100, 016014 (2019)

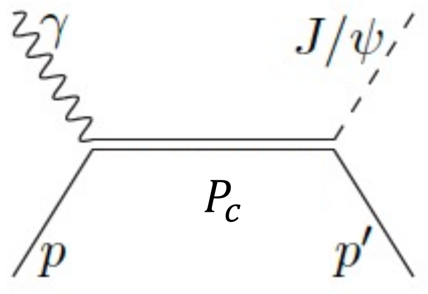
- Hadrocharmonium Eides et al., Mod. Phys. Lett. A 35, 2050151 (2020)

  $J^P = 1/2^+$ for Pc(4312) as χ_{c0}-N bound state, $1/2^-$ for Pc(4440) , $3/2^-$ for Pc(4457) as $\psi(2S)$-N bound states

All models reproduce Pc masses (more or less); measurement of spin-parity is important to test the models

P_c signals in other processes

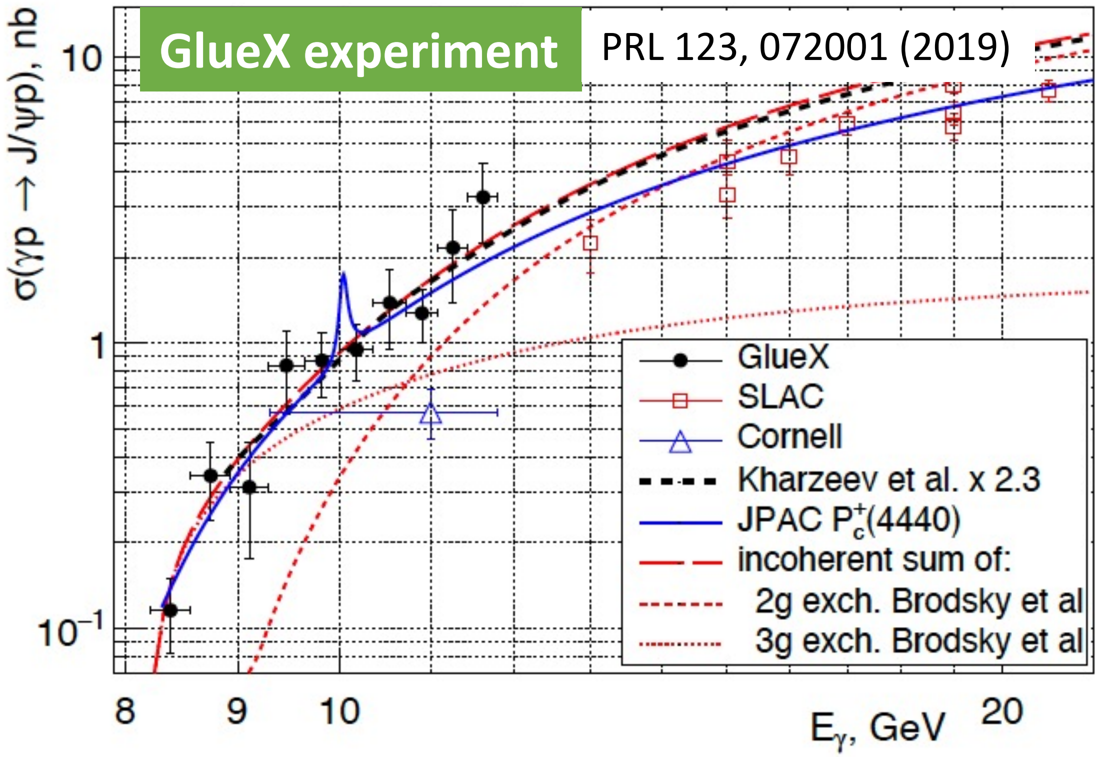
Important to establish Pc as hadronic states



J/ψ photoproduction

Wang et al., PRD 92, 034022 (2015) , etc.

Advantage : No kinematical effect to mimic Pc


GlueX experiment

PRL 123, 072001 (2019)

No Pc signals, why ?

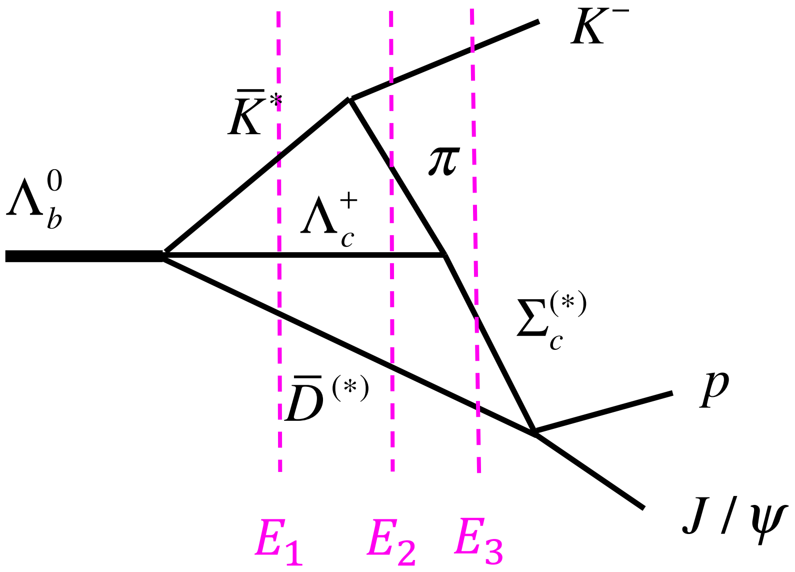
- photo-coupling of Pc is weak

  → higher statistics data might find a signal

- Pc in $\Lambda_b^0 \to J/\psi\, p\, K^-$ is a kinematical effect

  → but no such mechanism has been found

Kinematics closest to double triangle leading singularity condition



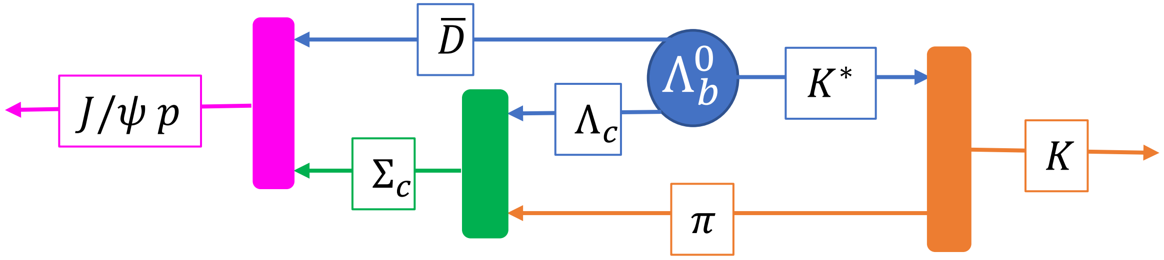
- $m_{\Lambda_b} = E = E_2 = E_3 \neq E_1$

(On-shell condition)

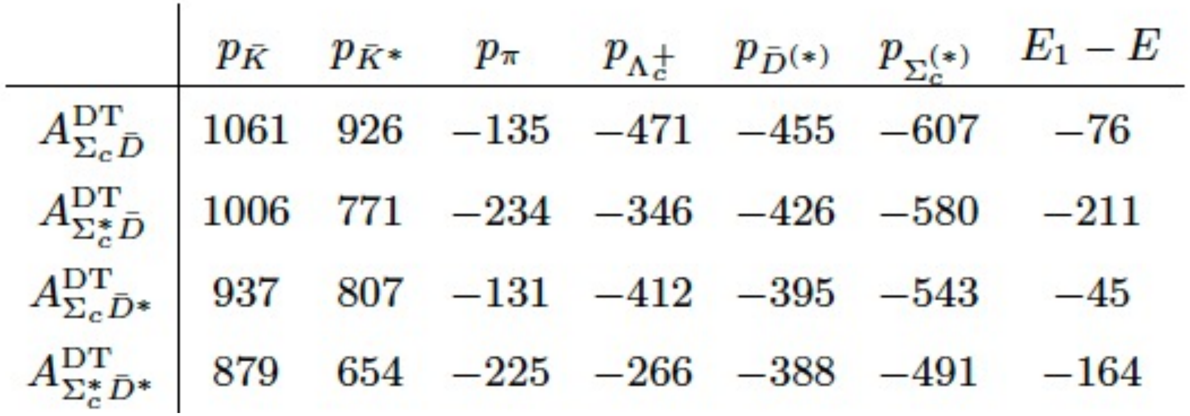
- $|E - E_1|$: minimum

  Criteria of leading singularity : $|E - E_1| \lesssim \Gamma_{K^*}$

- Collinear internal momenta ($p_{\bar{K}}$ taken along positive axis)
- $v_{\bar{D}} \geq v_{\Lambda_c}$, $v_{\pi} \geq v_{\Lambda_c}$, $v_{\Sigma_c} \geq v_{\bar{D}}$

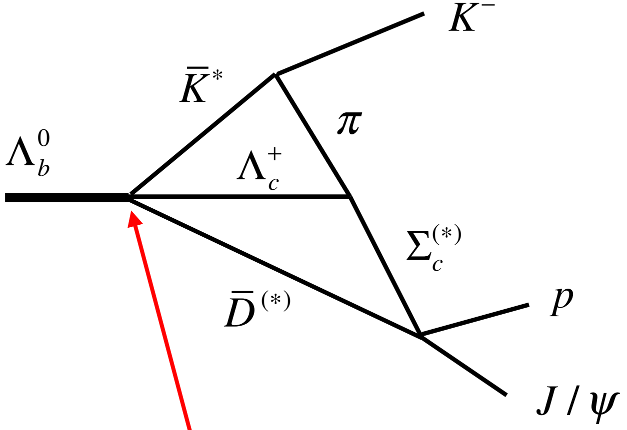


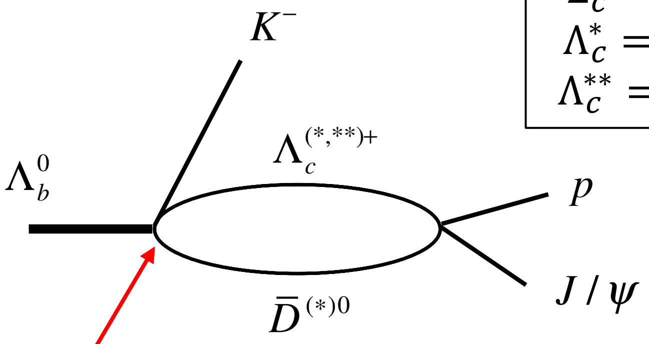
Internal momenta (MeV) in CM frame satisfying above



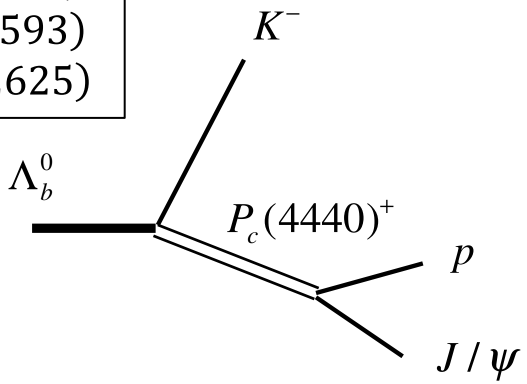
	$p_{\bar{K}}$	$p_{\bar{K}^*}$	p_π	$p_{\Lambda_c^+}$	$p_{\bar{D}^{(*)}}$	$p_{\Sigma_c^{(*)}}$	$E_1 - E$
$A^{\rm DT}_{\Sigma_c \bar{D}}$	1061	926	−135	−471	−455	−607	−76
$A^{\rm DT}_{\Sigma_c^* \bar{D}}$	1006	771	−234	−346	−426	−580	−211
$A^{\rm DT}_{\Sigma_c \bar{D}^*}$	937	807	−131	−412	−395	−543	−45
$A^{\rm DT}_{\Sigma_c^* \bar{D}^*}$	879	654	−225	−266	−388	−491	−164

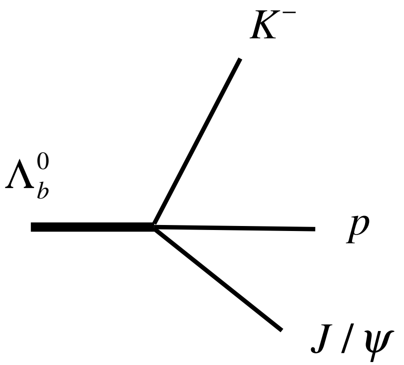
When Σ_c (Σ_c^*) propagates, the DT amplitude has the leading (lower-order) singularity



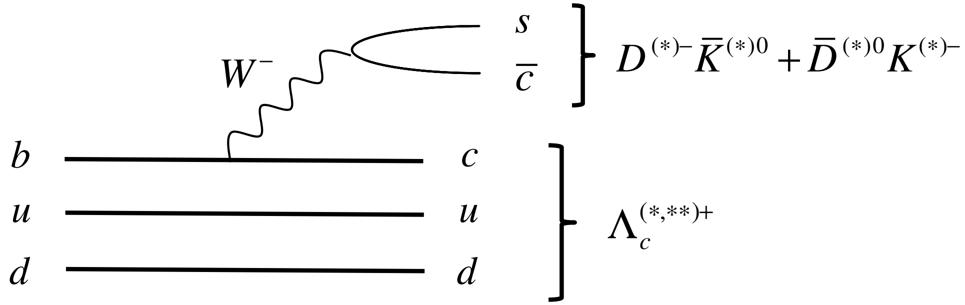


$\Sigma_c^* = \Sigma_c(2520)$
$\Lambda_c^* = \Lambda_c(2593)$
$\Lambda_c^{**} = \Lambda_c(2625)$

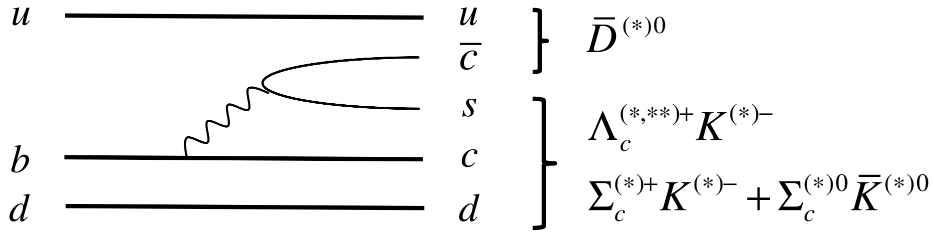


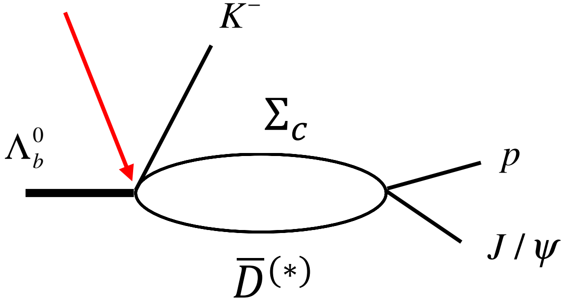


Assume dominance of color-favored weak vertices →



Previous models often used color-suppressed vertices →





Cannot explain Pc production rates ? Burns and Swanson, PRD 100, 114033 (2019)

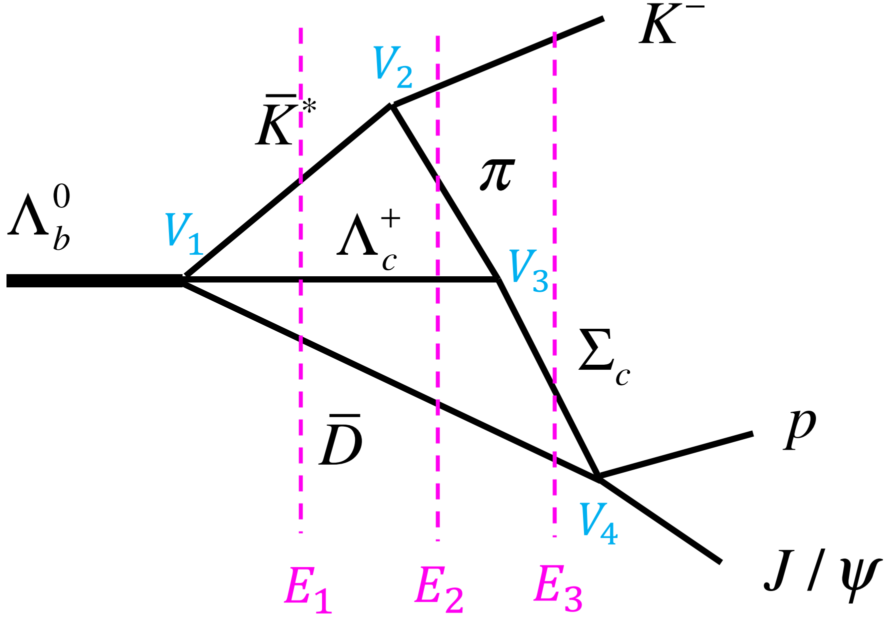
Color-suppressed decay cannot explain Pc production rates ? Burns and Swanson, PRD 100, 114033 (2019)

Generally, color suppression is difficult to predict Du et al., arXiv:2102.07159

We still assume dominance of color-favored decay

← color-suppressed mechanisms are redundant to fit only $M_{J/\psi\, p}$ distribution data

Double triangle amplitudes



$$V_1 = c_{\Lambda_c \bar{D} \bar{K}^*, \Lambda_b} \left(\frac{1}{2} t_{\bar{D}} \frac{1}{2} t_{\bar{K}^*} \middle| 00 \right) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\bar{K}^*}$$

$$V_2 = c_{\bar{K}\pi, \bar{K}^*} \left(1 t_\pi \frac{1}{2} t_{\bar{K}} \middle| \frac{1}{2} t_{\bar{K}^*} \right) \boldsymbol{\epsilon}_{\bar{K}^*} \cdot (\boldsymbol{p}_{\bar{K}} - \boldsymbol{p}_\pi)$$

$$V_3 = c_{\Lambda_c \pi, \Sigma_c} \boldsymbol{\sigma} \cdot \boldsymbol{p}_\pi$$

$$V_4 = c^{1/2^-}_{\psi p, \Sigma_c \bar{D}} \left(1 t_{\Sigma_c} \frac{1}{2} t_D \middle| \frac{1}{2} t_p \right) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_\psi$$

Dipole form factor is multiplied to each vertex (cutoff 1 GeV as default)

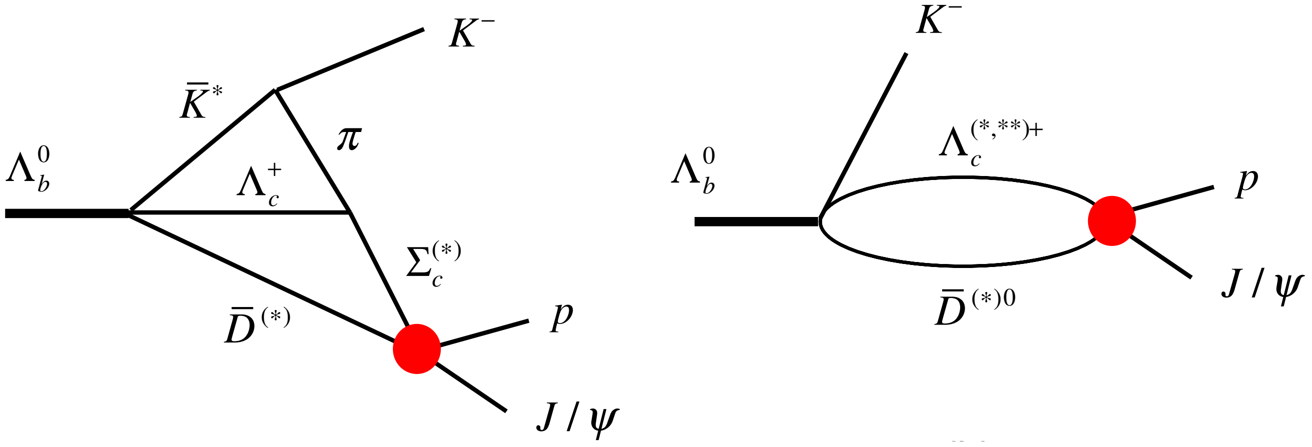
$$A_{DT} = \iint d^3 p_\pi \; d^3 p_{\bar{D}} \; V_4 \frac{1}{E - E_3} V_3 \frac{1}{E - E_2} V_2 \frac{1}{E - E_1} V_1$$

$$E_1 = E_{K^*} + E_{\Lambda_c} + E_{\bar{D}} - i \frac{\Gamma_{K^*}}{2}$$

… etc.

$Y_c\bar{D}^{(*)}$ final state interactions

$Y_c = \Lambda_c^{(*,**)}, \Sigma_c^{(*)}$



Non-perturbative treatment required

for $Y_c\bar{D}^{(*)}$ coupled-channel system

- Reasonable approach

$Y_c\bar{D}^{(*)}$ coupled-channel scattering model ← HQSS-constrained interactions + pion-exchange mechanism

- Simplified approach employed in this work

$Y_c\bar{D}^{(*)}$ single-channel scattering model with a contact interaction (elastic unitarity)

other possible coupled-channel effect → absorbed by couplings fitted to data

$Y_c\bar{D}^{(*)}$ final state interactions

Justification of the simplified treatment to describe $M_{J/\psi\, p}$ distribution of $\Lambda_b^0 \to J/\psi\, p\, K^-$

In our model, Pc structures (other than Pc(4440)) are described by kinematical effect

not directly by poles from $Y_c\bar{D}^{(*)}$ scattering ; even perturbative $Y_c\bar{D}^{(*)} \to J/\psi\, p$ can fit Pc peaks fairly well

→ Data can only loosely constrain $Y_c\bar{D}^{(*)}$ interactions

→ Details of $Y_c\bar{D}^{(*)}$ interactions do not play a major role

The simplification is not valid to describe possible Pc structure in $M_{\Sigma_c\bar{D}}$ distribution of $\Lambda_b^0 \to \Sigma_c\bar{D}\, K^-$

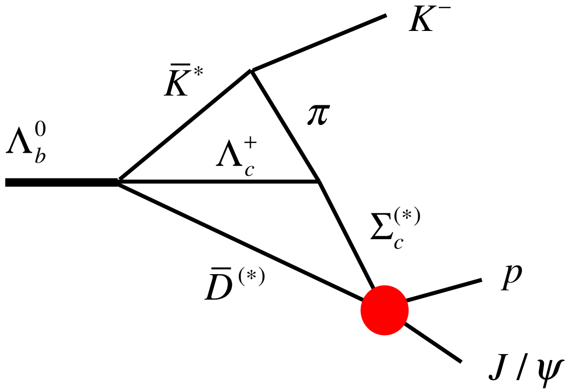
In contrast, for $Y_c\bar{D}^{(*)}$ molecule model, the simplification is not valid because:

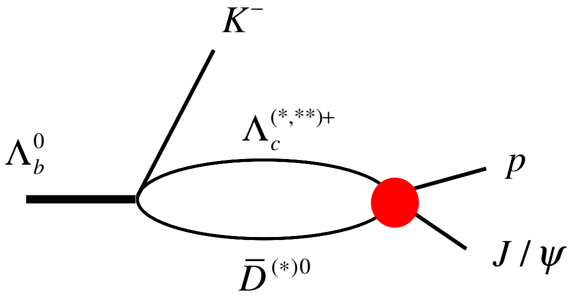
$Y_c\bar{D}^{(*)}$ interactions need fine-tuning → Pc poles at exact positions are generated

→ Details of $Y_c\bar{D}^{(*)}$ interactions do matter

Analysis of LHCb data

Setup





$Y_c\bar{D}^{(*)}$ interaction   Examine if the fit favors attraction or repulsion for each $Y_c\bar{D}^{(*)}(J^P)$

Attraction : $\Sigma_c\bar{D}(1/2^-),\ \Sigma_c^*\bar{D}(3/2^-),\ \Sigma_c\bar{D}^*(1/2^-),\ \Sigma_c\bar{D}^*(3/2^-),\ \Lambda_c(2593)\bar{D}\ (1/2^+),\ \Lambda_c(2625)\bar{D}\ (3/2^+)$

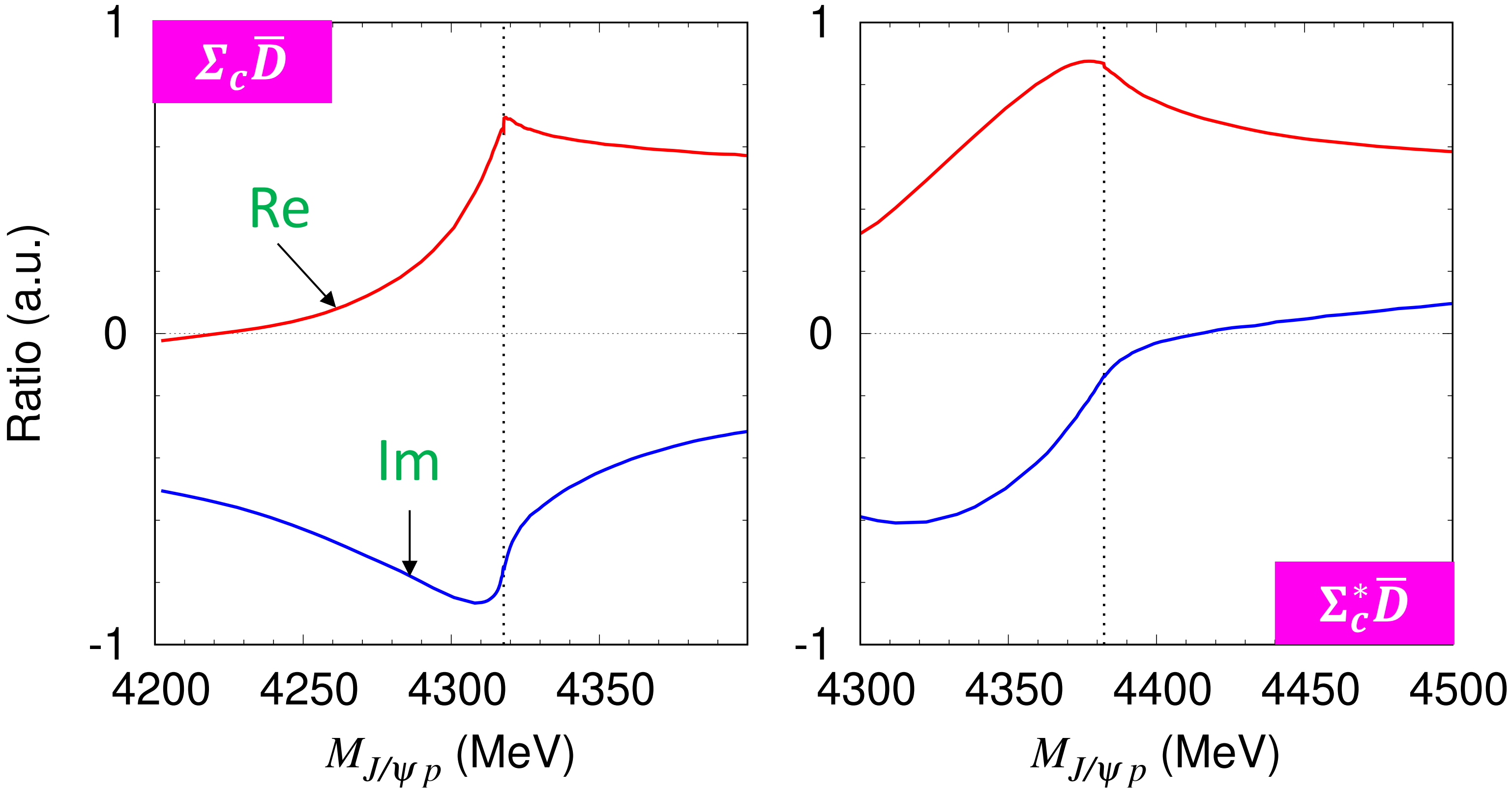
All interaction strengths are fixed so that $a \approx 0.5$ fm ; $p \cot\delta \sim 1/a + \mathcal{O}(p^2)$

Repulsion : $\Sigma_c^*\bar{D}^*(1/2^-),\ \Sigma_c^*\bar{D}^*(3/2^-), \Lambda_c\bar{D}^*\ (1/2^-)$ ← common interaction strength is used

$\Lambda_c\bar{D}^*\ (1/2^-)$ interaction strength is fitted to LHCb data → $a = -0.4 \sim -0.05$ fm for $\Lambda = 0.8 - 2$ GeV

Note: Pc-like peak positions are NOT sensitive to a values

Singular behavior of double triangle amplitude

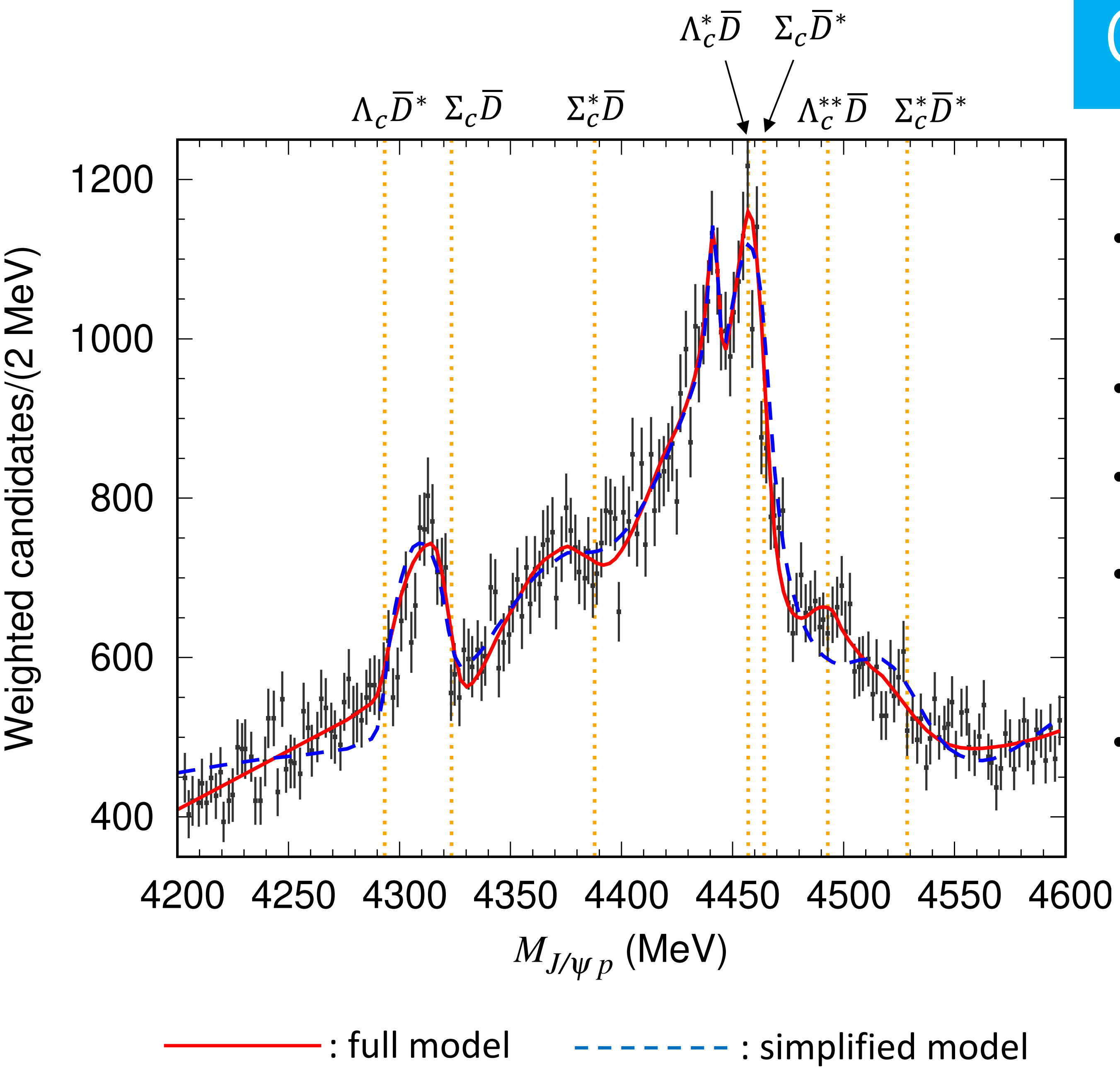


- $A_{\mathrm{DT}}/A_{\mathrm{1L}}$ shows how DT amplitude behaves differently from threshold cusp
- Singular behavior remains in $\mathrm{Re}[A_{\mathrm{DT}}/A_{\mathrm{1L}}]$

First (second) derivate of in $\mathrm{Re}[A_{\mathrm{DT}}/A_{\mathrm{1L}}]$ for $\Sigma_c\bar{D}$ $(\Sigma_c^*\bar{D})$ seems divergent

→ qualitatively different singular behaviors between leading and lower-order singularity

Comparison with LHCb data



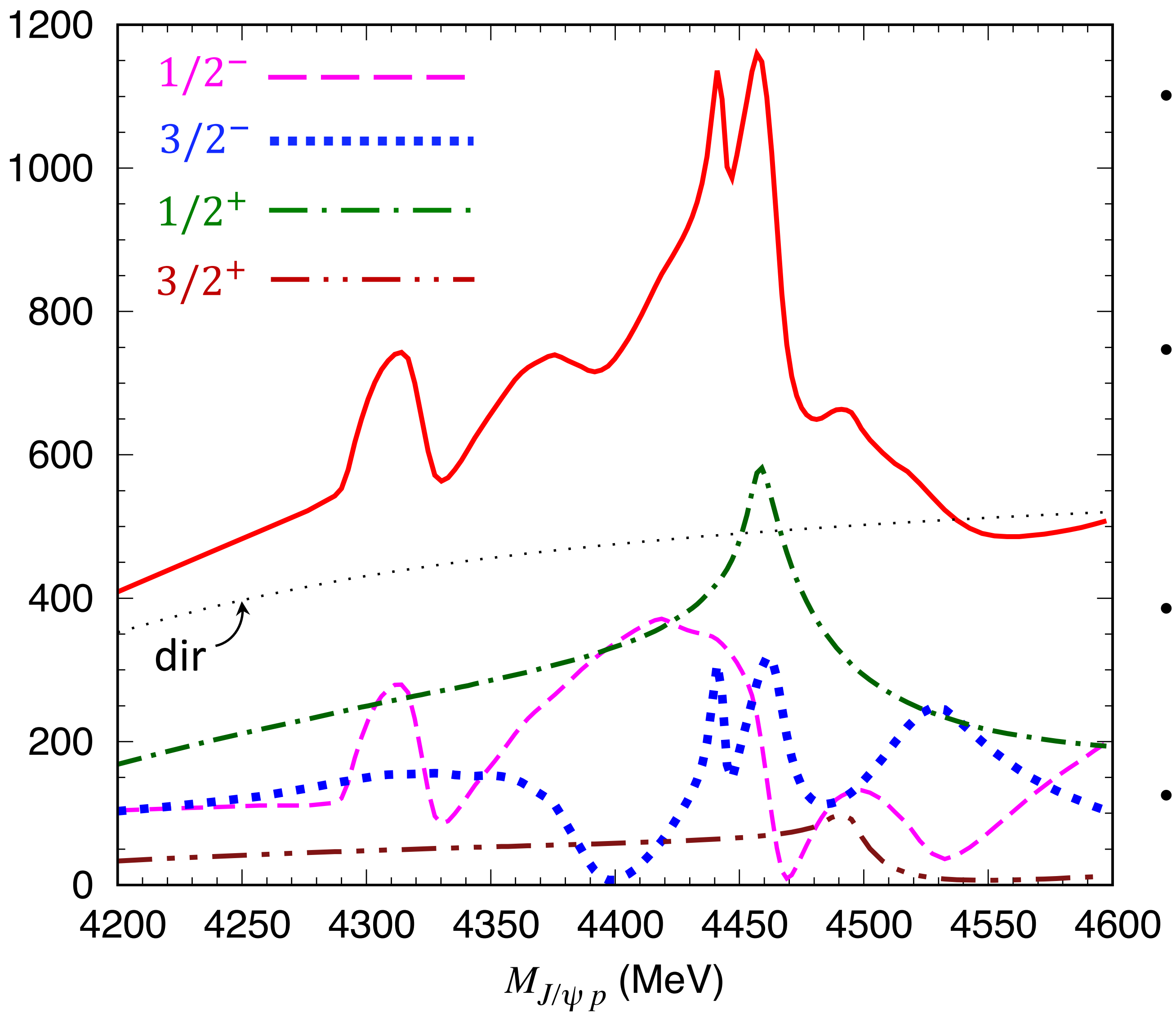
(smeared by exp. resolution)

- Pc(4312), Pc(4380), Pc(4457) peaks are well described by kinematical effects; not by poles
- $\Lambda_c\bar{D}^*$ and $\Lambda_c(2625)\bar{D}$ threshold cusps fit the data
- Pc(4440) requires a resonance pole ($J^P = 3/2^-$ in figure)
- Similar fit quality when changing cutoff over $0.8 - 2$ GeV and changing $J^P = 1/2^\pm, 3/2^\pm$ for Pc(4440)
- Simplified model works fairly well

  $J^P = 1/2^+, 3/2^+$ amplitudes omitted

  perturbative treatment of $Y_c\bar{D}^{(*)} \to J/\psi\, p$

Partial wave decomposition



- Interference of DT, direct-decay, and one-loop amplitudes

  → Pc(4312), Pc(4380), and Pc(4457) peak structures in $1/2^-$ and $3/2^-$ contributions

- Constructive interference between $\Lambda_c(2593)\bar{D}$ one-loop and direct-decay amplitudes → relatively large $1/2^+$contribution ($\Lambda_c\bar{D}^*$ >> $\Lambda_c(2593)\bar{D}$ one-loop amplitudes in magnitude)

- Direct-decay amplitudes (not fitted to data) alone give phase-space-like distribution

- Limited experimental information ($M_{J/\psi\,p}$ distribution only)

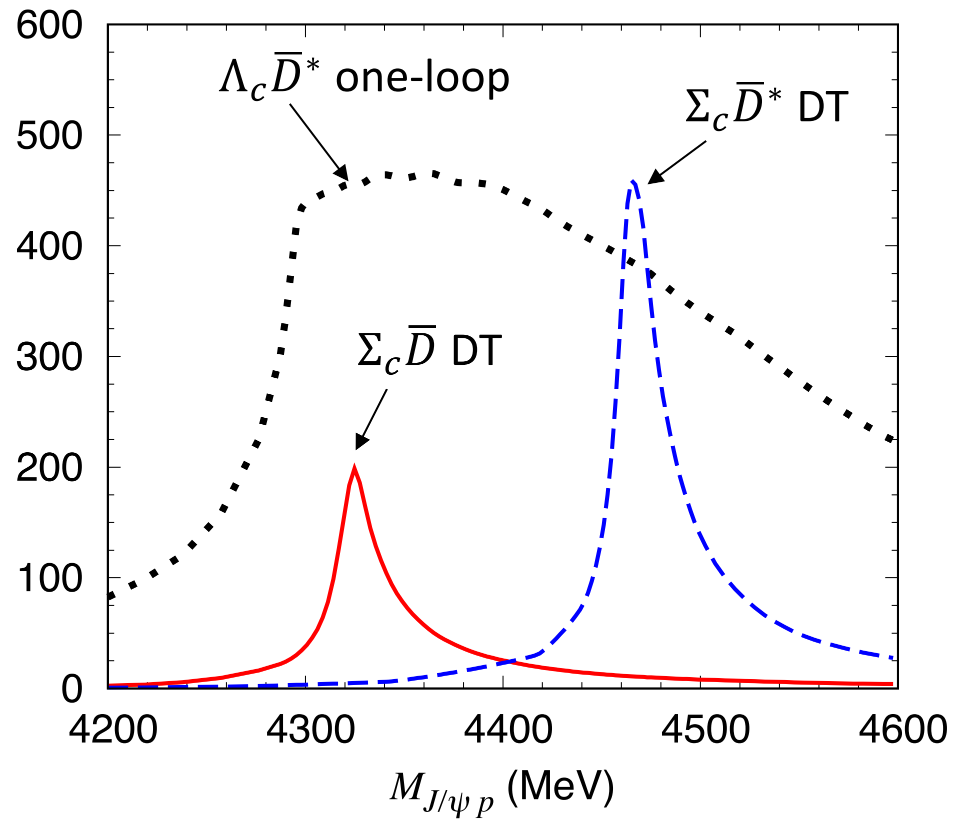
  → uncertainty in the partial wave decomposition

FAQ : Isn’t two-loop amplitude normally suppressed compared to one-loop ? (Therefore your model seems strange)

Ans. When a kinematical singularity occurs, the situation is not very normal.

→ something unusual can happen

One-loop and DT contributions (no interference)



At singularity peaks, DT are comparable to one-loop contribution

Otherwise, DT is suppressed compared to one-loop, as usual

Coupling ratio of $\Sigma_c\bar{D}^*$ DT to $\Lambda_c\bar{D}^*$ one-loop

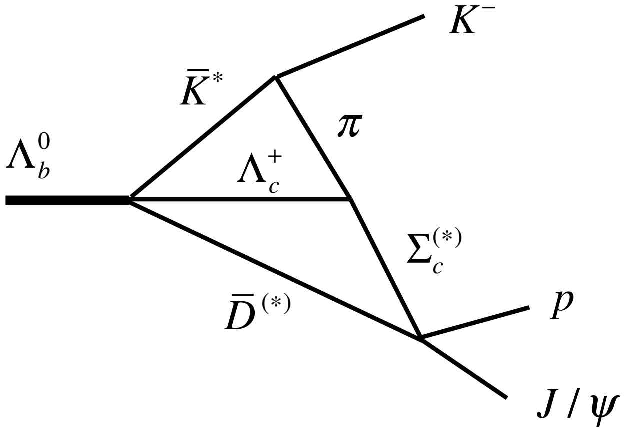
$$R \equiv \left| \frac{c_{\Lambda_c\,\bar{D}\bar{K}^*,\Lambda_b} \times c^{1/2^-}_{\psi p,\Sigma_c\bar{D}}}{c_{\Lambda_c\,\bar{D}^*\bar{K},\Lambda_b} \times c^{1/2^-}_{\psi p,\Lambda_c\,\bar{D}^*}} \right| = 7.2 - 3.2 \quad \text{for} \quad \Lambda = 0.8 - 2\text{ GeV}$$

Unreasonably large coupling ($R \gg 1$) for DT amplitude is not used

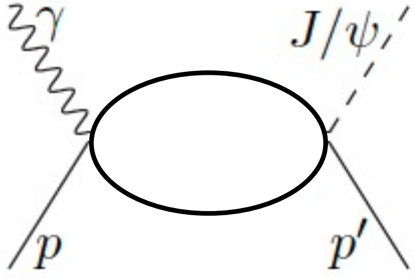
→ Comparable DT singularity peak and one-loop is not artifact

J/ψ photoproduction

DTS scenario of Pc can (partly) explain no Pc signals in J/ψ photoproduction data of GlueX



cannot be accommodated by



(no K^- in
the final state)

→ No Pc signal

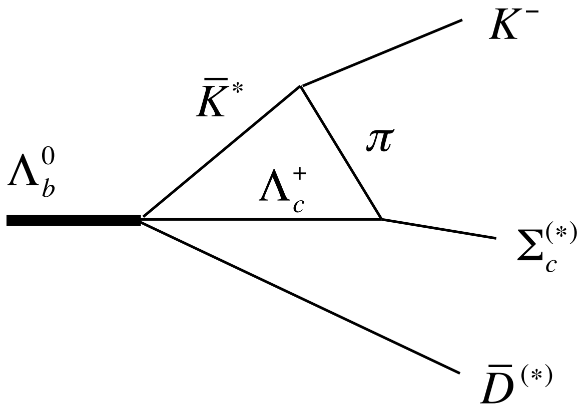
Pc(4440)

Pc(4440) width and strength extracted in this work are significantly smaller than those of LHCb analysis

→ Finding Pc(4440) signal in J/ψ photoproduction is more challenging than expected based on the LHCb result

Next step

- Study of $\Lambda_b^0 \rightarrow \Sigma_c^{(*)}\, \bar{D}^{(*)}\, K^-$ decays and Pc structures

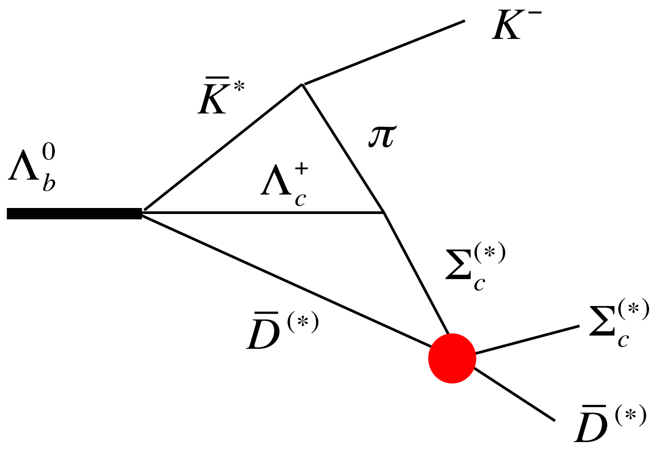


→

TS peak expected at

$M_{\Sigma_c K^-} \sim$ 3.4 GeV and

$M_{\Sigma_c^* K^-} \sim$ 3.2 GeV



Coupled-channel

$Y_c\ \bar{D}^{(*)}$ scattering

need developed

- Understand other resonance-like structures near thresholds with DTS

DTS should now be a possible option