Heavy quark expansion for inclusive semileptonic charm decays

10th International Workshop on Charm Physics

Matteo Fael | 2 June 2021
based on hep-ph/1910.05234, 2005.06487, 2011.11655, 2011.13654

Heavy Quark Expansion for B mesons

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From Inclusive Decays

- $\bar{B} \to X_c \ell \bar{\nu}$ with $X_c = D, D^*, D\pi, DKK, \ldots$
- Global fit in the kinetic and 1S scheme.

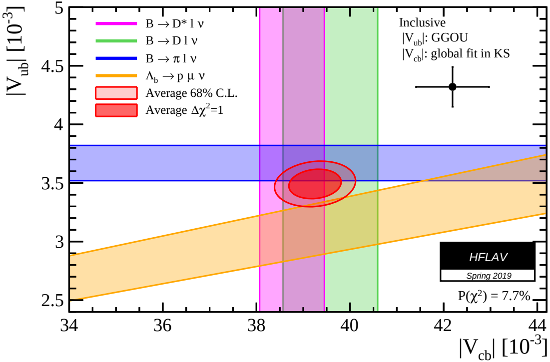
$$|V_{cb}^{\mathrm{inc}}| = (42.19 \pm 0.78) \times 10^{-3}$$

Gambino et al, PRL 114 (2015) 061802
Bauer, Ligeti, Manohar, Trott, Phys.Rev.D 70 (2004) 094017

From Exclusive Decays

- $\bar{B} \to D \ell \bar{\nu}$
- $\bar{B} \to D^* \ell \bar{\nu}$

$$|V_{cb}| = (39.25 \pm 0.56) \times 10^{-3} \quad \text{(LQCD, CLN)}$$



HFLAV '19

The Heavy-Quark Expansion

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$$\Gamma_{\mathrm{sl}} = \mathcal{C}_0 + \mathcal{C}_{\mu_\pi} \frac{\mu_\pi^2}{m_Q^2} + \mathcal{C}_{\mu_G} \frac{\mu_G^2}{m_Q^2} + \mathcal{C}_{\rho_D} \frac{\rho_D^3}{m_Q^3} + \mathcal{C}_{\rho_{LS}} \frac{\rho_{LS}^3}{m_Q^3} + \ldots$$

- Wilson coefficients $\mathcal{C}_i$ are calculable in pQCD
- Non-perturbative HQE parameters $\mu_\pi, \mu_G, \rho_D, \rho_{LS} \sim \langle H | \mathcal{O}_i | H \rangle$.
- Global fits of semileptonic B decays determine $|V_{cb}|$, m_b, m_c and the HQE parameters.

The Heavy-Quark Expansion

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	tree	α_s	α_s^2	α_s^3	
1	✓	✓	✓	✓	Jezabek, Kuhn, NPB 314 (1989) 1; Gambino et al., NPB 719 (2005) 77; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015. **MF, Schönwald, Steinhauser, hep-ph:2011.13654**;
$1/m_Q^2$	✓	✓	!		Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PRD 92 (2015) 054025. Becher, Boos, Lunghi, JHEP 0712 (2007) 062.
$1/m_Q^3$	✓	✓			Mannel, Pivovarov, PRD 100 (2019) 9.
$1/m_Q^{4,5}$	✓				Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087, JHEP 1011 (2010) 109 **MF, Mannel, Vos, JHEP 02 (2019) 177, JHEP 12 (2019) 067.**
$\overline{m}_Q - m_Q^{\mathrm{kin}}$		✓	✓	✓	Bigi et al, PRD 56 (1997) 4017; Czarnecki, Melnikov, Uraltsev, PRL 80 (1998) 3189. **MF, Steinhauser, Schönwald, PRL 125 (2020) 5; PRD 103 (2021) 014005**

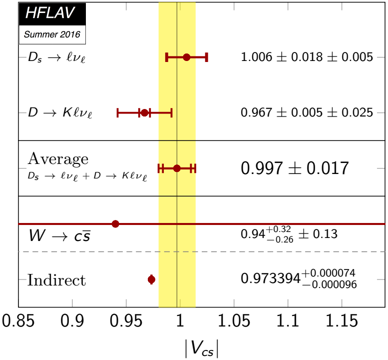
Can we play the same game with D mesons?

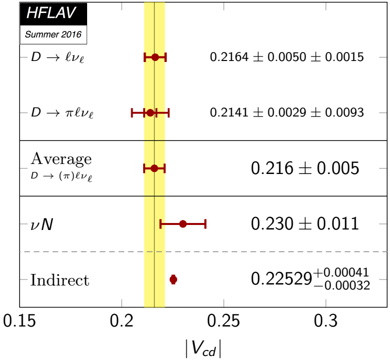
From Exclusive Decays

- $\bar{D} \to \pi \ell \bar{\nu}$
- $\bar{D} \to K \ell \bar{\nu}$

$$|V_{cs}| = 0.997 \pm 0.017$$
$$|V_{cd}| = 0.216 \pm 0.005$$

HFLAV 2016





Error in semilep. decays is dominated by the uncertainty of the form factors.

Inclusive Measurements

CLEO coll. PRD 81 (2010) 052007
BESIII, hep-ex/2104.07311

$D \to X\ell^+\nu_\ell$: Motivations

Pros:

- Exploit the full physics potential of BES III, LHCb and future STC factories.
- $D \to X\ell\nu$ is more sensitive to higher $1/m_Q$ corrections.
- Effective theory for $c \to q\ell\nu$ is more similar to $b \to u\ell\nu$ than $b \to c\ell\nu$.

End point region: $\Lambda_{\mathrm{QCD}}/(m_Q - 2E_\ell)$

- Important to constrain **Weak Annihilation (WA)** contributions and reduce the theoretical uncertainty in $B \to X_{d,s}\ell\bar{\ell}$ and $B \to X_u\ell\bar{\nu}_\ell$.

  Huber et al. JHEP 1910 (2019) 228

Contra: (aka very severe challenges!)

- Valance and non-valance WA operators at higher orders.
- Radiative corrections.
- Charm mass definition.

Can we apply HQE to charm?

- Lifetimes

	$D^0(\bar{u}c)$	$D^+(\bar{d}c)$	$D_s^+(\bar{s}c)$
mass [MeV]	1.86	1.87	1.97
τ [10^{-12} s]	0.4	1.0	0.5
$\tau/\tau(D^0)$	1	2.5	1.2

theory review A. Lenz, hep-ph/1305.3588, 1405.3601
see also Shifman, Voloshin, in *Heavy Quarks* ed. (1981); Shifman, Voloshin, Sov. J. Nucl. Phys. 41(1985) 12; Bilic, Guberina, Trampetic, NPB 248(1984) 261; Bigi, hep-ph/9311206; Bigi, Uraltsev, Z. Phys. C62 (1994) 623.

- Inclusive semileptonic decays

$$\Gamma(D^+ \to Xe^+\nu_e)/\Gamma(D^0 \to Xe^+\nu_e) = 0.985 \pm 0.015 \pm 0.024$$

$$\Gamma(D_s^+ \to Xe^+\nu_e)/\Gamma(D^0 \to Xe^+\nu_e) = 0.828 \pm 0.051 \pm 0.025$$

CLEO coll. PRD 81 (2010) 052007

$$\Gamma(D_s^+ \to Xe^+\nu_e)/\Gamma(D^0 \to Xe^+\nu_e) = 0.790 \pm 0.016 \pm 0.020$$

BESIII hep-ex/2104.07311

Can we apply HQE to charm?

- Breakdown of quark hadron duality?

  Blok, Dikeman, Shifman, hep-ph/9410293

- Applicability of HQE & non factorizable terms
  Falk, Luke, Savage, hep-ph/9511454;
  Voloshin hep-ph/0202028.

- Weak Annihilation (WA) estimates in semileptonic D meson decays
  Ligeti, Luke, Manohar, hep-ph/1003.1351;
  Gambino, Kamenik, hep-ph/1004.0114

- Ongoing pilot studies of inclusive decays on the lattice.

  Hashimoto, Gambino, Phys.Rev.Lett. 125 (2020) 032001
  Gambino, Melis, Simula, Phys.Rev.D 96 (2017) 014511
  Hansen, Meyer, Robaina, Phys.Rev.D 96 (2017) 9, 094513

Outlook

- Heavy Quark Expansion for Charm

  MF, Mannel, Vos, JHEP 12 (2019) 067

- The heavy-quark kinetic mass to $O(\alpha_s^3)$

  MF, Schönwald, Steinhauser, Phys.Rev.Lett. 125 (2020) 052003, Phys.Rev.D 103 (2021) 014005

- Semileptonic decays to third order in pQCD

  MF, Schönwald, Steinhauser, hep-ph/2011.13654

Heavy Quark Expansion for $D \to X\ell\nu$

- Goal:
  - semileptonic total rate $\Gamma_{\rm sl}\left[V_{\rm CKM}, m_c, m_q, \alpha_s, \mu_\pi, \mu_G, \ldots\right]$
  - moments of kinematic distributions $M_{\rm th}^{(n)}\left[m_c, m_q, \alpha_s, \mu_\pi, \mu_G, \ldots\right]$

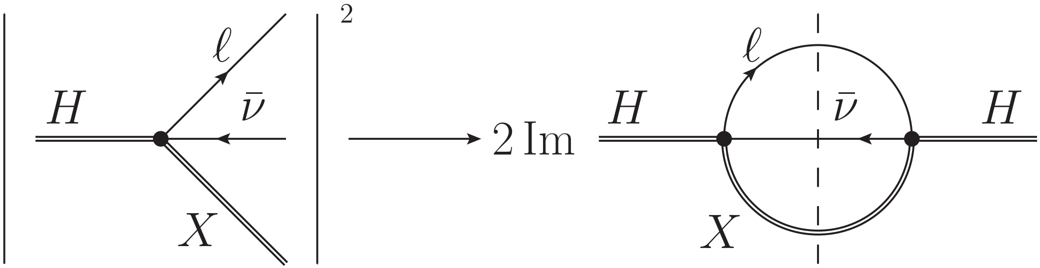
- $D \to X_q\ell\nu$ is not a copy of $B \to X_c\ell\nu$!
- How to systematically deal with IR sensitivity on m_q: Large

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 16\log\left(\frac{m_q}{m_Q}\right) + \ldots\right]\frac{\rho_D}{m_b^3}$$

well-known for $b \to u\ell\nu$, solution: include WA contribution

Bigi, uraltsev, hep-ph//9310285; Bigi, Uraltsev, Zwicky, hep-ph/0511158;
Breidenbach, Feldmann, Mannel, Turczyk, hep-ph/0805.0971; Bigi, Mannel, Turczyk, Uraltsev, hep-ph/09.11.3322;

Inclusive Decays



$$\sum_X |\langle X| \mathcal{H}_{\rm eff}(0) |H\rangle|^2 = 2\,\mathrm{Im} \int d^4x\, e^{-iq\cdot x} \langle H| T\{\mathcal{H}^\dagger_{\rm eff}(x), \mathcal{H}_{\rm eff}(0)\} |H\rangle$$

Operator Product Expansion

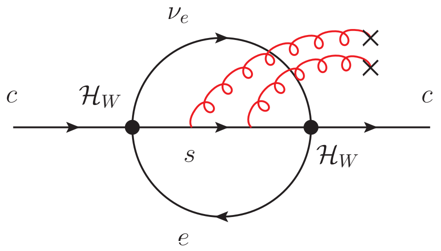
- Heavy-quark $p_Q \sim$ Heavy-Meson p_H

$$p_Q = m_Q v + k, \quad v = \frac{p_H}{m_H}$$

- Rephase heavy-quark field

$$Q(x) = e^{-i m_Q v \cdot x} Q_v(x)$$
$$iD_\mu Q(x) = (m_Q v_\mu + iD_\mu) Q_v(x)$$

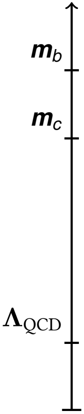
$2\,\mathrm{Im}$



- OPE: expand the propagator of interemdiate quark

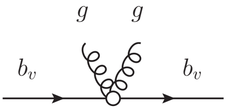
$$S = \frac{1}{m_Q \not{v} + \not{k} - \not{q} - m_q}$$

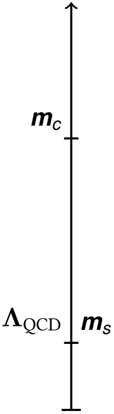
$b \to c\ell\bar{\nu}$ vs $c \to s\ell^+\nu_\ell$



$$\sum_{n,i} \frac{C_{i,n}(\frac{m_c}{m_b})}{m_b^n} \langle B | O^{2q} | B \rangle$$

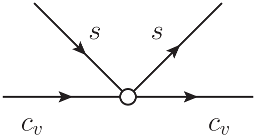
$C_2 \times$





$$\sum_{n,i} \frac{C_{i,n}(\frac{\mu}{m_c})}{m_c^n} \langle D | O^{2q} | D \rangle$$

$$+ \frac{C_{j,n}}{m_c^n} \langle D | O^{4q} | D \rangle$$



HQE for charm decays

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$$\Gamma = \Gamma_0 + \Gamma_{(2,1)} \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^2 + \Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + \Gamma_3 \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^3 + \Gamma_{(4,1)} \left(\frac{\Lambda_{\mathrm{QCD}}}{m_c}\right)^4 + \Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

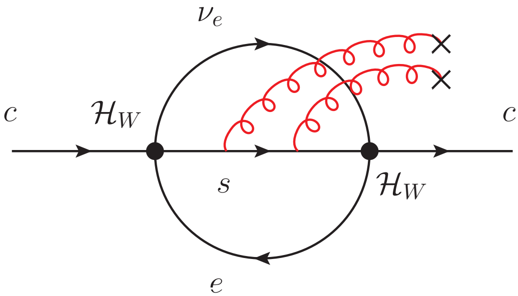
- Higher power corrections $\mu_G, \mu_\pi, \rho_D, \rho_{LS}$ etc.
- Weak annihilation contributions: $\langle D | O^{4q} | D \rangle$.
- Phase-space logarithms $m_s^n \log(m_s/m_c)$ reabsorbed into WA.

Bauer, Falk, Luke hep-ph/9604290

Calculation of higher power corrections

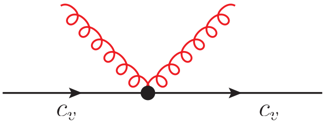
We calculate the HQE directly for total width $\Gamma_{\rm sl}$ and kinematic moments $\langle M^{(n)} \rangle$

$2\,\mathrm{Im}$

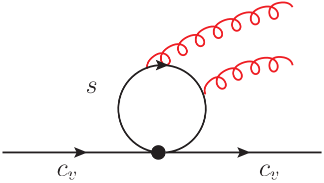


The imaginary part of such diagram has an OPE of the form

$C^{2q}(\mu) \times$

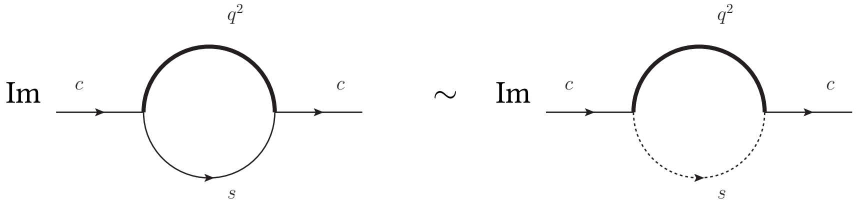


$+\, C^{4q}(\mu) \times$



- To perform the OPE we borrow the formalism of expansion-by-region.

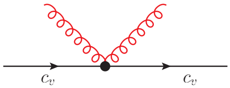
Beneke, Smirnov, NPB 522 (1998) 321; Smirnov Springer Tracts Mod. Phys. 250 (2010)

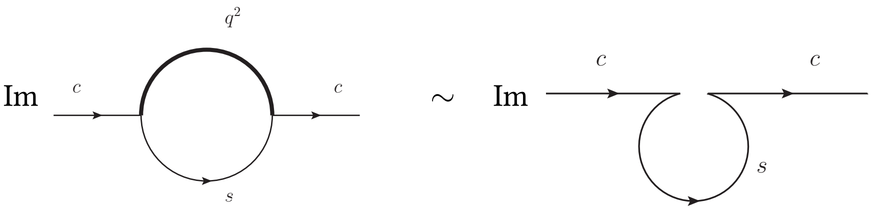


- The strange quark momentum is a **large scale** in $p_s \sim m_c \gg m_s$.

$$\int d^d p_s \frac{1}{(p_c - p_s)^2 - q^2} \frac{1}{p_s^2 - m_s^2} \sim \int d^d p_s \frac{1}{(p_c - p_s)^2 - q^2} \frac{1}{p_s^2} \sum_{n \geq 0} \left(\frac{m_s^2}{p_s^2} \right)$$

- The computation proceeds as for $b \to q \ell \nu$ with $m_q = 0$.
- The expansion gives power corrections in $\Lambda_{\rm QCD}/m_c$ and m_s/m_c.



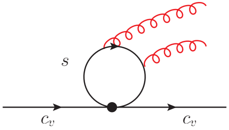


- Close to the q^2 end-point, the strange quark momentum is a **soft scale** in $p_s \sim m_s \ll m_c$.

$$\int d^d p_s \frac{1}{(p_c - p_s)^2 - q^2} \frac{1}{p_s^2 - m_s^2} \sim \frac{1}{p_c^2 - q^2} \int d^d p_s \frac{1}{p_s^2 - m_s^2}$$

- Taking the imaginary part we obtain $\delta(p_c^2 - q^2)$.
- It gives one-loop matrix elements of WA four-quark operators.
- Dimension-six $(1/m_c^3)$:

$$O_1 = (\bar{c}_v \slashed{v} P_L s)\,(\bar{s} \slashed{v} P_L c_v) \qquad O_2 = (\bar{c}_v \gamma^\mu P_L s)\,(\bar{s} \gamma_\mu P_L c_v)$$



- Four more operators at dimension-seven ($1/m_c^4$)

$$O_3 = \frac{1}{2}\Big[(\bar{c}_v\gamma^\mu P_L s)(v\cdot i\partial\, \bar{s}\gamma_\mu P_L c_v) - (v\cdot i\partial\, \bar{c}_v\gamma^\mu P_L s)(\bar{s}\gamma_\mu P_L c_v)\Big]$$

$$O_4 = (\bar{c}_v \not{v} P_L s)(i\partial^\mu\, \bar{s}\gamma_\mu P_L c_v) - (i\partial^\mu\, \bar{c}_v\gamma_\mu P_L s)(\bar{s}\not{v} P_L c_v)$$

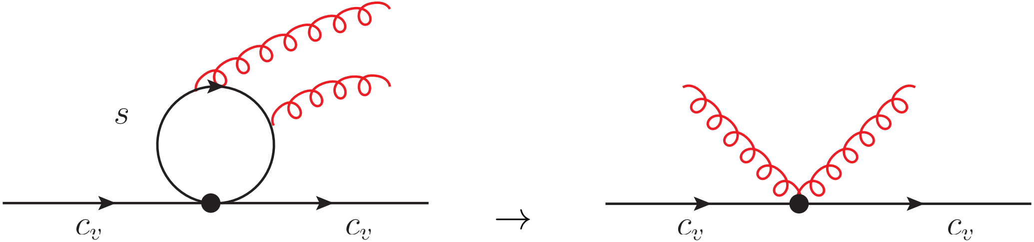
$$O_5 = \frac{1}{2}\Big[(\bar{c}_v\not{v} P_L s)(v\cdot i\partial\, \bar{s}\not{v} P_L c_v) - (v\cdot i\partial \bar{c}_v\not{v} P_L s)(\bar{s}\not{v} P_L c_v)\Big]$$

$$O_6 = \frac{1}{2}(-i\varepsilon_{\mu\nu\rho\alpha} v^\alpha)\Big[(\bar{c}_v\gamma^\mu P_L s)(i\partial^\rho\, \bar{s}\gamma^\nu P_L c_v) - (i\partial^\rho\, \bar{c}_v\gamma^\mu P_L s)(\bar{s}\gamma^\nu P_L c_v)\Big]$$

Operator Mixing

- We explicitly proved that all the logs come from WA operators mixing into two-quark operators:
- $\log(m_c/m_b)$ in $B \to X\ell\nu$ corresponds to $\log(\mu/m_c)$ in $D \to X\ell\nu$.

$$C_i^{2q}(\mu) = C_i^{2q}(m_c) + \log\left(\frac{\mu}{m_c}\right)\sum_j \hat{\gamma}_{ij}^T C_j^{4q}(m_c)$$



Additional HQE parameters for $c \to q$

- Hadronic matrix elements: $T_i \equiv \frac{1}{2m_D} \langle D| O_i^{4q} |D\rangle$
- RGE invariant combination:
  - Up to $1/m_c^3$:

$$\tau_0 = 128\pi^2 \left(T_1 - T_2 - 2\frac{T_3}{m_c} + \frac{T_4}{m_c} \right) + \log\left(\frac{\mu^2}{m_c^2}\right) \left[8\rho_D^3 + \frac{1}{m_c} \left(4\delta\rho_D^4 + \frac{16}{3} r_G^4 - \frac{16}{3} r_E^4 + \frac{8}{3} s_E^4 - \frac{1}{3} s_{qB}^4 - 12 m_s^4 \right) \right]$$

  - Up to $1/m_c^4$:

$$\tau_n = -128\pi^2 (T_3 - T_5) + \log\left(\frac{\mu^2}{m_c^2}\right) \left(+r_G^4 - 4r_E^4 + s_B^4 + \frac{2}{3} s_E^4 + \frac{1}{6} s_{qB}^4 - 3m_s^4 - 2m_s^2\mu_G^2 \right)$$

$$\tau_\epsilon = -64\pi^2 T_6 + \log\left(\frac{\mu^2}{m_c^2}\right) \left(+r_G^4 + \frac{4}{3} r_E^4 - \frac{1}{3} s_B^4 - \frac{2}{3} s_E^4 + \frac{1}{6} s_{qB}^4 - 3m_s^4 - 2m_s^2\mu_G^2 \right)$$

Total Semileptonic Rate

$$\Gamma_{\rm sl} = \frac{G_F^2 m_c^5 |V_{\rm CKM}|^2}{192\pi^3}\left[\left(1 - 8\frac{m_s^2}{m_c^2}\right)\mu_3 + \left(-2 - 8\frac{m_s^2}{m_c^2}\right)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3}\right.$$
$$\left. + \frac{16}{9}\frac{r_G^4}{m_c^4} - \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}\right]$$

- Total rate and q^2 moments depend on a smaller set of HQE parameter due to RPI.
- Up to $1/m_c^3$ ($1/m_c^4$) one needs 4 instead of 5 (10 instead of 15) operators

  Mannel, Vos, hep-ph/1802.09409, Fael, Mannel, Vos, hep-ph/1812.07472

- We computed moments of various spectra. Similar structure for the HQE as for the total semileptonic rate.

Size of the WA operators

- Vacuum saturation approximation:

$$
\begin{aligned}
T_1 - T_2 &= (g^{\mu\nu} - v^\mu v^\nu) \langle D| (\bar{c}\gamma_\mu P_L s)(\bar{s}\gamma_\nu P_L c) |D\rangle \\
&\sim (g^{\mu\nu} - v^\mu v^\nu) \langle D| \bar{c}\gamma_\mu P_L s |0\rangle \langle 0| \bar{s}\gamma_\nu P_L c |D\rangle \\
&\sim (g^{\mu\nu} - v^\mu v^\nu) \times \frac{1}{4} f_D^2 M_D^2 v_\mu v_\nu = 0
\end{aligned}
$$

- The HQE for semileptonic *D* decays is not spoiled by WA contributions.

  Mannel, Moreno, Pivovarov, hep-ph/2103.02058

- Global fit of CLEO data gives WA contributions compatible with zero.
  Ligeti, Luke, Manohar, hep-ph/1003.1351;
  Gambino, Kamenik, hep-ph/1004.0114

Higher α_s corrections to $D \to X\ell^+\nu$

$$\Gamma_{\rm sl} = \frac{G_F^2 |V_{\rm CKM}|^2 (m_c^{\rm OS})^5}{192\pi^3} \left[1 - 2.41\left(\frac{\alpha_s}{\pi}\right) - 23.4\left(\frac{\alpha_s}{\pi}\right)^2 - 321.5\left(\frac{\alpha_s}{\pi}\right)^3\right] + O\left(\frac{1}{m_c^2}\right)$$

See: Bigi, Shifman, Uraltsev, Vainshtein PRD 50 (1994) 2234; Beneke, Braun, NPB 426 (1994) 301; Ball, Beneke, Braun, PRD 52 (1995) 3929; Melnikov, van Ritbergen, PLB 482 (2000) 99.

- Mass scheme change: $m_Q^{\rm OS} \to \tilde{m}_Q \left(1 + c\frac{\alpha_s}{\pi}\right)$

$$\Gamma_{\rm sl} \propto (\tilde{m}_Q)^n \left[1 + (nc + a_1)\left(\frac{\alpha_s}{\pi}\right) + \left(\frac{n(n+1)}{2}c^2 + nc\, a_1 + a_2\right)\left(\frac{\alpha_s}{\pi}\right)^2 + \dots\right]$$

- Can we resum the power enhanced $(n\alpha_s)^k$ terms (with $n = 5$)?

Meson-quark mass relation

$$m_Q = M_H - \overline{\Lambda} - \frac{\mu_\pi^2}{2m_Q} + \dots$$

- $\overline{\Lambda}$: the heavy-meson binding energy.
- μ_π: the kinetic energy induced by the residual motion of the heavy quark.

The relevant parameter in $\Gamma_{\rm sl}$ is $\boldsymbol{m_Q^5}$, **not** $\boldsymbol{M_H^5}$:

$$\Gamma_{\rm sl} \simeq \frac{G_F^2 |V_{\rm CKM}|^5}{192\pi^3} (M_H - \overline{\Lambda})^5$$

Bigi, Shifman, Uraltsev, Vainshtein, PRD 56 (1997) 4017

The kinetic mass

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$$m_Q^{\text{kin}}(\mu) = m_Q^{\text{OS}} - [\overline{\Lambda}(\mu)]_{\text{pert}} - \frac{[\mu_\pi^2(\mu)]_{\text{pert}}}{2m_Q^{\text{kin}}(\mu)} - \ldots$$

Bigi, Shifman, Uraltsev, Vainshtein, PRD 56 (1997) 4017.
see also: Czarnecki, Melnikov, Uraltsev, PRL 80 (1998) 3189;
Gambino, JHEP 09 (2011) 055;

- In pQCD, we can *peel off* the IR renormalon from the on-shell mass identifying:

$$m_Q(\mu) \to m_Q^{\text{kin}}(\mu) \qquad \overline{M}_H \to m_Q^{\text{OS}}$$

$$\overline{\Lambda}(\mu) \to [\overline{\Lambda}(\mu)]_{\text{pert}} \qquad [\mu_\pi^2(\mu)] \to [\mu_\pi^2(\mu)]_{\text{pert}}$$

- The perturbative version of $\overline{\Lambda}$ and μ_π^2 are obtained from Small Velocity sum rules.

The kinetic mass to $O(\alpha_s^3)$

$$\frac{m^{\rm kin}}{m^{\rm OS}} = 1 - \frac{\alpha_s^{(n_l)}}{\pi} C_F \left(\frac{4}{3}\frac{\mu}{m^{\rm OS}} + \frac{1}{2}\frac{\mu^2}{(m^{\rm OS})^2}\right) + \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^2 C_F \left\{\frac{\mu}{m^{\rm OS}}\left[C_A\left(-\frac{215}{27} + \frac{2\pi^2}{9} + \frac{22}{9} l_\mu\right) + n_l T_F \left(\frac{64}{27} - \frac{8}{9} l_\mu\right)\right]\right.$$
$$\left. + \frac{\mu^2}{(m^{\rm OS})^2}\left[C_A\left(-\frac{91}{36} + \frac{\pi^2}{12} + \frac{11}{12} l_\mu\right) + n_l T_F \left(\frac{13}{18} - \frac{1}{3} l_\mu\right)\right]\right\} + \left(\frac{\alpha_s^{(n_l)}}{\pi}\right)^3 C_F \left\{\frac{\mu}{m^{\rm OS}}\left[C_A^2\left(-\frac{130867}{1944}\right.\right.\right.$$
$$+ \frac{511\pi^2}{162} + \frac{19\zeta_3}{2} - \frac{\pi^4}{18} + \left(\frac{2518}{81} - \frac{22\pi^2}{27}\right) l_\mu - \left.\frac{121}{27} l_\mu^2\right) + C_A n_l T_F \left(\frac{19453}{486} - \frac{104\pi^2}{81} - 2\zeta_3\right.$$
$$\left. + \left(-\frac{1654}{81} + \frac{8\pi^2}{27}\right) l_\mu + \frac{88}{27} l_\mu^2\right) + C_F n_l T_F \left(\frac{11}{4} - \frac{4\zeta_3}{3} - \frac{2}{3} l_\mu\right) + n_l^2 T_F^2 \left.\left(-\frac{1292}{243} + \frac{8\pi^2}{81} + \frac{256}{81} l_\mu - \frac{16}{27} l_\mu^2\right)\right]$$
$$+ \frac{\mu^2}{(m^{\rm OS})^2}\left[C_A^2\left(-\frac{96295}{5184} + \frac{445\pi^2}{432} + \frac{57\zeta_3}{16} - \frac{\pi^4}{48} + \left(\frac{2155}{216} - \frac{11\pi^2}{36}\right) l_\mu - \frac{121}{72} l_\mu^2\right) + C_A n_l T_F \left(\frac{13699}{1296} - \frac{23\pi^2}{54}\right.\right.$$
$$\left.\left.\left. - \frac{3\zeta_3}{4} + \left(-\frac{695}{108} + \frac{\pi^2}{9}\right) l_\mu + \frac{11}{9} l_\mu^2\right) + C_F n_l T_F \left(\frac{29}{32} - \frac{\zeta_3}{2} - \frac{1}{4} l_\mu\right) + n_l^2 T_F^2 \left(-\frac{209}{162} + \frac{\pi^2}{27} + \frac{26}{27} l_\mu - \frac{2}{9} l_\mu^2\right)\right]\right\}, \quad (4)$$

MF, Schönwald, Steinhauser, PRL 125 (2020) 052003

- Wilsonian cutoff μ separates perturbative and non-perturbative regimes.
- The mass relation is written in terms of $\alpha_s^{(n_l)}$.
- n_l = number of **massless quarks**, $l_\mu = \log(2\mu/\mu_s)$.

Charm mass in the kinetic scheme

- $m_c^{\text{kin}}(0.5\text{ GeV})$:

$$
\begin{aligned}
m_c^{\text{kin}}(0.5\text{ GeV}) &= 993 + 191 + 100 + 52\text{ MeV} = 1336\text{ MeV}\,,\\
m_c^{\text{kin}}(0.5\text{ GeV}) &= 1099 + 163 + 76 + 34\text{ MeV} = 1372\text{ MeV}\,,\\
m_c^{\text{kin}}(0.5\text{ GeV}) &= 1279 + 84 + 30 + 11\text{ MeV} = 1404\text{ MeV}\,.
\end{aligned}
$$

- $m_c^{\text{kin}}(1\text{ GeV})$:

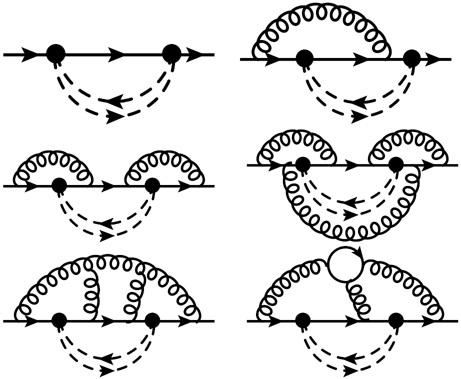
$$
\begin{aligned}
m_c^{\text{kin}}(1\text{ GeV}) &= 993 + 83 + 35 + 14\text{ MeV} = 1125\text{ MeV}\,,\\
m_c^{\text{kin}}(1\text{ GeV}) &= 1099 + 37 + 2 - 3\text{ MeV} = 1135\text{ MeV}\,,\\
m_c^{\text{kin}}(1\text{ GeV}) &= 1279 - 73 - 61 - 17\text{ MeV} = 1128\text{ MeV}\,,
\end{aligned}
$$

where from top to bottom we use as input
$\overline{m}_c(\mu_s)$ and $\alpha_s^{(3)}(\mu_s)$ at $\mu_s = 3\text{ GeV}, 2\text{ GeV}$ and $\overline{m}_c$.

N3LO corrections to semileptonic decay of heavy quark

$$\Gamma_{\rm sl} = \frac{G_F^2 m_Q^5 |V_{\rm CKM}|^2}{192\pi^3} \left[X_0 + C_F \sum_{n\geq 1} \left(\frac{\alpha_s}{\pi}\right)^n X_n \right]$$

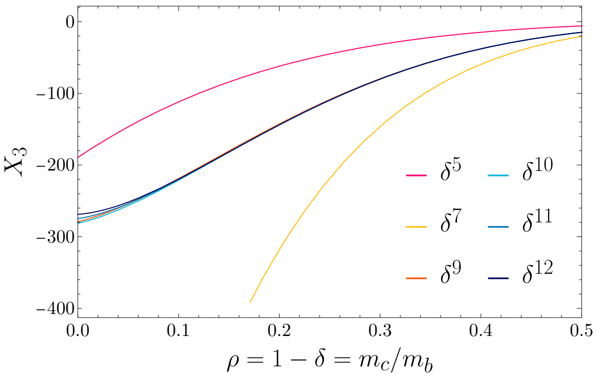
MF, Schönwald, Steinhauser, hep-ph/2011.13654



- We computed the third order corrections X_3 in pQCD.
- Expansion around equal mass limit

$$\rho = \frac{m_c}{m_b} \sim 0.25 \qquad \boxed{\delta = 1 - \frac{m_c}{m_b} \sim 0.75}$$

- Crucial factorisations in the $m_q \sim m_Q$ limit considerably simplify the calculation.
- We computed the first 8 terms in the δ-expansion.



MF, Schönwald, Steinhauser, hep-ph/2011.13654

$$C_F X_3 = \delta^5 \left[\frac{533858}{1215} - \frac{20992 a_4}{81} + \frac{8744\pi^2\zeta_3}{135} - \frac{6176\zeta_5}{27} - \frac{16376\zeta_3}{135} - \frac{2624 l_2^4}{243} + \frac{5344\pi^2 l_2^2}{1215} + \frac{179552\pi^2 l_2}{405} - \frac{39776\pi^4}{6075} - \frac{1216402\pi^2}{3645} \right] + O(\delta^6)$$

$$X_3^{b\to c} = -68.4 \pm 0.3$$
$$X_3^{b\to u} = -202 \pm 20$$
$$X_3^{c\to q} = -243 \pm 24$$

Charm semileptonic decay in the kinetic and 1S scheme

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- Kinetic scheme
  - $m_c^{\text{kin}}(0.5\text{ GeV}) = 1336\text{ MeV}$

$$\Gamma/\Gamma_0 = 1 + 1.09\frac{\alpha_s(m_c^{\text{kin}})}{\pi} + 13.7\left(\frac{\alpha_s(m_c^{\text{kin}})}{\pi}\right)^2 + 155(32)\left(\frac{\alpha_s(m_c^{\text{kin}})}{\pi}\right)^3$$

  - $m_c^{\text{kin}}(0.25\text{ GeV}) = 1456\text{ MeV}$

$$\Gamma/\Gamma_0 = 1 - 0.79\frac{\alpha_s(m_c^{\text{kin}})}{\pi} - 2.10\left(\frac{\alpha_s(m_c^{\text{kin}})}{\pi}\right)^2 + 20(32)\left(\frac{\alpha_s(m_c^{\text{kin}})}{\pi}\right)^3$$

- 1S scheme $m_c^{\text{1S}} = 1565\text{ MeV}$

$$\Gamma/\Gamma_0 = 1 - 1.22\frac{\alpha_s(m_c^{\text{1S}})}{\pi} - 2.42\left(\frac{\alpha_s(m_c^{\text{1S}})}{\pi}\right)^2 + 69(32)\left(\frac{\alpha_s(m_c^{\text{1S}})}{\pi}\right)^3$$

with $\Gamma_0 = G_F^2 m_c^5 |V_{CKM}|^2/(192\pi^3)$.

Conclusions

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- We studied the Heavy Quark Expansion for semileptonic charm decays.
- The OPE for charm requires new HQE parameters to take into account WA contributions.
  These can be combined into RGE invariant quantities.
- WA contributions are suppressed in semileptonic decays and should not spoil the original OPE, as in non-leptonic decays.
- We studied the charm quark mass in the kinetic scheme.
- Our calculation of N3LO corrections to $B \to X\ell\nu$ is applicable also to charm decay.
- The charm semileptonic width points towards the use of $1S$ scheme, however the third order corrections seem large but still have a large uncertainty.
- Kinetic scheme is a viable option at the price of choosing a very low value
  for the Wilsonian cutoff 0.25 GeV.