Lattice QCD inputs for leptonic and semileptonic charm decays + $R(D^{(*)})$

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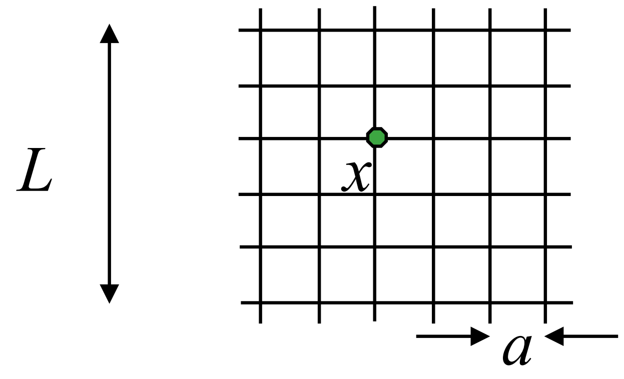
Outline

- Lattice QCD Introduction
- Leptonic D, D_s-meson decays
  - Decay constants
  - $|V_{cd}|$ and $|V_{cs}|$
- Semileptonic D, D_s-meson decays
  - Lattice QCD form factors
  - $|V_{cd}|$ and $|V_{cs}|$
  - 2nd row CKM unitarity test
- LFU ratios: $R(D), R(D^*), R(D_s), R(D_s^*), R(\Lambda_c), R(J/\psi)$
- Summary and Outlook

Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



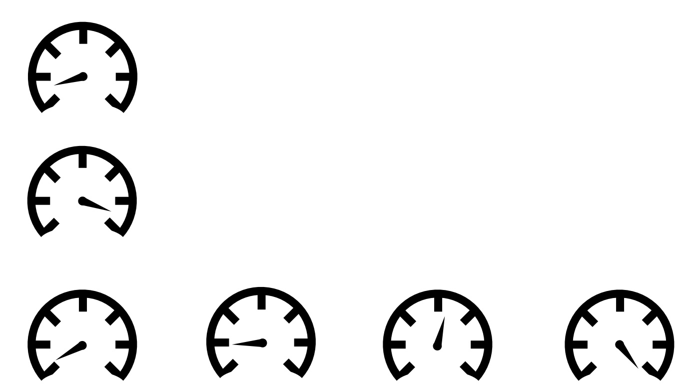
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➞ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$

  tune using hadron masses $m_f \to m_{f,\rm phys}$

  extrapolations/interpolations



m_{ud} m_s m_c m_b

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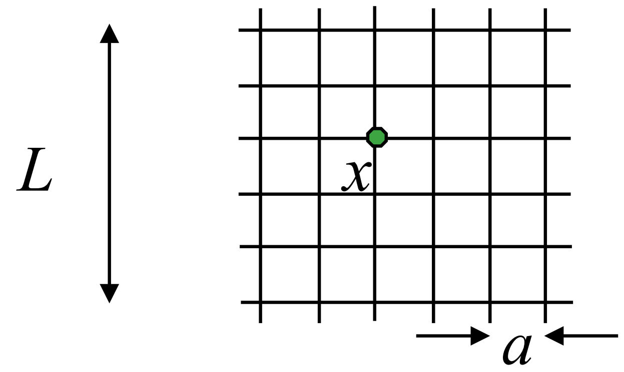

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Integrals are evaluated numerically using monte carlo methods.

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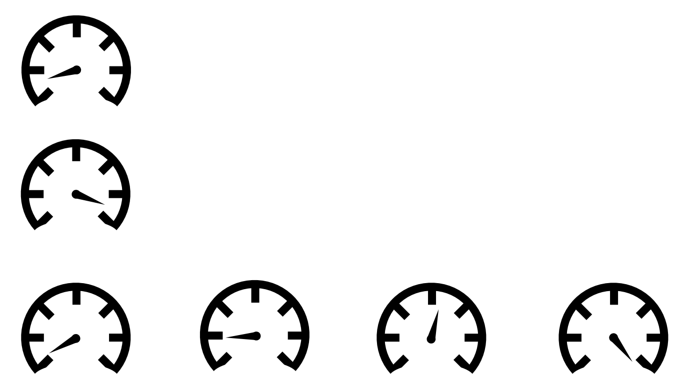

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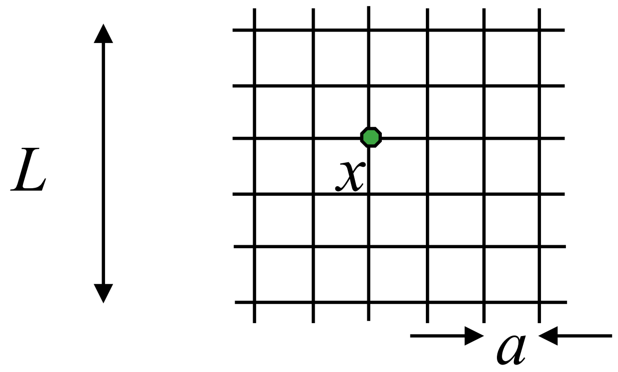

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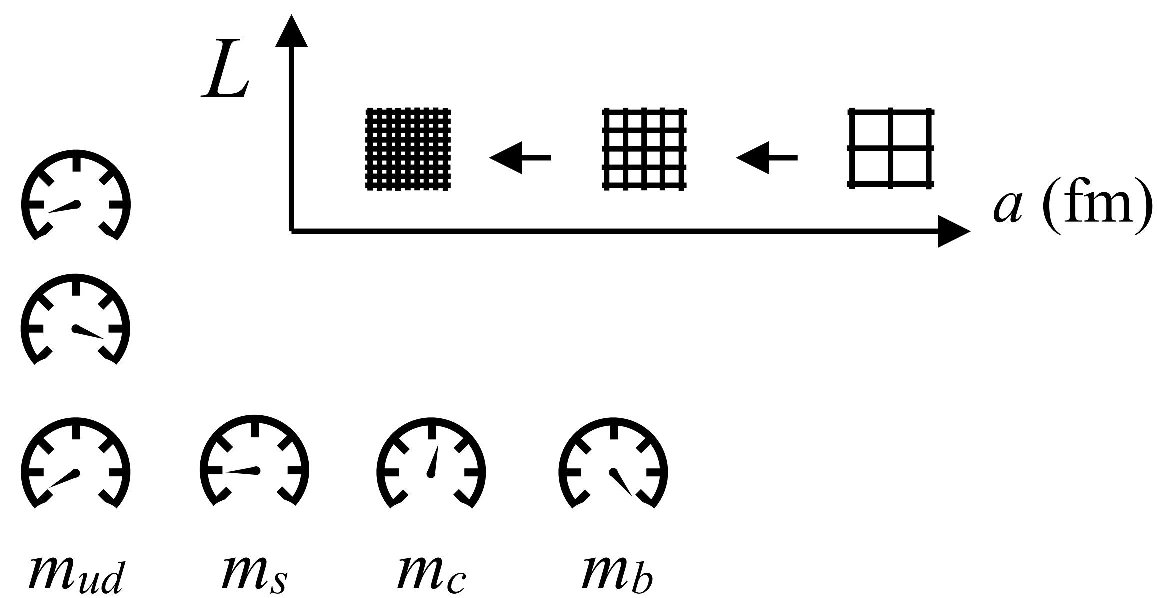
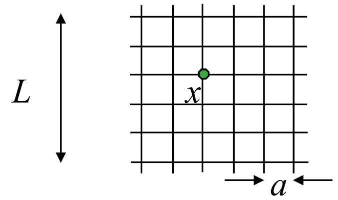
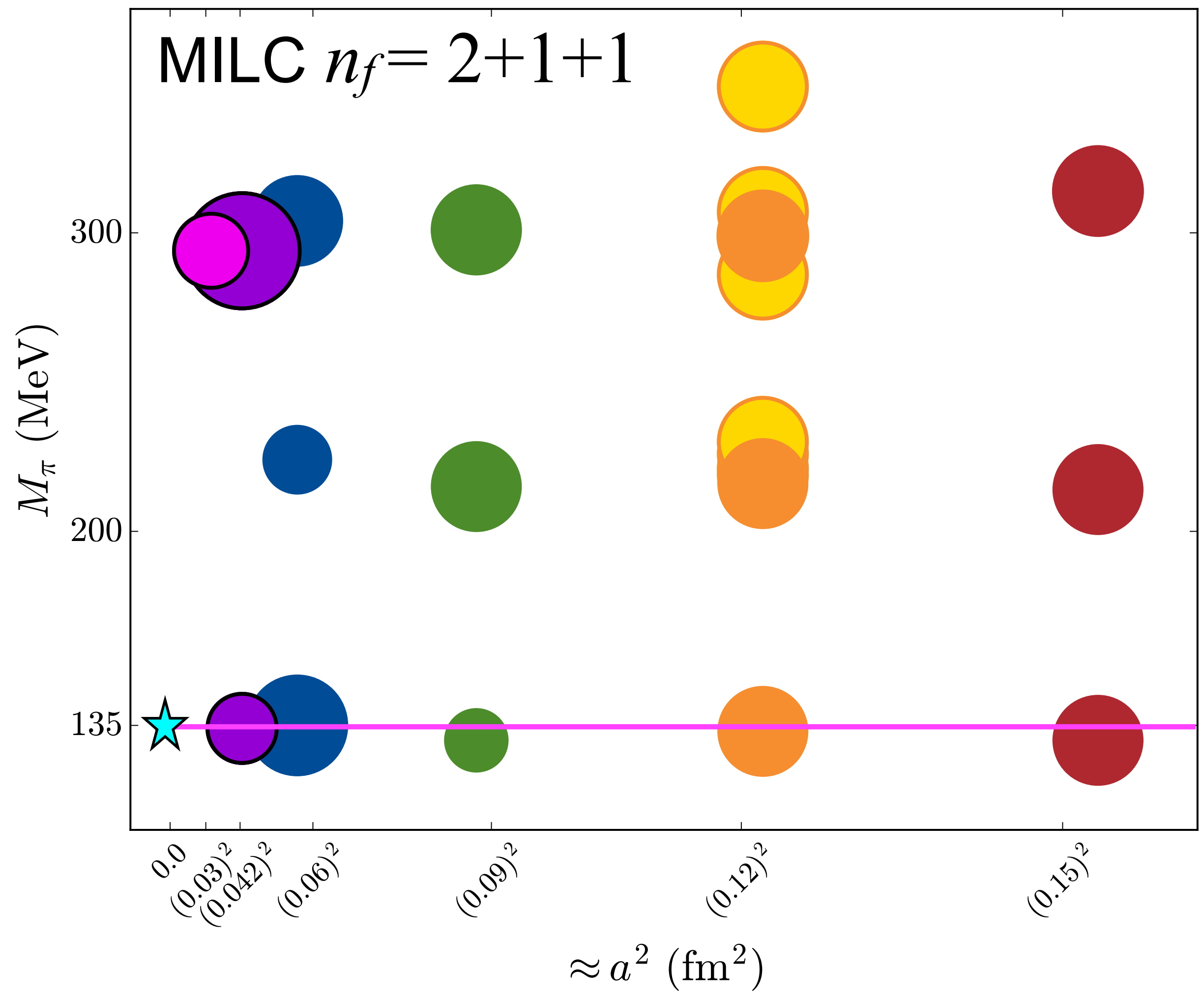


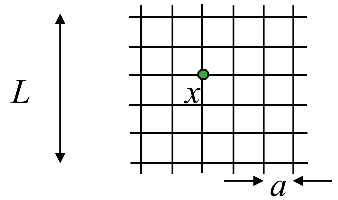


Lattice QCD Introduction

combined chiral-continuum interpolation/extrapolation



Growing number of collaborations have generated sets of ensembles that include sea quarks **with physical light-quark masses and use improved lattice actions:** PACS-CS, BMW, MILC, RBC/UKQCD, ETM,…



Lattice QCD Introduction

The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleons and other baryons
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance effects, …
- QED effects
- radiative decay rates …

Lattice QCD: Overview

[inspired by A. Kronfeld]

$a_\mu^{\text{HVP LO}}$ a_μ^{HLbL}

g_A, g_T, g_S

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K$ …

$\Lambda_b \to p, \Lambda_c, \Lambda$

nucleon form factors, ..

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi}$ $f_{+,0,T}^{B\to\pi}$ …

$f_{K^\pm}$ $f_{B_{(s)}}$ … $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\Delta M_K,\ \epsilon_K$

$\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

MEs for light nuclei

$B \to K^* \ell\ell \to K\pi\, \ell\ell$ …

$K^+ \to \ell^+ \nu\, (\gamma)$ …

$K^+ \to \pi^+ \ell^+ \ell^-$ …

$K^+ \to \pi^+ \nu\bar{\nu}$

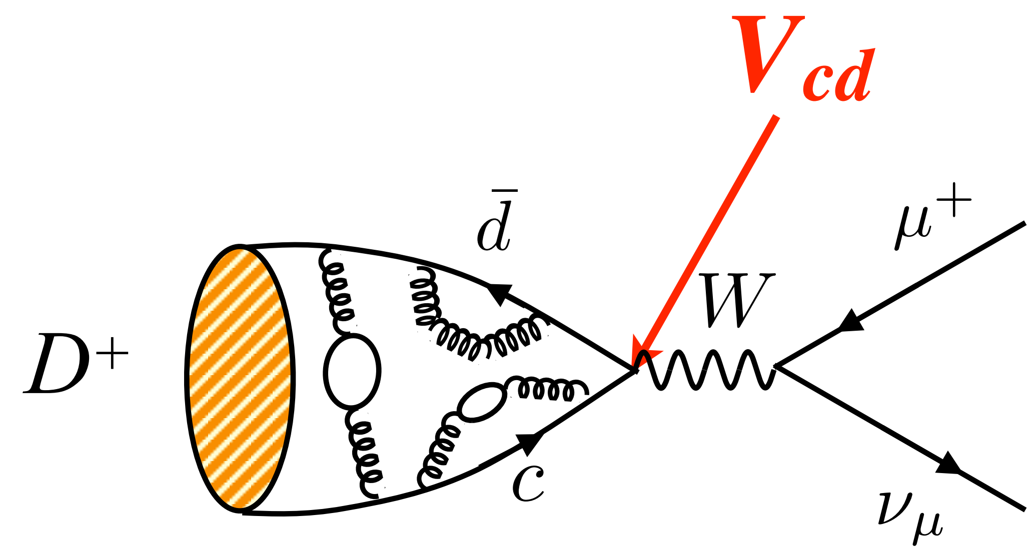
$B \to X_c \ell \nu$, other inclusive decay rates, …

Complexity →

LQCD flagship results	Complete LQCD results, large(ish) errors	First results, physical params, incomplete systematics	new methods, pilot projects, unphysical kinematics	new ideas, first studies

Leptonic D, D_s meson decay

example: $D^+ \to \mu^+ \nu_\mu$



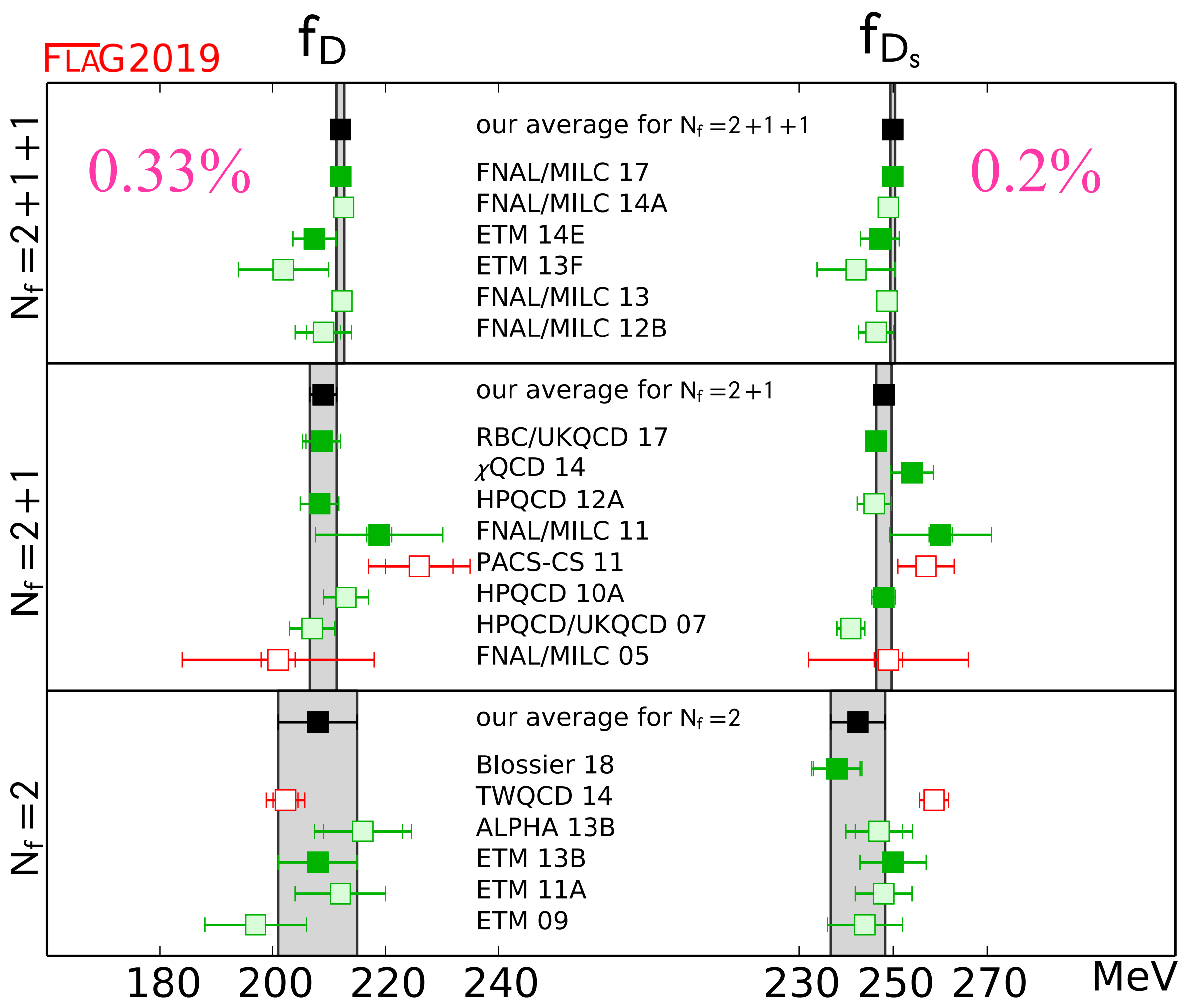
$$\Gamma(D^+ \to \mu^+ \nu_\mu (\gamma)) = (\text{known}) \times S_{\rm EW} (1 + \delta_{\rm EM}) \times |V_{cd}|^2 \times f_{D^+}^2$$

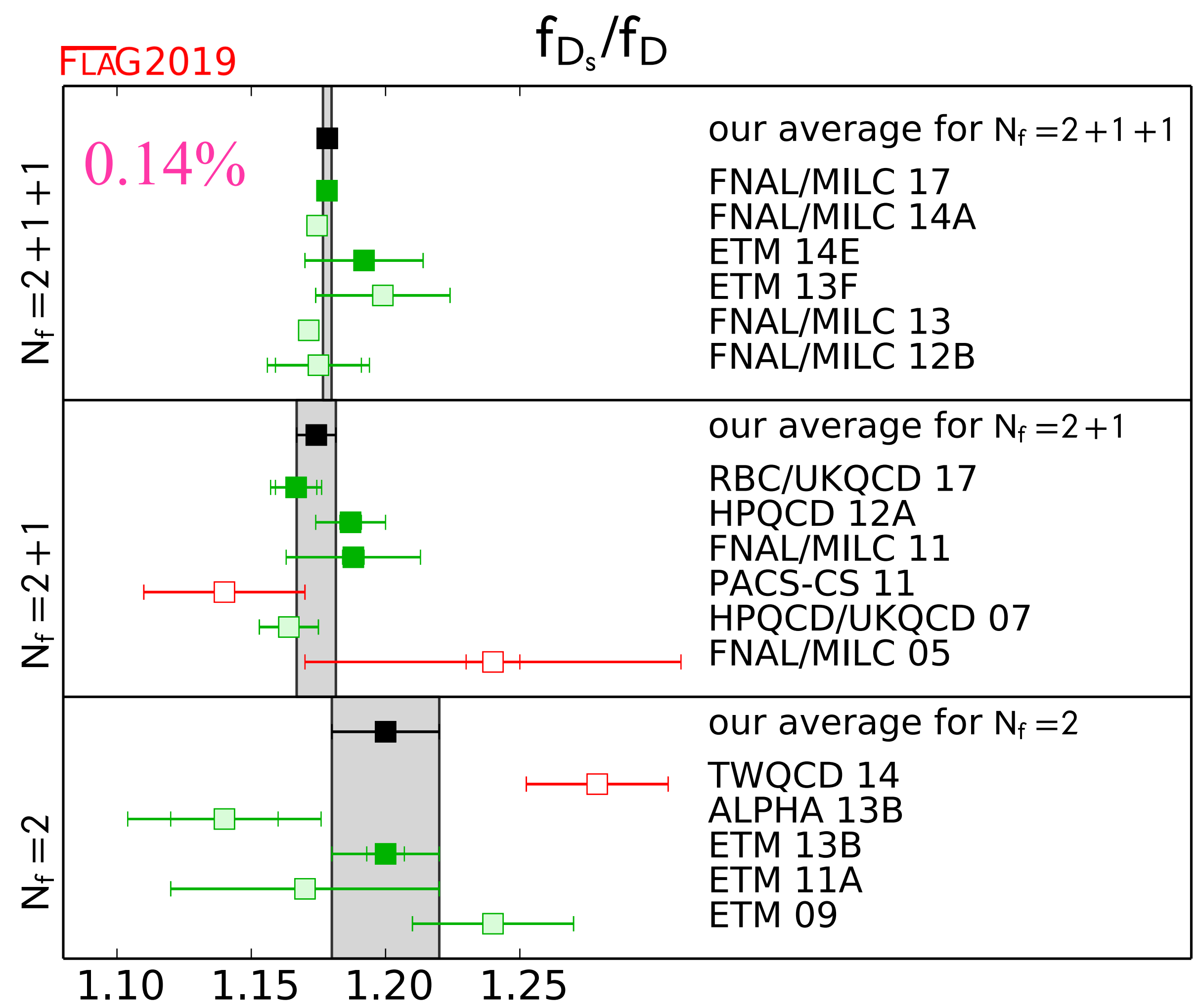
- use experiment + LQCD input (f_{D^+}) for determination of CKM element
- account for EW+EM corrections in the experimental rate
  - EW: [Sirlin, Nuc. Phys. 1982] ~ 1.8%
  - EM: Structure dependent: [Dobrescu+Kronfeld, PRL 2008] ~ 1%
    depends on photon energy cut
    Long distance: [Kinoshita, PRL 1959] ~ 2.4%
    ➠ removed with PHOTOS

PDG: 2.8% uncertainty

D, D_s meson decay constants

[S. Aoki et al FLAG 2019 review, 1902.08191, webupdate: flag.unibe.ch/2019/]





Small errors due to:

- **physical light quark masses**
- improved light-quark actions
- small lattice spacings
- NPR or no renormalization

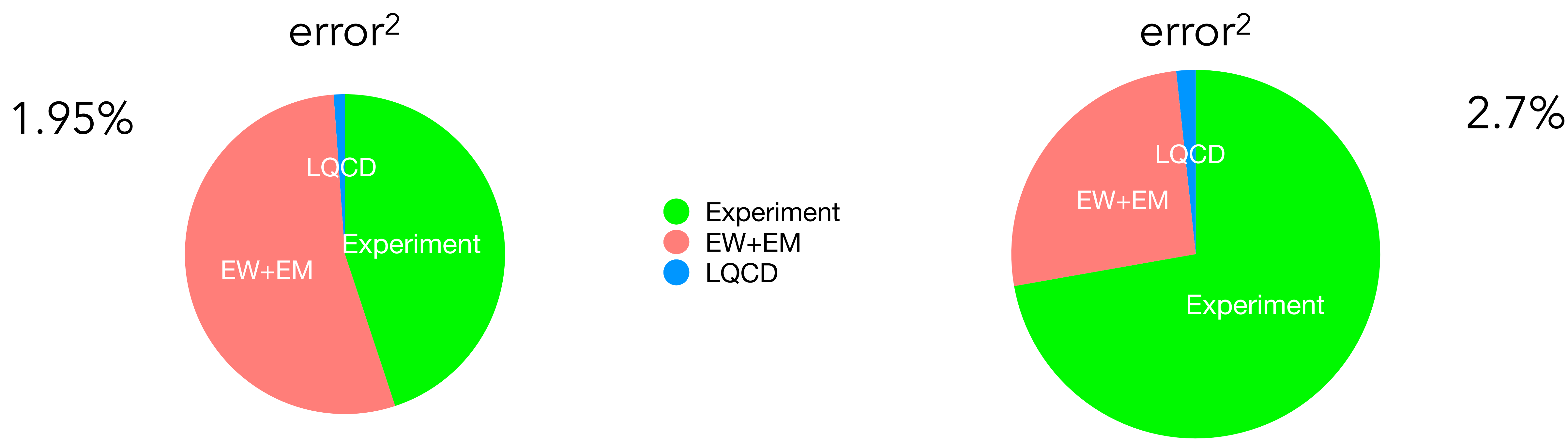
Consider strong isospin breaking effects to obtain f_{D^+}

Leptonic D, D_s meson decay

experimental averages [PDG 2019, Rosner, Stone, Van de Water]:

$$|V_{cs}| f_{D_s} = 245.7\,(3.1)_{\rm exp}(3.4)_{\rm (EW+EM)}\ {\rm MeV} \qquad |V_{cd}| f_{D^+} = 46.2\,(1.0)_{\rm exp}(0.6)_{\rm (EW+EM)}\ {\rm MeV}$$

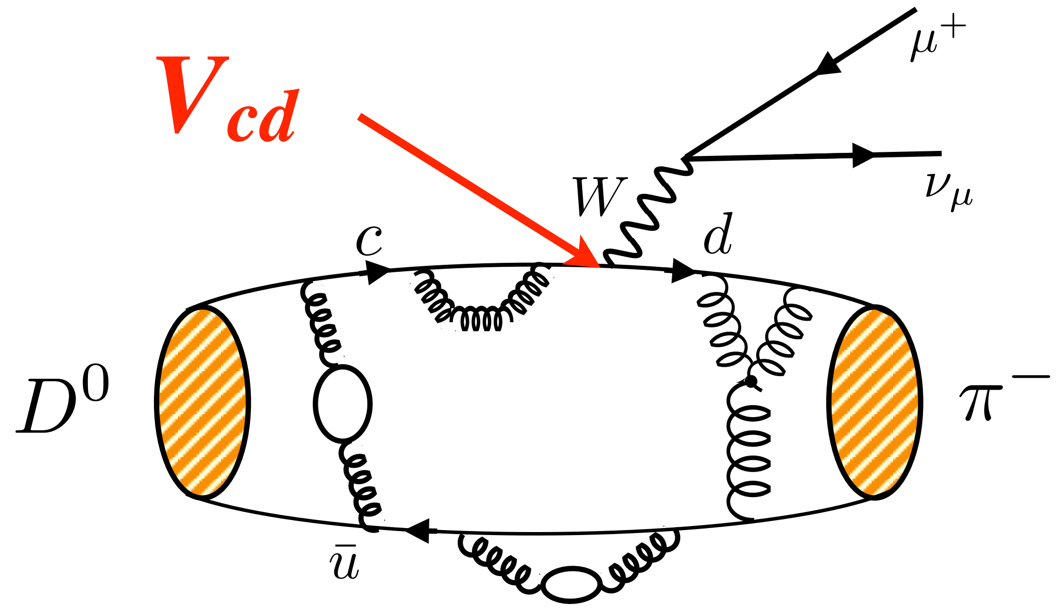
$$|V_{cs}| = 0.983\,(13)(14)(2) \qquad |V_{cd}| = 0.217\,(5)(3)(1)$$



2nd row CKM unitarity test: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.016(37)$

Semileptonic D, D_s meson decay

example: $D^0 \to \pi^- \mu^+ \nu_\mu$

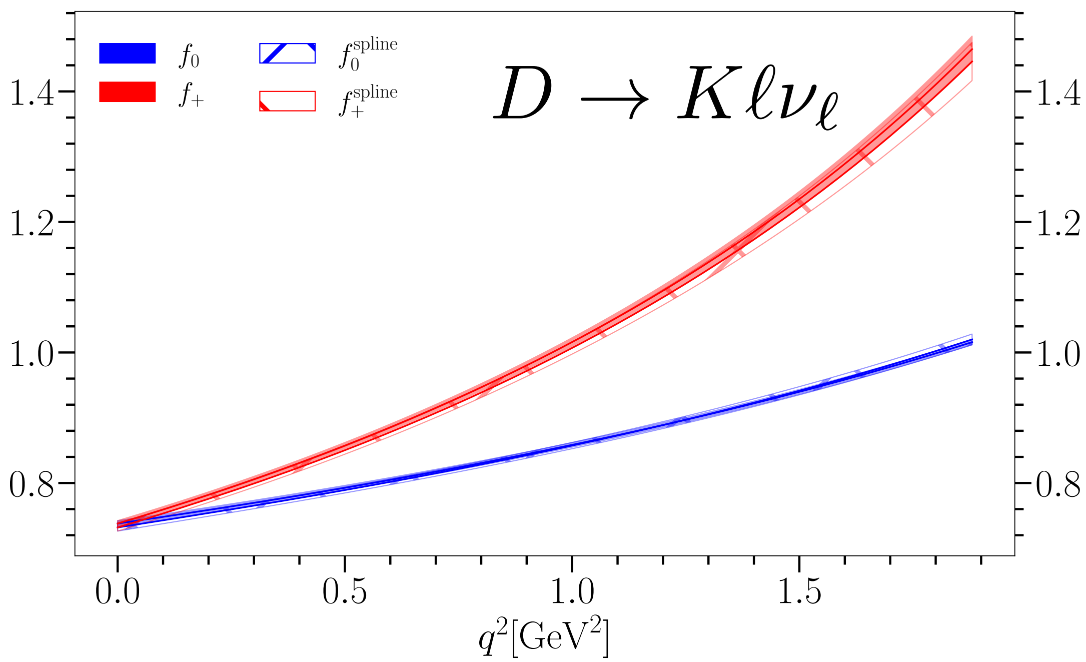


$$\frac{d\Gamma(D^0 \to \pi^- \mu^+ \nu_\mu (\gamma))}{dq^2} = (\text{known}) \times S_{\text{EW}} (1 + \delta_{\text{EM}}) \times |V_{cd}|^2 \times f_+(q^2)^2$$

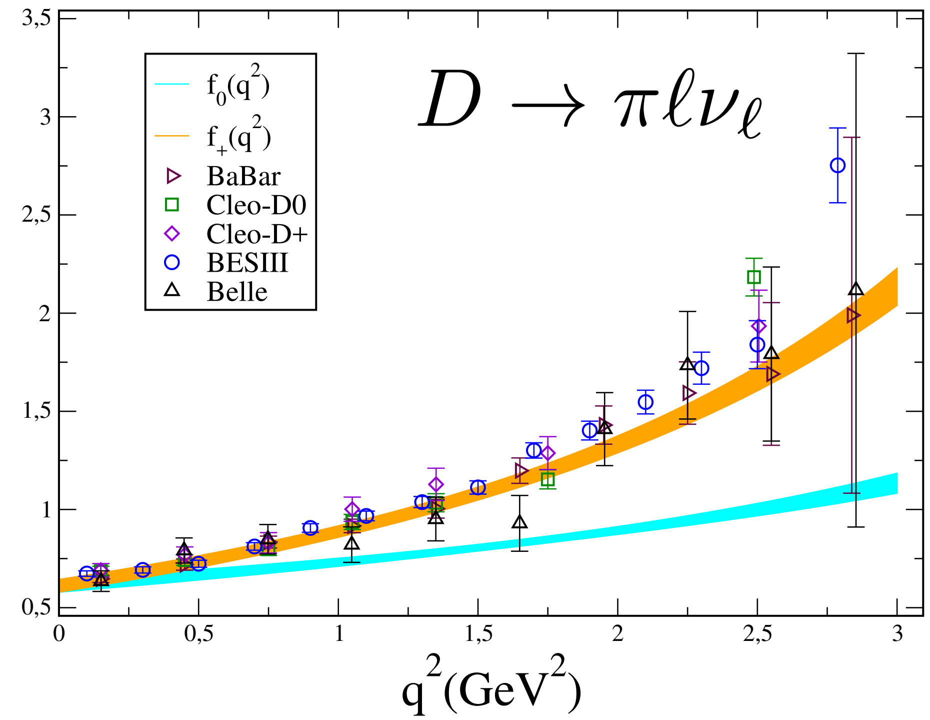
- calculate the form factors over entire q^2 range + model-independent parametrization of shape (z-expansion).
- account for EW+EM corrections in experimental rate
  - EW: [Sirlin, Nuc. Phys. 1982] ~ 1.8%
  - EM: Structure dependent: use guidance from $K_{\ell 3}$? ~ 1% ?
    depends on photon energy cut
    Long distance: [Kinoshita, PRL 1959] ~ 2.4%
    ➠ removed with PHOTOS

Semileptonic D meson decay form factors

HPQCD [arXiv:2104.09883]



ETM [arXiv:1706.03017, PRD 2017;
arXiv:1706.03657, EPJC 2017]



- ☆ Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$
- ☆ can also extract CKM elements from exp. average of $|V_{cq}| f_+(0)$
- ☆ similar analysis with Λ_c decay form factors [Meinel, arXiv:1611.09696, 2017 PRL].
- ☆ also: D-meson tensor form factors [ETM, arXiv:1803.04807, 2018 PRD]
- ☆ ongoing work by FNAL/MILC, JLQCD, RBC/UKQCD, ALPHA,…

Semileptonic D meson decay

For illustration: experimental averages [HFLAV 2019, arXiv:1909.12524, EPJC2021]:

$$[S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cs}|f_+^{DK}(0) = 0.7180\,(33)_{\rm exp} \quad [S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cd}|f_+^{D\pi}(0) = 0.1426\,(18)_{\rm exp}$$

From joint exp + LQCD fits:

HPQCD [arXiv:2104.09883]

$$|V_{cs}| = 0.9663\,(39)_{\rm exp}(53)_{\rm LQCD}(19)_{\rm EW}(40)_{\rm EM}$$

ETM [arXiv:1706.03657, EPJC 2017]

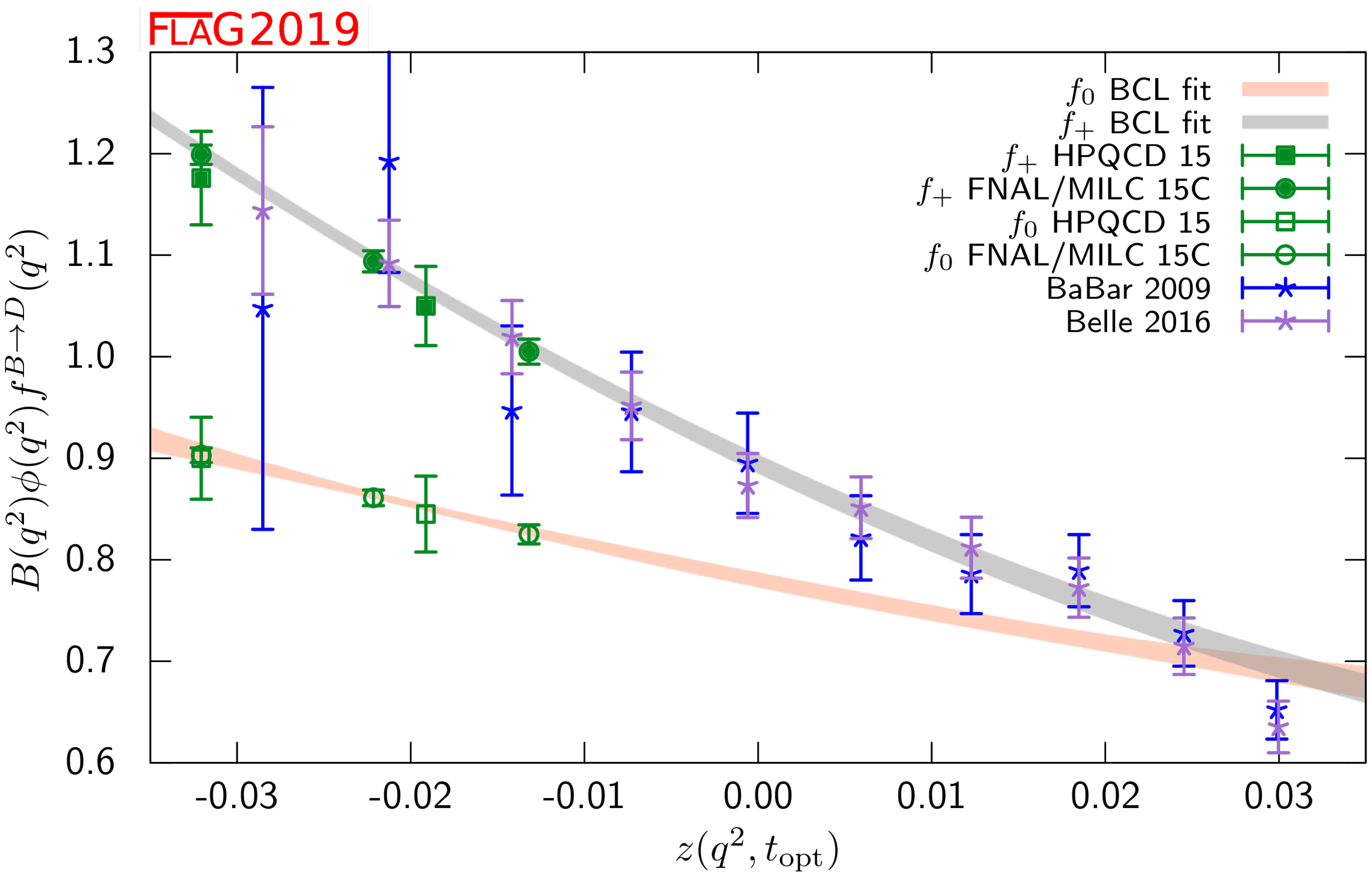
$$|V_{cd}| = 0.2341(74)_{\rm exp+LQCD}$$

0.83% error^2

error^2 3.2%

2nd row CKM unitarity test: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.0174\,(157)$

Form factors for $B \to D\,\ell\nu_\ell$



S. Aoki et al
FLAG 2019 review, 1902.08191
webupdate: flag.unibe.ch/2019/

HPQCD [arXiv:1406.2279, PRD 2014]
FNAL/MILC [arXiv:1505.03925, PRD 2015]

★The form factors obtained from the combined exp/lattice fit are well determined over entire recoil range.

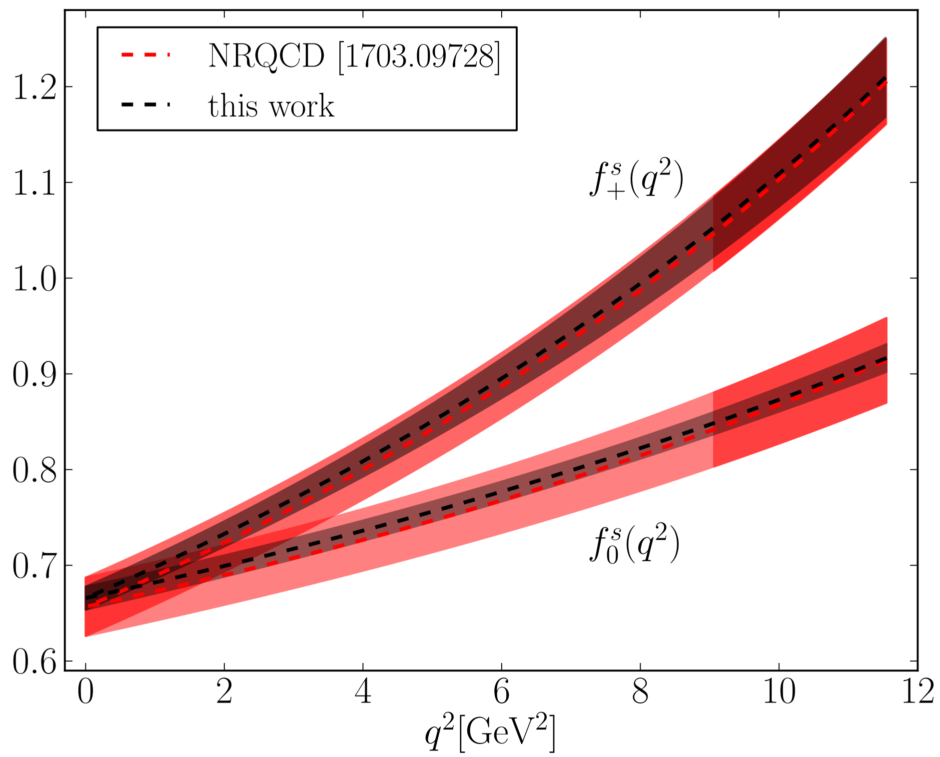
★Can be used for an improved SM prediction of *R*(*D*).

★Ongoing work by FNAL/MILC, JLQCD, RBC/UKQCD, HPQCD

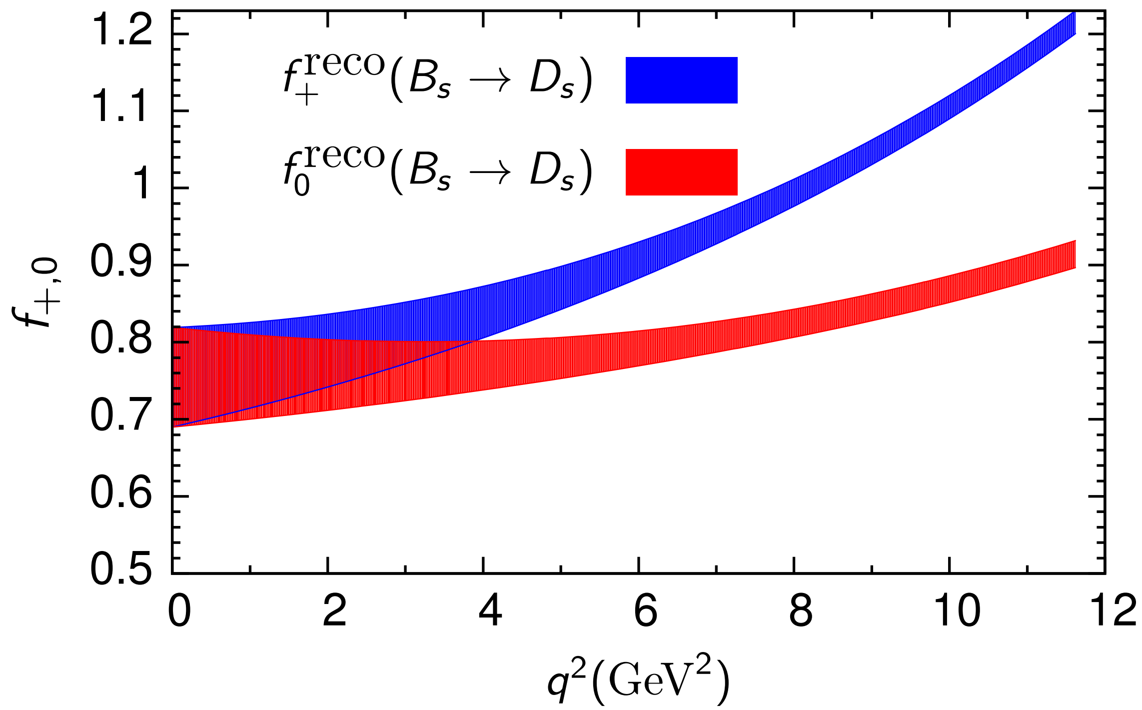
★Also: form factors for $\Lambda_b \to \Lambda_c \ell\nu$ Detmold+Meinel [arXiv:1503.01421, 2015 PRD]

Form factors for $B_s \to D_s \ell \nu_\ell$

HPQCD [arXiv:1906.00701, PRD 2020]



FNAL/MILC [arXiv:1901.02561, PRD 2019]



Reconstructed from $B \to D$ form factors [1505.03925] and B_s/B ratio [1403.0635]

- ★Can be used to predict $R(D_s)$.
- ★New: experimental measurements of differential decay rate by LHCb
- ★Ongoing work by FNAL/MILC, JLQCD, RBC/UKQCD, HPQCD

Form factors for $B \to D^* \ell \nu_\ell$ and $|V_{cb}|$

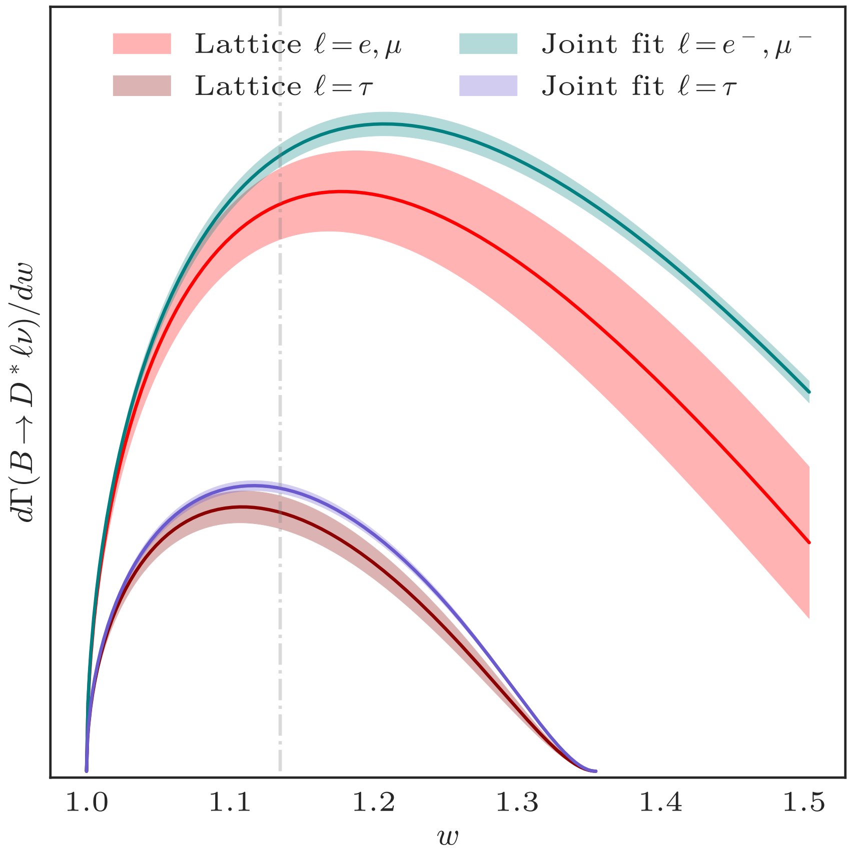
$$\frac{d\Gamma}{dw} = (\text{known}) \times \eta_{\text{EW}}^2 (1 + \delta_{\text{EM}}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} \times \chi(w) |\mathcal{F}(w)|^2$$

$$w = v_B \cdot v_{D^*}$$

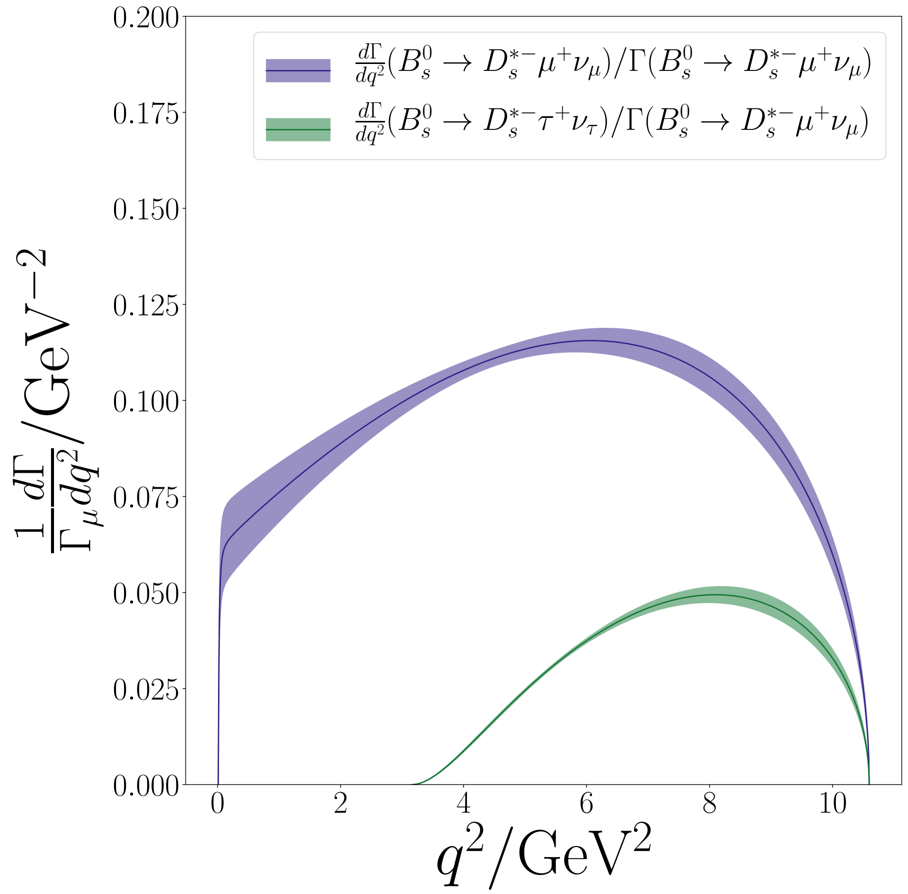
- ⋆ $\mathcal{F}(w) = f[h_{A_1}(w), h_V(w), h_{A_2}(w), h_{A_3}(w)]$
- ⋆ results for form factor at zero recoil:
  FNAL/MILC [arXiv:1403.0635, 2014 PRD], HPQCD [arXiv:1711.11013, 2018 PRD]
- ⋆ result for $\mathcal{F}^{B_s \to D_s^*}(1)$: HPQCD [arXiv:1904.02046, 2019 PRD]
- ⋆ New: non-zero recoil form factors:
  $B \to D^*$: FNAL/MILC [arXiv:2105.14019] $B_s \to D_s^*$: HPQCD [arXiv:2105.11433]
- ⋆ ongoing efforts by
  JLQCD [T. Kaneko @APLAT 2020 conference, arXiv:1912.11770]
  LANL/SWME [Bhattacharya et al, arXiv:2003.09206]
  FNAL/MILC [A. Vaquero & A. Lytle @ Lattice 2021]
- ⋆ new constraints/LQCD inputs:
  [Martinelli et al, arXiv:2105.08674, arXiv:2105.07851]

Form factors for $B_{(s)} \to D^*_{(s)} \ell \nu_\ell$

[FNAL/MILC, arXiv:2105.14019]



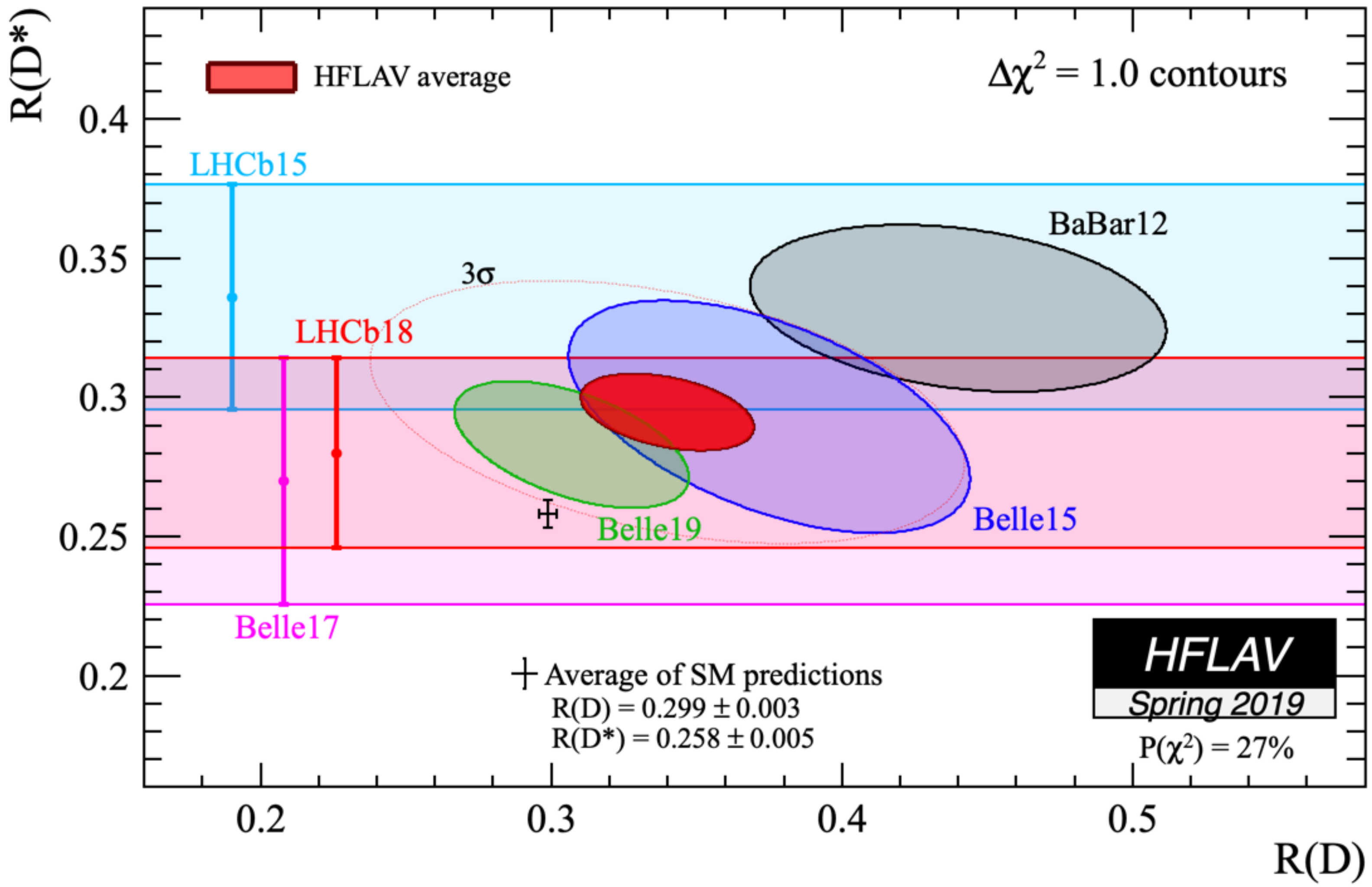
[HPQCD, arXiv:2105.11433]



- ★Results for $h_{A_1}(w), h_{A_2}(w), h_{A_3}(w), h_V(w)$.
- ★Can be used to calculate $R(D^*_{(s)})$ (lattice-only)
- ★Can be used in joint fits with experimental data to determine $|V_{cb}|$ and $R(D^*_{(s)})$ (lattice + exp)

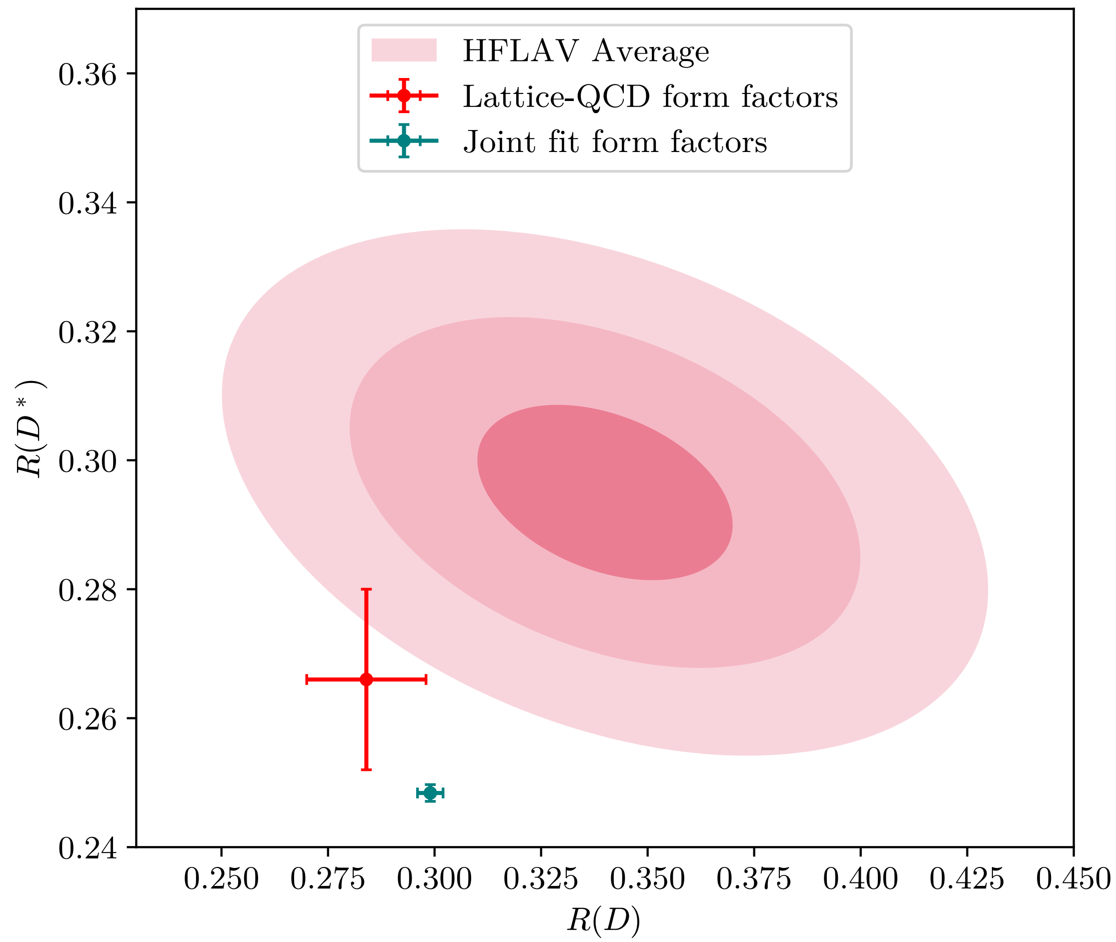
Phenomenology: LFU τ/ℓ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$



Phenomenology: LFU τ/ℓ

FNAL/MILC [arXiv:2105.14019]



HPQCD:

$R(D_s^*) = 0.2442\,(79)_{\rm lat}(35)_{\rm EM}$

[arXiv:2105.14019]

$R(D_s) = 0.2987\,(46)$

[arXiv:1906.00701, 2020 PRD]

$R(J/\psi) = 0.2582\,(38)$ ~ 2σ below LHCb

[arXiv:2007.06956, 2020 PRL]

LHCb:

$R(J/\psi) = 0.71\,(17)(18)$

[arXiv:1711.05623, 2018 PRL]

Meinel+Detmold:

$R(\Lambda_c) = 0.332\,(10)$

[arXiv:1503.01421, 2015 PRD]

Can also use the lattice form factors to study how observables change under NP scenarios.

Summary and Outlook

- ☆ D, D_s-meson decay constants known from LQCD with ~0.2-0.3% precision.
  - ➠ exp. uncertainties dominate in $|V_{cq}|$ determination
  - smaller exp. errors: ➠ big impact on $|V_{cq}|$ and CKM unitarity
- ☆ including EW correction resolves ~2σ tension CKM unitarity test [PDG]
- ☆ D, D_s-meson form factors known from LQCD with ~1-3% precision
  - -First LQCD calculation of $f_{+,0}^{D\to K}(q^2)$ with ~0.5% precision [HPQCD]
    (see parallel talk by W. Parrott on Friday, 13:10, for more details)
  - -focus of ongoing LQCD efforts is on full q^2 dependence
- ☆ EW + EM corrections significant (dominant) source of uncertainty; need to be better quantified
  LQCD calculations of radiative corrections, radiative decay [Desidero et al, arXiv:2006.05358, 2020 PRD, Kane et al, arXiv:1907.00279,Di Carlo et al, arXiv:1904.0873, 2019 PRD,….]
- ☆ New: LQCD results for $B \to D^*$ [FNAL/MILC] and $B_s \to D_s^*$ [HPQCD] form factors @ nonzero q^2.
  - ➠ new results for $R(D^*), R(D_s^*)$ and related
- ☆ scope of LQCD calculations continues to increase (new methods, new formulations, new quantities)
  - ➠ meeting the growing precision needs of the experimental program

Thank you!

Farah Willenbrock

Appendix

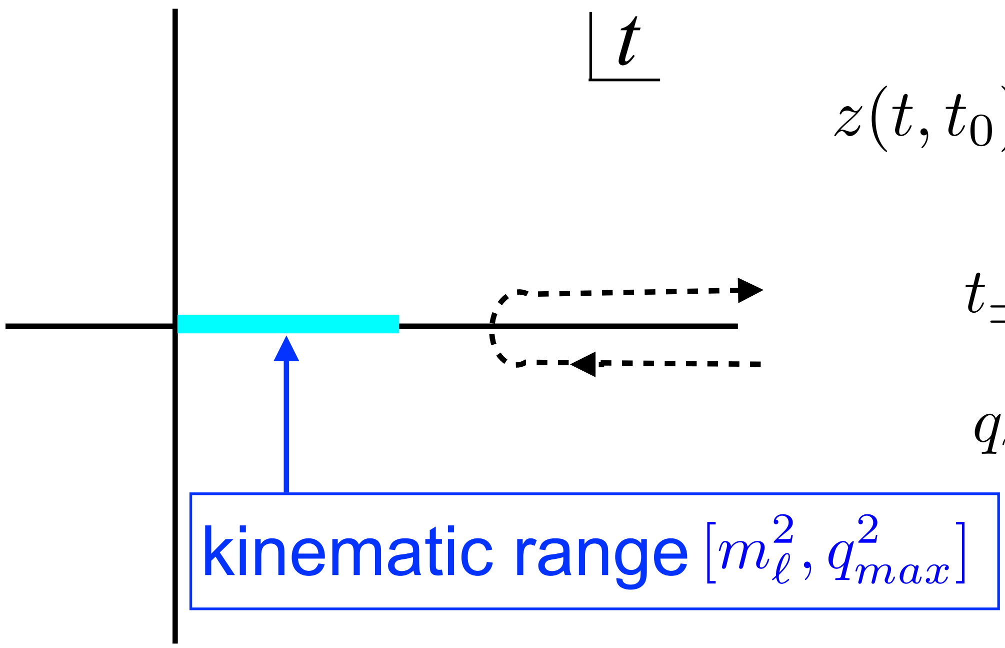
Heavy Quarks

- For light quark ($m_q \ll \Lambda_{\rm QCD}$) quantities, the leading discretization errors $\sim (a\Lambda)^2$ — if the fermion action is $O(a)$ improved.
- Using the same action for heavy quarks ($m_Q > \Lambda_{\rm QCD}$) results in leading discretization errors $\sim (am_Q)^2$. The effects are large, if $am_q \not< 1$, which is true for b quarks on most available ensembles.

➠Two classes of solutions:

1. avoid $\sim (am_Q)^2$ effects using EFT (HQET, NRQCD)
   but: nontrivial matching and renormalization
   - rel. heavy quarks (Fermilab, Columbia,..): matching rel. lattice action via HQET to continuum
   - lattice NRQCD, HQET: use EFT to construct lattice action
2. brute force: use the same lattice action for heavy quarks as for light quarks
   - generate gauge ensembles with a small enough so that $(am_b) < 1$
   - supplement with HQET inspired extrapolation and/or static limit

The z-expansion

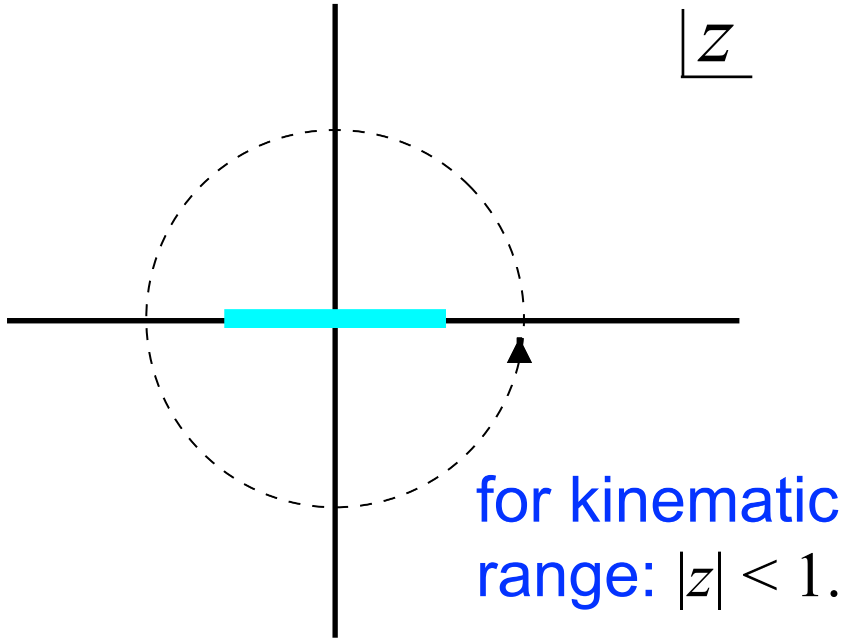


$$t = q^2$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



The form factor can be expanded as:

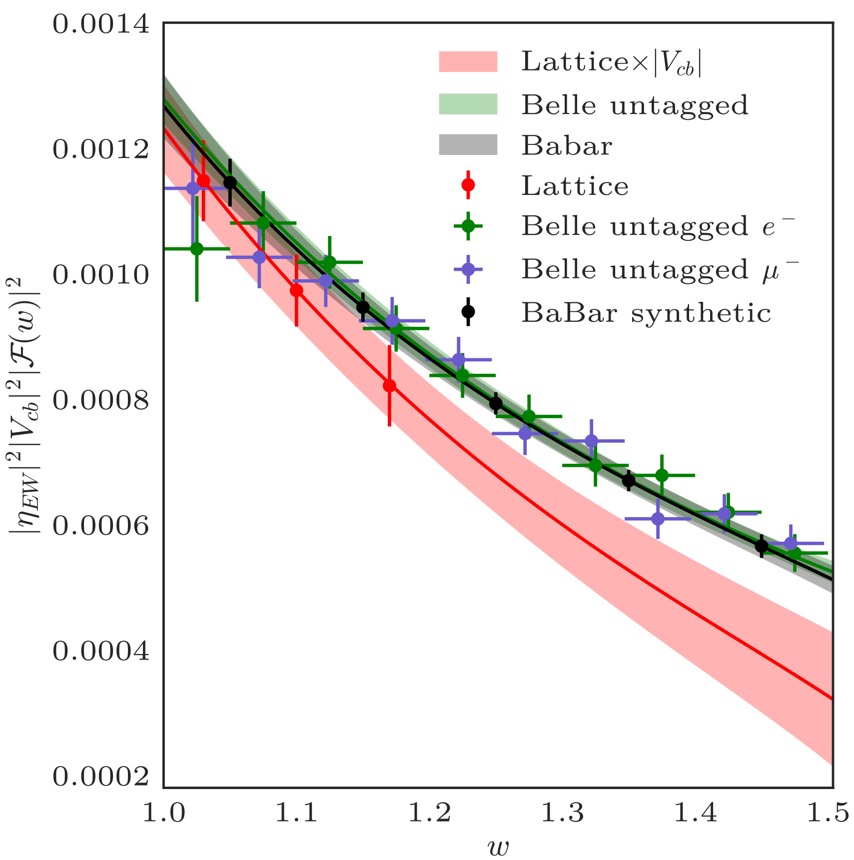
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0} a_k(t_0) z(t,t_0)^k$$

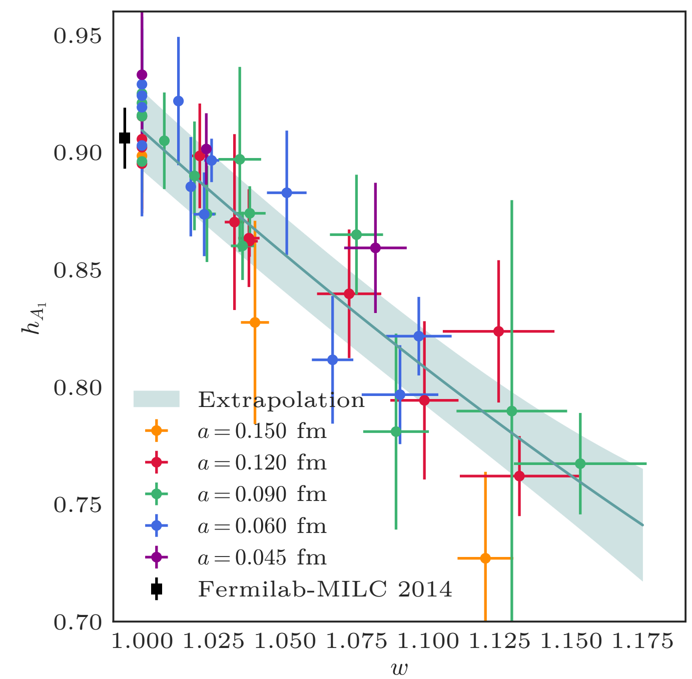
Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd, Grinstein, Lebed (hep-ph/9412324, PRL 95; hep-ph/9504235, PLB 95; hep-ph/9508211, NPB 96; hep-ph/9705252, PRD 97)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

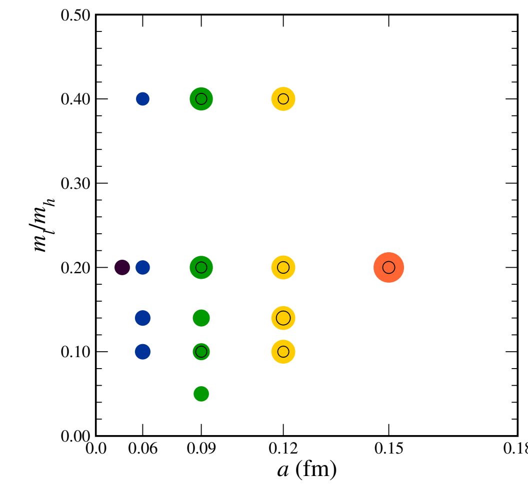
- $P(t)$ removes poles in $[t_-,t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

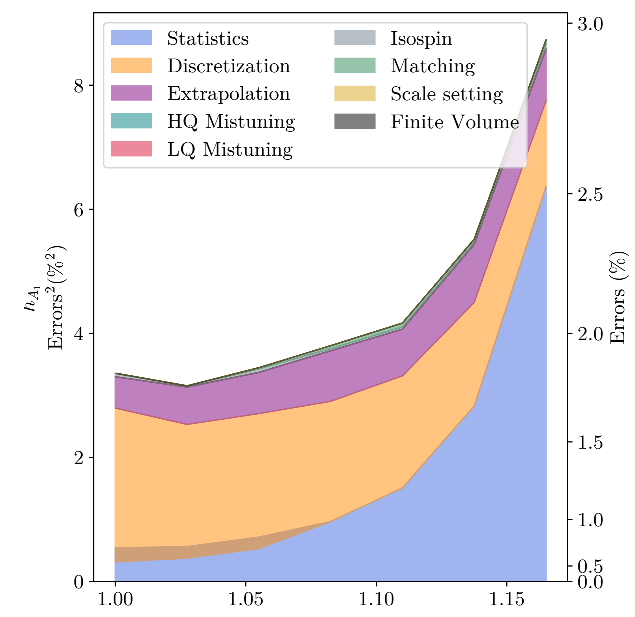
Form factors for $B \to D^* \ell \nu_\ell$

[FNAL/MILC, arXiv:2105.14019]









A. Vaquero

★Results for $h_{A_1}(w), h_{A_2}(w), h_{A_3}(w), h_V(w)$.

★Can be used to calculate $R(D^*)$ (lattice-only)

★Can be used in joint fits with experimental data from BaBar and Belle to determine $|V_{cb}|$ and $R(D^*)$ (lattice + exp)

$$|V_{cb}| = (38.57 \pm 0.70_{\rm th} \pm 0.34_{\rm exp}) \times 10^{-3}$$