Charmonium Production: Theory in Comparison with Experiment

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Focus

- Inclusive charmonium production
- Interesting theoretical and experimental work that will not be discussed:
  - Exclusive charmonium production
  - Diffractive charmonium production
  - Associated charmonium production (*e.g.*, $J/\psi + Z$)
  - Double charmonium production
  - Bottomonium production

Outline

- Brief Review of NRQCD Factorization
- Status of a Proof of NRQCD Factorization
- Why is large p_T important?
- J/ψ Cross Sections
- J/ψ Polarization
- χ_{c1} and χ_{c2} Production
- χ_{c1} and χ_{c2} Polarizations
- J/ψ Energy Fraction in a Jet
- Outstanding Problems
- Conclusions
- Future Directions

Brief Review of NRQCD Factorization

- NRQCD Factorization Conjecture (GTB, Braaten, Lepage hep-ph/9407339):
  The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle .$$

- The $F_n(\Lambda)$ are the “short-distance” coefficients (SDCs).
  - The SDCs are essentially the partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
- The $\langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle$ are the NRQCD long-distance matrix elements (LDMEs).
  - The LDMEs are the probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium.

- The SDCs depend on the production process.
  They can be calculated in QCD perturbation theory.
- The LDMEs are nonperturbative, but they are conjectured to be universal (process independent).
- The LDMEs have a known scaling with the heavy-quark velocity v.
  - $v^2 \approx 0.23$ for the J/ψ. $v^2 \approx 0.1$ for the $\Upsilon(1S)$.
  - The sum in the factorization formula is a v expansion.
- In phenomenology, the v expansion in the factorization formula is truncated at a particular order in v.
- A key feature of NRQCD factorization: Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
  - The color-singlet production LDMEs are simply related to color-singlet decay LDMEs.
  - The color-octet LDMEs must be determined from fits to measured production cross sections.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).

- The current phenomenology of production of S-wave quarkonia (J/ψ, $\psi(2S)$, and $\Upsilon(nS)$) makes use of LDMEs through relative order v^4:

$$
\begin{array}{ll}
\langle \mathcal{O}^H({}^3S_1^{[1]})\rangle & (O(v^0)), \\
\langle \mathcal{O}^H({}^1S_0^{[8]})\rangle & (O(v^3)), \\
\langle \mathcal{O}^H({}^3S_1^{[8]})\rangle & (O(v^4)), \\
\langle \mathcal{O}^H({}^3P_J^{[8]})\rangle & (O(v^4)).
\end{array}
$$

  - Calculations show that the ${}^3S_1^{[1]}$ contributions are negligible for J/ψ hadroproduction.
  - The $\langle \mathcal{O}^H({}^3P_J^{[8]})\rangle$ ($J = 0, 1, 2$) are related by the heavy-quark spin symmetry.
  - Three color-octet LDMEs need to be determined phenomenologically for each state.

- The current phenomenology of production of P-wave quarkonia (χ_{cJ}) makes use of LDMEs through relative order v^4:

$$\langle \mathcal{O}^H(^3P_J^{[1]}) \rangle \quad (O(v^4)),$$
$$\langle \mathcal{O}^H(^3S_1^{[8]}) \rangle \quad (O(v^4)).$$

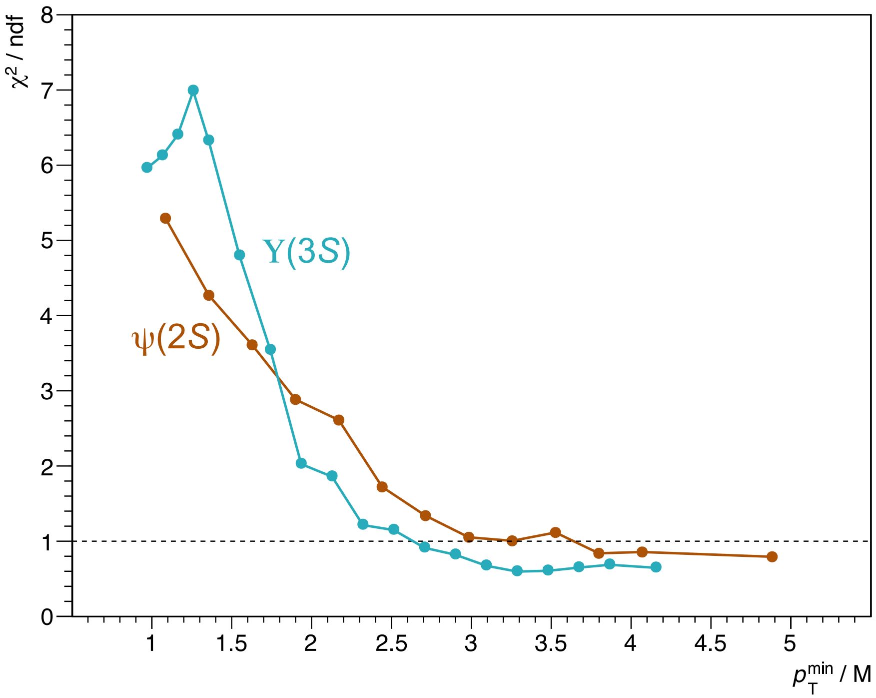
  – The $\langle \mathcal{O}^H(^3P_J^{[1]}) \rangle$ ($J = 0, 1, 2$) are related by the heavy-quark spin symmetry. They can be determined from potential models or quarkonium decays.
  – Only one LDME ($\langle \mathcal{O}^H(^3S_1^{[8]}) \rangle$) has to be determined from phenomenology.

Status of a Proof of NRQCD Factorization

- Nayak, Qiu, Sterman (hep-ph/0501235, hep-ph/0509021, hep-ph/0608088): Factorization holds through two loops, up to corrections of relative order m_Q^2/p_T^2.
- GTB, Chung, Ee, Kim (1910.05497): Confirmed the two-loop result using covariant, rather than light-front, methods.
- Zhang, Meng, Ma, Chao (2011.04905): Confirmed the two-loop result as part of a calculation of gluon fragmentation into a ${}^3P_J^{[1,8]}$ quark pair.
- It is not known if this result generalizes to higher orders in α_s.
- An all-orders proof is essential to establish that soft (nonperturbative) gluons can be factored into universal LDMEs.
- In the absence of further theoretical progress, we must rely on experiment to prove or to disprove NRQCD factorization.

Why is large p_T important?

- Two-loop verifications of NRQCD factorization require $m_Q^2/p_T^2 \ll 1$.
- Factorization-violating corrections $\propto (m_Q^2/p_T^2)^n$ get out of control when $p_T \sim m_Q$.
- This picture is supported by the analysis of Faccioli, Knünz, Lourenço, Seixas, and Wöhri (1403.3970) for quarkonium production in pp collisions.
- They adjusted the NRQCD LDMEs to obtain the best fits of NLO cross-section calculations to LHC data.



- No feeddown contributions for $\Upsilon(3S)$ and $\psi(2S)$: Simplifies the analysis.
- The $\chi^2/\mathrm{d.o.f.}$ increases unacceptably if the minimum p_T is less then $3m_{\mathrm{quarkonium}}$.

J/ψ Cross Sections

- Three groups have carried out complete NLO calculations.
  - PKU group: Ma, Wang, Chao
  - Hamburg group: Butenschön, Kniehl
  - IHEP group: Gong, Wan, Wang, Zhang
- In addition, the
  - ANL-PKU group: GTB, Chao, Chung, Kim, Lee, Ma

  has computed NLO + leading-power fragmentation contributions (logs of p_T^2/m_c^2) to all orders.
- All the groups agree on the NLO SDCs.
- The fragmentation contributions computed by the ANL-PKU group have a big effect on the shape of the SDC for the ${}^3P_J^{[8]}$ channel because of a cancellation between the LO and NLO contributions.

- The groups extract very different NRQCD LDMEs and make different predictions.
- The PKU group (1009.3655, 1012.1030) fit the CDF J/ψ data for $p_T > 7$ GeV. Experimental data were used to subtract feeddown contributions. They were able to determine only 2 linear combinations of LDMEs unambiguously:

$$M_{0,r_0} = \langle O^{J/\psi}(^1S_0^{[8]})\rangle + (r_0/m_c^2)\langle O^{J/\psi}(^3P_0^{[8]})\rangle = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,r_1} = \langle O^{J/\psi}(^3S_1^{[8]})\rangle + (r_1/m_c^2)\langle O^{J/\psi}(^3P_0^{[8]})\rangle = (0.05 \pm 0.02) \times 10^{-2}\ \text{GeV}^3.$$

$r_0 = 3.9$ and $r_1 = -0.56$.

- The Hamburg group (1105.0920) determined all 3 color-octet LDMEs by making a global fit to data with $p_T > 3$ GeV from the Tevatron, LHC, RHIC, HERA, LEP II, KEKB.
  - They made use of their computations of NLO corrections to $p\bar{p}$, pp, ep, $\gamma\gamma$, and e^+e^- production. Uncorrected for feeddown.
  - Their LDMEs are very different from those of the PKU group:

$$M_{0,r_0} = (2.17 \pm 0.56) \times 10^{-2}\ \text{GeV}^3,$$
$$M_{1,r_1} = (0.62 \pm 0.08) \times 10^{-2}\ \text{GeV}^3.$$

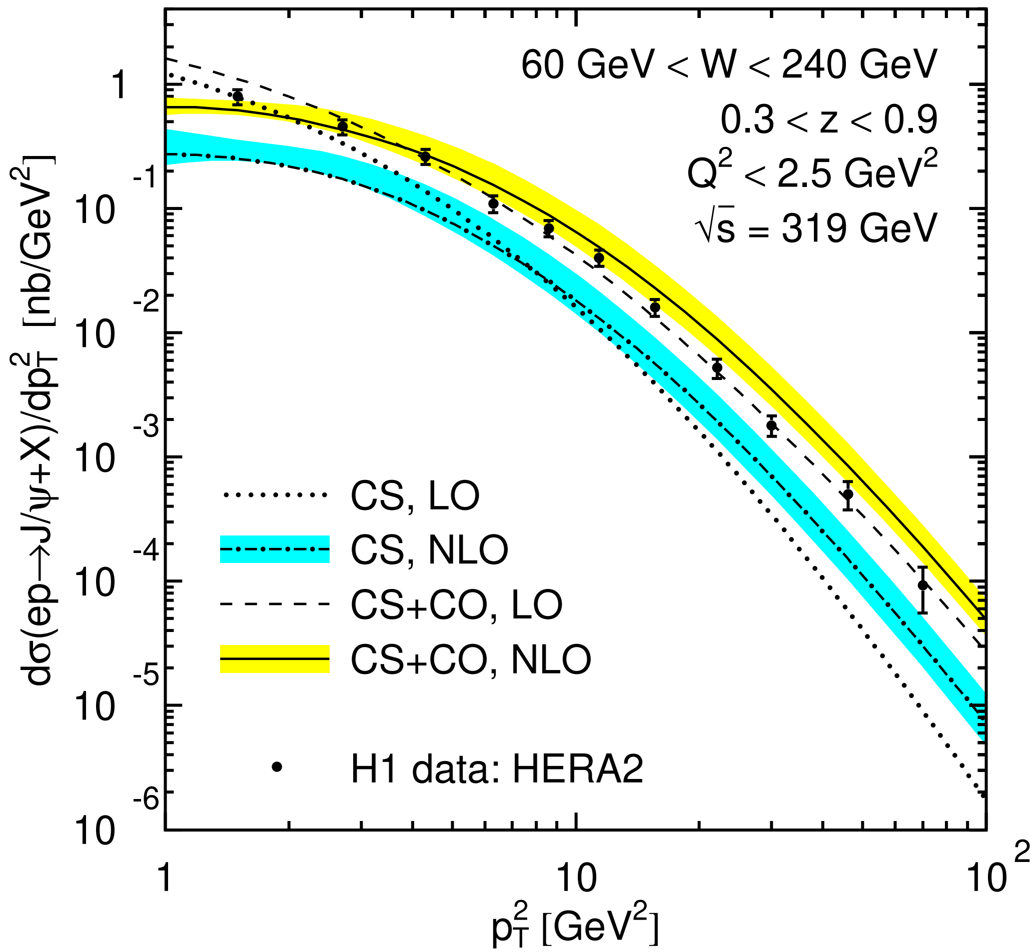
- The IHEP group (1205.6682) fit the CDF J/ψ, $\psi(2S)$, and χ_{cJ} cross sections for $p_T > 7$ GeV.
  - They included NLO feeddown contributions from $\psi(2S)$ and χ_{cJ} in their fit.
  - They were able to determine all 3 color-octet LDMEs.
  - They obtained a quality of fit and a result for the LDME linear combinations that is similar to that of the PKU group:

$$M_{0,r_0} = (6.00 \pm 0.98) \times 10^{-2} \text{ GeV}^3,$$
$$M_{1,r_1} = (0.07 \pm 0.02) \times 10^{-2} \text{ GeV}^3.$$

- The ANL-PKU group (1509.07904) fit the CDF and CMS J/ψ, $\psi(2S)$, and χ_{cJ} cross sections for $p_T > 10$ GeV.
  - They included NLO feeddown contributions from $\psi(2S)$ and χ_{cJ} in their fit.
  - They were able to determine all 3 color-octet LDMEs.
  - Their results for M_{1,r_1} agree with those of the PKU and IHEP groups, but they obtain a negative value for M_{0,r_0}:

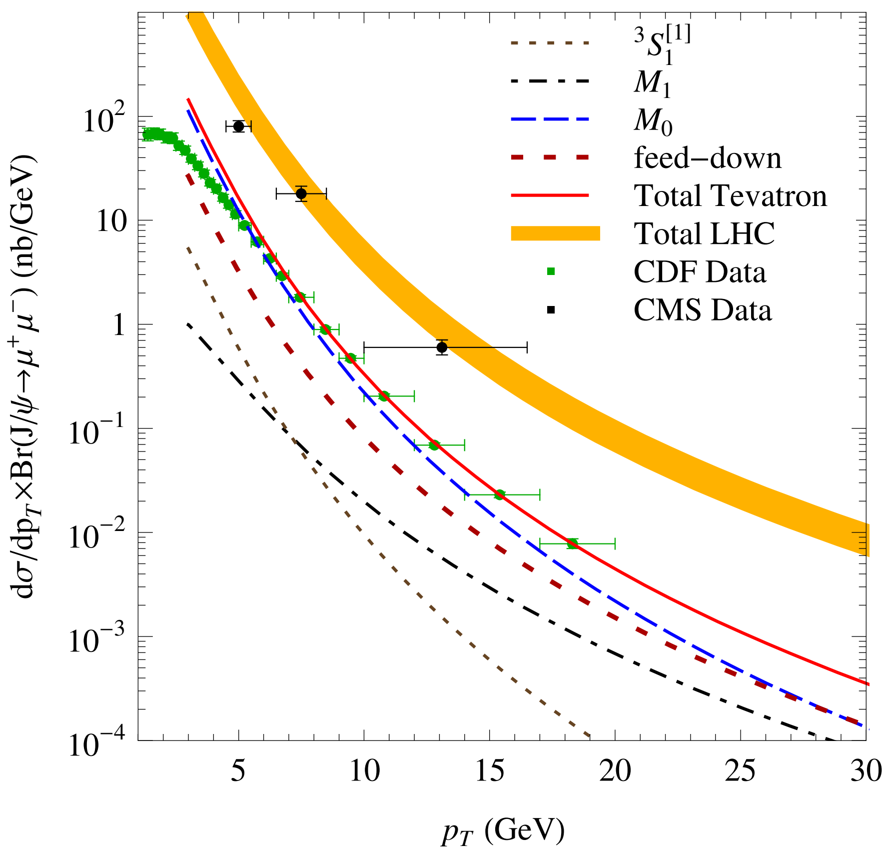
$$M_{0,r_0} = (9.78 \pm 1.52) \times 10^{-2} \text{ GeV}^3,$$
$$M_{1,r_1} = (-0.54 \pm 0.37) \times 10^{-2} \text{ GeV}^3.$$

- Most of the difference between the Hamburg-group fit and the others comes from the use of low-p_T data.

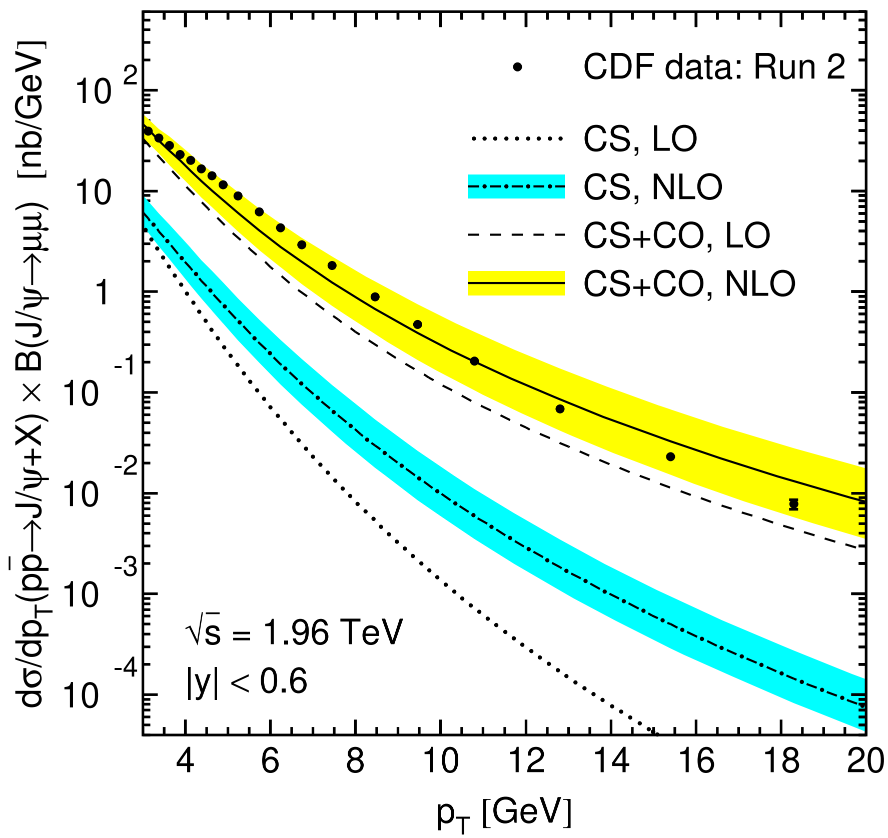


- In particular, they use the HERA H1 photoproduction data (hep-ex/0205064, hep-ex/0510016).
- The H1 data lie at $p_T \lesssim 8$ GeV.
- Is NRQCD factorization valid at such low values of p_T?

- Although the Hamburg-group fits agree with the data, within uncertainties, there are tensions in the shapes.
- The shape of the PKU-group fit agrees with the CDF data (hep-ex/0412071) better than the shape of the Hamburg-group global fit.

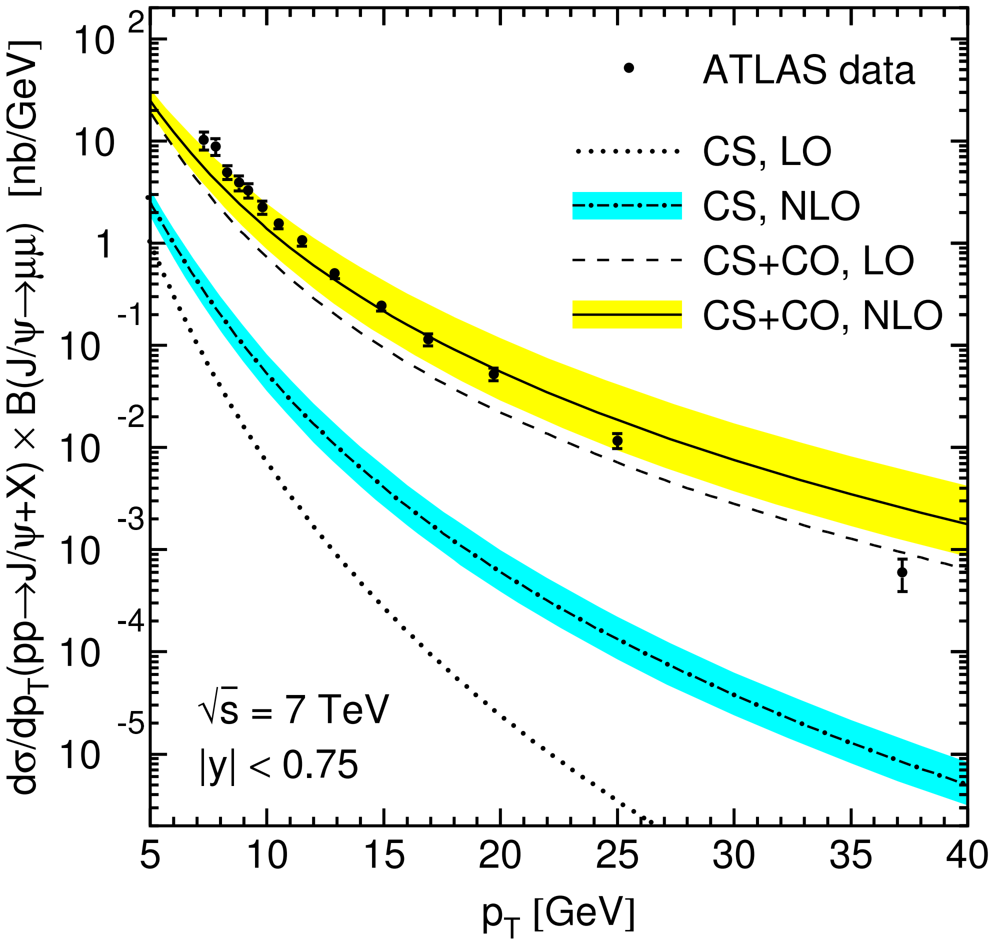


PKU group



Hamburg group

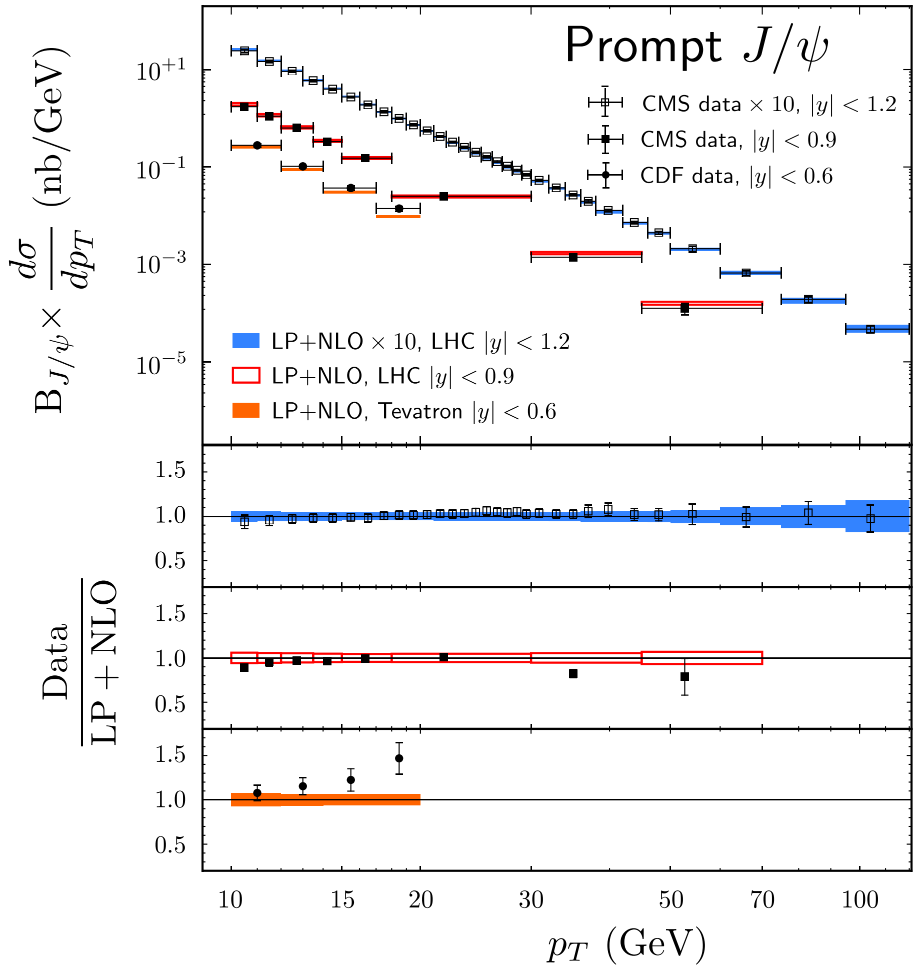
- The shape discrepancy between the Hamburg-group prediction and the data becomes more apparent at high p_T.



- ATLAS data (1104.3038).
- Not included in the Hamburg-group global fit.

- The theory uncertainties come from varying the factorization/renormalization scales and shift the predictions without changing the shape significantly.

- Most of the differences between the ANL-PKU group and the PKU group come from the fragmentation contributions included by the ANL-PKU group.
- The ANL-PKU group is able to fit the CMS data (1111.1557, 1502.04155) well over a large p_T range:

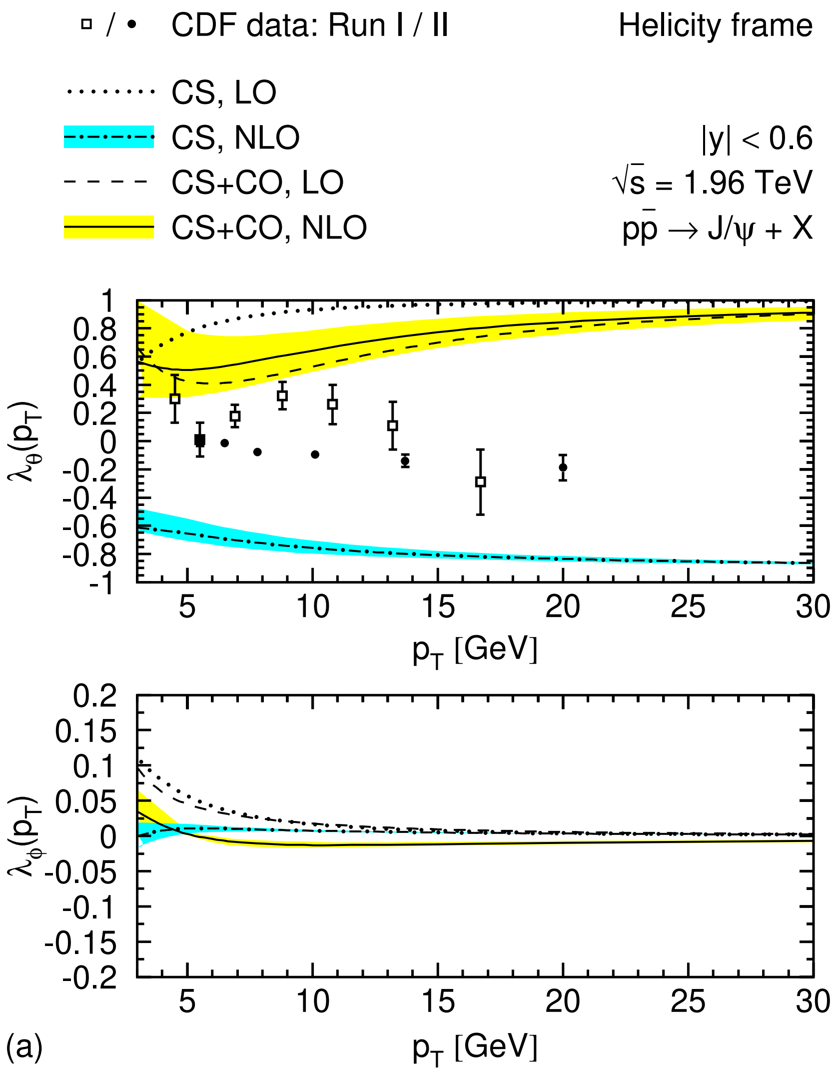


- There is some tension in the fit to the CDF data (hep-ex/0412071).

J/ψ Polarization

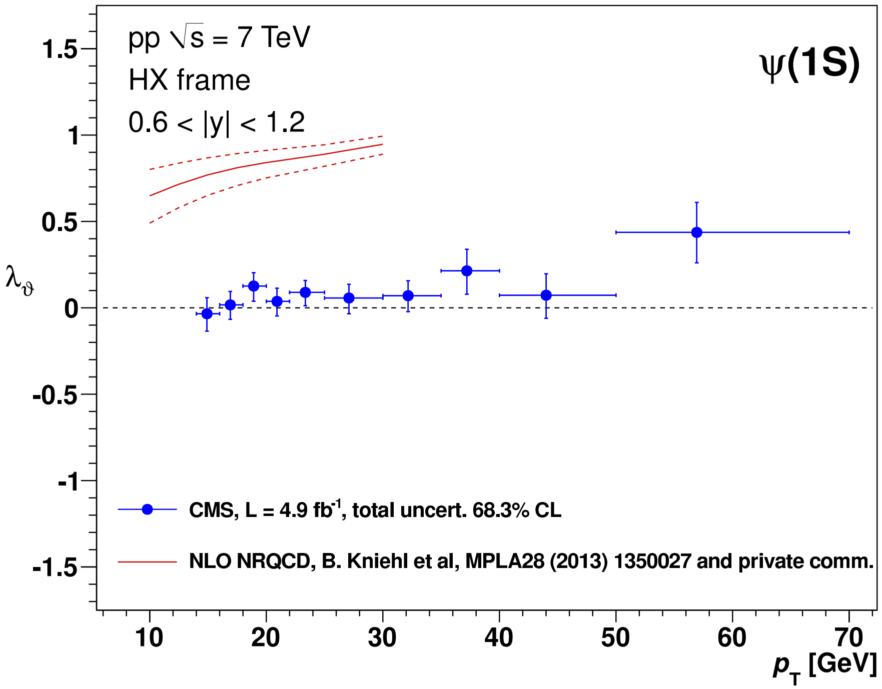
- At high p_T, the ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels are mostly transversely polarized.
- Large NLO corrections to the ${}^3P_J^{[8]}$ channel give it the same shape as the ${}^3S_1^{[8]}$ channel at high p_T.
- The contribution from the ${}^3P_J^{[8]}$ channel could cancel the contribution from the ${}^3S_1^{[8]}$ channel if the LDMEs have opposite signs.
- Since the ${}^1S_0^{[8]}$ channel is unpolarized, ${}^1S_0^{[8]}$ dominance would result in near-zero polarization.

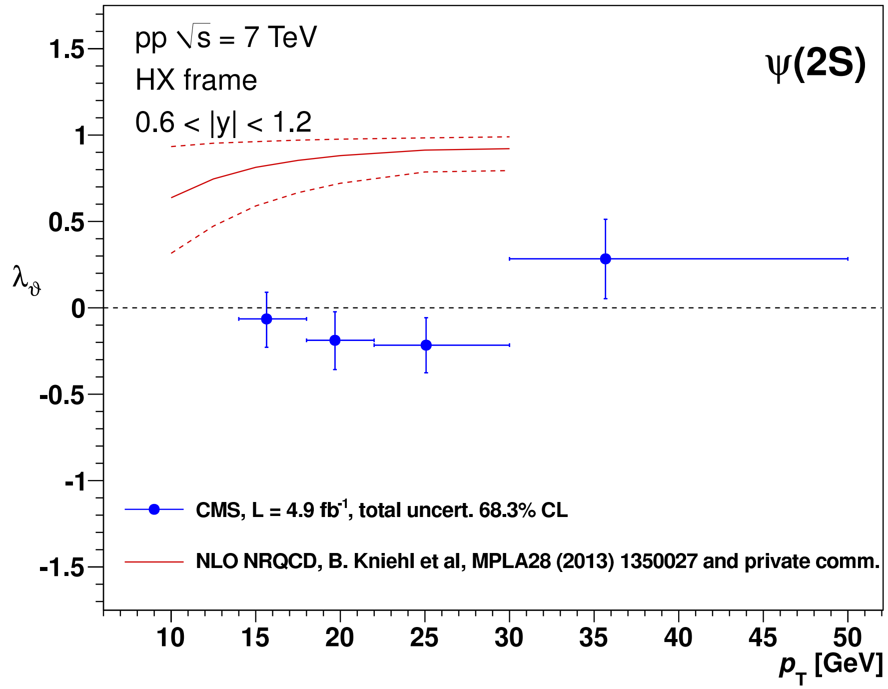
Hamburg group (1201.3862) NLO Prediction for J/ψ Polarization



- Uses the LDMEs from their global fit to the J/ψ production cross sections, corrected to remove estimated feeddown contributions.
- The contributions of the ${}^3P_J^{[8]}$ and ${}^3S_1^{[8]}$ channels add to produce substantial polarization at high p_T.
- The prediction is in disagreement with the CDF (hep-ex/0004027, 0704.0638) data.

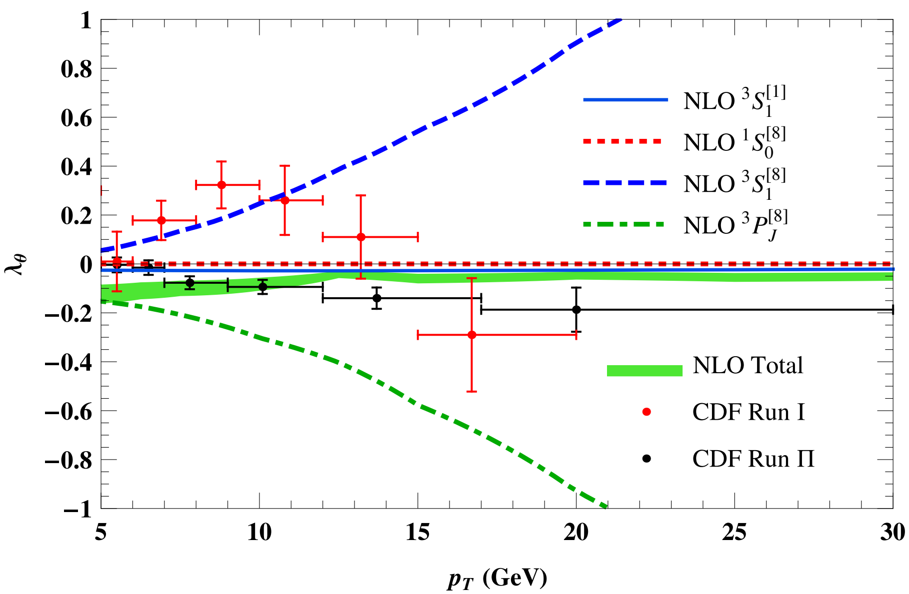
- The prediction based on the Hamburg-group LDMEs is also in disagreement with the CMS (1307.6070) data.





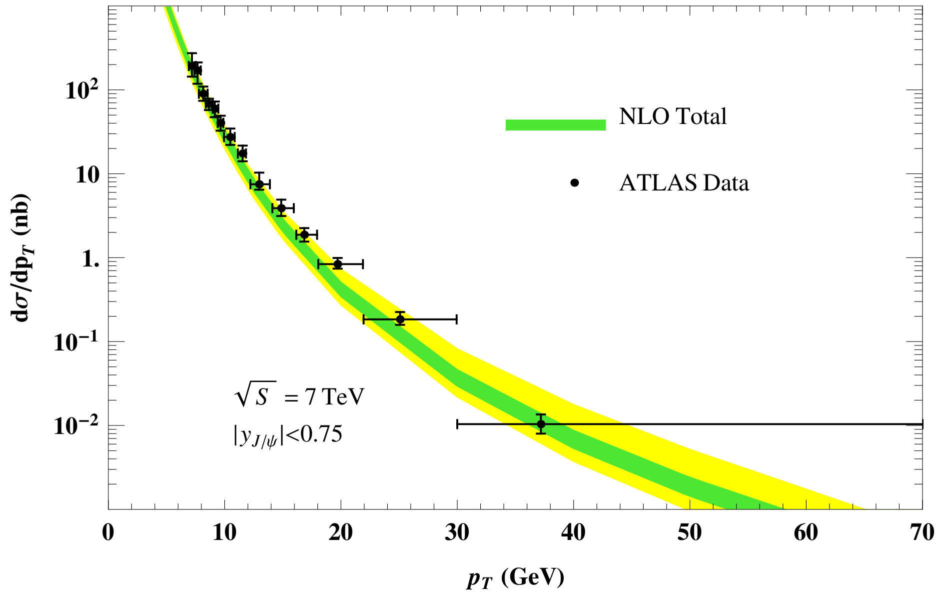
Chao, Ma, Shao, Wang, Zhang (Chao *et al.*) NLO Fit to J/ψ Polarization (1201.2675)

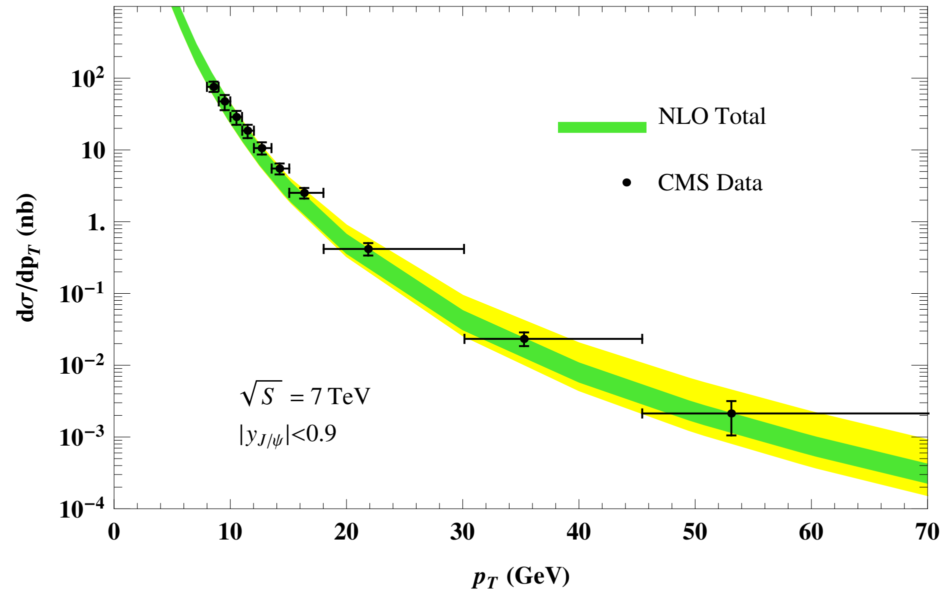
- Two LDME combinations are insufficient to predict the polarization.
- Fix all three LDMEs by including the CDF Run II J/ψ polarization measurement in the fit, as well as the CDF Run II measurements of $d\sigma/dp_T$.

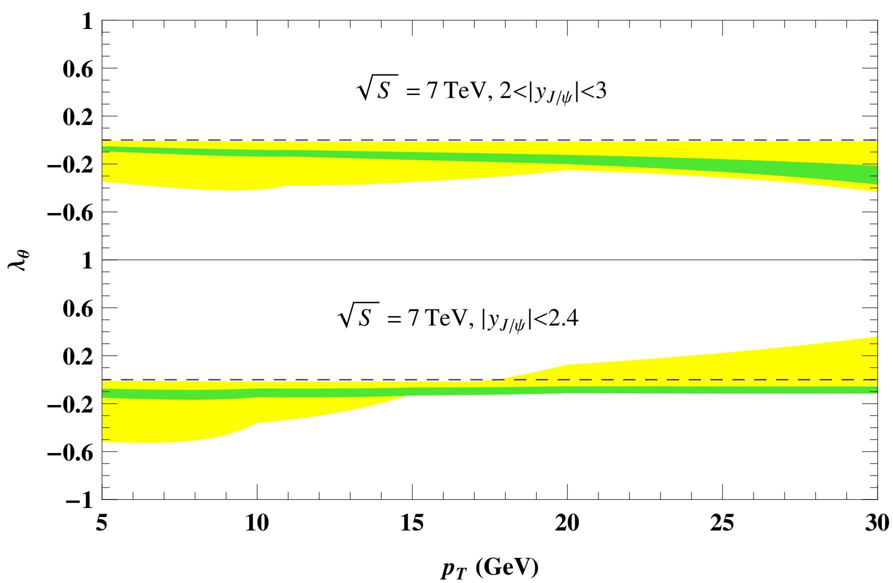


- The $^3S_1^{[8]}$ and $^3P_J^{[8]}$ contributions largely cancel.
- $^1S_0^{[8]}$ dominance $\Rightarrow$ near-zero polarization.

• The Chao *et al.* LDMEs still give reasonable predictions for the LHC p_T spectra.



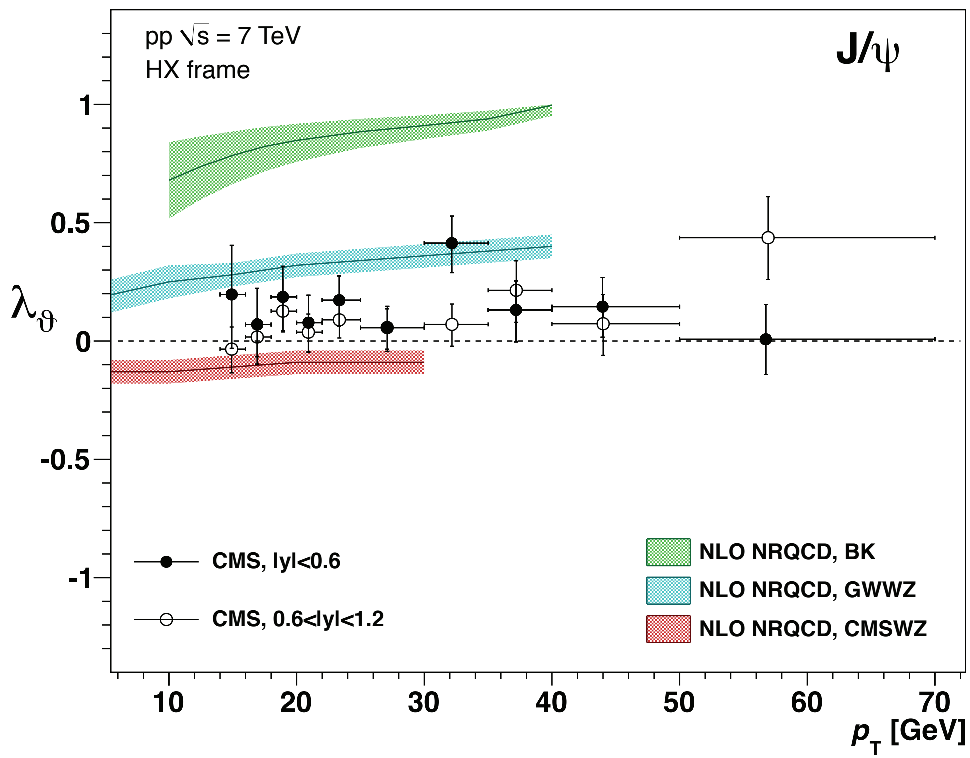




• The Chao *et al.* LDMEs predict a slightly longitudinal polarization at the LHC.

IHEP group (1205.6682) NLO Prediction for J/ψ polarization

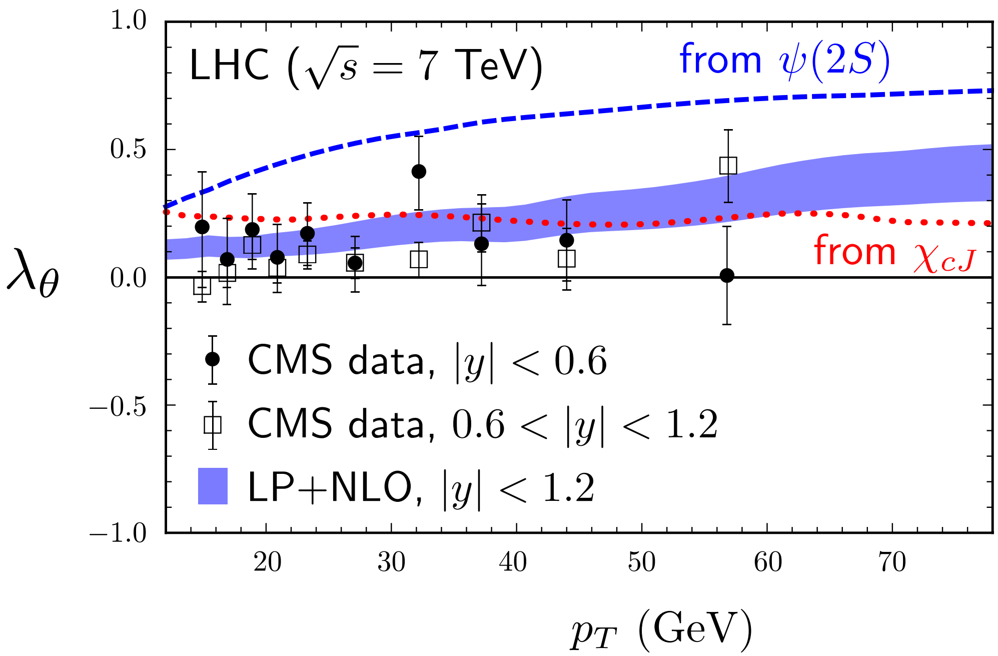
- Makes use of the LDMEs from a fit to the CDF (PRL 79, 578; hep-ex/0412071; 0905.1982) and LHCb (1103.0423, 1204.1258, 1204.1462) J/ψ, χ_{cJ}, and $\psi(2S)$ production cross sections.
- Effects of feeddown from χ_{cJ} and $\psi(2S)$ states calculated and included in fits and polarization predictions.



- The IHEP-group prediction (blue band) shows less transverse polarization than the Hamburg-group prediction (green band).
- Still in disagreement with the CMS data.
- The red band is the Chao *et al.* prediction, which lies below the CMS data.

ANL-PKU Group (1509.07904) NLO Prediction for J/ψ Polarization

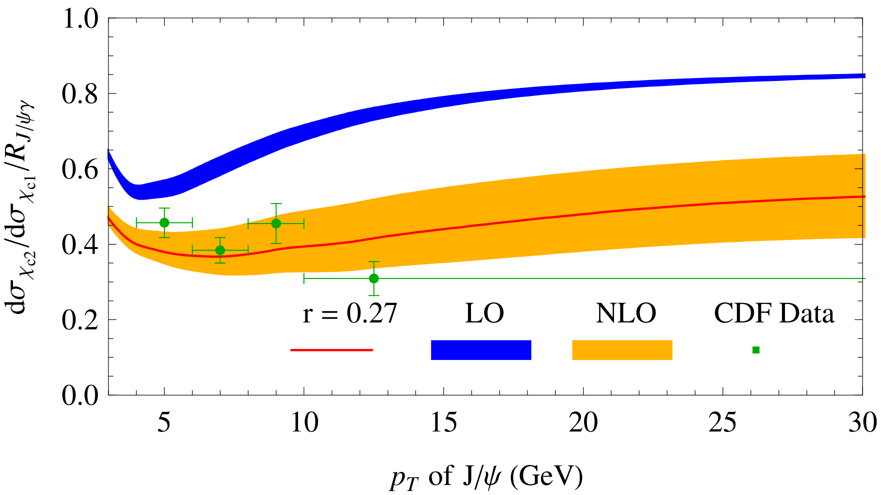
- Used its LDMEs from the fits to cross sections to predict the polarization, with no adjustments.



- Good agreement with the CMS data.
- First prediction of polarization to agree with data.
- ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels largely cancel, leaving the unpolarized contribution of the ${}^1S_0^{[8]}$ channel.
- ${}^1S_0^{[8]}$ dominance

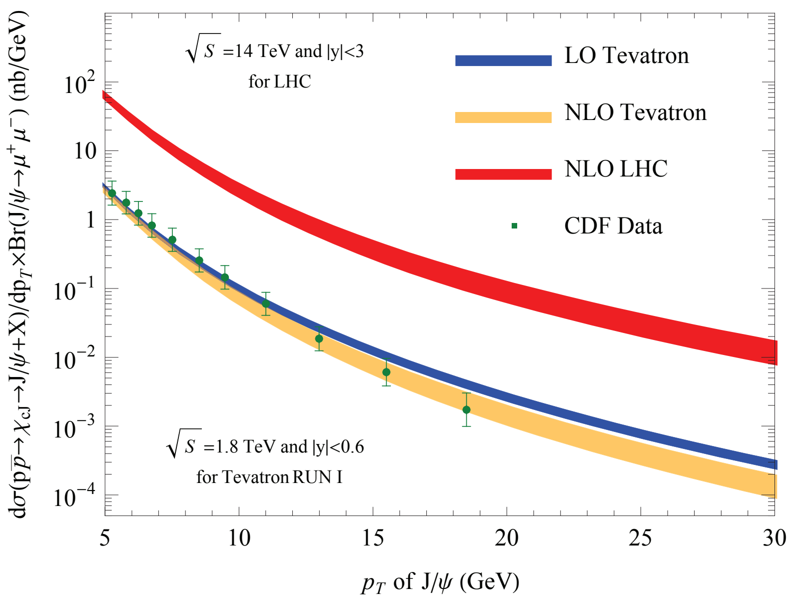
χ_{c1} and χ_{c2} Production

- Ma, Wang, Chao (1002.3987) fit the ratio $d\sigma_{\chi_{c1}}/d\sigma_{\chi_{c2}}$:



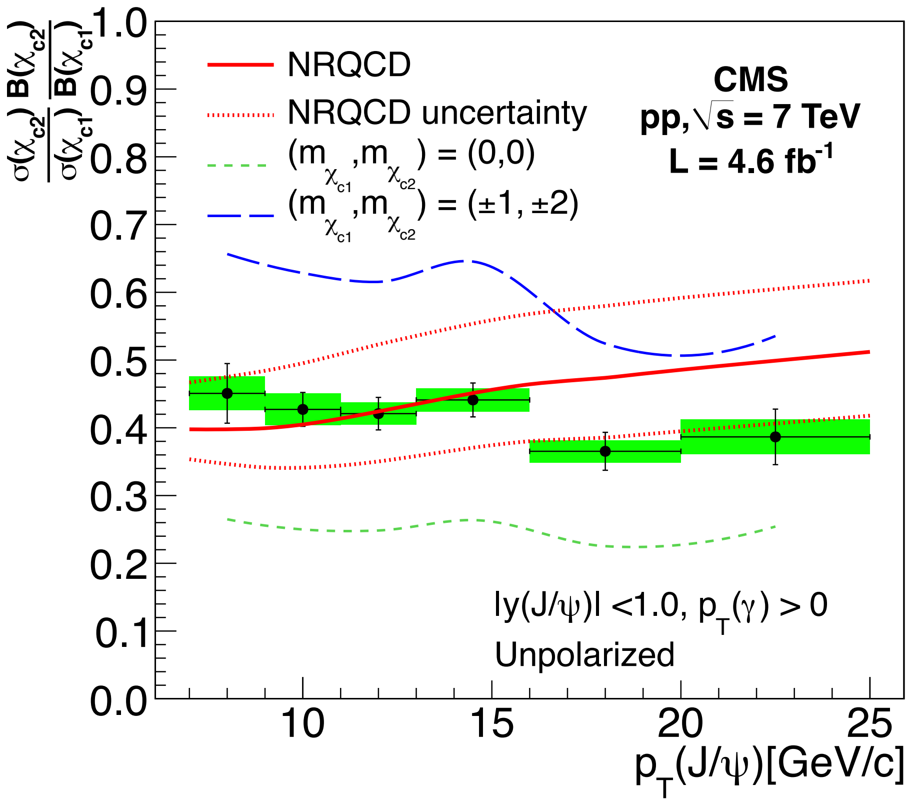
- Used CDF (hep-ex/0703028) data.
- Fit, plus a potential-model value of $\langle {}^3P_J^{[1]} \rangle$ (Eichten, Quigg (PRD 52, 1726)), allowed them to fix $\langle {}^3S_1^{[8]} \rangle$.

- That allowed them to predict the cross section:



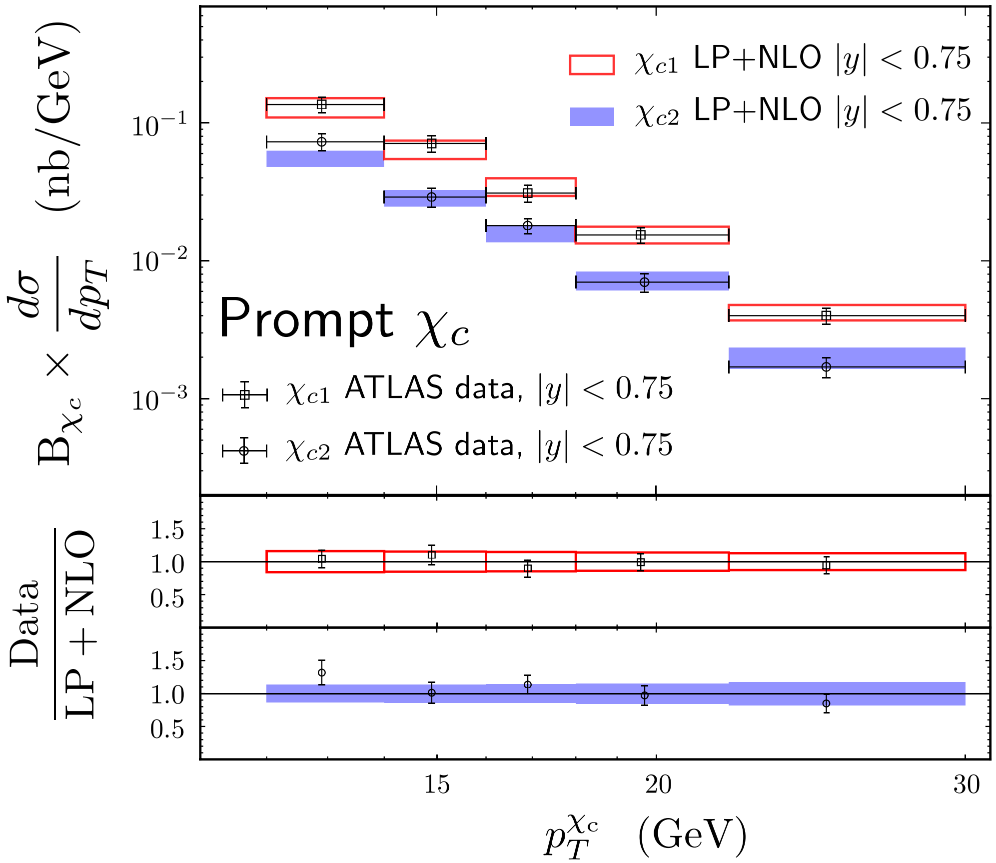
- In good agreement with the CDF data.

• The prediction of Ma, Wang, Chao also agrees with the CMS data (1210.0875):



• The dashed green and blue lines are the shifts in the ratio for extreme values of the χ_{cJ} polarizations.

- The ANL-PKU group fit the ATLAS (1404.7035) data:

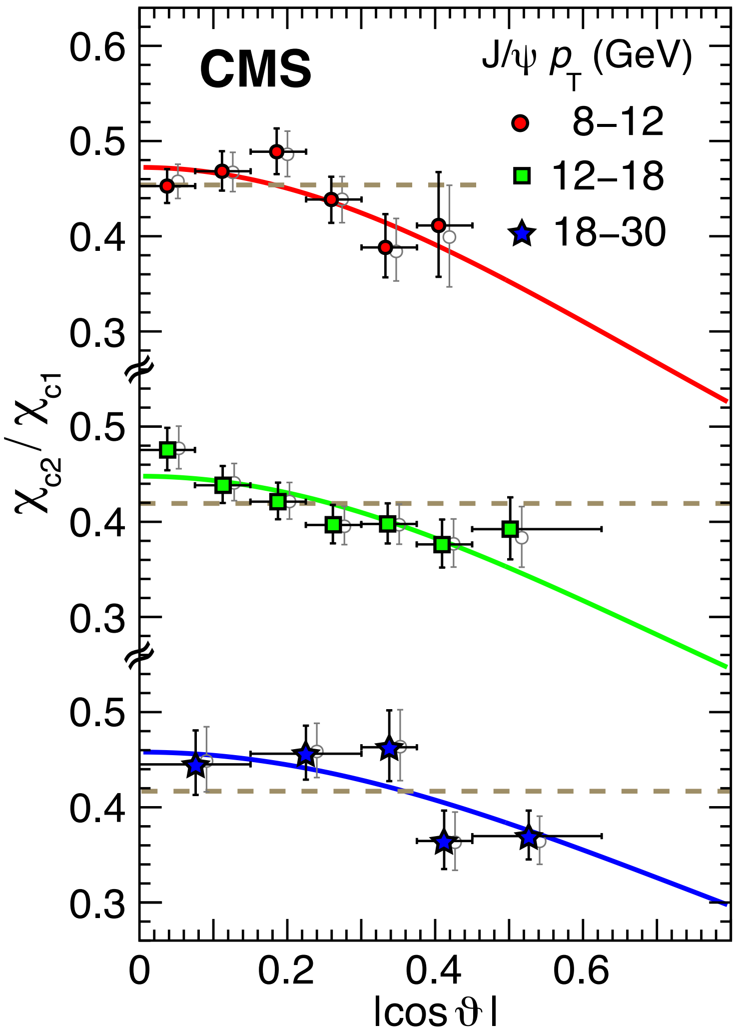


- The fitted value of the color-singlet LDME:
  $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = (7.9 \pm 2.4) \times 10^{-2}$ GeV^5.
- Potential-model value [Eichten, Quigg (PRD 52, 1726)]:
  $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = 10.7 \times 10^{-2}$ GeV^5.
- Value from two-photon decays of the χ_{c1} and χ_{c2} [Chung, Lee, Yu (0808.1625)]:
  $\langle \mathcal{O}^{\chi c1}(^3P_J^{[1]})\rangle = 6.0^{+4.3}_{-2.9} \times 10^{-2}$ GeV^5.

- Good agreement among LDMEs from different physical processes strongly suggests that NRQCD factorization is more than just curve fitting.

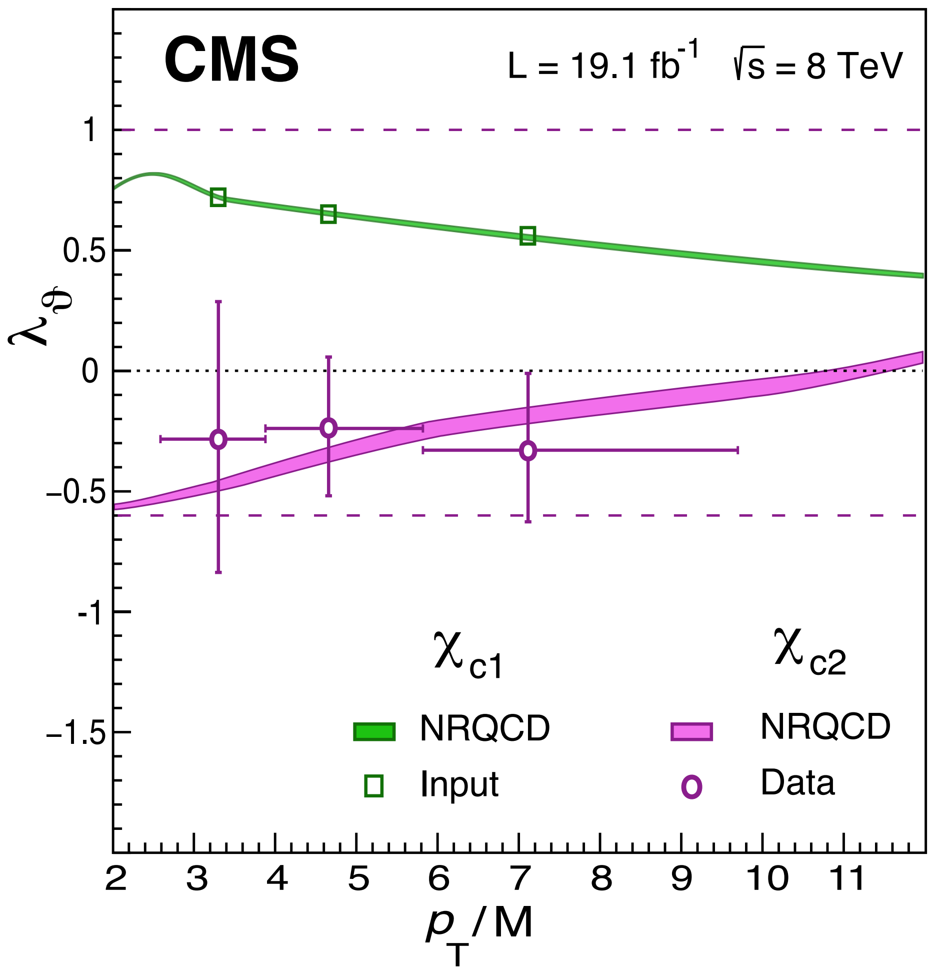
χ_{c1} and χ_{c2} Polarizations

- The muon angular distribution in $\chi_{cJ} \to J/\psi \to \mu^+\mu^-$ is a proxy for the χ_{cJ} polarization.



- In order to reduce systematics, CMS measures the ratio of angular distributions.
- The NRQCD curves [Faccioli, Lourenço, Araújo, Seixas, Krätschmer, Knünz (1802.01106)] use the SDCs from the PKU group.

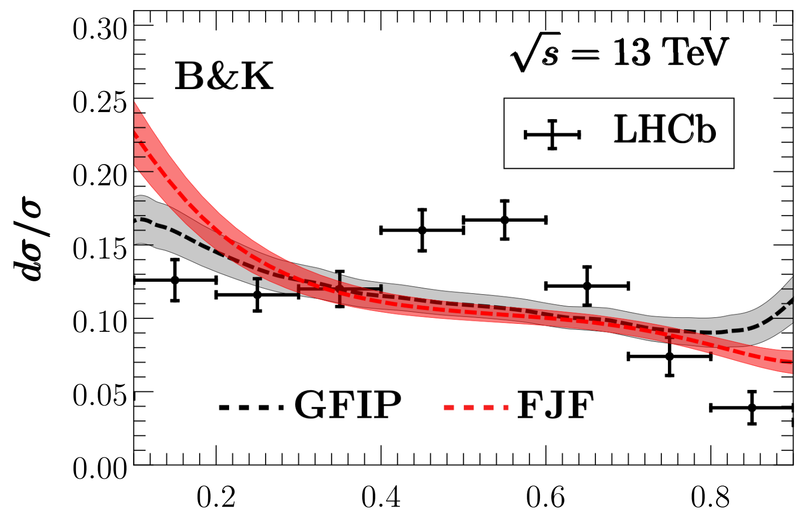
- Faccioli et al. used the CMS measurement of $d\sigma_{\chi_{c2}}/d\sigma_{\chi c1}$ to fix the ratio of NRQCD LDMEs and used the PKU-group SDCs to predict the polarizations.

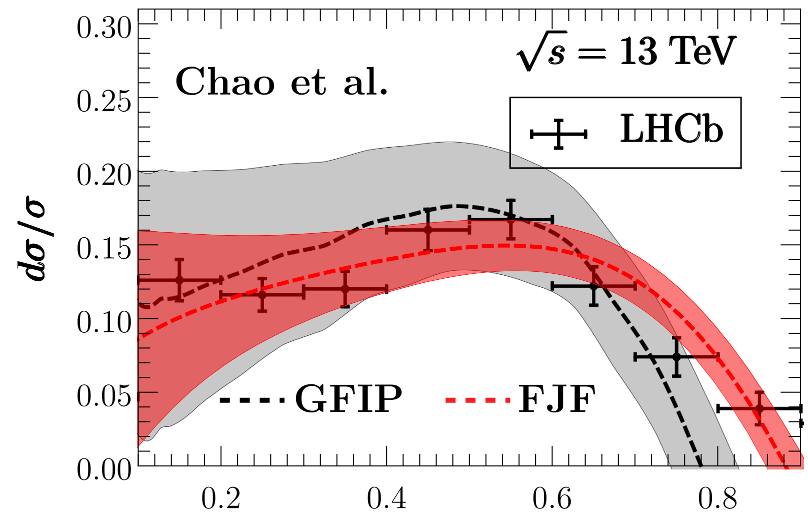


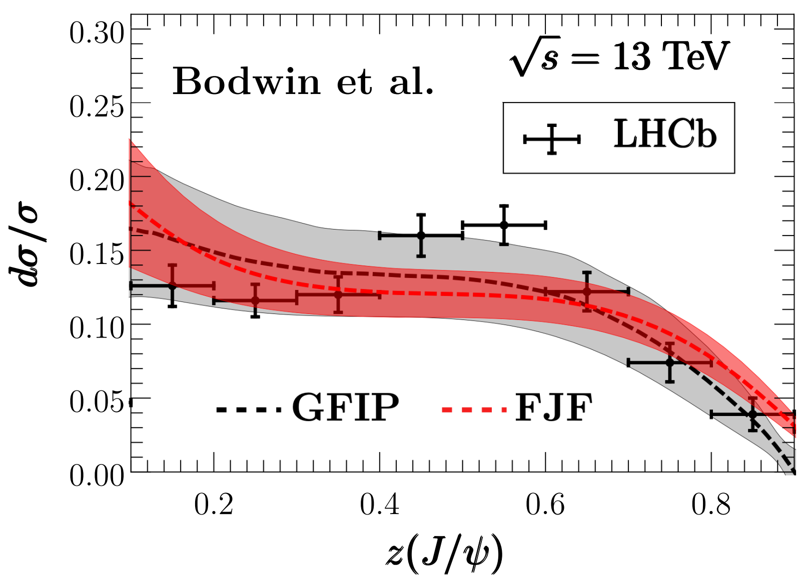
- They determined the χ_{c2} polarization (purple "data" points) by using
  - the prediction for the χ_{c1} polarization,
  - the measured ratio of angular distributions.
- "Out of the box" prediction of NRQCD works well.

J/ψ Energy Fraction in a Jet

- Bain, Dai, Makris, Leibovich, Mehen (1702.05525) used NRQCD to compute $z(J/\psi)$, the energy of the J/ψ divided by the energy of the accompanying jet.
- This is a measure of the gluon radiation that is expected to accompany the quarkonium in color-octet production channels.
- Computed using a corrected version of Pythia (GFIP) and the fragmenting jet function (FJF).
- Compared with the LHCb (1701.05116) data with predictions using different LDME sets.

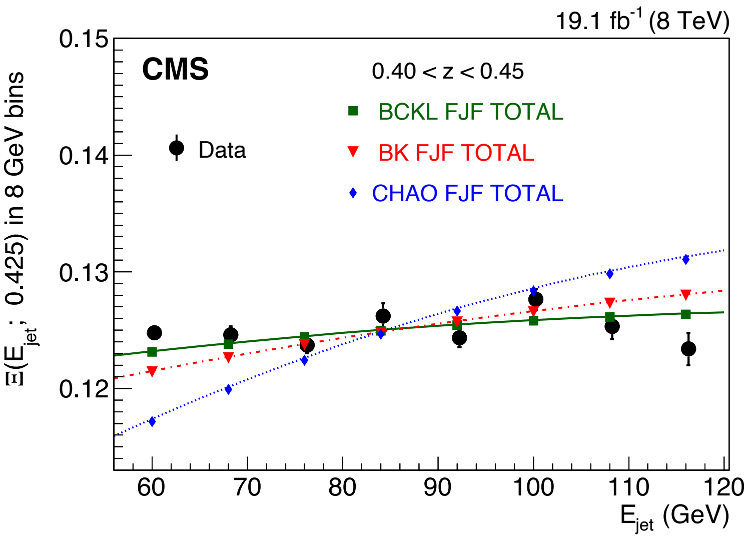


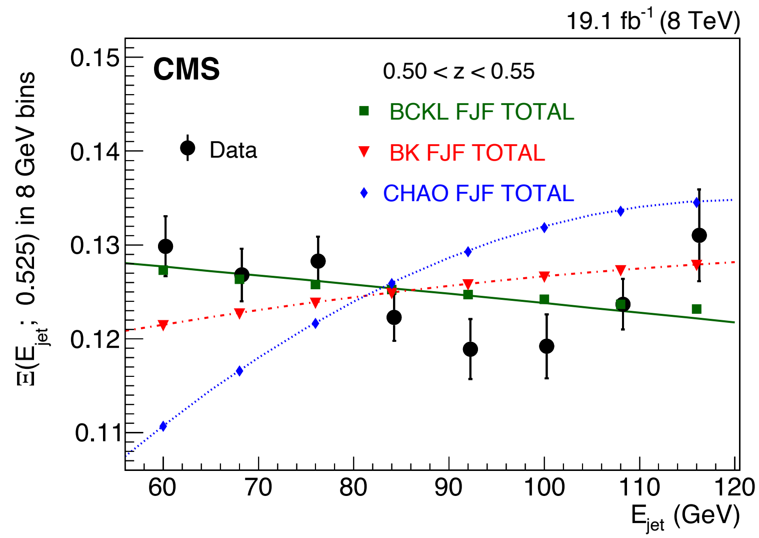


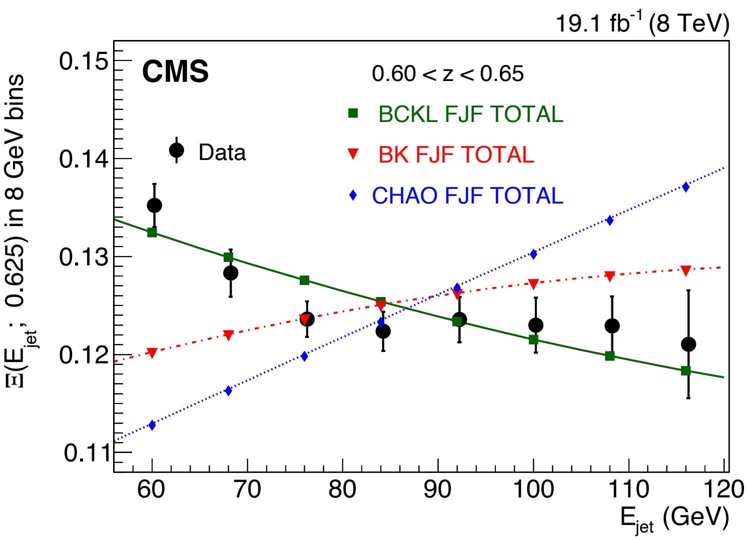


- BK is the Hamburg Group.
- Chao, Ma, Shao, Wang, Zhang (Chao *et al.*) LDMEs are from the polarization-constrained fit (1201.2675).
- Bodwin, Chung, Kim, Lee (BCKL) LDMEs (1403.3612) are similar to those of the ANL-PKU group, but don't separate feeddown contributions.
- The Chao *et al.* and BCKL LDMEs give reasonable fits.

- CMS (1910.01686) measured events in three z bins as a function of $E_{\rm jet}$.







- $\Xi(E_{\rm jet}; z)$ is number of events in a z bin divided by number of events with $0.3 \leq z \leq 0.8$.
- z bins at $z = 0.425$, 0.525, 0.635.
- Only the BCKL LDMEs give a good fit.

Outstanding Problems

J/ψ Photoproduction at HERA

- The Chao *et al.* polarization-constrained LDMEs are incompatible with the H1 photoproduction data.

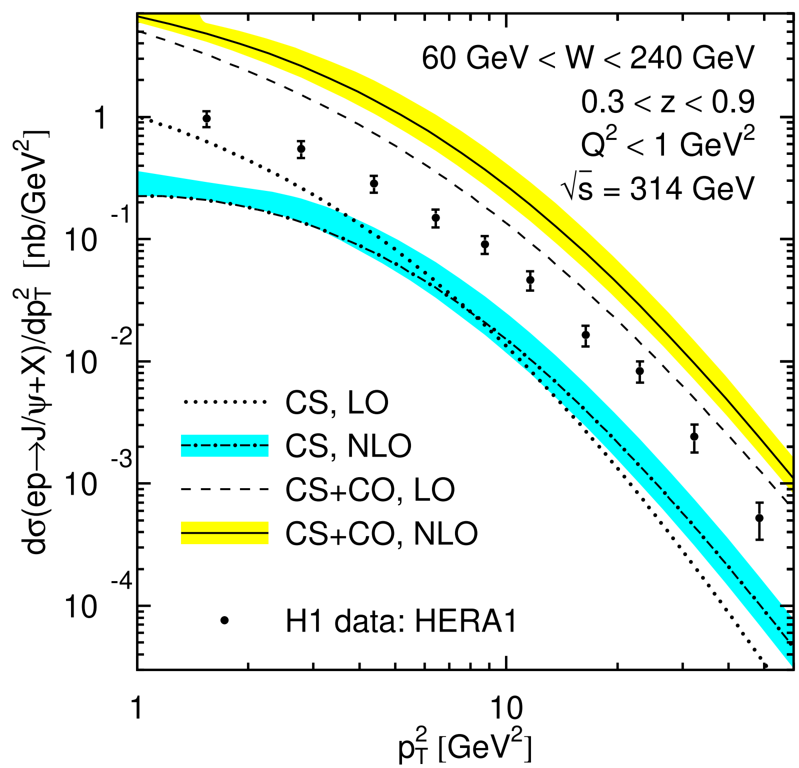
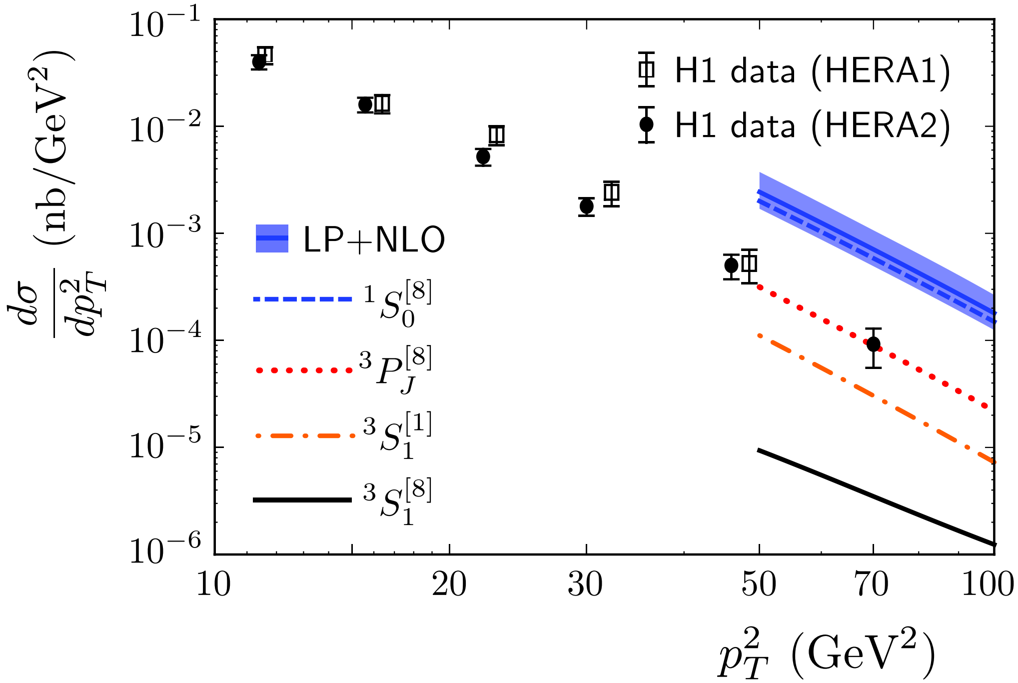


Figure courtesy of Mathias Butenschön.

- Is NRQCD factorization valid at such low values of p_T?

- GTB, Chung, Kim, Lee (1504.06019): Fragmentation corrections do not resolve the discrepancy.



- The p_T of the highest measured point is only about 8 GeV.
- But theory and data are not trending toward each other as p_T increases.

Constraints from $e^+e^- \to J/\psi + X_{\text{non}-c\bar{c}}$ at the B factories.

- Zhang, Ma, Wang, Chao (0812.5106, 0911.2166) computed the cross section for $e^+e^- \to c\bar{c}g$ through the ${}^1S_0^{[8]}$ and ${}^3P_0^{[8]}$ channels at NLO.
- Comparison with the Belle (0901.2775) measurement of $\sigma(e^+e^- \to J/\psi + X_{\text{non}-c\bar{c}})$ leads to a bound on the color-octet LDMEs:

$$M_{4.0} = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + 4.0\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) < (2.0 \pm 0.6) \times 10^{-2}\ \text{GeV}^3$$

- Bound comes from neglecting the color-singlet contribution, which saturates the measured cross section by itself.
- Bound is in conflict with the LDMEs extracted from the hadron-collider data, except for the Hamburg LDMEs:

$$M_{3.9}^{\text{PKU}} = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3 \qquad M_{3.9}^{\text{Hamburg}} = (2.17 \pm 0.56 \times 10^{-2}\ \text{GeV}^3$$
$$M_{3.9}^{\text{IHEP}} = (6.00 \pm 0.98) \times 10^{-2}\ \text{GeV}^3 \qquad M_{3.9}^{\text{ANL-PKU}} = (9.78 \pm 1.52) \times 10^{-2}\ \text{GeV}^3$$

- But, the Belle (0901.2775) measurement

$$\sigma(e^+e^- \to J/\psi + X) = (1.17 \pm 0.02 \pm 0.07) \text{ pb}$$

is more than a factor 2 smaller than the BaBar (hep-ex/0106044) measurement

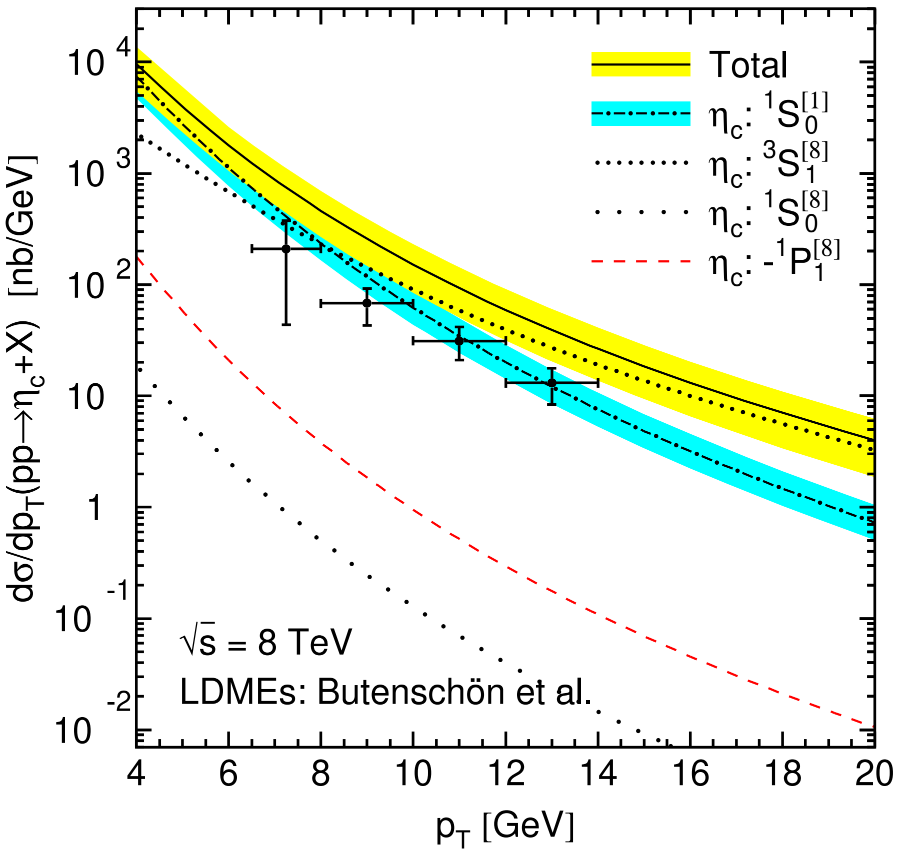
$$\sigma(e^+e^- \to J/\psi + X) = (2.52 \pm 0.21 \pm 0.21) \text{ pb.}$$

- Most of the data are at $p_T \lesssim 3$ GeV.
- Is p_T too small for NRQCD factorization to be valid?

η_c Production at LHCb

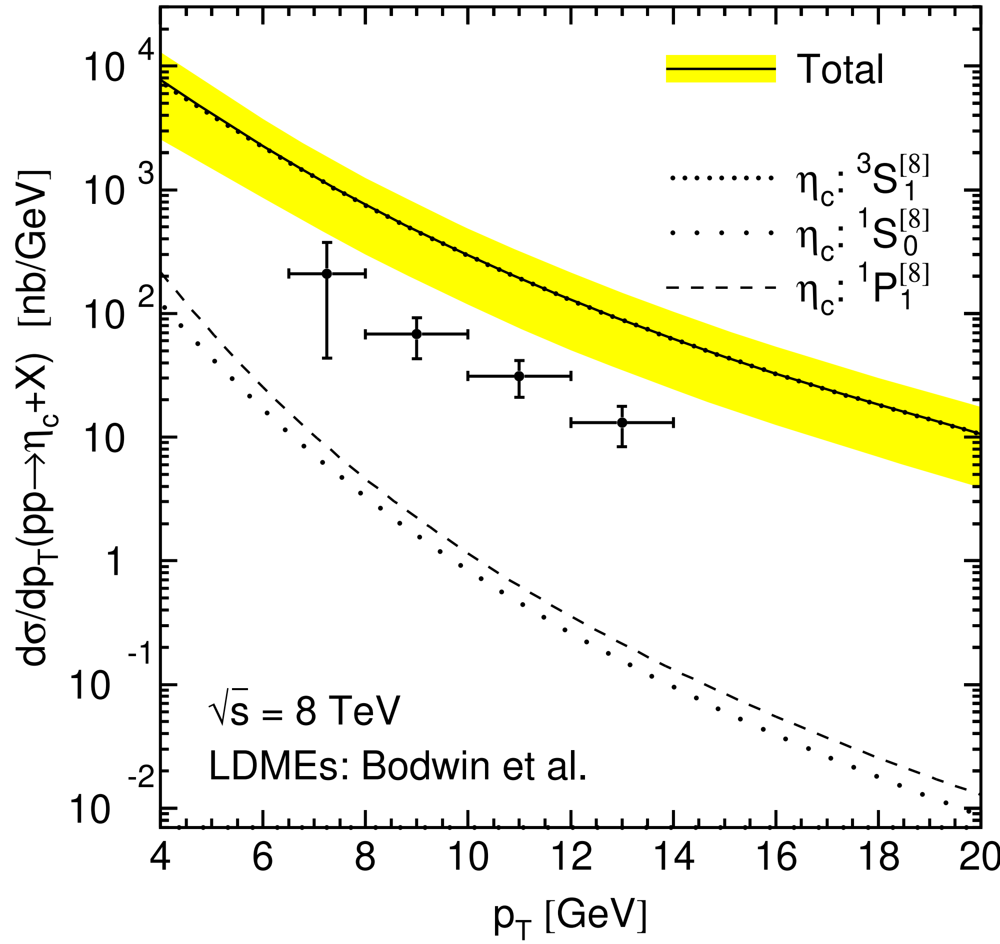
Butenschön and Kniehl (1411.5287)

- The NLO prediction for the η_c cross section overshoots the LHCb measurement (1409.3612) by a factor of about 6.



- The η_c LDMEs are fixed by using the heavy-quark spin symmetry of NRQCD to relate them to the J/ψ LDMEs.
  Good up to corrections of relative order v^2.
- The color-singlet contribution alone accounts for the measured cross section.

- Use of the BCKL LDMEs from NLO + fragmentation fits to the J/ψ cross-section makes the situation worse.

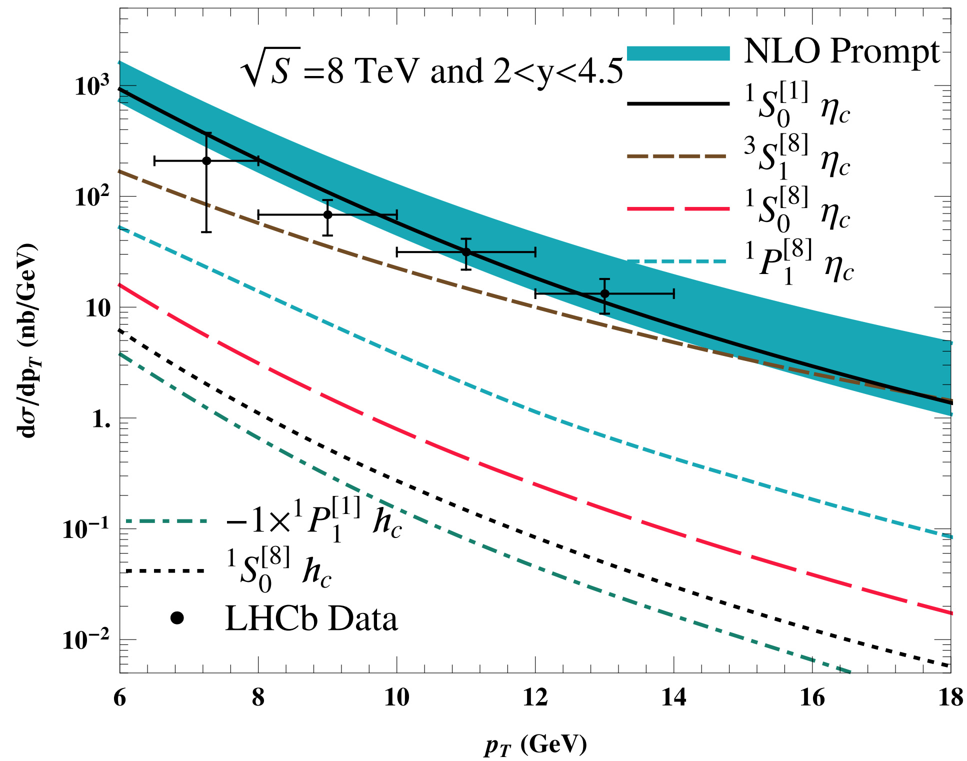


Han, Ma, Meng, Shao, Chao (1411.7350)

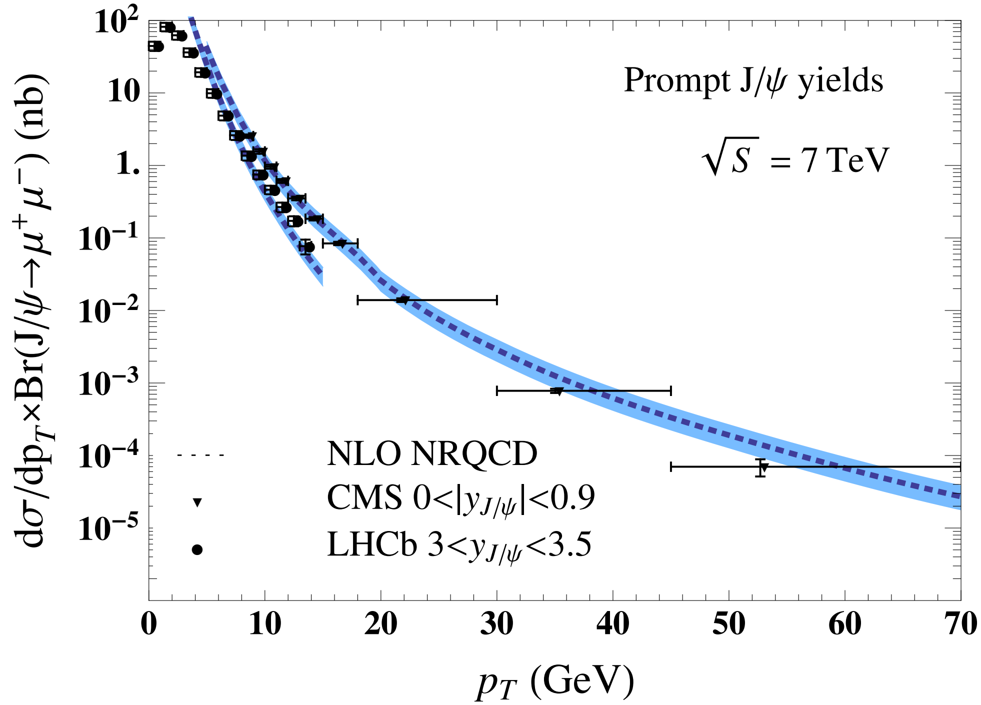
- Apply an additional constraint to the PKU LDME fit (1009.3655, 1012.1030):

$$0 < \langle \mathcal{O}^{\eta_c}(^3S_1^{[8]}) \rangle < 0.0146 \text{ GeV}^3 \quad \Longrightarrow \quad 0 < \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle < 0.0146 \text{ GeV}^3$$

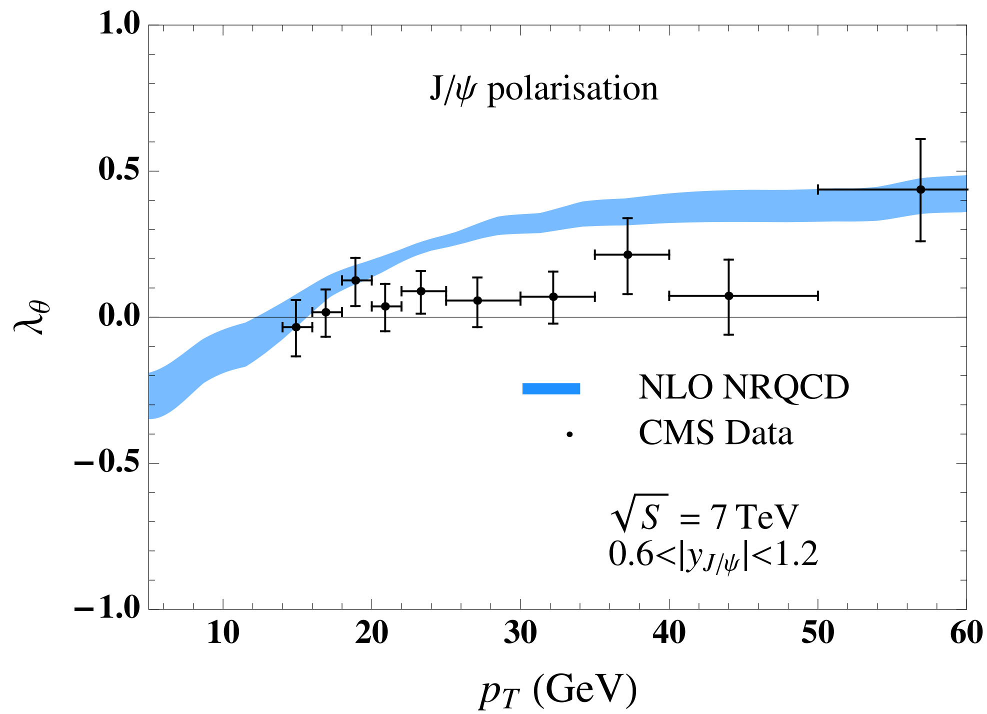
- Obtain reasonable agreement with the η_c cross-section data.



- The fit to the J/ψ cross-section data is still reasonable, as well.



- However, there is tension between the data and the prediction for the J/ψ polarization.



A Possible Difficulty with the Measurement

- LHCb measures the relative rates of the η_c and J/ψ in the $p\bar{p}$ channel.
- $\mathrm{BF}(\eta_c \to p\bar{p})$ was determined from a global fit to BFs that has a marginal $\chi^2/\mathrm{d.o.f.} = 121.6/81$.
- Direct measurements of $\mathrm{BF}(\eta_c \to p\bar{p})$ have large uncertainties.
- A 2σ deviation to the low side would boost the cross section by a factor 3.
- A new BESIII measurement may decide this issue.

Conclusions

- A fairly consistent theoretical picture has emerged for charmonium production at $p_T > 10$ GeV.
  - NRQCD factorization at NLO + all-orders fragmentation describes most of the data well.
  - ${}^1S_0^{[8]}$ dominance is a feature of S-wave charmonium production.
- High-quality, high p_T measurements from the LHC have enabled meaningful comparisons with theory:
  - J/ψ cross sections and polarization,
  - $\psi(2S)$ cross sections and polarization,
  - χ_{cJ} cross sections and polarization,
  - J/ψ jet energy fraction.

- There are still problematic areas:
  - The NLO + fragmentation prediction for J/ψ photoproduction is well above the H1 measurement.
  - Constraints from on the LDMES from the Belle measurement of $\sigma(e^+e^- \to J/\psi + X_{\mathrm{non}-c\bar{c}})$ are inconsistent with many of the LDME sets from hadron-collider cross sections.
  - The NLO + fragmentation prediction for η_c production overshoots the LHCb measurement by an order of magnitude.
- But, the HERA and Belle measurements are at $p_T < 8$ GeV and $p_T < 5$ GeV.
- There is no obvious explanation for the discrepancy in η_c production.
  - An independent measurement of the η_c cross section is urgently needed.
  - Very difficult to measure any of the η_c decay channels at CMS and ATLAS.

Future Directions

- New CMS measurements of χ_{cJ} polarizations
- New NLO calculations of gluon fragmentation to quarkonium:
  - ${}^1S_0^{[1,8]}$: Artoisenet and Braaten (1810.02448)
  - ${}^1S_0^{[1,8]}$: Feng, Jia (1810.04138)
  - ${}^3P_J[1,8]$: Zhang, Meng, Ma, Chao (2011.04905)
  - Should be incorporated into new fits of theory to data.
- It is important to take into account correlations between polarization and acceptance [Faccioli, Lourenço, Araújo, Seixas, Krätschmer, Knünz (1802.01106)]. New fits in progress.
- For bottomonium (including χ_{bJ}) make more stringent tests of NRQCD factorization:
  - Include fragmentation contributions in theory predictions,
  - Measure cross sections and polarizations at high p_T with high statistics.

- Brambilla, Chung, Vairo (2007.07613): pNRQCD can be used to relate LDMEs, reduce the number of parameters to be fit.
- Improved theory and new experimental measurements for additional production processes might help to understand production mechanisms.
  - Double-charmonium production
    Need to understand double-parton scattering PDFs from first principles (lattice?).
  - Theoretical progress on $J/\psi + Z$, $J/\psi + W^{\pm}$ final states.
    Need high-p_T, high-precision experimental measurements.
  - $J/\psi +$ jet
    Additional measures of jet substructure might help [Kang, Qiu Zing, Zhang (1702.03287)]
- Soft-gluon factorization [Ma, Chao (1703.08402)]
  Alternative to NRQCD that resums higher orders in v^2 and α_s.