# $m_c$ (and $m_b$) from the lattice.

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Has there been significant progress since CHARM 2015?

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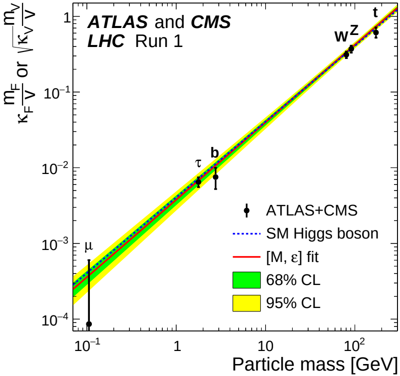
Absolutely!

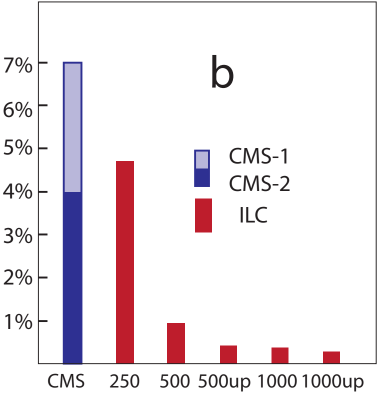
# Intro & Motivation

- Quark masses – fundamental parameters of the Standard Model.
- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial widths from $m_b$, $m_c$, $\alpha_s$. [1404.0319]
- Focus on precision results using independent methods.

# Higgs couplings

[1606.02266] [1312.4974]




Estimated final ILC precision in $hc\bar{c}$ coupling: $\sim 0.7\%$.

# Outline

- Background
  - Lattice simulations
  - Mass determinations
- Progress and Summary plots
- Quark mass methods
  - Current-current correlator moments
  - Regularisation Invariant (RI) methods
  - Minimal renormalon subtraction (MRS) masses
- Summary & Outlook

# Tuning mass input parameters

Bare quark masses are input parameters to lattice simulations. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud,0} \to m_\pi^2$
- $m_{s,0} \to m_K^2$
- $m_{c,0} \to m_{\eta_c}$

Tuning performed at multiple lattice spacings, defining a continuum trajectory for which $a^2 \to 0$ limit can be taken.

- Rest of physics is then prediction of QCD.
- Parameters can be varied away from physical values.. understand effect of quark mass, quantify systematics, etc.

# A note on quark mass definitions

- Quarks are not asymptotic (physical) states due to confinement – mass cannot be measured directly.

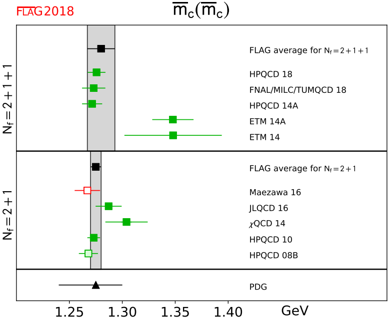
- Quark masses are scheme and scale dependent, $m_q^{\text{scheme}}(\mu)$.

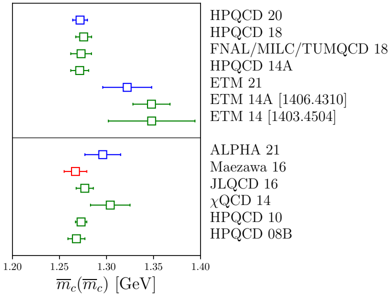
- Generally will quote results $m_q^{\overline{\text{MS}}}(\mu_{\text{ref}})$.

- Lattice input quark masses are non-universal (depend on discretisation), but can be connected to quark masses defined in a continuum scheme.

# FLAG and updates

1902.08191, flag.unibe.ch




New results:

- 2005.01845 - HPQCD, $n_f = 2+1+1$, add (quenched) QED.
- 2101.02694 - ALPHA, $n_f = 2+1$, based on SF techniques.
- 2104.13408 - ETM, $n_f = 2+1+1$, based on RI/MOM.

# FLAG and updates

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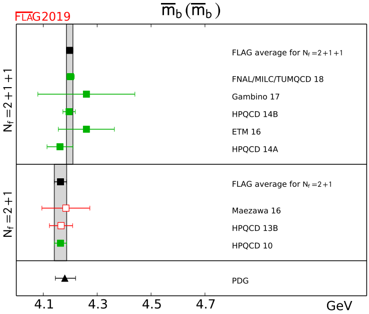
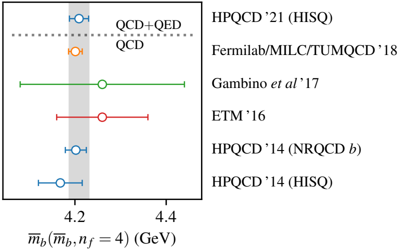



FLAG average ($n_f = 2+1+1$):
$m_b^{\overline{\rm MS}}(m_b) = 4.198(12)$ GeV $\to$ 0.3% uncertainty(!)

New results:

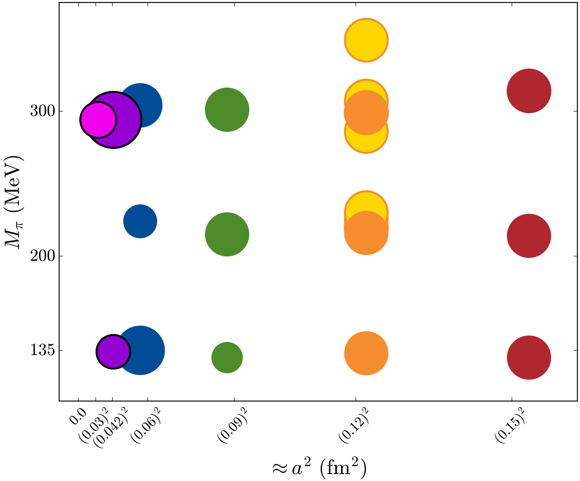
- 2102.09609 - HPQCD, $n_f = 2+1+1$, SMOM + $\frac{m_b}{m_c}$ + QED.

# MILC ensembles [1004.0342, 1212.4768]

- HISQ fermion action.
  - Discretization errors begin at $\mathcal{O}(\alpha_s a^2)$.
  - Designed for simulating heavy quarks ($m_c$ and higher at current lattice spacings).
- Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]
- Multiple lattice spacings down to $\sim 0.045$ (now 0.03) fm.
- Effects of $u/d$, $s$, and $c$ quarks in the sea.
- Multiple light-quark input parameters down to physical pion mass.
  - Chiral fits.
  - Reduce statistical errors.

# MILC ensemble parameters

1712.09262

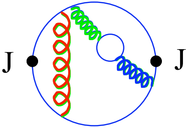


# $\langle JJ \rangle$-correlator moments

# Current-current correlators

Calculate time-moments of $J_5 \equiv \bar{\psi}_h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle J_5(t,\mathbf{x}) J_5(0,0) \rangle$$




- Currents are absolutely normalized (no $Z$s required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\text{cont}} = G(t)_{\text{latt}} + \mathcal{O}(a^2)$.

# Moments

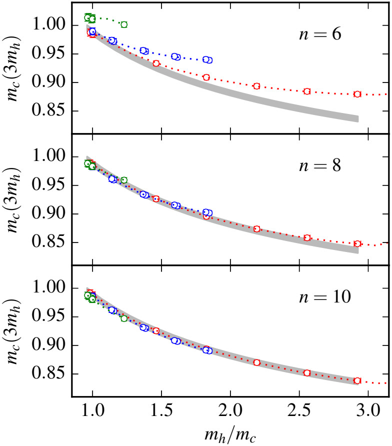
The time-moments $G_n = \sum_t (t/a)^n G(t)$ have been computed in perturbation theory to $\mathcal{O}(\alpha_s^3)$. For $n \geq 4$,

$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}, \mu)}{am_h(\mu)^{n-4}} \, .$$

Basic strategy:

1. Calculate $G_{n,\mathrm{latt}}$ for a variety of lattice spacings and $m_{h0}$.
2. Compare continuum limit $G_{n,\mathrm{cont}}$ with $G_{n,\mathrm{pert}}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\mathrm{MS}}}(m_h), m_h(m_h)$.

# Results for $n_f = 4$ [1408.4169]

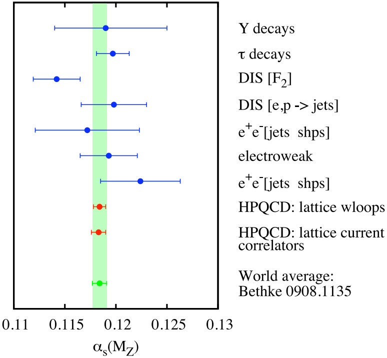



$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{\rm MS}}, \mu = 3m_h)}{R_n}$$

- Discretization effects grow with $am_h$ and decrease with $n$.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

$$\boxed{m_c^{\overline{\rm MS}}(3\ \text{GeV}) = 0.9851(63)\ \text{GeV}}$$

# $\alpha_s^{\overline{\mathrm{MS}}}(m_Z)$ [1408.4169]




HPQCD $\langle JJ \rangle$ result:

- $\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1182(7)$
- Agrees with $n_f = 3$ result.
- Agrees well with world average.

Precise lattice result from ALPHA collaboration using Schrödinger Functional and step-scaling:

$$\alpha_s^{\overline{\mathrm{MS}}}(m_Z) = 0.1185(8) \text{ [1706.03821]}$$

RI/SMOM

# NPR method

Trying to determine $Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)$ st

$$m^{\overline{\mathrm{MS}}}(\mu) = Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)\, m_0$$

Options:

- Lattice perturbation theory. – difficult!
- Alternatively, use two steps:
  latt $\leftrightarrow$ intermediate(continuum-like) $\leftrightarrow$ $\overline{\mathrm{MS}}$

# NPR method

General idea is to renormalize operators using a scheme that is well-defined both in the continuum and on the lattice, e.g. the RI schemes:

Calculate off-shell Green's functions of operator-of-interest with external quark states.

$$G_\Gamma^{ij}(p) = \langle q^i(p) \left( \sum_x \bar{q}(x) \Gamma q(x) \right) \bar{q}^j(-p) \rangle_{\mathrm{amp}}$$

Require that the trace of the renormalized operator takes its tree-level value:

$$\Lambda_\Gamma(p) \equiv \frac{1}{12} \mathrm{Tr} \left[ \Gamma \, G_\Gamma(p) \right] \simeq \frac{Z_q(p)}{Z_\Gamma(p)}$$

# NPR method (cont.)

The RI (and $\overline{\text{MS}}$) schemes satisfy $Z_m = Z_S^{-1} = Z_P^{-1}$. $Z_m$ can be extracted from the scalar correlator provided

$$\Lambda_{\text{QCD}} \ll |p| \ll \pi/a$$

After determining $Z_m^{RI}(p)$, a perturbative calculation can be used to convert $Z^{\overline{\text{MS}}}(p) = C^{\overline{\text{MS}} \leftarrow RI}(p)\, Z_m^{RI}(p)$.

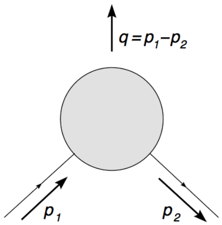
# RI/SMOM scheme

0901.2599

- Momentum flow suppresses infrared effects.
  $p_1^2 = p_2^2 = (p_1 - p_2)^2$

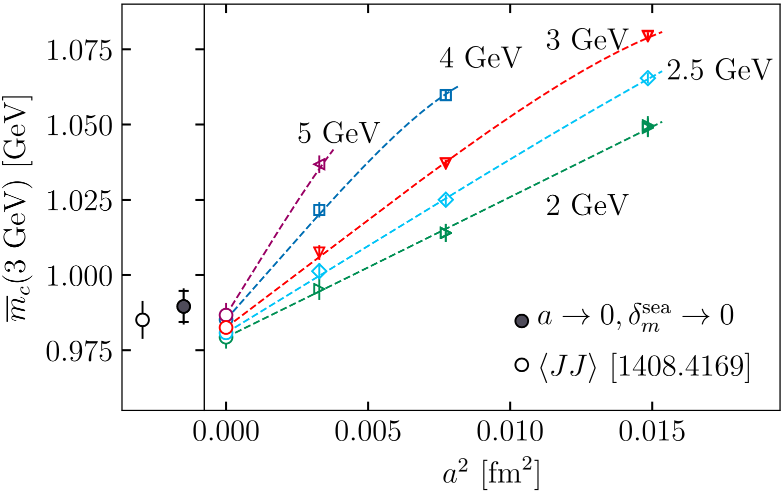
- $p_1 \sim (x, x, 0, 0)$,
  $p_2 \sim (0, x, x, 0)$ for
  $x = 2, 3, 4$

- Other advantages:
  - Reduced mass dependence.
  - SMOM $\to \overline{\text{MS}}$ matching factors closer to 1.



# Continuum extrapolations

1805.06225



# Note on perturbative matching

- Perturbative SMOM $\to \overline{\text{MS}}$ conversion factor, in 2018, known at $\mathcal{O}(\alpha_s^2)$, was a leading source of uncertainty.
- Uncertainty from $c_\alpha \alpha_s^3$ was estimated from fit result to be 0.22%.
- Subsequently this term was calculated in 2002.10894, Kniehl and Veretin and 2002.12758, Bednyakov and Pikelner:

$$C_m^{\overline{\text{MS}}/\text{SMOM}}(n_f = 4, 3\ \text{GeV}) = 1 - 0.01307 - 0.00269 - 0.00196$$

- Fit update in 2005.01845 gives new $c_\alpha \alpha_s^4$ uncertainty at 0.1%

# Adding quenched QED

2005.01845

At some point, absence of QED effects becomes a dominant uncertainty.

- Potential size of EM effects $\alpha$, $\sim 1\%$.
- Include in simulation what is expected to be the bulk effect - multiply QED field into the $SU(3)$ field.
- Neglects terms of size $\alpha_s^2\alpha$, so maybe 10% of an overall tiny correction.
- Neglects strong isospin breaking.

# Adding QED

2005.01845, 2102.09609

Three steps required using RI/SMOM:

1. Retune bare charm mass to physical $J/\psi$ mass.
2. Include EM field in RI/SMOM renormalization.

   $m_c$ decreases by 0.18(2)%:

   $m_c^{\overline{\rm MS}}(3\ \text{GeV}) = 0.9858(51) \to 0.9841(51)\ \text{GeV}$

3. Retune quark mass ratio $m_b/m_c$:

   $m_b/m_c$ increases by 0.17(3)%:

   $m_b^{\overline{\rm MS}}(3\ \text{GeV}) = 4.513(26)\ \text{GeV}$

# Renormalon subtracted (MRS) masses

# HQET masses

Mass of a heavy meson $H$ in heavy quark effective theory (HQET)

$$M_H = m_Q + \overline{\Lambda} + \frac{\mu_\pi^2}{2m_Q} - \frac{\mu_G^2(m_Q)}{2m_Q} + \cdots ,$$

where

- $m_Q$: Pole mass of the heavy quark $Q$
- $\overline{\Lambda}$: Energy of light quarks and gluons
- $\frac{\mu_\pi^2}{2m_Q}$: Kinetic energy of heavy quark
- $\frac{\mu_G^2(m_Q)}{2m_Q}$: Hyperfine energy due to heavy quark spin

Want to relate pole mass to $\overline{\mathrm{MS}}$ mass,

Meson mass $\leftrightarrow$ quark pole mass $\leftrightarrow$ quark $\overline{\mathrm{MS}}$ mass

# $m_Q \leftrightarrow m^{\overline{\text{MS}}}$ [1701.00347]

Perturbative series connecting the pole mass to the $\overline{\text{MS}}$ mass (known to four loops) diverges due to renormalons,

$$m_{\text{pole}} = \overline{m}\left(1 + \sum_{n=0}^{\infty} r_n \, \alpha_s^{n+1}(\overline{m})\right),$$

with

$$r_n \propto (2\beta_0)^n \Gamma(n+b+1) \text{ as } n \to \inf$$

but can be interpreted using Borel summation. After subtracting the (leading) renormalon from the pole mass, there is a well-behaved connection between the subtracted mass and the $\overline{\text{MS}}$ mass.

$$m_{\text{pole}} \to m_{\text{MRS}} + \mathcal{O}(\Lambda_{\text{QCD}})$$

## $m_Q \leftrightarrow m^{\overline{\mathsf{MS}}}$

[1712.04983]

$$m_{\text{pole}} + \overline{\Lambda} = \overline{m}\left(1 + \sum_{n=0}^{\infty} r_n\, \alpha_s^{n+1}(\overline{m})\right) + \overline{\Lambda} \rightarrow$$

$$\overline{m}\left(1 + \sum_{n=0}^{\infty} [r_n - R_n]\, \alpha_s^{n+1}(\overline{m})\right) + J_{\text{MRS}}(\overline{m}) + \left[\delta_m + \overline{\Lambda}\right]$$

$$= m_{\text{MRS}} + \overline{\Lambda}_{\text{MRS}}$$

$$r_n = (0.4244, 1.0351, 3.6932, 17.4358, \dots)$$

$$R_n = (0.5350, 1.0691, 3.5966, 17.4195, \dots)$$

$$r_n - R_n = (-0.1106, -0.0340, 0.0966, 0.0162, \dots)$$

# MRS - calculation

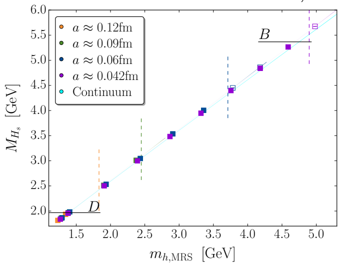
1802.04248

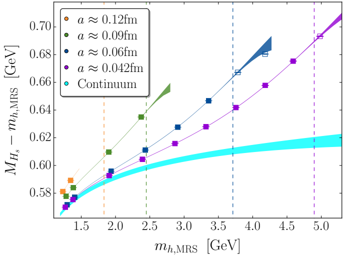
Measure meson mass $M_{Hs}$ varying heavy input mass $am_{h,0}$.

$$m_h^{\overline{\mathrm{MS}}}(\mu) = m_r^{\overline{\mathrm{MS}}}(\mu)\frac{am_{h,0}}{am_{r,0}} + \mathcal{O}(a^2)\,,$$

with $m_r^{\overline{\mathrm{MS}}}(\mu)$ treated as a fit parameter.

- Fit data including discretization artifacts as as well as HQET parameters $\overline{\Lambda}_{\mathrm{MRS}}$, $\mu_\pi^2$, $\mu_G^2(\mu)$.
- Evaluate fit at $M_{D_s}$,$M_{B_s}$ to obtain $\overline{m}_c$, $\overline{m}_b$.



# MRS - results



$$m_s^{\overline{\text{MS}}}(2\text{ GeV}) = 92.47(39)_{\text{stat}}(18)_{\text{sys}}(52)_{\alpha_s}(11)_{f_{\pi,\text{PDG}}}\text{ MeV}$$
$$\overline{m_c} = 1273(4)_{\text{stat}}(1)_{\text{sys}}(10)_{\alpha_s}(0)_{f_{\pi,\text{PDG}}}\text{ MeV}$$
$$\overline{m_b} = 4201(12)_{\text{stat}}(1)_{\text{sys}}(8)_{\alpha_s}(1)_{f_{\pi,\text{PDG}}}\text{ MeV}$$

These results can be compared e.g. with current-correlator results:

$$m_s^{\overline{\text{MS}}}(2\text{ GeV}) = 93.6\,(8)\text{ MeV} \qquad [1408.4169]$$
$$\overline{m_c} = 1271\,(10)\text{ MeV}$$
$$\overline{m_b} = 4196\,(23)\text{ MeV} \qquad [1408.5768]$$

# Comparing methods

- Quark-level diagrams on the lattice (RI/SMOM)
  - Perturbative and IR (condensate) uncertainties decrease with lattice spacing.
  - Main uncertainty comes from tuning uncertainties - need improved determinations of lattice spacings and input masses.
- Time-moments of heavyonium correlators
  - Main uncertainty from perturbation theory.
  - Finer lattice means reference scale $am_h$ can be increased.
- MRS subtracted masses –
  Heavy-light meson masses and HQET
  - Calculation already includes $a \sim 0.045$, 0.03 fm lattices.
  - Uncertainty in $\alpha_s$ is a major source of error.

# Summary & Conclusion

# Summary & Conclusion

- Since CHARM 2015 - Highlights
  - Many new results: 13 since 2015, 4 since FLAG19.
    $m_c$: $6 \to 13$, $n_f = 3$: $5 \to 9$
    $m_b$: $4 \to 9$, $n_f = 4$: $5 \to 13$
  - From more collaborations $\to$ differing discretizations/lattice artifacts under control.
  - New techniques with different sources of systematic uncertainty.
- QED corrections are small ($\lesssim 0.2\%$ for charm).
- Precision continues to improve.
- Complementary techniques - good agreement at sub-percent level.

Thank you!

# Meson masses – summary plot