A combined description of $D^+ \to \pi^+\pi^+K^-$ and $D_s \to K^+K^+\pi^-$ decays

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Work in Coll. with R. Escribano & P. Masjuan

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Outline

- Naive factorization approach to $D^+ \to \pi^+\pi^+K^-$ decays
- Interlude: $D^+ \to \pi^+K^-\ell^+\nu$ decays
- Back to $D^+ \to \pi^+\pi^+K^-$ decays
- Bonus: $D_s \to K^+K^+\pi^-$ decays
- Conclusions

Section 1

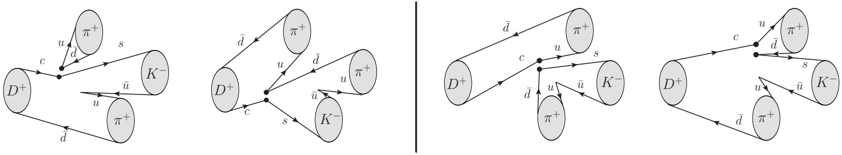
Naive factorization approach to $D^+ \to \pi^+\pi^+K^-$ decays

Naive factorization for $D^+ \to \pi^+\pi^+K^-$ decays: recipe

- Based on BE[1]: The weak effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2] + \text{h.c.}, \qquad \mathcal{O}_{1(2)} = 4[\bar{s}_L^i \gamma^\mu c_L^{i(j)}][\bar{u}_L^j \gamma^\mu d_L^{j(i)}],$$

- Their contribution from $\mathcal{O}_{1,2}$ (each with $\mathcal{O}(N_c^1, N_c^0)$ hadronization modes)



- Can be expressed as $[a_1 = C_1 + \frac{C_2}{N_c},\ a_2 = C_2 + \frac{C_1}{N_c}]$

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \Big[a_1 \langle K^-\pi_1^+ | \bar{s}\gamma^\mu(1-\gamma^5)c | D^+\rangle \langle \pi_2^+ | \bar{u}\gamma_\mu(1-\gamma^5)d | 0\rangle + a_2 \langle K^-\pi_1^+ | \bar{s}\gamma^\mu(1-\gamma^5)d | 0\rangle \langle \pi_2^+ | \bar{u}\gamma_\mu(1-\gamma^5)c | D^+\rangle \Big] + (\pi_1^+ \leftrightarrow \pi_2^+),$$

[1] D. R. Boito & R. Escribano, PRD80, (2009)

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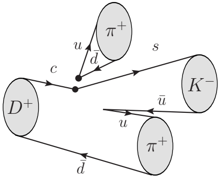

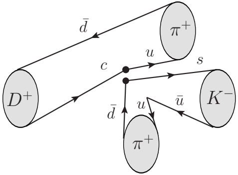

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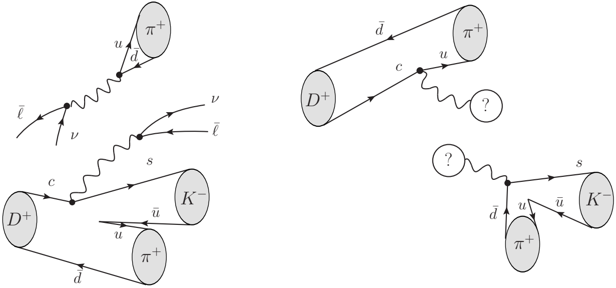

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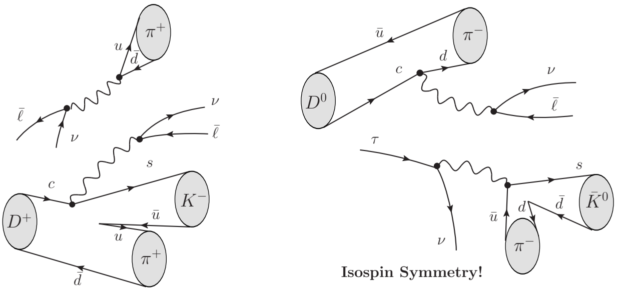

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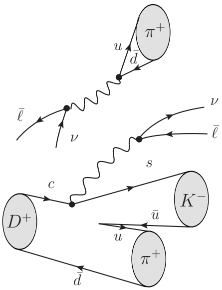

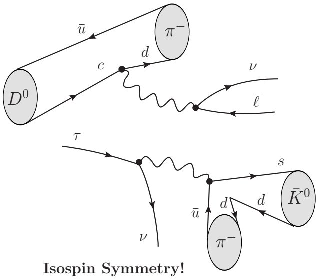

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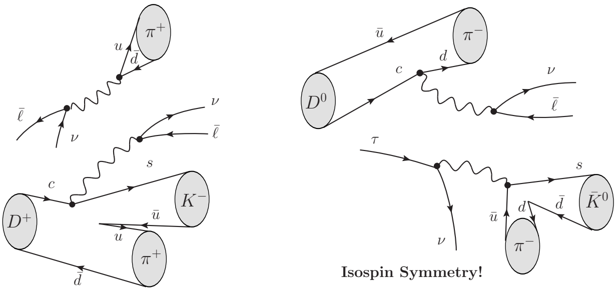

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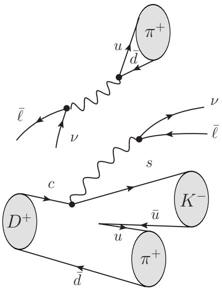

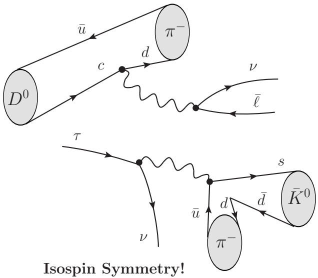

- Factorization: $D^+ \to \pi^+\pi^+K^- \sim$ semileptonic decays + spectator π^+
- Requires a closer look into $D^+ \to \pi^+K^-\bar{\ell}\nu$ decays (interesting *per se*)

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Section 2

Interlude: $D^+ \to \pi^+K^-\ell^+\nu$ decays

$D^+ \to \pi^+K^-\ell^+\nu$ decays: general features

- The matrix element reads

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{cs}^* \langle \pi^+ K^- |\, \bar{s}\gamma^\mu(1-\gamma^5)c \,| D^+ \rangle \, [\bar{u}_\nu \gamma_\mu (1-\gamma^5) v_\ell],$$

- The involved form factors

$$\langle K^- \pi^+ |\, \bar{s}\gamma^\mu(1-\gamma^5)c \,| D^+ \rangle = i w_+ p^\mu + i w_- \bar{p}^\mu + i r q^\mu - h \epsilon^{\mu q p \bar{p}}$$

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$$\langle K^-\pi^+ | \bar{s}\gamma^\mu(1-\gamma^5)c | D^+\rangle = i\boldsymbol{w}_+(p^\mu - q^\mu \frac{p\cdot q}{q^2}) + i\boldsymbol{w}_-(\bar{p}^\mu - q^\mu \frac{\bar{p}\cdot q}{q^2}) + \frac{i\tilde{\boldsymbol{r}}}{q^2} q^\mu - \boldsymbol{h}\epsilon^{\mu q p \bar{p}}$$

- With divergence $\partial_\mu \bar{s}\gamma^\mu(1-\gamma^5)c$ appearing in $\tilde{r}$ alone

$$\tilde{r} = -(m_c + m_s)\langle K^-\pi^+ | \bar{s} i\gamma^5 c | D^+\rangle \qquad \lim_{q^2\to 0}[(p\cdot q)\boldsymbol{w}_+ + (\bar{p}\cdot q)\boldsymbol{w}_- - \tilde{\boldsymbol{r}}] = 0$$

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- In terms of (exp. used) helicity form factors

$$F_1 = \frac{1}{X}\left(X^2\boldsymbol{w}_+ + [(p\cdot q)(\bar{p}\cdot q) - q^2(p\cdot\bar{p})]\boldsymbol{w}_-\right) \quad F_2 = \ldots \quad F_3 = \ldots \quad F_4 = \tilde{\boldsymbol{r}}$$

$$\lim_{q^2\to 0}[F_1(q^2, p^2, \bar{p}\cdot q) - F_4(q^2, p^2, \bar{p}\cdot q)] = 0$$

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- It is the divergence ($\tilde{r}/F_4$) that we need! Irrelevant for $m_\ell = 0$, but...

$D^+ \to \pi^+K^-\ell^+\nu$ decays: model (I) P-wave

- If mediated by a narrow-width(NW) single resonance [3]

$$\langle \pi^+K^- |\, \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle\, |_{P\text{-wave}} = \langle \pi^+K^- | \bar{K}^*\rangle\, P_{\bar{K}^*}\, \langle \bar{K}^* |\, \bar{s}\gamma^\mu(1-\gamma^5)c\, |D^+\rangle$$
$$= \chi_V^{\bar{K}^*}(p_K - p_\pi)^\alpha \varepsilon_\alpha P_{\bar{K}^*}\Big(A\epsilon^{\mu\nu qp} - i\Big[B(g^{\mu\nu} - \tfrac{q^\mu q^\nu}{q^2}) + Cq^\nu(p^\mu - \tfrac{q\cdot p}{q^2}q^\mu) + \tfrac{\tilde{D}}{q^2}q^\mu q^\nu\Big]\Big) m_{\bar{K}^*}\varepsilon_\nu$$

[3] D. R. Boito & R. Escribano, PRD80, (2009)
[4] R. Escribano *et. al* JHEP 09, 042 (2014)

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$$= \chi_V^{\text{eff}}(p_K-p_\pi)^\alpha\varepsilon_\alpha F_+^{K\pi}(p^2)\Big(A\epsilon^{\mu\nu qp} - i\Big[B(g^{\mu\nu}-\tfrac{q^\mu q^\nu}{q^2}) + Cq^\nu(p^\mu - \tfrac{q\cdot p}{q^2}q^\mu) + \tfrac{\tilde{D}}{q^2}q^\mu q^\nu\Big]\Big)m_{\bar{K}^*}\varepsilon_\nu$$

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$$F_1 = -\,\chi_{\bar{K}^*}F_+^{K\pi}(p^2)\beta_{K\pi}\cos\theta_{K\pi}\,\frac{X^2C(0)+(q\cdot p)B(0)}{1-q^2/m_{D_{s1}}^2},\quad F_2=...,\quad F_3=...$$

$$F_4 = -\,\chi_{\bar{K}^*}F_+^{K\pi}(p^2)\,\frac{N(p^2)}{2}\,\frac{B(0)+\frac{m_D^2-p^2}{2}C(0)}{1-q^2/m_{D_s}^2}$$

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- From analiticity unitarity[4] ($\lambda_1 = 0.0025$)

$$F_+^{K\pi}(s) = \exp\left[\lambda_1\frac{s}{m_\pi^2} + G_+(s)\right], \quad G_+(s) = \frac{s^2}{\pi}\int_{s_{th}}^{\infty} d\eta\,\frac{\delta_1^{1/2}(\eta)}{\eta^2(\eta-s)}.$$

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- Get $\chi^{\rm eff}_{A,B,C} = \chi_{\bar{K}^*}\{A,B,C\}$ from semileptonic decays

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$D^+ \to \pi^+K^-\ell^+\nu$ decays: model (II) S-wave

- If NW singly resonance mediated

$$\langle\pi^+K^-|\,\bar{s}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle\,|_{S\text{-wave}} = \langle\pi^+K^-|\bar{K}_0^*\rangle\, P_{\bar{K}_0^*}\,\langle\bar{K}_0^*|\,\bar{s}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle$$
$$= i\chi_{\bar{K}_0^*}P_{\bar{K}_0^*}\left[w_+^{\bar{K}_0^*}(q^2)\left(p^\mu_{\bar{K}_0^*} - \frac{q\cdot p_{\bar{K}_0^*}}{q^2}q^\mu\right) + q^\mu\,\frac{\tilde{r}^{\bar{K}_0^*}(q^2)}{q^2}\right]$$

[5] V. Bernard *et. al* PLB638, 480 (2006)

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- From which the form factor with std (charmed) res. saturation assumption

$$F_1 = \frac{2\chi_S^{\text{eff}} X}{1-q^2/m^2_{D_{s1}}} F_0^{K\pi}(p^2) \qquad F_4 = \frac{\chi_S^{\text{eff}}(m_D^2-p^2)}{1-q^2/m^2_{D_s}} F_0^{K\pi}(p^2)$$

- Where use of $\lim_{q^2\to 0}[F_1 - F_4] = 0$ and high-energy behavior has been made

[5] V. Bernard *et. al* PLB638, 480 (2006)

$D^+ \to \pi^+K^-\ell^+\nu$ decays: model (II) S-wave

- If NW singly resonance mediated

$$\langle \pi^+K^- |\, \bar{s}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle\,|_{S\text{-wave}} = \langle \pi^+K^-|\bar{K}_0^*\rangle\, P_{\bar{K}_0^*}\, \langle \bar{K}_0^*|\, \bar{s}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle$$
$$= i\chi_S^{\text{eff}} F_0^{K\pi}(p^2)\left[w_+^{\bar{K}_0^*}(q^2)\left(p^\mu_{\bar{K}_0^*} - \frac{q\cdot p_{\bar{K}_0^*}}{q^2}q^\mu\right) + q^\mu\,\frac{\tilde{r}^{\bar{K}_0^*}(q^2)}{q^2}\right]$$

- From which the form factor with std (charmed) res. saturation assumption

$$F_1 = \frac{2\chi_S^{\text{eff}}X}{1-q^2/m^2_{D_{s1}}} F_0^{K\pi}(p^2) \qquad F_4 = \frac{\chi_S^{\text{eff}}(m_D^2-p^2)}{1-q^2/m^2_{D_s}} F_0^{K\pi}(p^2)$$

- Where use of $\lim_{q^2\to 0}[F_1 - F_4] = 0$ and high-energy behavior has been made
- Based on unitarity + analiticity[5]

$$F_0^{K\pi}(s) = \exp\left[\frac{s[\ln C_{K\pi} + G_0(s)]}{\Delta_{K\pi}}\right], \quad G_0(s) = \frac{\Delta_{K\pi}(s-\Delta_{K\pi})}{\pi}\int_{s_{th}}^{\infty} d\eta\, \frac{\delta_0^{1/2}(\eta)}{\eta(\eta-\Delta_{K\pi})(\eta-s)}$$

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$D^+ \to \pi^+K^-\ell^+\nu$ decays: model (II) S-wave

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- But not NW dominated $\longrightarrow$ same phase-shift, different subtractions!

$$F_0^{K\pi}(s) \to F_0^{D_{\ell 4}}(s) \qquad (e.g., \ln C_{K\pi} \to \ln C_{D_{\ell 4}})$$

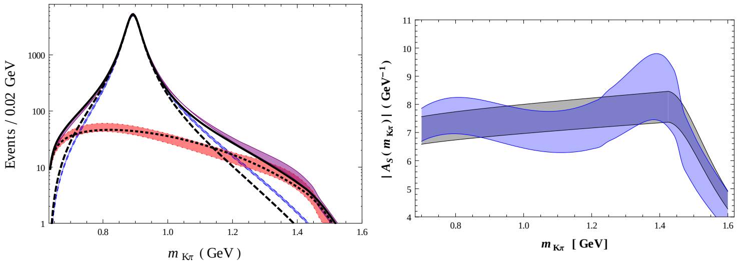
[5] V. Bernard *et. al* PLB638, 480 (2006)

$D^+ \to \pi^+K^-\ell^+\nu$ decays: results

- Data not available $\longrightarrow$ fit to BES-3 model for FFs[6]

Scalar: $\chi_S^{\rm eff} = -2.23(16)\ \mathrm{GeV}^{-1}$ $\quad \ln C_{D_{\ell 4}} = 0.152(11)$

Vector: $\chi_A^{\rm eff} = -3.52(16)\ \mathrm{GeV}^{-3}$ $\quad \chi_B^{\rm eff} = 8.87(23)\ \mathrm{GeV}^{-1}$ $\quad \chi_C^{\rm eff} = -1.72(12)\ \mathrm{GeV}^{-3}$



- Our model could be used to improve exp. description
- Ideally preferred option: fit data (not available) rather than exp. models
- Model for F_4 form factor ready: go back to $D^+ \to \pi^+\pi^+K^-$ decays

[6] BES3 Coll., PRD94, 032001 (2016)

Section 3

Back to $D^+ \to \pi^+\pi^+K^-$ decays

Results for $D^+ \to \pi^+\pi^+K^-$ decays

- We first compare to P-wave BR (1.06%)

$\Rightarrow$ Consistent within errors; matching central value requires $D_{\ell 4}$ input rescaling

$$1.24(6)_{\mathrm{B3}}(3)_{PDG}(12)_{a_1}(16)_{a_2}(5)_{F^{D\pi}}[21]_{\mathrm{Total}}$$

- For S-wave, some additions to fit data and full BR:

$\Rightarrow$ Extra $(180-65)^\circ$ phase as in BE/exp[a]

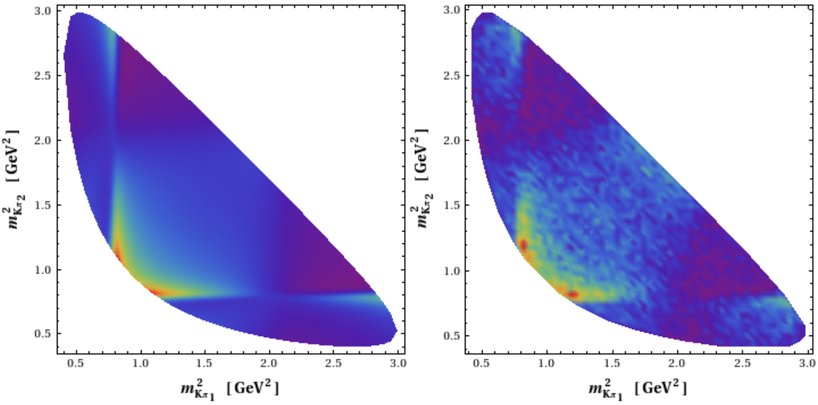
$\Rightarrow$ Additional $D_{\ell 4}$ S-wave rescaling

$$1.55(10)_{\mathrm{B3}}(6)_{P}(2)_{BR}(0)_{a_1}(26)_{a_2}(5)_{F^{D\pi}}[30]_{\mathrm{Total}}$$

[a] D. R. Boito & R. Escribano, PRD80, (2009); CLEO Coll., PRD78, 052001 (2008)

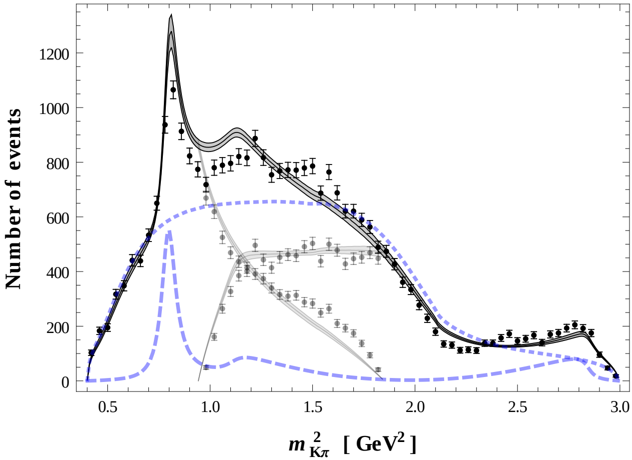
Results for $D^+ \to \pi^+\pi^+K^-$ decays

- Data from E791 Coll., PRD 73, 032004 (2006)



Results for $D^+ \to \pi^+\pi^+K^-$ decays

- Data from E791 Coll., PRD 73, 032004 (2006)



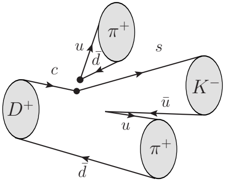
Section 4

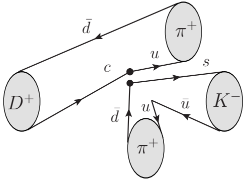
Bonus: $D_s \to K^+K^+\pi^-$ decays

Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: recipe

- Connected to $D^+ \to \pi^+\pi^+K^-$ via $d \leftrightarrow s$ exchange

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}^*\Big[a_1\,\langle K^-\pi_1^+|\,\bar{s}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle\,\langle\pi_2^+|\,\bar{u}\gamma_\mu(1-\gamma^5)d\,|0\rangle +$$
$$a_2\,\langle K^-\pi_1^+|\,\bar{s}\gamma^\mu(1-\gamma^5)d\,|0\rangle\,\langle\pi_2^+|\,\bar{u}\gamma_\mu(1-\gamma^5)c\,|D^+\rangle\Big] + (\pi_1^+ \leftrightarrow \pi_2^+)$$

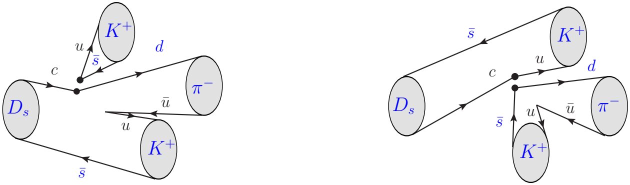




Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: recipe

- Connected to $D^+ \to \pi^+\pi^+K^-$ via $d \leftrightarrow s$ exchange

$$i\mathcal{M} = -i\frac{G_F}{\sqrt{2}}V_{us}V_{cd}^*\Big[a_1 \langle \pi^- K_1^+ | \bar{d}\gamma^\mu(1-\gamma^5)c\,|D^+\rangle \langle K_2^+ | \bar{u}\gamma_\mu(1-\gamma^5)s\,|0\rangle + a_2 \langle \pi^- K_1^+ | \bar{d}\gamma^\mu(1-\gamma^5)s\,|0\rangle \langle K_2^+ | \bar{u}\gamma_\mu(1-\gamma^5)c\,|D^+\rangle\Big] + (K_1^+ \leftrightarrow K_2^+)$$



Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: recipe

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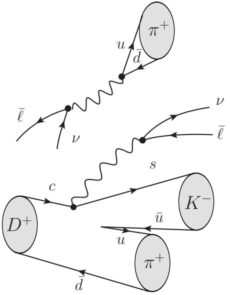

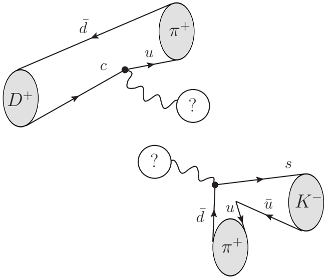

Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: recipe

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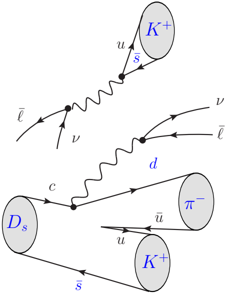

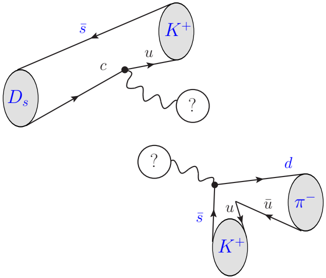

Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: recipe

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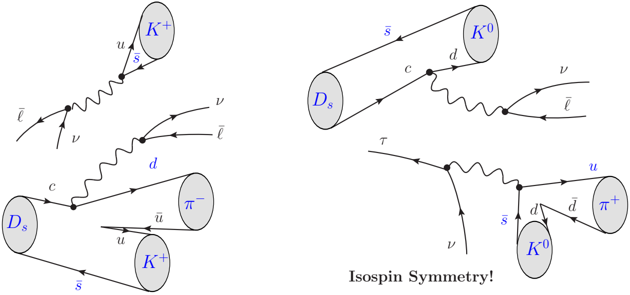

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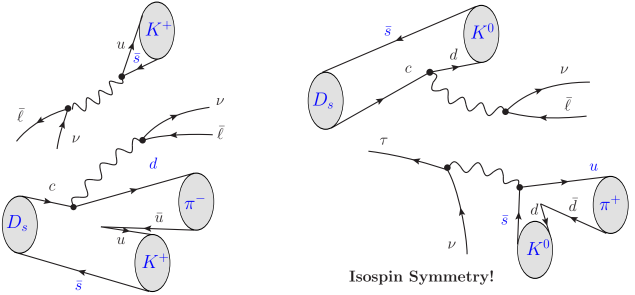
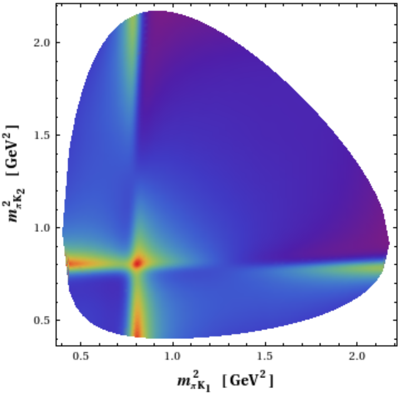
$$a_2 \langle \pi^- K_1^+|\,\bar{d}\gamma^\mu(1-\gamma^5)s\,|0\rangle \langle K_2^+|\,\bar{u}\gamma_\mu(1-\gamma^5)c\,|D^+\rangle\Big] + (K_1^+ \leftrightarrow K_2^+)$$

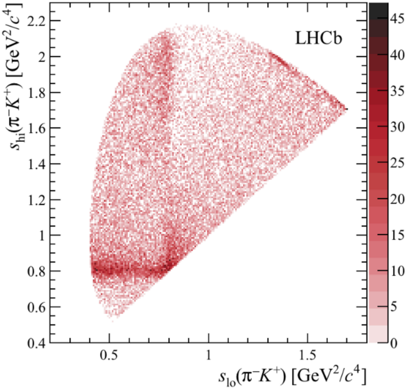


- Changes:
  - $f_\pi \Rightarrow f_K$ (exp/latt)
  - $D^0 \to \pi^-\bar{\ell}\nu \Rightarrow D_s \to K^0\bar{\ell}\nu$ (exp)
  - $D^+ \to \pi^+K^-\bar{\ell}\nu \Rightarrow D_s \to \pi^-K^+\bar{\ell}\nu$ (U-spin symm/exp)

Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: results

- We obtain for the BR $1.44(13) \times 10^{-4}$ vs exp $1.28(4) \times 10^{-4}$
- Bottom-left depletion in Dalitz plot: correct S- & P-wave interference





- Data from LHCb Coll., JHEP 03, 176 (2019)

Naive factorization for $D_s^+ \to K^+K^+\pi^-$ decays: results

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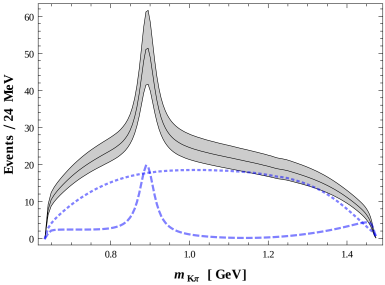

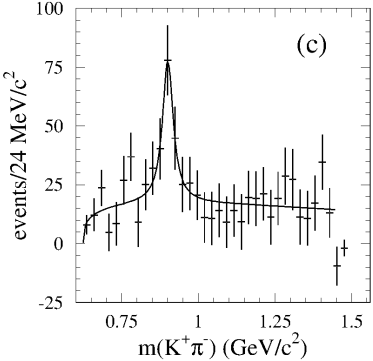

- Data from BaBar Coll., PRD 83, 052001 (2011)

Section 5

Conclusions

Conclusions

- Investigated naive factorization in $D^+ \to \pi^+\pi^+K^-$ & $D_s^+ \to K^+K^+\pi^-$
- Result in terms of semileptonic decays form factors
- Improved $D^+ \to \pi^+K^-\bar{\ell}\nu$ description as necessary ingredient
- Studied $D^+ \to \pi^+\pi^+K^-$ first

$\Rightarrow$ Nice "out of the box" description for P-wave BRs

$\Rightarrow$ With relative S/P-wave phase and S-wave rescaling nice Dalitz Plot

- Imported to $D_s^+ \to K^+K^+\pi^-$: nice predictions
- Nice naive factorization prediction