



THE OHIO STATE UNIVERSITY

Triangle singularities in production of $X(3872)$

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Outline

- **Brief review of X(3872)**
- **Charm-meson triangle singularity**
- **Production of X(3872):**
 - ◆ **e⁺e⁻ annihilation** [PRD100, 031501(2019), PRD101, 014021(2020), PRD 101, 096020(2020)]
 - ◆ **B meson decays** [PRD100, 074028(2019)]
 - ◆ **hadron colliders** [PRD100, 094006(2019)]
- **Summary**
 - ◆ Triangle singularity produces **peaks in reaction rates**
 - ◆ The observation of the peaks would definitely **resolve the nature of X(3872)**

Brief review of $X(3872)$ ($\equiv \chi_{c1}(3872)$)

- ✓ **discovery at e^+e^- collider [Belle (2003)]:**

$$B^+ \rightarrow K^+ + X$$

$$X \rightarrow J/\psi \pi^+ \pi^-$$

- ✓ **confirmation at $p\bar{p}$ collider [CDF (2003)]:**

$$p\bar{p} \rightarrow X + \text{anything}$$

- **quantum numbers [LHCb (2013)]:**

$$J^{PC} = 1^{++}$$

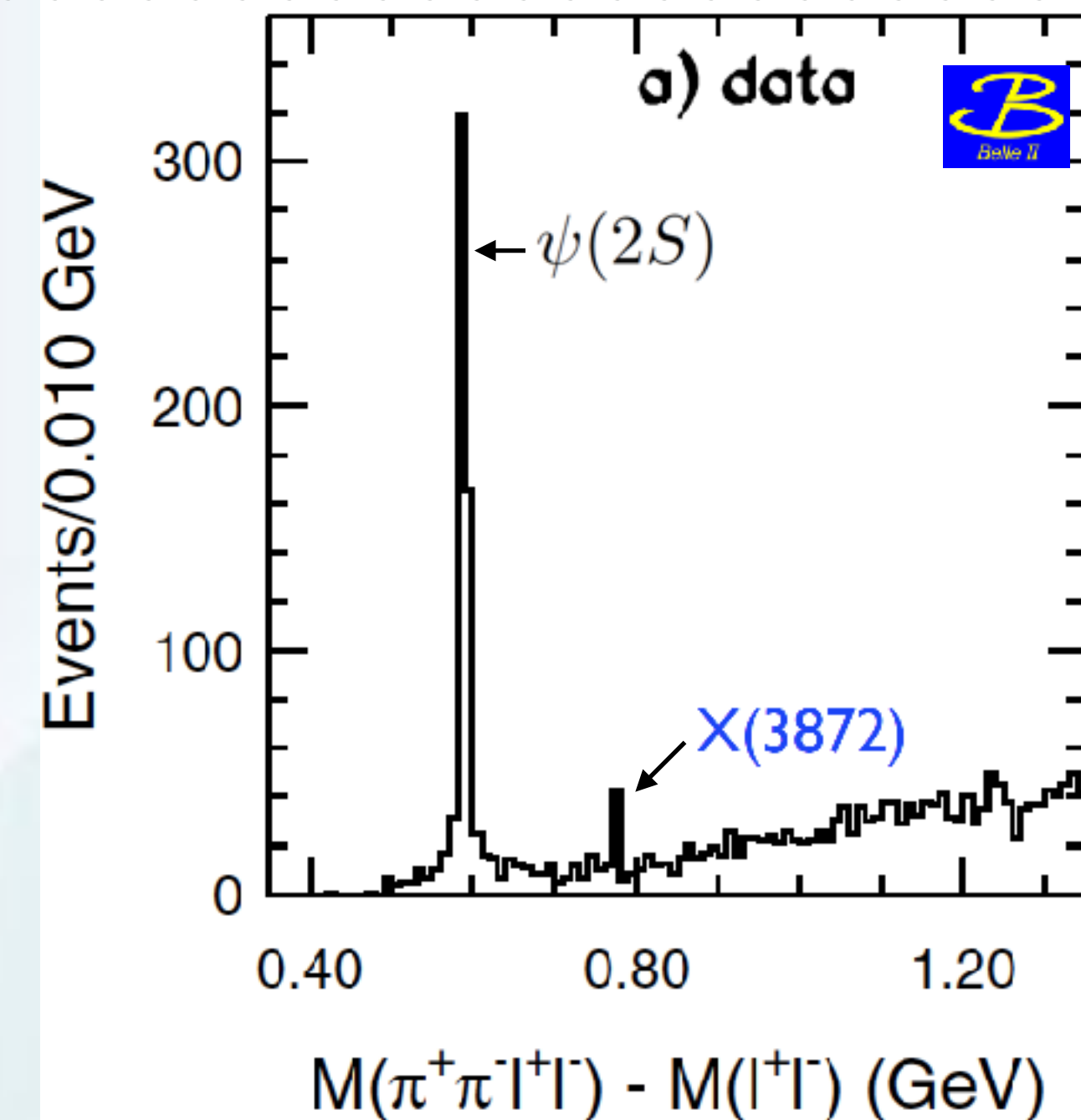
- **mass [LHCb (2020)]:**

$$E_X = M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12) \text{ MeV} \quad |E_X| < 0.22 \text{ MeV at 90\% CL}$$

- **first measurement of width (Breit-Wigner) [LHCb (2020) average]:**

$$\Gamma_X = (1.19 \pm 0.19) \text{ MeV}$$

- **7 observed decay modes: $J/\psi \pi^+ \pi^-$, $J/\psi \pi^+ \pi^- \pi^0$, $J/\psi \gamma$, $\psi(2S)\gamma$, $D^0 \bar{D}^0 \pi^0$, $D^0 \bar{D}^0 \gamma$, $\chi_{c1} \pi^0$**



see also F.-K. Guo's talk on Monday

Brief review of $X(3872)$ ($\equiv \chi_{c1}(3872)$)

What is the $X(3872)$?

$J^{PC} = 1^{++}$ \rightarrow S-wave coupling to $D^{*0}\bar{D}^0/\bar{D}^{*0}D^0$

$|E_X| < 0.22$ MeV \rightarrow resonant coupling

S-wave loosely bound **charm-meson molecule!!**

$$X = \frac{1}{\sqrt{2}} (D^{*0}\bar{D}^0 + D^0\bar{D}^{*0})$$

other components of wave functions have small probabilities:

- at long distances: $D^0\bar{D}^0\pi^0$
- at short distances:
 - ♦ $\chi_{c1}(2P)$?
 - ♦ charged charm mesons
 - ♦ compact tetraquark $[cq][\bar{c}\bar{q}]$?

Universal properties determined by the binding energy $|E_X|$

* large scattering length: $|a| = 1/\sqrt{2\mu|E_X|}$, $|a| \gg \text{range}$

* large mean separation: $\langle r \rangle = a/2$, $|E_X| < 0.22$ MeV implies $\langle r \rangle > 5$ fm

XEFT

effective field theory for charm mesons and pions

Fleming, Kusunoki, Mehen & van Kolck [PRD 76, 034006(2007)]

Galilean-invariant XEFT

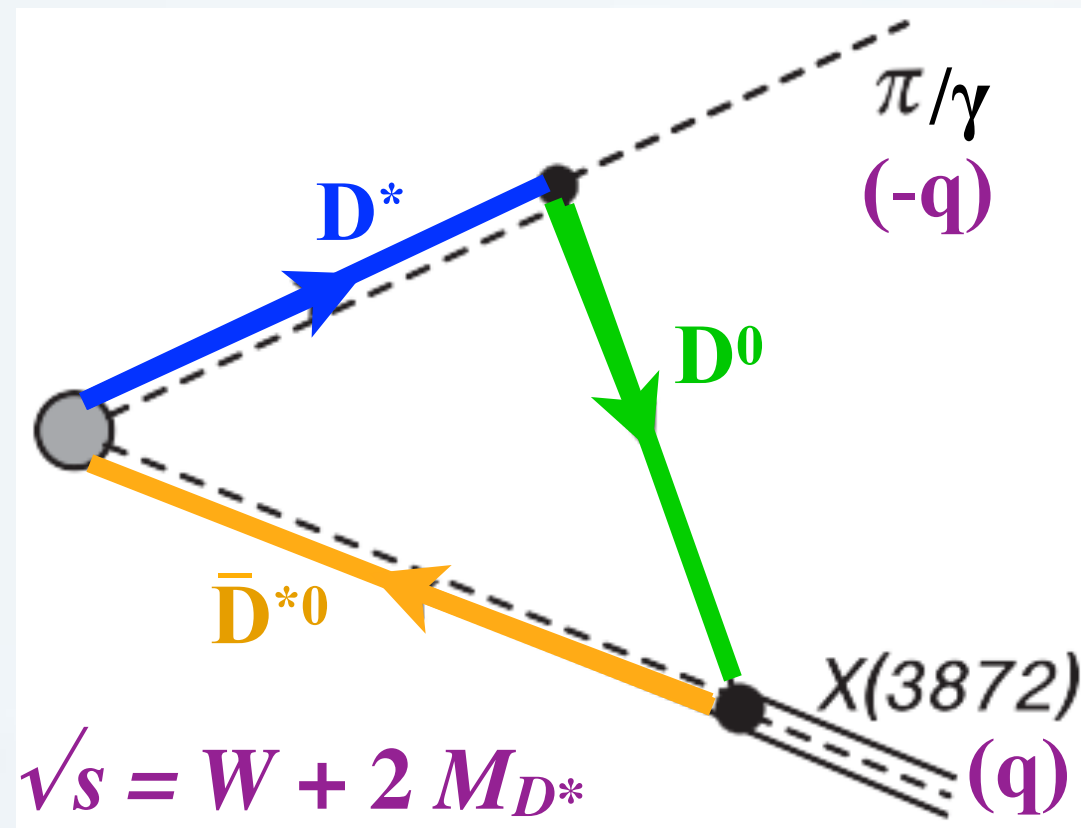
Braaten [PRD 91, 114007(2015)]

Braaten, He & Jiang [PRD 103, 036014(2021)]

Charm-meson triangle singularity

three charm mesons can be on shell simultaneously \rightarrow

log² divergence in reaction rate



loop amplitude near singularity:

$$F(W) \propto \log \frac{\sqrt{M_* W} + (M_*/M_X)q}{\sqrt{M_* W} - (M_*/M_X)q}$$

($M_* = M_{D^*}$)

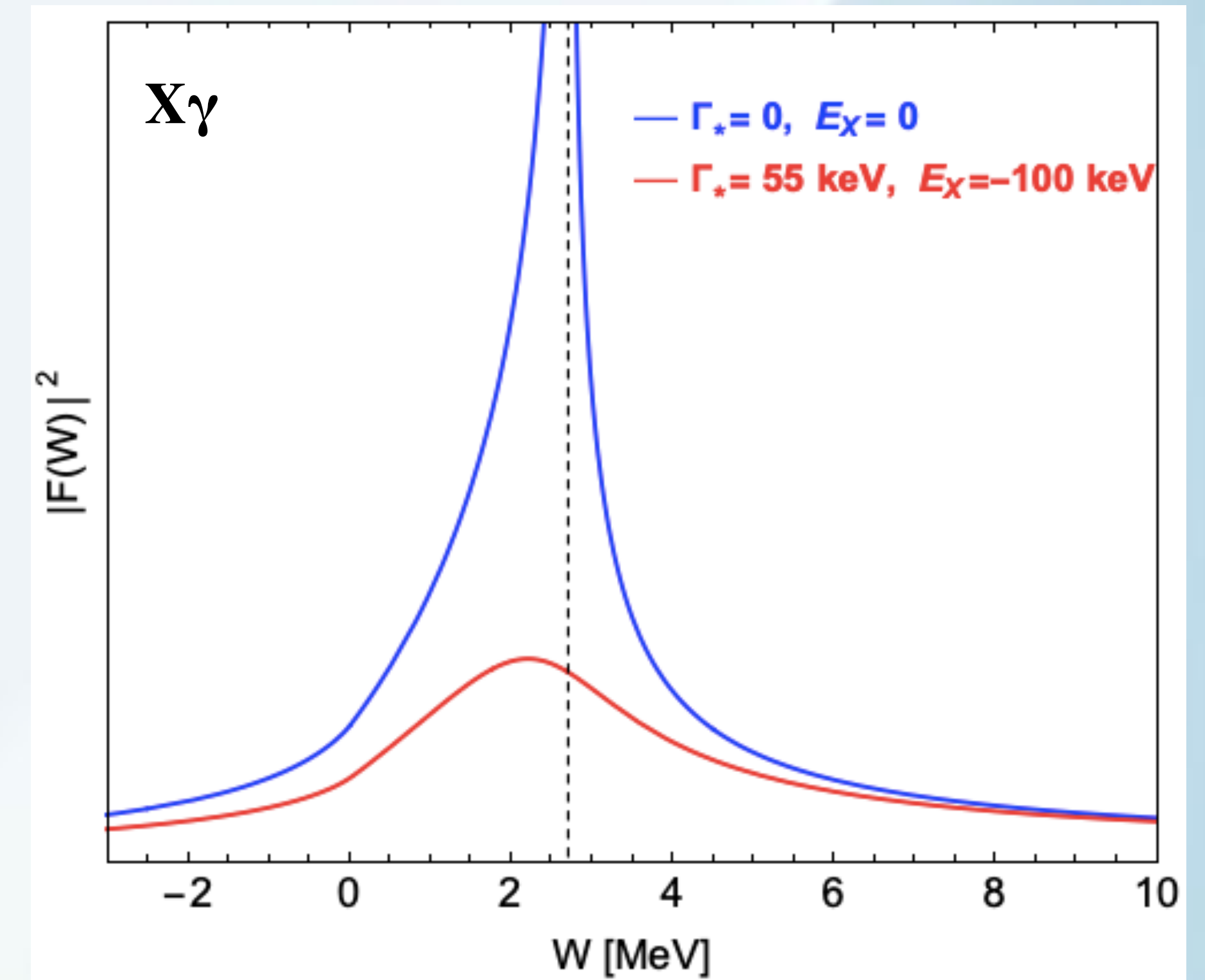
divergence at energy W above $D^*\bar{D}^*$ threshold:

- ❖ $X\gamma$: $(M_{D^{*0}}/M_X^2)(M_{D^{*0}} - M_{D^0})^2 = 2.7$ MeV
- ❖ $X\pi^0$: $(m_{\pi^0}/2M_{D^0})(M_{D^{*0}} - M_{D^0} - m_{\pi^0}) = 0.3$ MeV
- ❖ $X\pi^\pm$: $(m_{\pi^0}/2M_{D^0})(M_{D^{*+}} - M_{D^0} - m_{\pi^+}) = 0.2$ MeV

- BUT**
- * nonzero decay width for D^*
 - * nonzero binding energy ($-E_X$) for X

$$F(W) \propto \log \frac{\sqrt{2\mu E_X + i\mu\Gamma_*} + \sqrt{M_*(W + i\Gamma_*)} + (M_*/M_X)q}{\sqrt{2\mu E_X + i\mu\Gamma_*} + \sqrt{M_*(W + i\Gamma_*)} - (M_*/M_X)q}$$

narrow peak in reaction rate



review on TS: Guo, Liu, Sakai [Prog. Part. Nucl. Phys. 112, 103757 (2020)]

e^+e^- : production of $X(3872)$ and a photon

Experimental observation:

BESIII: $e^+e^- \rightarrow X\gamma$, $X \rightarrow J/\psi \pi^+\pi^-$, $J/\psi \omega$
 [PRL122,232002 (2019)]

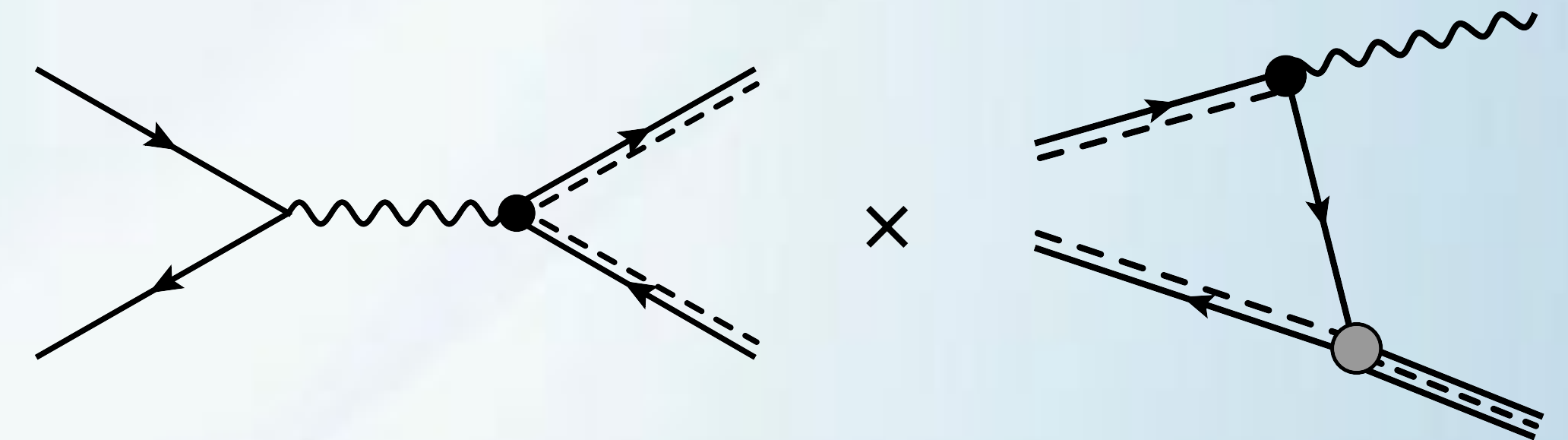
First theoretical calculation:

Dubynskiy & Voloshin [PRD 74, 094017 (2006)]

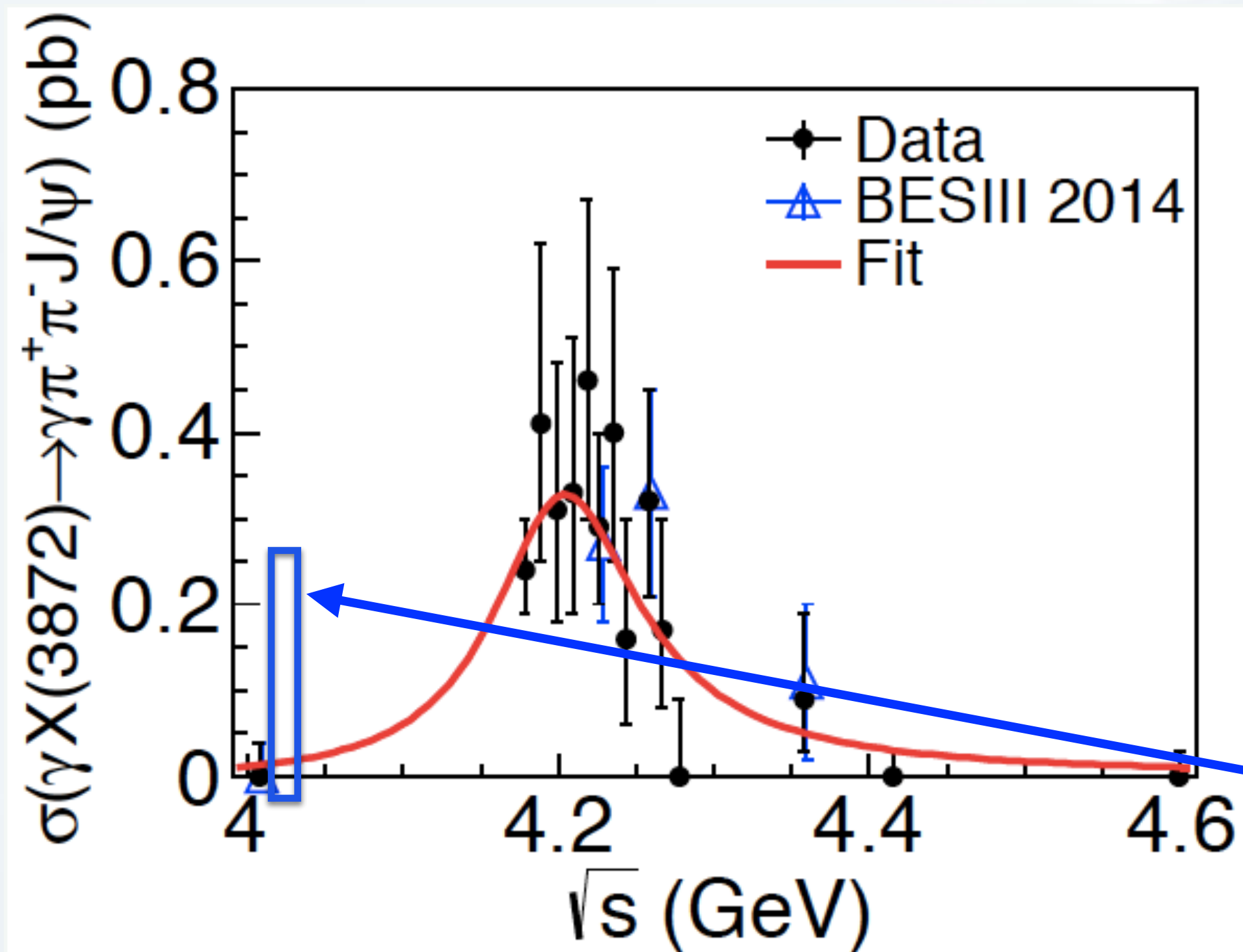
absorptive contribution only:

$e^+e^- \rightarrow D^{*0}\bar{D}^{*0}$ (P-wave) $\rightarrow X\gamma$

- e^+e^- annihilation creates $D^{*0}\bar{D}^{*0}$ (P-wave)
- rescattering of **real** $D^{*0}\bar{D}^{*0}$ into $X\gamma$



- ❖ Line shape of $X\gamma$ has narrow peak a few MeV above $D^{*0}\bar{D}^{*0}$ threshold
- ❖ $\sigma[X\gamma]$: of order **1pb** near the **peak**

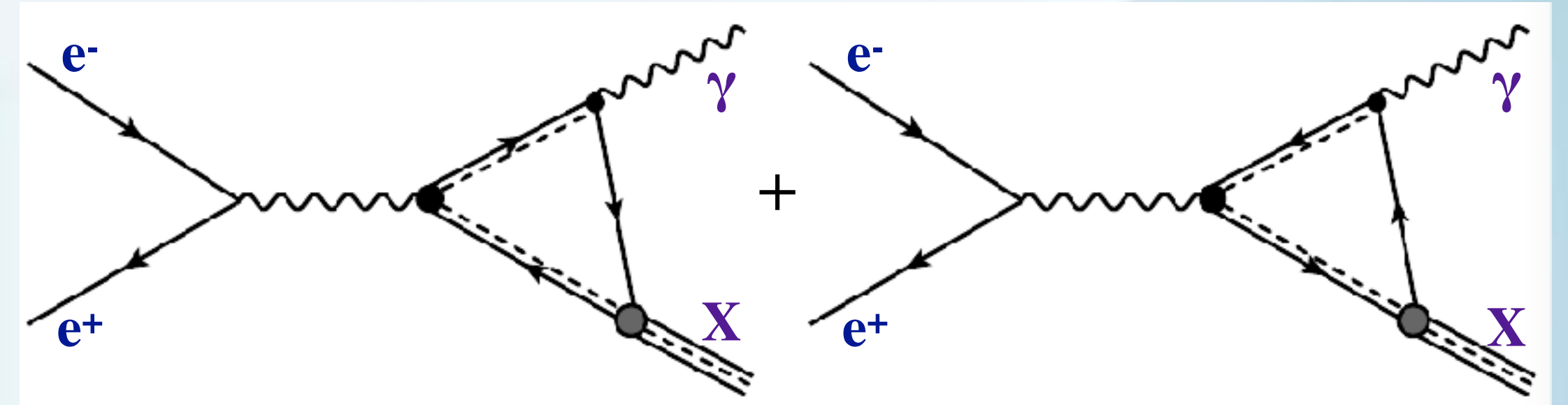


e^+e^- : production of $X(3872)$ and a photon

Braaten, He & Ingles [PRD 100, 031501(2019), PRD 101, 014021(2020)]

$$e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \text{ (P-wave)} \rightarrow X\gamma$$

- e^+e^- annihilation creates $D^{*0}\bar{D}^{*0}$ (P-wave)
- rescattering of virtual $D^{*0}\bar{D}^{*0}$ into $X\gamma$



* **improvements** over Dubynskiy & Voloshin:

- ❖ include $\text{Re}[M]$ as well as $\text{Im}[M]$
- ❖ include **decay width of D^{*0}**
- ❖ **normalize** cross section using $\sigma[D^{*+}D^{*-}]$
Uglov *et al.* (JETP Lett. 105,1 (2017))

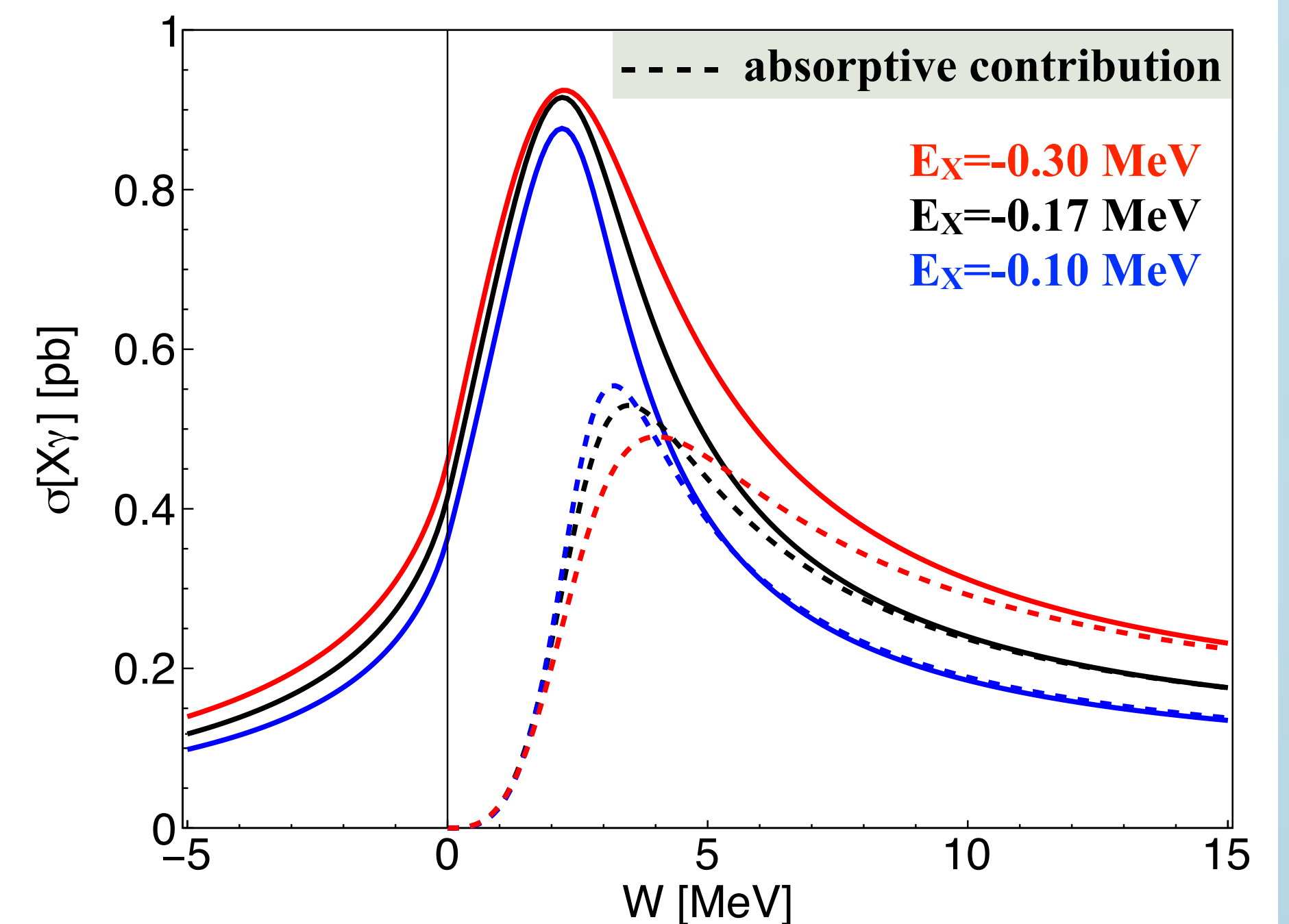
* **cross section:**

- ❖ **triangle singularity** gives narrow **peak at 2.2 MeV** above $D^{*0}\bar{D}^{*0}$ threshold at 4013.7 MeV
- ❖ **position of peak** insensitive to **binding energy**
- ❖ may be observable by **BESIII detector!**

absorptive contribution only is not a good approximation!

cross section for $X\gamma$

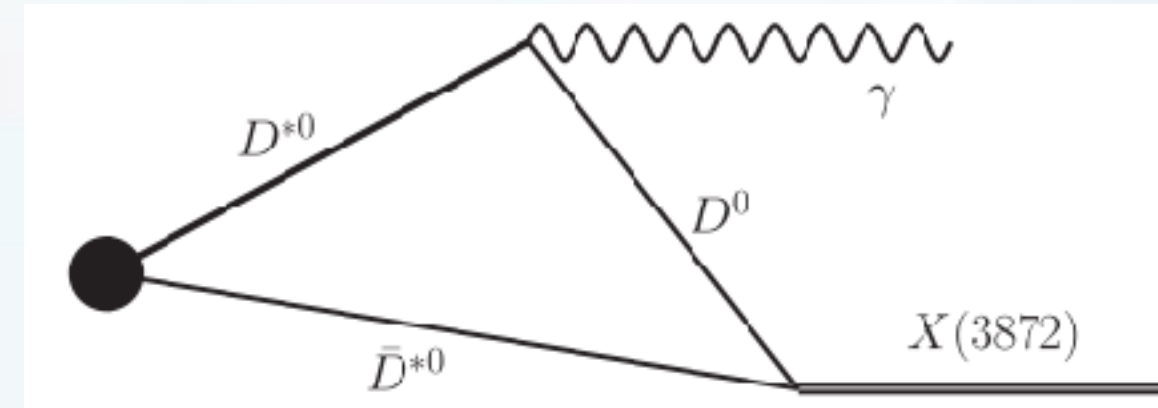
$$\sqrt{s} = W + 2M_{D^*}$$



e^+e^- : production of $X(3872)$ and a photon

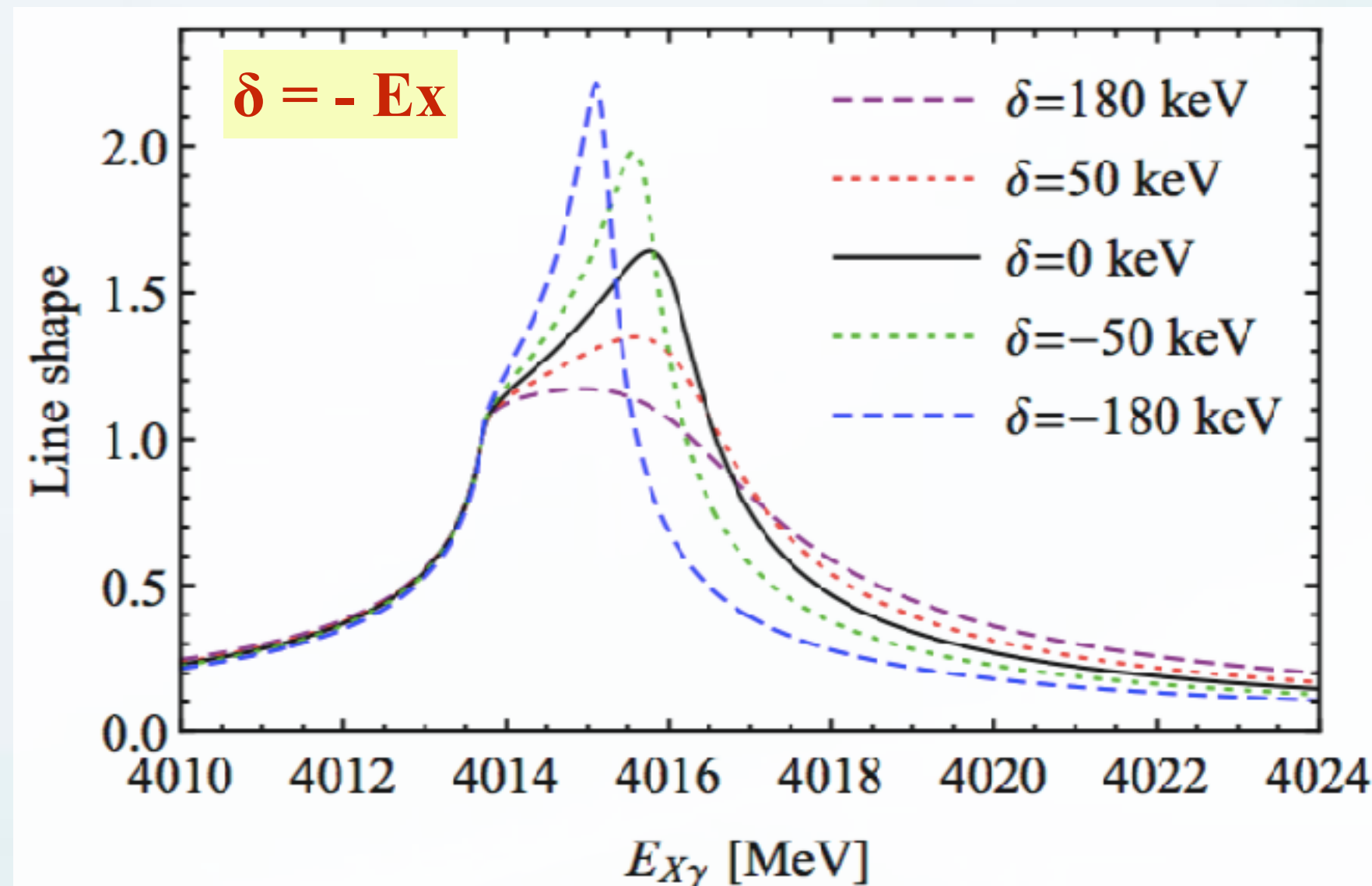
• Guo [PRL 112, 202002 (2019)]

- creation of $D^{*0}\bar{D}^{*0}$ (S-wave) at short distance
- rescattering of virtual $D^{*0}\bar{D}^{*0}$ into $X\gamma$



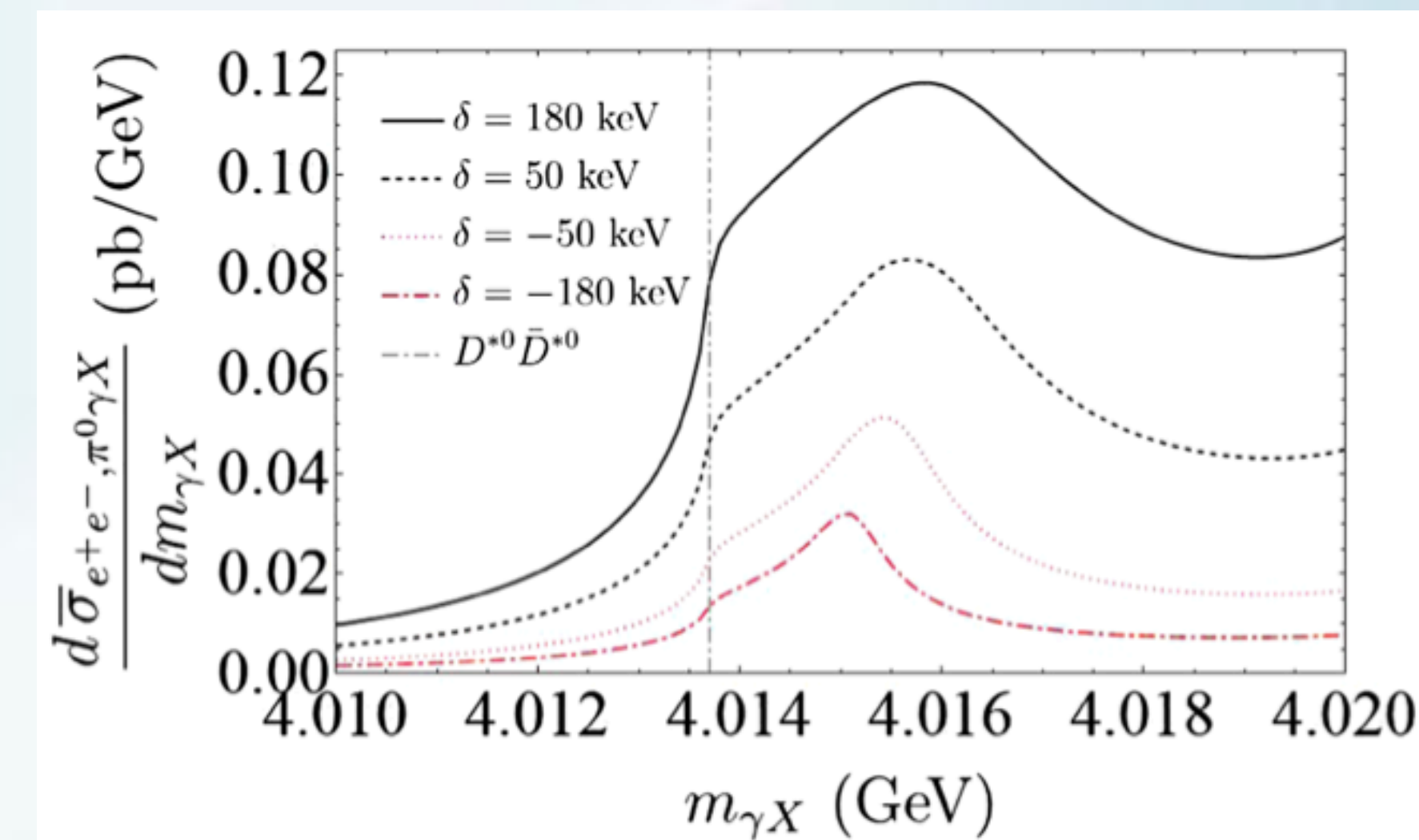
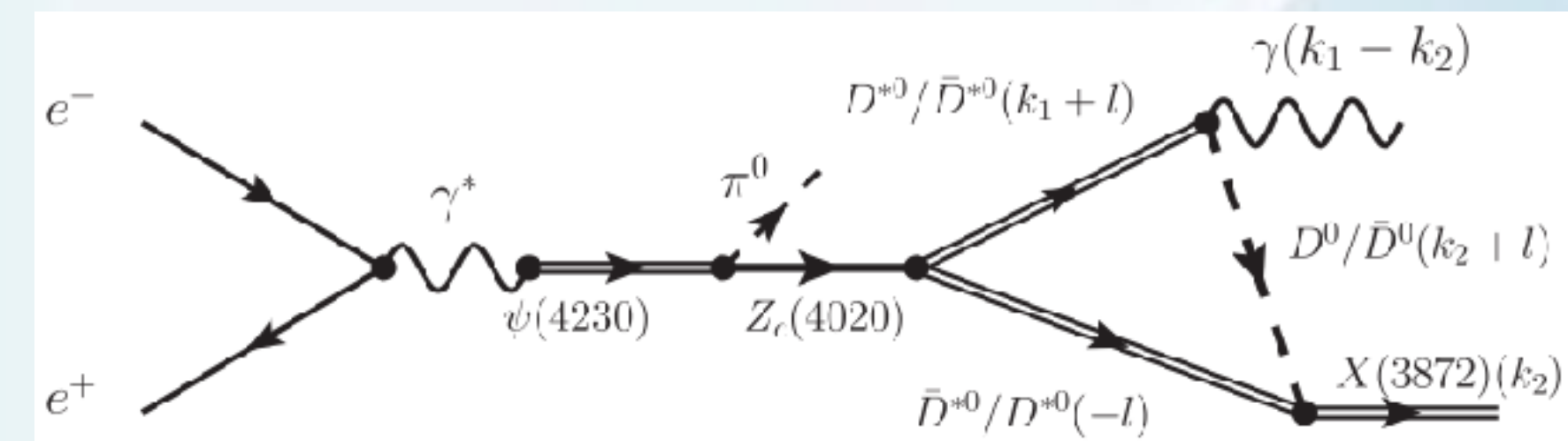
Line shape in $X\gamma$:

- ❖ peak a few MeV above $D^{*0}\bar{D}^{*0}$ threshold
- ❖ can be used to measure E_X



• Sakai, Jing & Guo [PRD 102, 114041(2020)]

$$e^+e^- \rightarrow Z_c(4020) \pi^0, Z_c(4020) \rightarrow D^{*0}\bar{D}^{*0}(\text{S-wave}) \rightarrow X\gamma$$

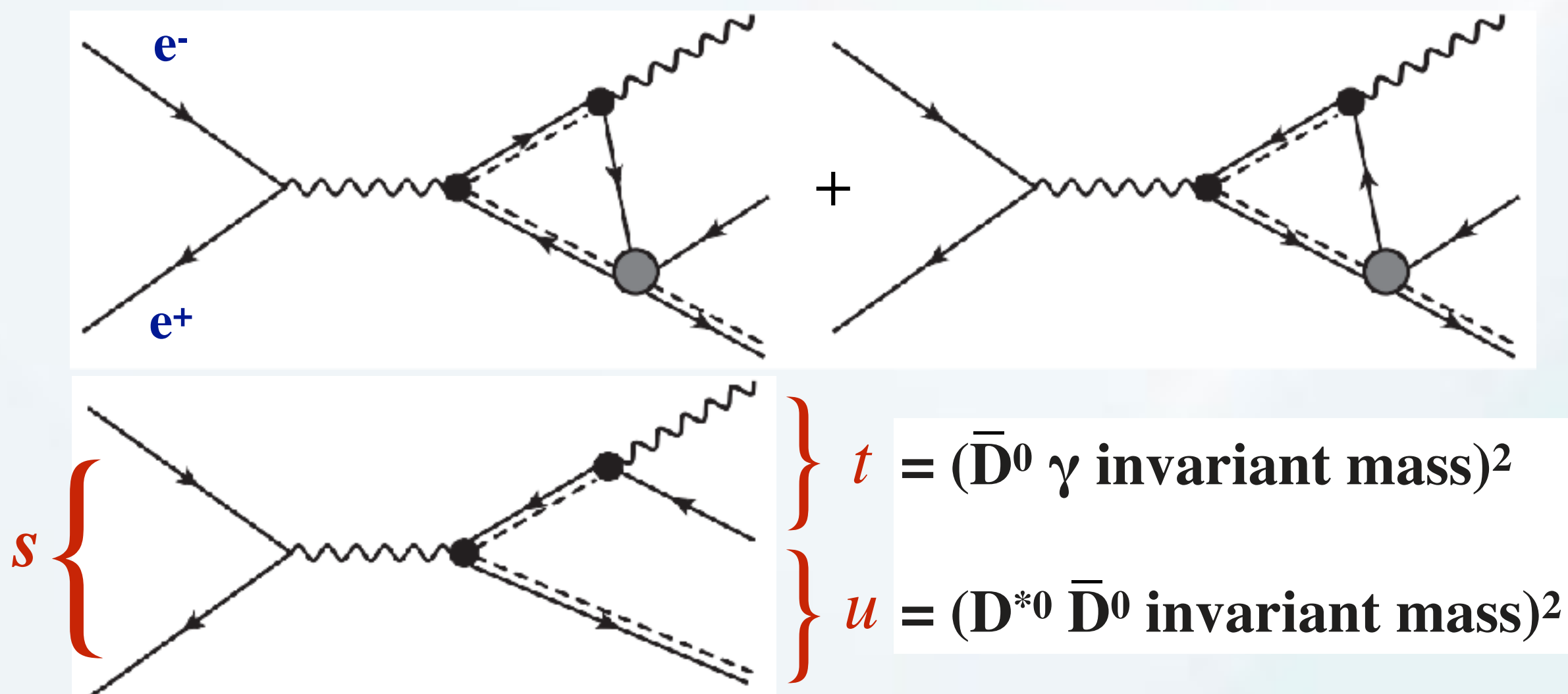


see also F.-K. Guo's talk on Monday

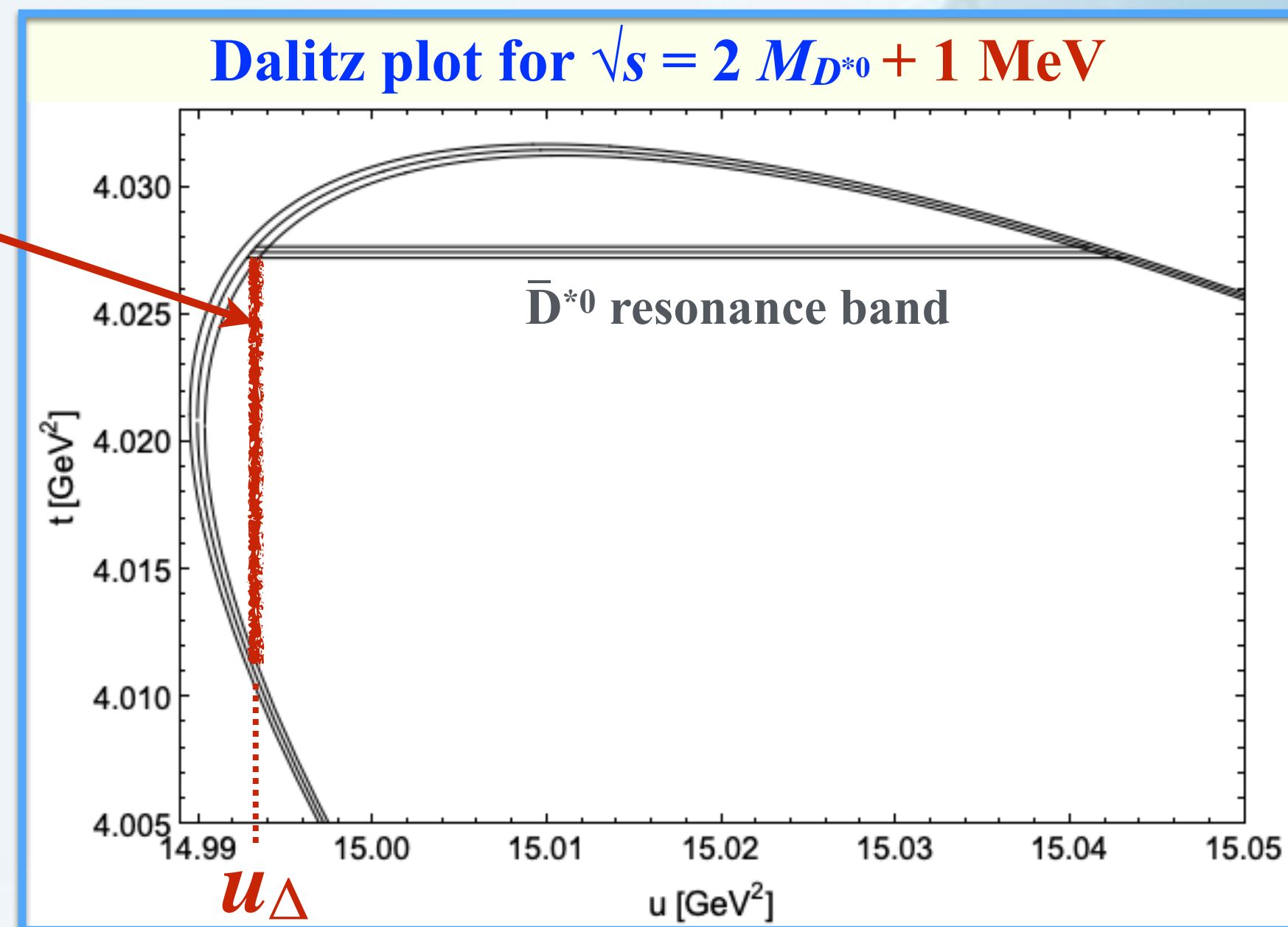
- BESIII [arXiv:2101.00644]: no significant signal
- $e^+e^- \rightarrow Z_c(4020) \pi^0, Z_c(4020) \rightarrow D^{*0}\bar{D}^{*0}(\text{S-wave}) \rightarrow X\gamma$

e^+e^- : production of $D^{*0}\bar{D}^0$ and a photon

Braaten, He, Ingles & Jiang [PRD 101, 096020(2020)]



triangle singularity at $u = u_\Delta$



* Schmid cancellation:

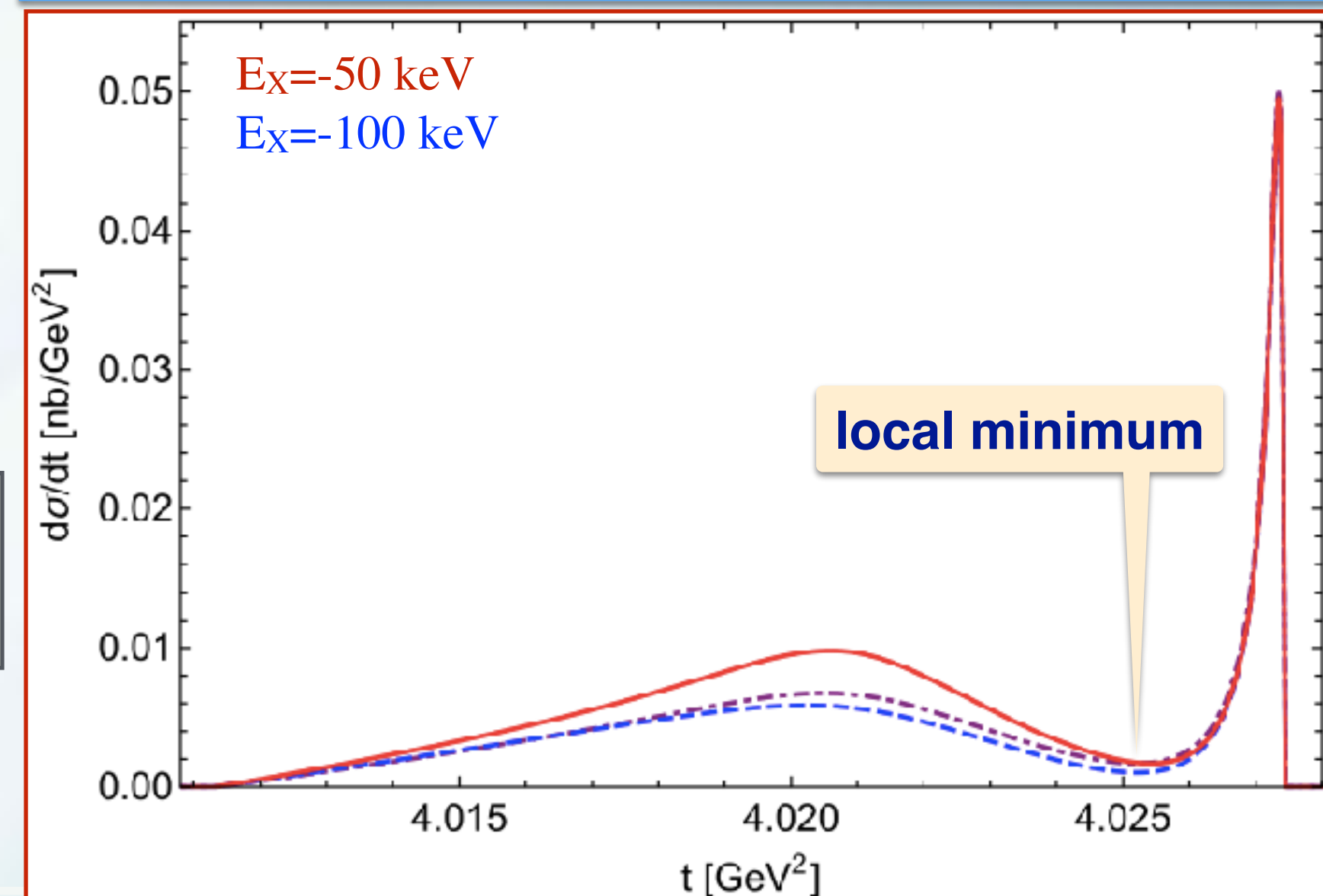
Schmid [PR154, 1363(1967)]

Anisovich & Anisovich [PLB345, 321(1995)]

- $d\sigma/(du dt)$ at fixed t : **\log^2 divergence**
- $d\sigma/du$ integrated over t : **log divergence**

* indirect way to observe triangle singularity:

$d\sigma/dt$ integrated over $u < u_\Delta$ has **local minimum in t**



B meson decay: production of X(3872) and a pion

Belle [PRD 91, 051101 (2015)]

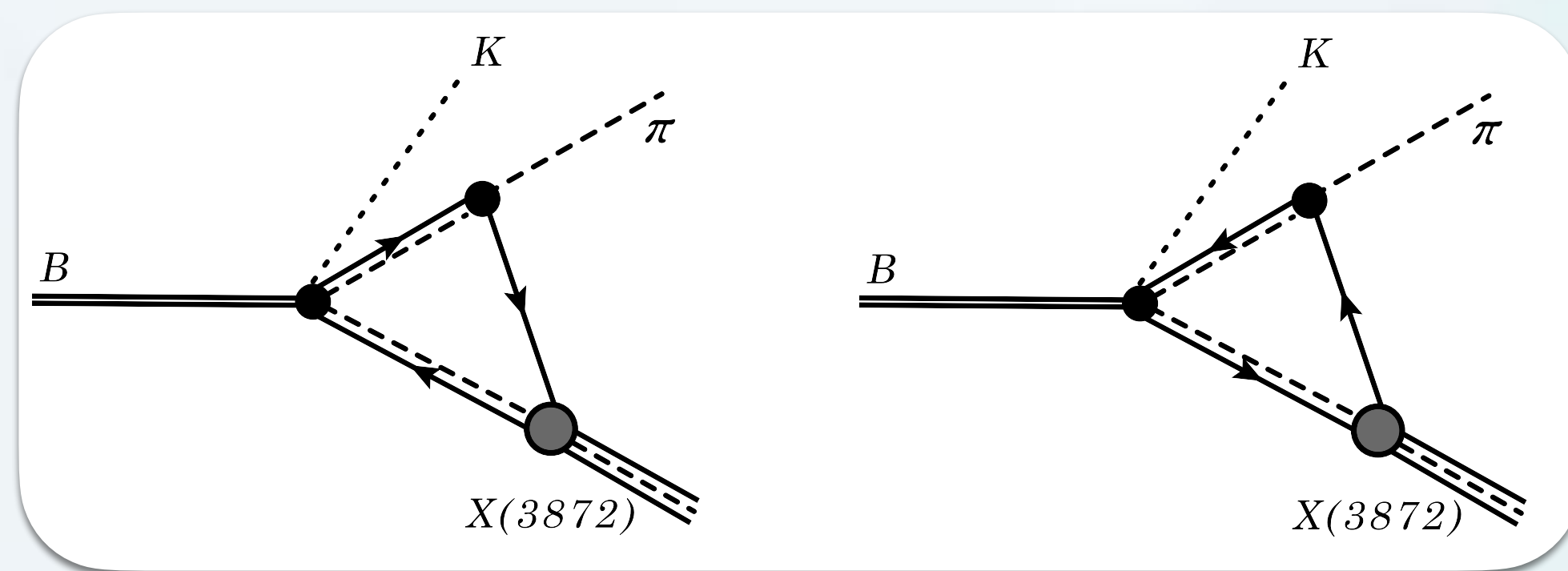
first observation of $B^0 \rightarrow K^+ \pi^- X$, $B^+ \rightarrow K^0 \pi^+ X$

34% of $B^0 \rightarrow K^+ \pi^- X$ from $B^0 \rightarrow K^{*0}(892) X$

Braaten, He, Ingles [PRD 100, 074028(2019)]

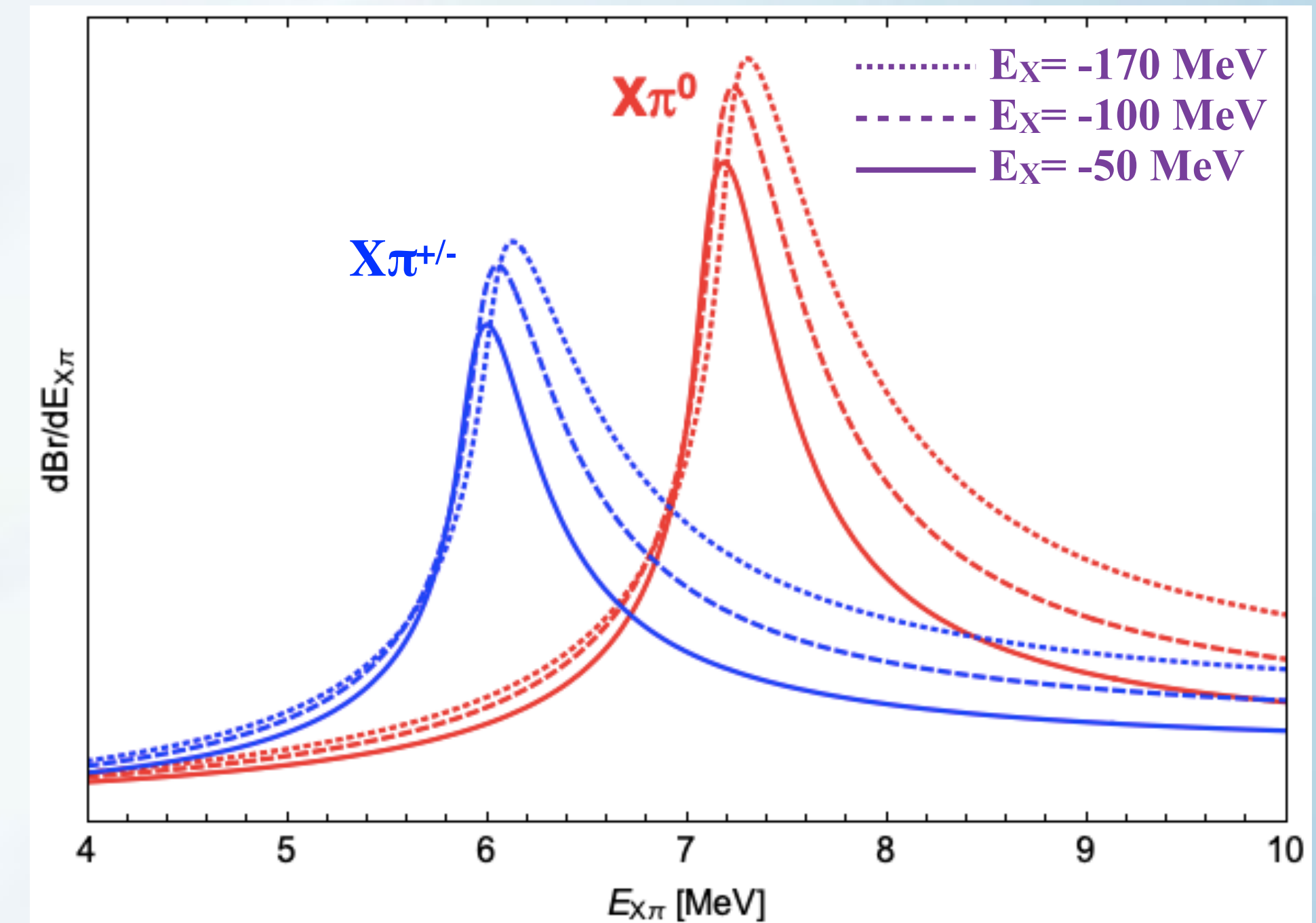
$$B \rightarrow K D^* \bar{D}^* \rightarrow K X \pi$$

decay of B meson into $K + D^* \bar{D}^*$, rescattering of virtual $D^* \bar{D}^*$ into $X \pi$



triangle singularity produces narrow peaks in $dBr[B \rightarrow K X \pi]$

- ❖ $X\pi^\pm$: near 6.1 MeV above $X\pi^+$ threshold
- ❖ $X\pi^0$: near 7.3 MeV above $X\pi^0$ threshold

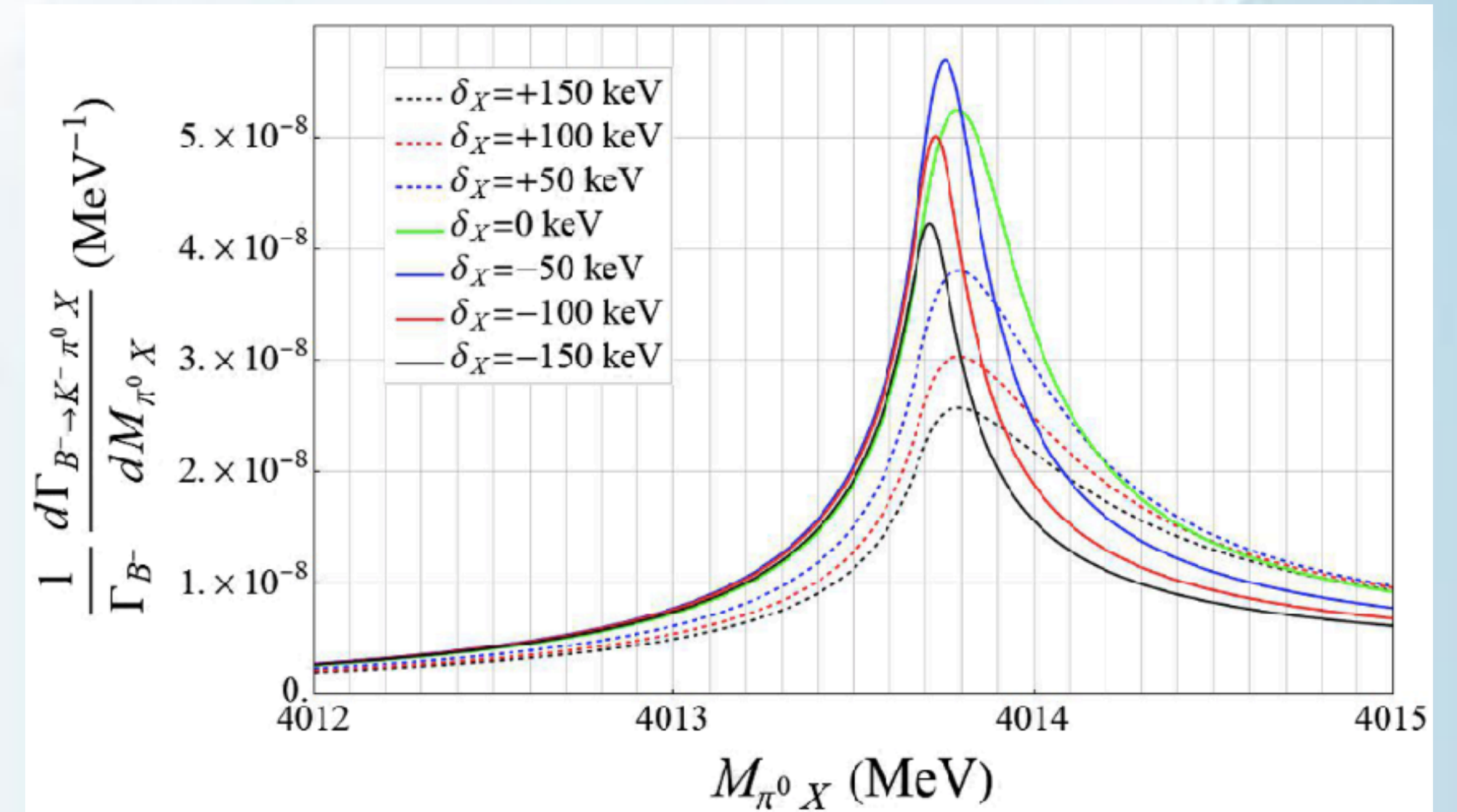


B meson decay: production of X(3872) and a pion

- Sakai, Oset & Guo [PRD 101, 054030(2020)]



$E_X (= -\delta_X)$ may be extracted from the asymmetry of the $X\pi$ line shape



- Nakamura [PRD 102, 074004(2020)]



triangle singularity could produce narrow peak in $J/\psi\rho$ invariant mass near 3872 MeV even without X(3872) resonance

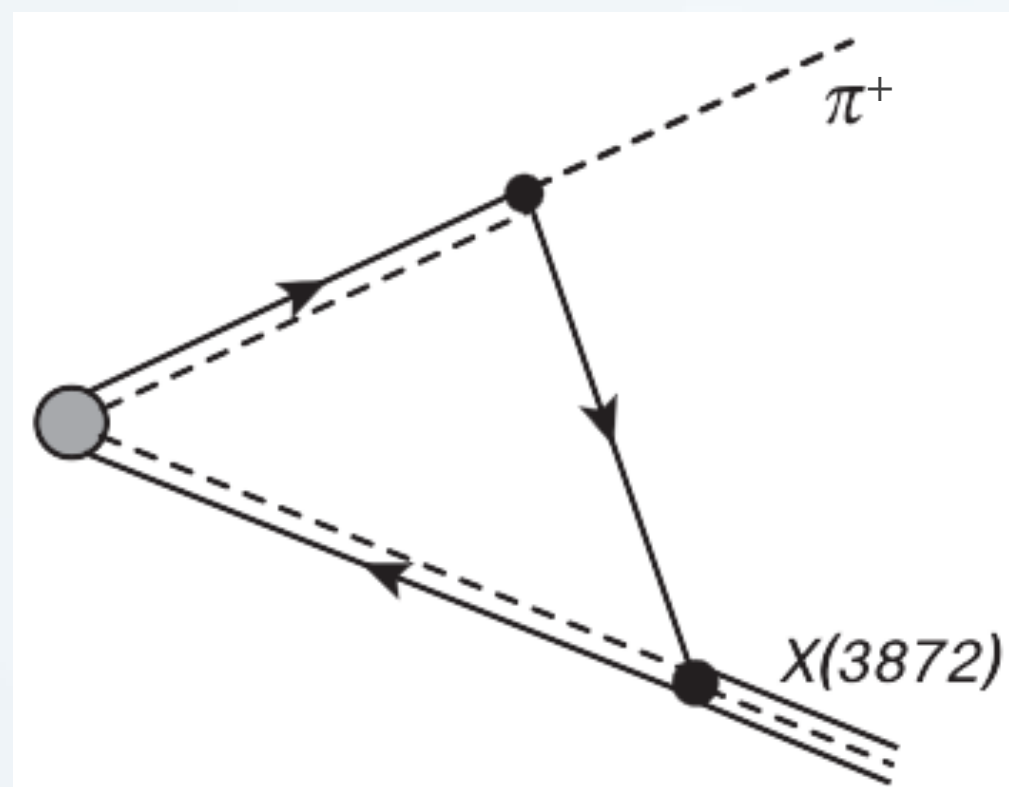
- Molina & Oset [EPJC 80, 451(2020)]



triangle singularity in decay of X

Hadron colliders: prompt production of X(3872) and a pion

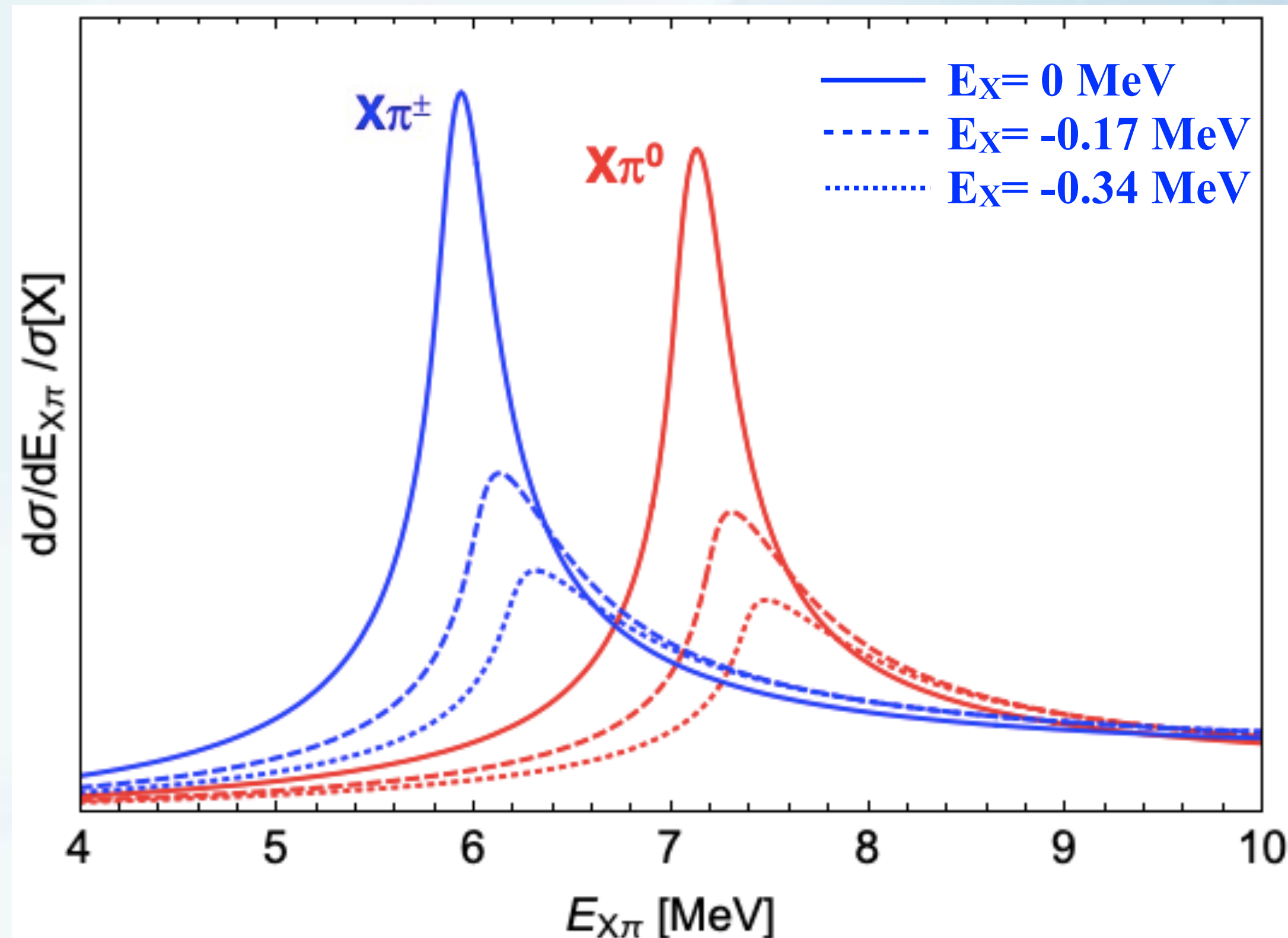
- Braaten, He & Ingles [PRD 100, 094006(2019)]



- ❖ creation of $D^{*+}\bar{D}^{*0}$ at short distance
- ❖ rescattering of **virtual** $D^{*+}\bar{D}^{*0}$ into $X\pi^+$

triangle singularity produces narrow peak in $X\pi^\pm$ invariant mass

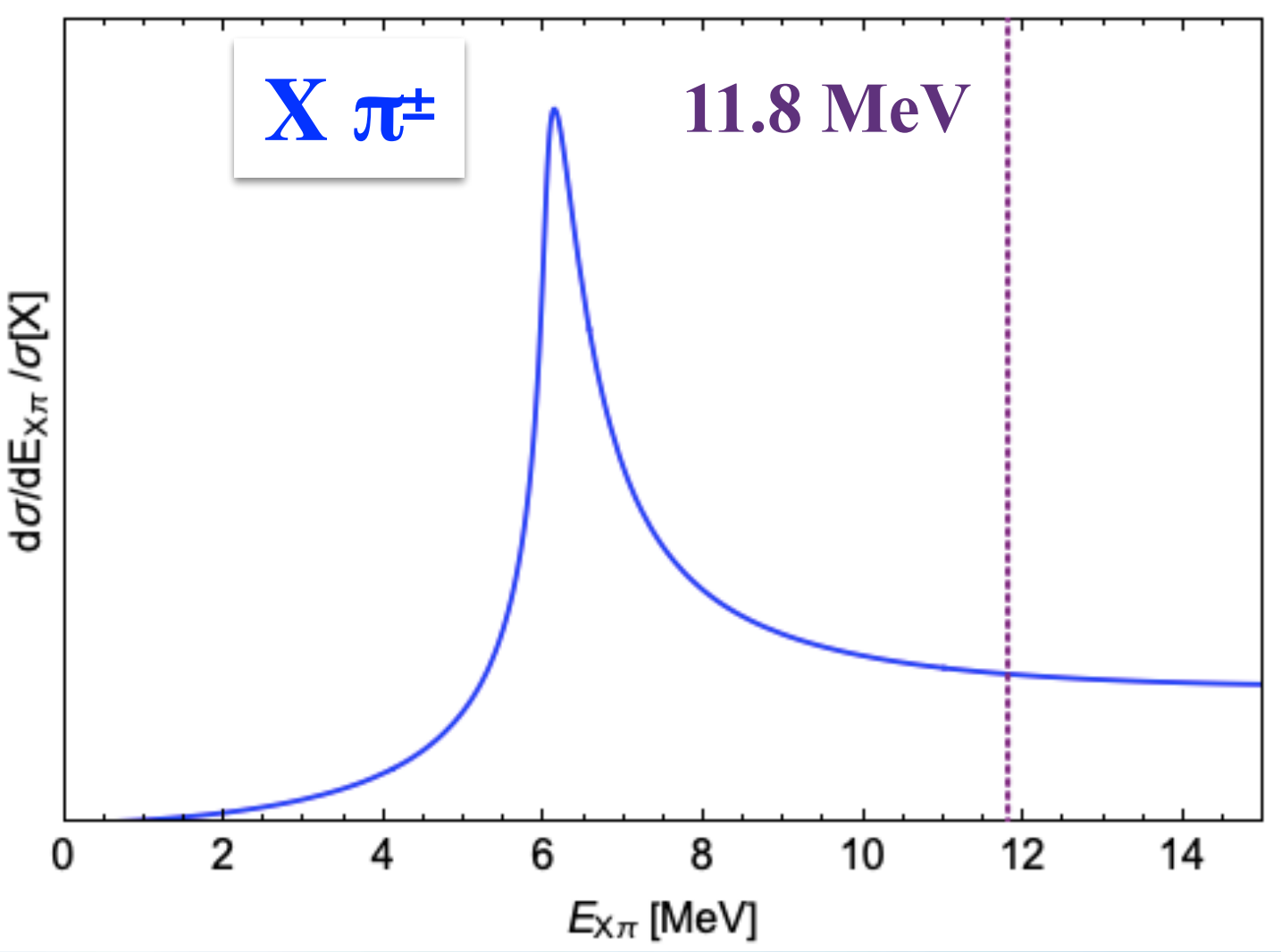
peak near 6.1 MeV above $X\pi^+$ threshold



Experimental observation: production of X(3872) and a pion

contributions from the triangle peak: $E_X = -0.17 \text{ MeV}$

- **prompt production:** $\frac{\sigma[(X\pi^\pm)_\Delta]}{\sigma[X]} \approx 14\%$
- **B-meson decay:** $\frac{\text{Br}[B^0 \rightarrow K^+(X\pi^-)_\Delta]}{\text{Br}[B^0 \rightarrow K^0 X]} \approx 14\%$



● **D0 Collaboration [PRD 102, 072005 (2020)]**

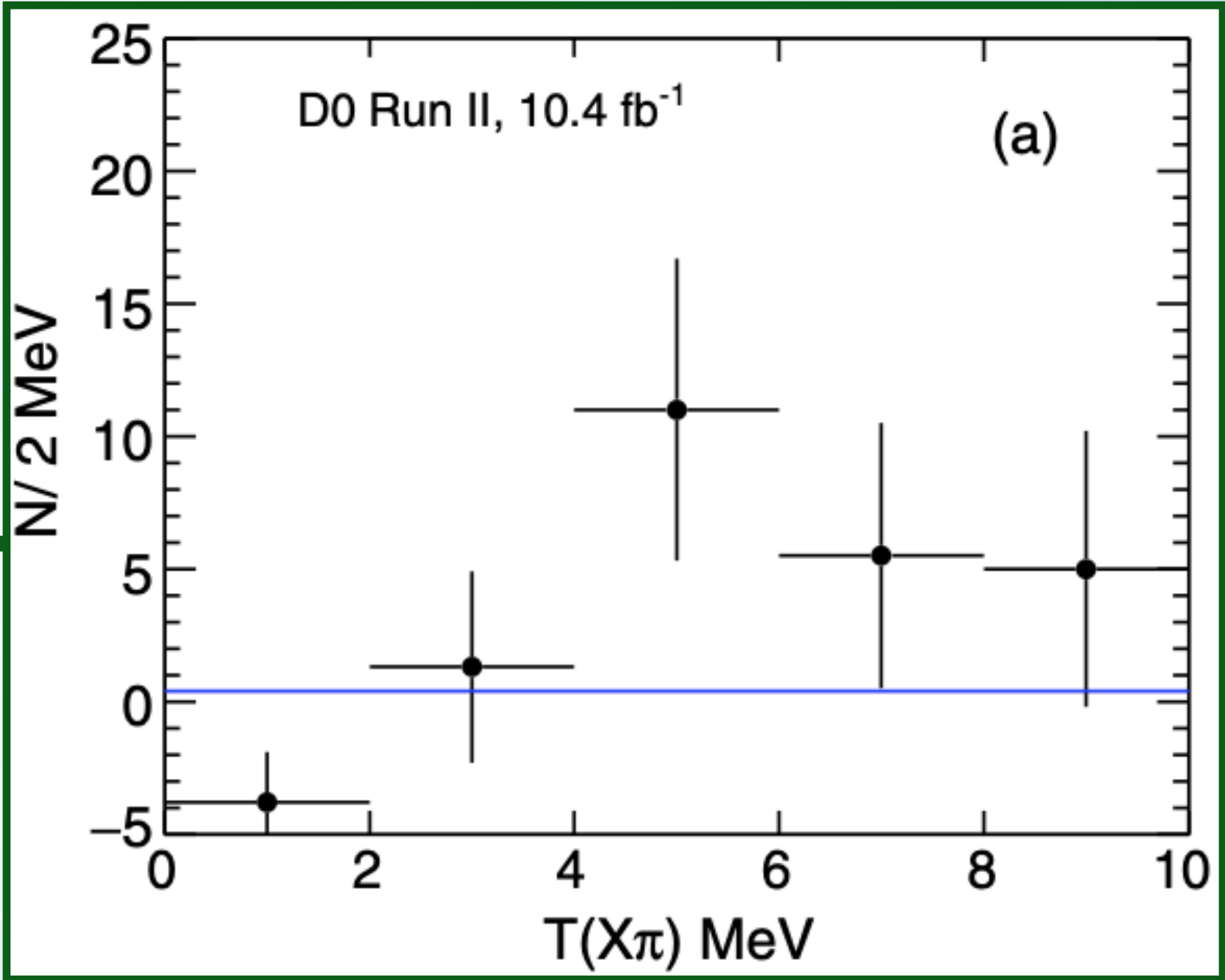
prompt and non prompt production of X(3872) + soft π±

$T(X\pi) < 11.8 \text{ MeV}$

observed events expected events

- **prompt production:** 12 ± 16 $245 - 730$
- **b-decay:** 25 ± 12 $30 - 90$

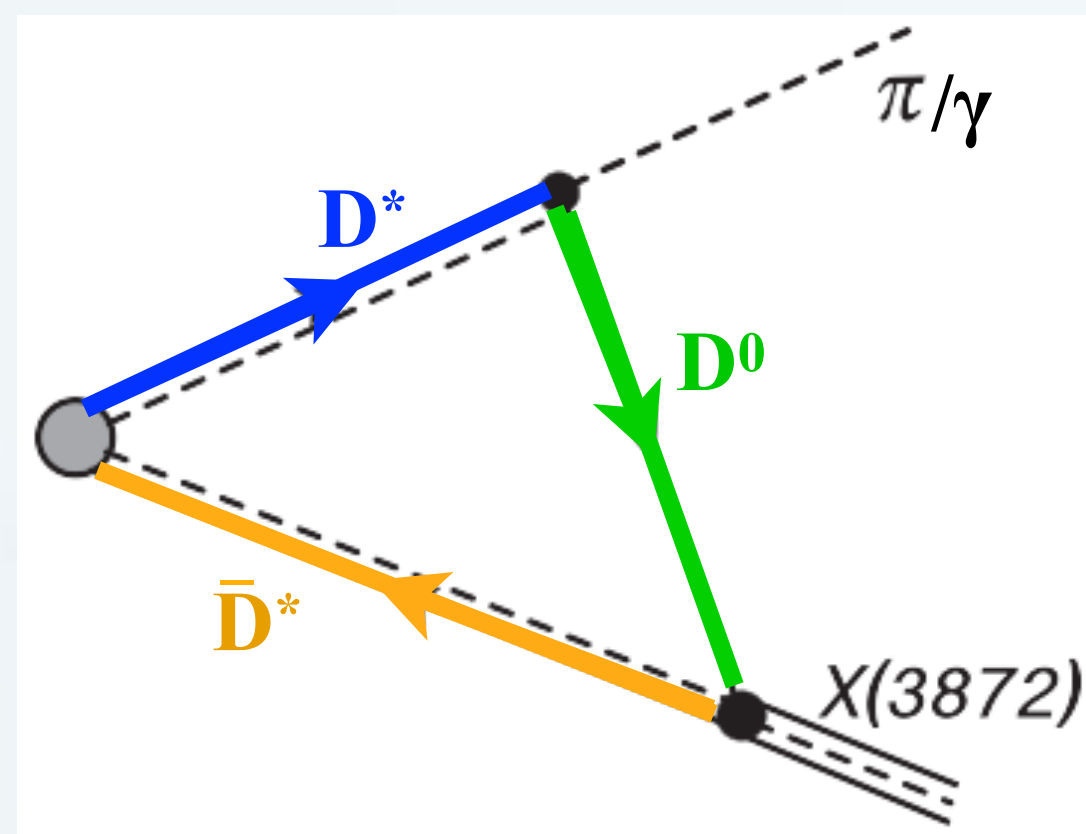
- * **prompt production : no evidence for the accompanying π±**
- * **b-decay: agreement with the expectation, significance: 2σ**



Summary

Production of $X+\gamma$ or $X+\pi$

charm meson triangle singularity produces **narrow peaks** just above $D^*\bar{D}^*$ threshold



■ e^+e^- annihilation

- ◇ $\sigma[X\gamma]$: narrow peak at 4015.9 MeV
- ◇ peak is in region not yet measured by BESIII

■ B meson decay

- ◇ $d\text{Br}[X\pi^0]/dE_{X\pi}$: peak near 7.3 MeV above $X\pi^0$ threshold
- ◇ $d\text{Br}[X\pi^\pm]/dE_{X\pi}$: peak near 6.1 MeV above $X\pi^+$ threshold

■ Hadron colliders

- ◇ $d\sigma[X\pi^\pm]/dE_{X\pi}$: peak near 6.1 MeV above $X\pi^+$ threshold

Thank you!

The observation of the peaks would definitely resolve the nature of X(3872)

Backup

B meson decay

$$\frac{d\Gamma}{d^3q} [B^+ \rightarrow K^+ X \pi^0] = \frac{|\mathcal{A}[K^+ X \pi^0]|^2}{4|\mathcal{A}[K^0 X \pi^0]|^2} \frac{d\Gamma}{d^3q} [B^0 \rightarrow K^0 X \pi^0], \quad (36a)$$

$$\frac{d\Gamma}{d^3q} [B^+ \rightarrow K^0 X \pi^+] = \frac{d\Gamma}{d^3q} [B^0 \rightarrow K^+ X \pi^-]. \quad (36b)$$

$$\text{Br}[B^0 \rightarrow K^+ (X\pi^-)_\Delta] \approx (2.4 \times 10^{-7}) \left(\frac{|E_X|}{0.17 \text{ MeV}} \right)^{1/2} \times \left[2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$

$$\text{Br}[B^0 \rightarrow K^0 (X\pi^0)_\Delta] < (8 \times 10^{-8}) \left(\frac{|E_X|}{0.17 \text{ MeV}} \right)^{1/2} \times \left[2.82 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$

Hadron collider

$$d\sigma[D^{*0} \bar{D}^{*0}] \approx d\sigma[X(3872)] \frac{12\pi\mu}{\gamma_X \Lambda^2} \frac{d^3k}{(2\pi)^3 M_{*0}}$$

$$\frac{\sigma[(X\pi^0)_\Delta]}{\sigma[X]} \approx 0.049 \left(\frac{m_\pi}{\Lambda} \right)^2 \left[2.82 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right],$$

$$\frac{\sigma[(X\pi^+)_\Delta]}{\sigma[X]} \approx 0.028 \left(\frac{m_\pi}{\Lambda} \right)^2 \left[2.64 - \log \frac{|E_X|}{0.17 \text{ MeV}} \right].$$