Charmonium properties from lattice QCD+QED: hyperfine splitting, J/ψ leptonic width and a_μ^c

Christine Davies
University of Glasgow
HPQCD collaboration
christine.davies@glasgow.ac.uk

CHARM2021,
June 2021

Lattice QCD enables precision tests of the Standard Model when accurate theory and experiment results can be brought together.

The ground-state charmonium mesons (η_c, J/ψ) provide such an opportunity.

D. Hatton, CD, B. Galloway, J. Koponen, P.Lepage, A.Lytle, 2005.01845

To achieve high precision we have:

E. Follana et al, HPQCD, hep-ph/0610092

1) Used HPQCD's Highly Improved Staggered Quark (HISQ) action - designed as a very accurate discretisation of the Dirac equation.

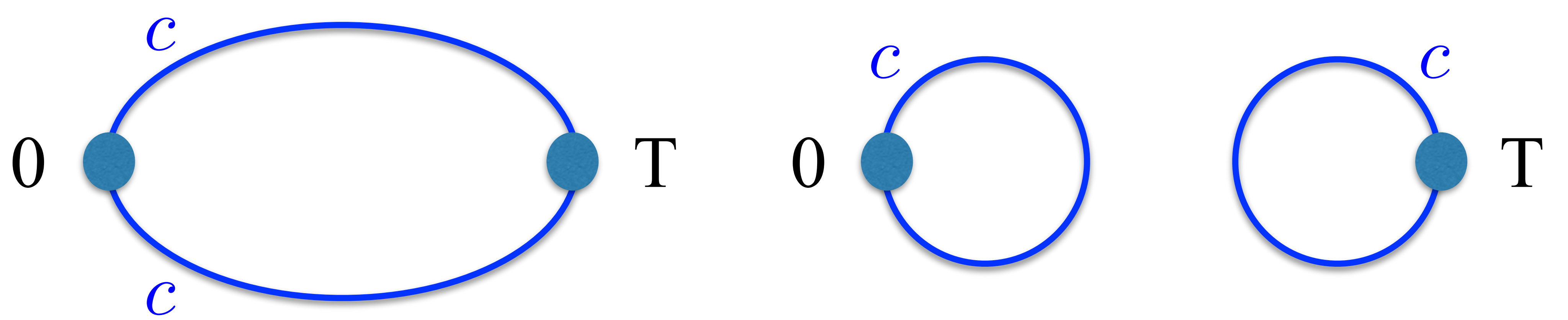
2) Used gluon field configs that include u, d, s and c sea quarks with the HISQ action (generated by MILC). Lattice spacing (a) values range from 0.15fm to 0.03fm. u/d sea quark masses from ms/5 to physical.

3) Included 'quenched' QED to allow for effects from electric charge of c quark. (Random photon field incorporated with gluon when solving Dirac eq.)

More calculational details

Tune m_c so that J/ψ mass correct for every gluon ensemble (a fixed from w_0/f_π).

Calculate 'connected' correlators only - η_c annihilation to gluons NOT included.
Key question: What impact does this have?

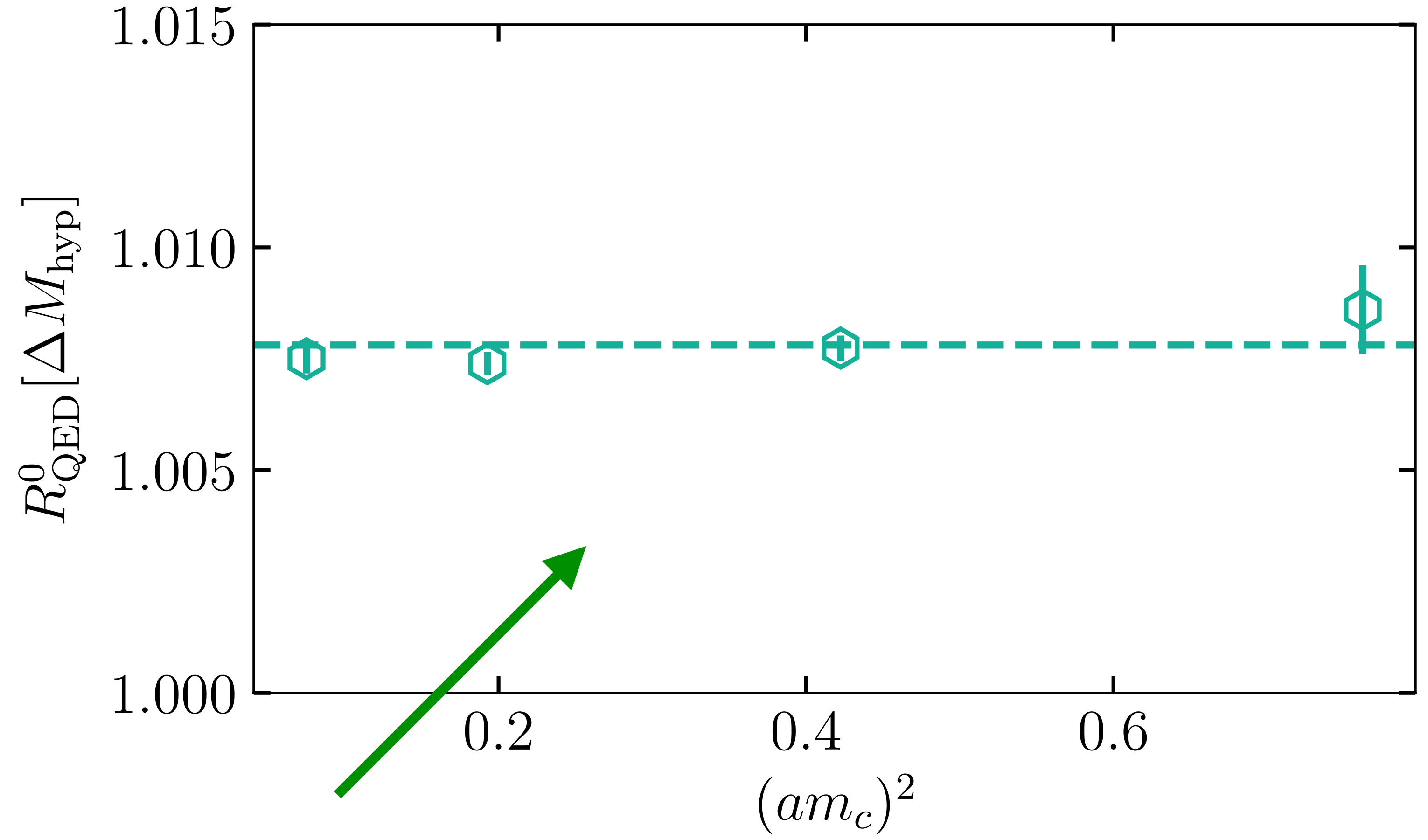


included for 0^- and 1^-

not included

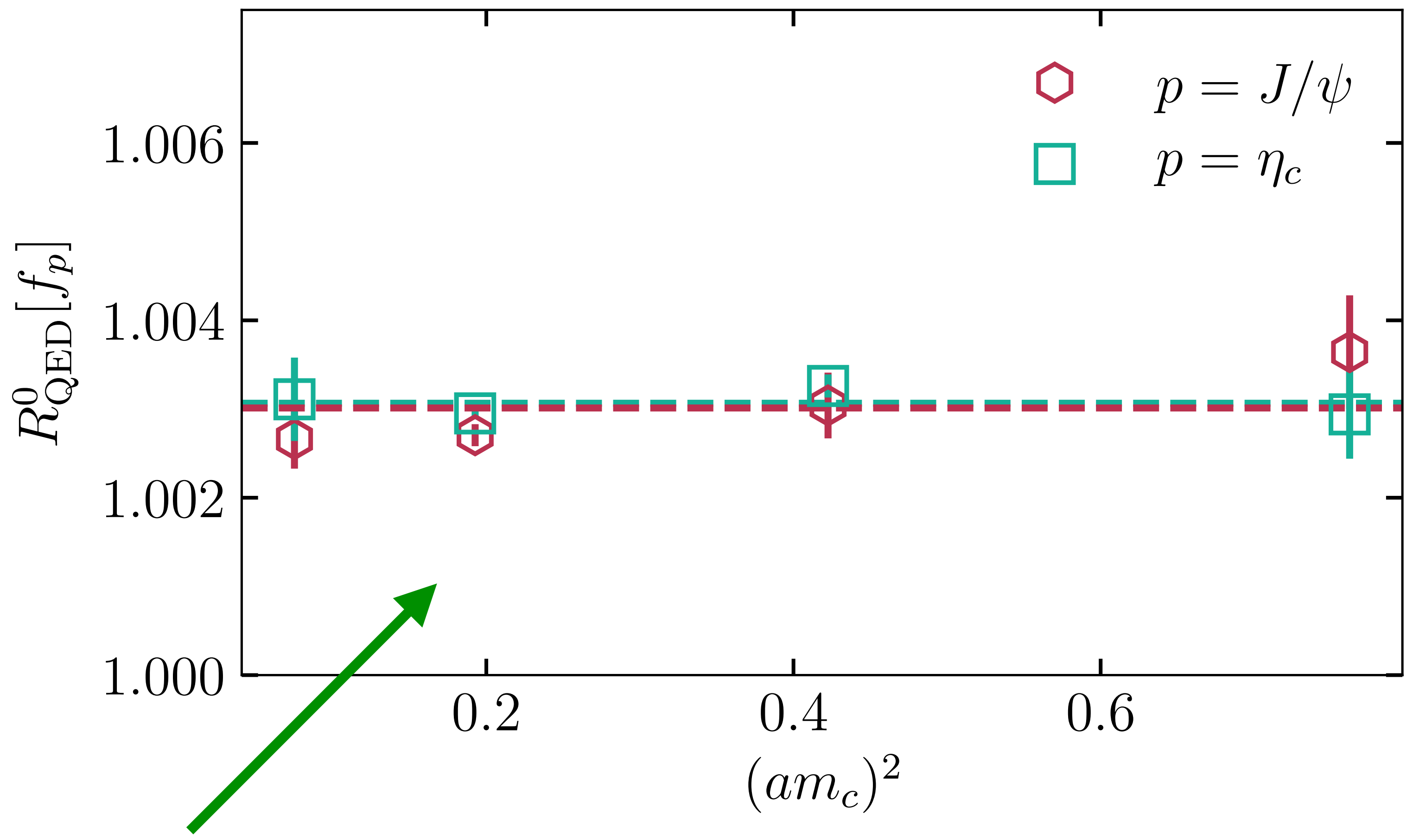
Fit correlators to obtain ground-state masses and amplitudes = decay constants

$$R^0 = \frac{\text{QCD} + \text{QED}}{\text{QCD}}\Big|_{\text{fixed am}}$$



Impact of QED on hyperfine splitting

Impact of QED on decay constants



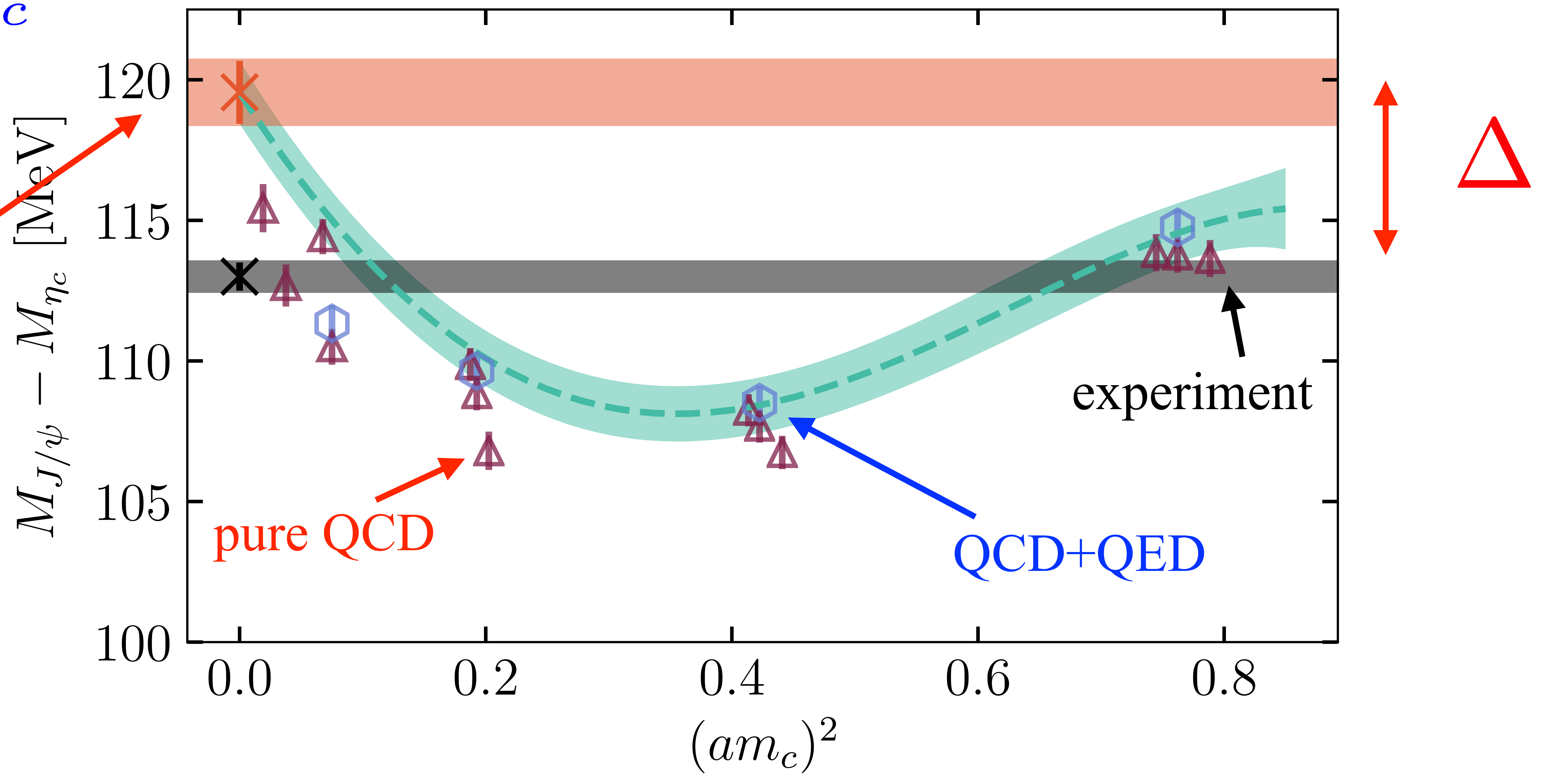
Including QED INCREASES the hyperfine splitting by 0.8% (0.7% direct and 0.1% from retuning m_c)

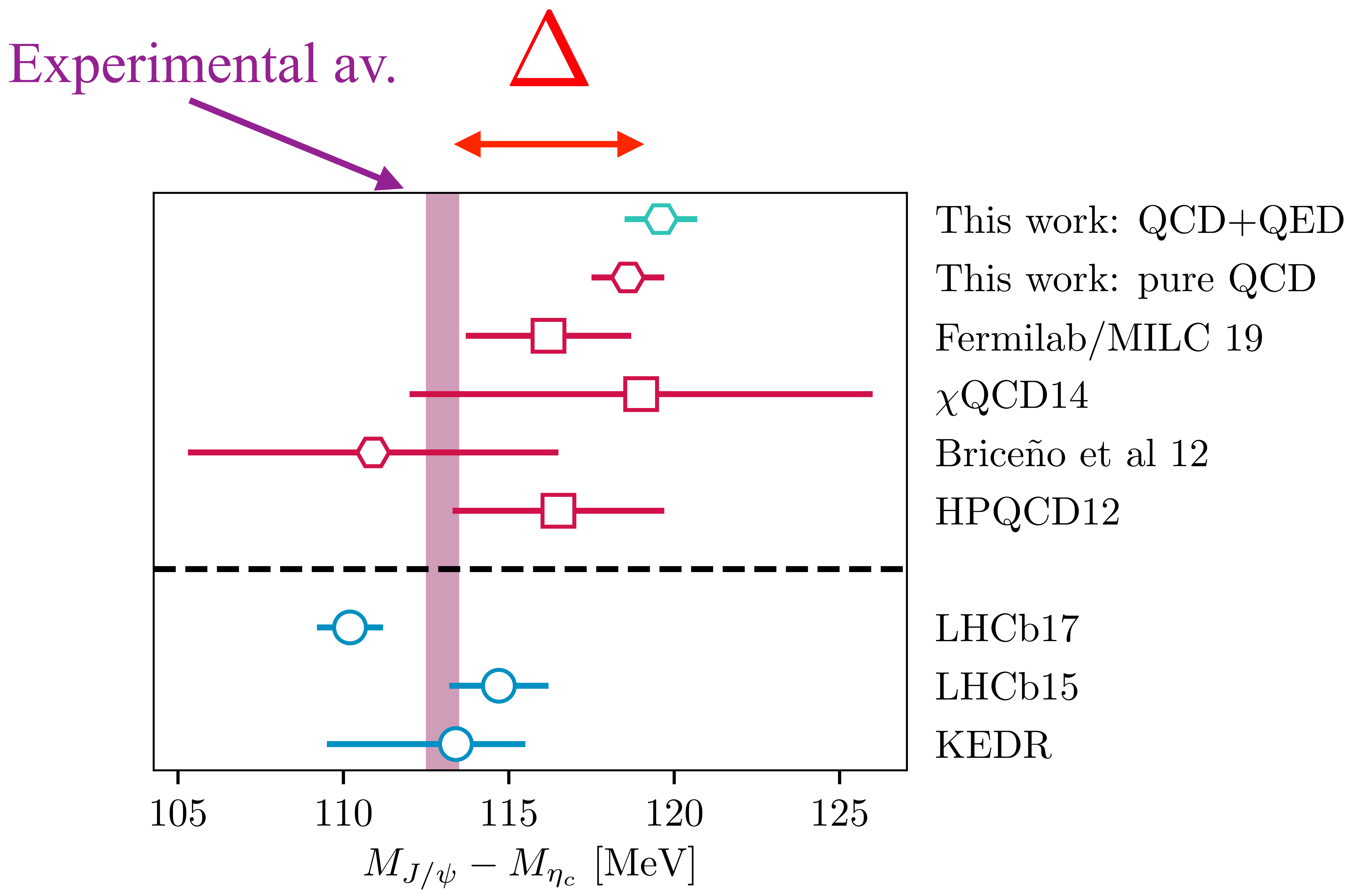
There is an additional QED contribution from J/ψ annihilation to a photon - estimate with pert. th. as +0.7 MeV.

Including QED INCREASES the decay constants by 0.2% (0.3% direct and -0.1% from retuning m_c)

Hyperfine Splitting: $M_{J/\psi} - M_{\eta_c}$

Hyperfine splitting in QCD+QED - extrapolate to a=0 and physical u/d masses in sea. (Note: a^2 effects are small for HISQ; allow a^{2n} terms up to a^{10} in extrapolation.)





Conclude: lattice QCD+QED connected calculation of hyperfine splitting disagrees with experiment. If this is because of missing η_c annihilation, then

$$\Delta M_{\eta_c}^{\text{annihiln}} = +7.3(1.2)\ \text{MeV}$$

Leading order NRQCD pert. th. gives -3 MeV, but subleading terms could easily change sign. Not (yet) calculable directly in lattice QCD.

Decay constants of η_c and J/ψ

$$\langle 0|\overline{\psi}\gamma_5\gamma_0\psi|\eta_c(\vec{p}=0)\rangle = f_{\eta_c}M_{\eta_c}$$

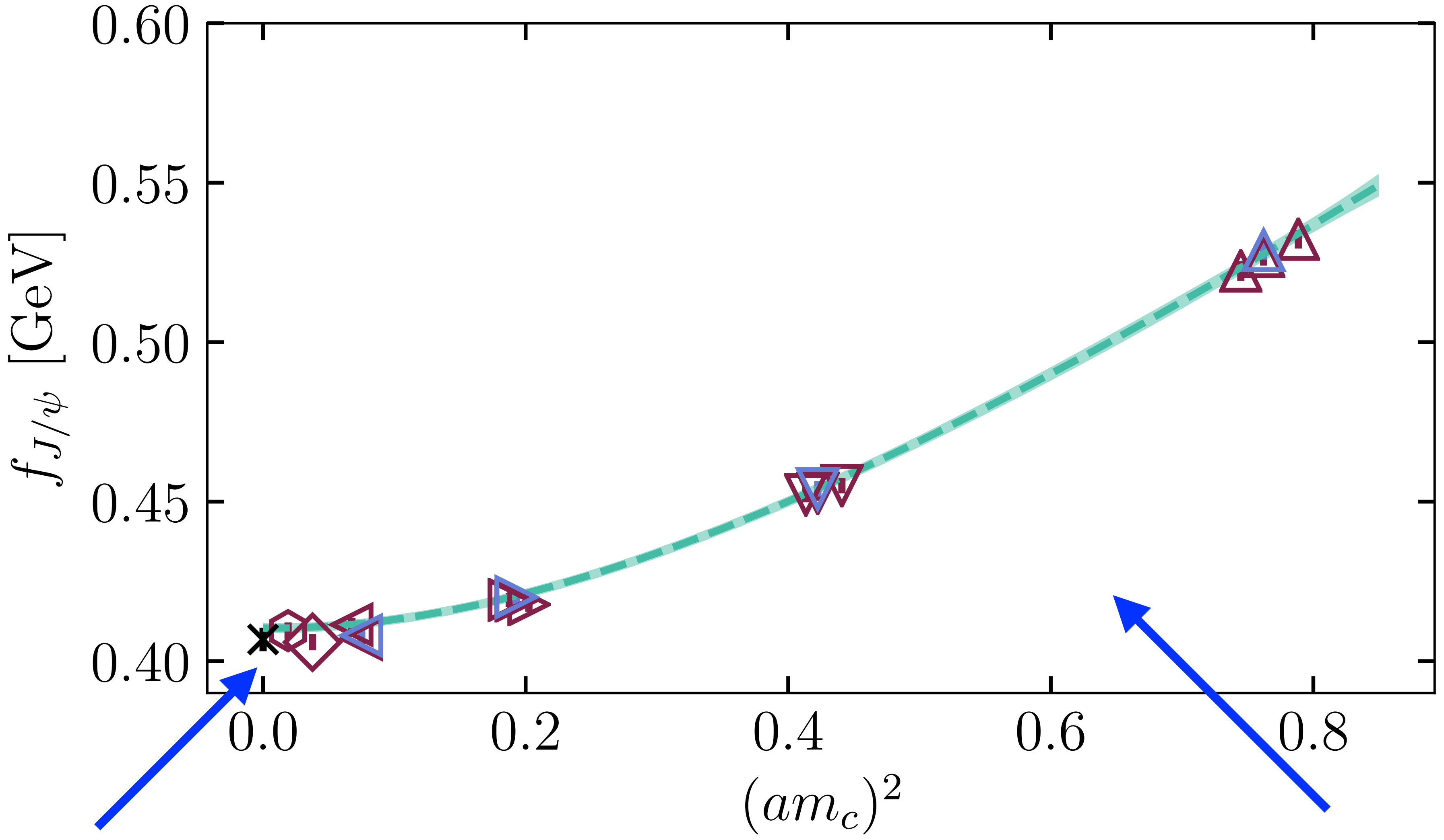
$$\langle 0|\overline{\psi}\gamma_\mu\psi|J/\psi(\vec{p}=0)\rangle = f_{J/\psi}M_{J/\psi}\varepsilon_\mu$$

Must normalise lattice vector current carefully - use RI-SMOM since respects Ward identity on lattice.

D. Hatton et al, HPQCD, 1909.00756

$$\Gamma(J/\psi \to \ell^+\ell^-) = \frac{4\pi}{3}\alpha_{\rm QED}^2 Q_c^2 \frac{f_{J/\psi}^2}{M_{J/\psi}}$$

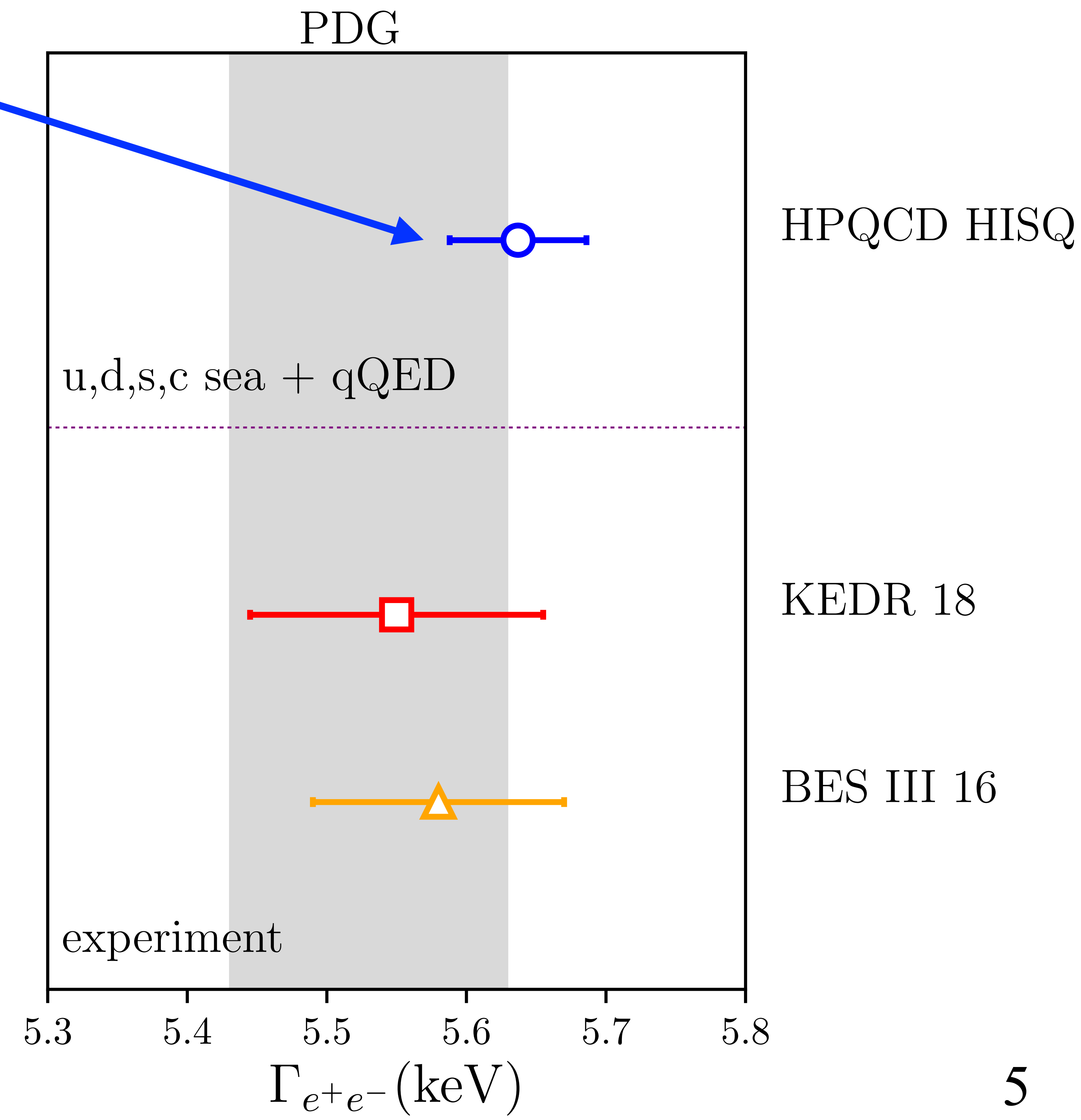
Hadronic parameter for annihilation to a photon



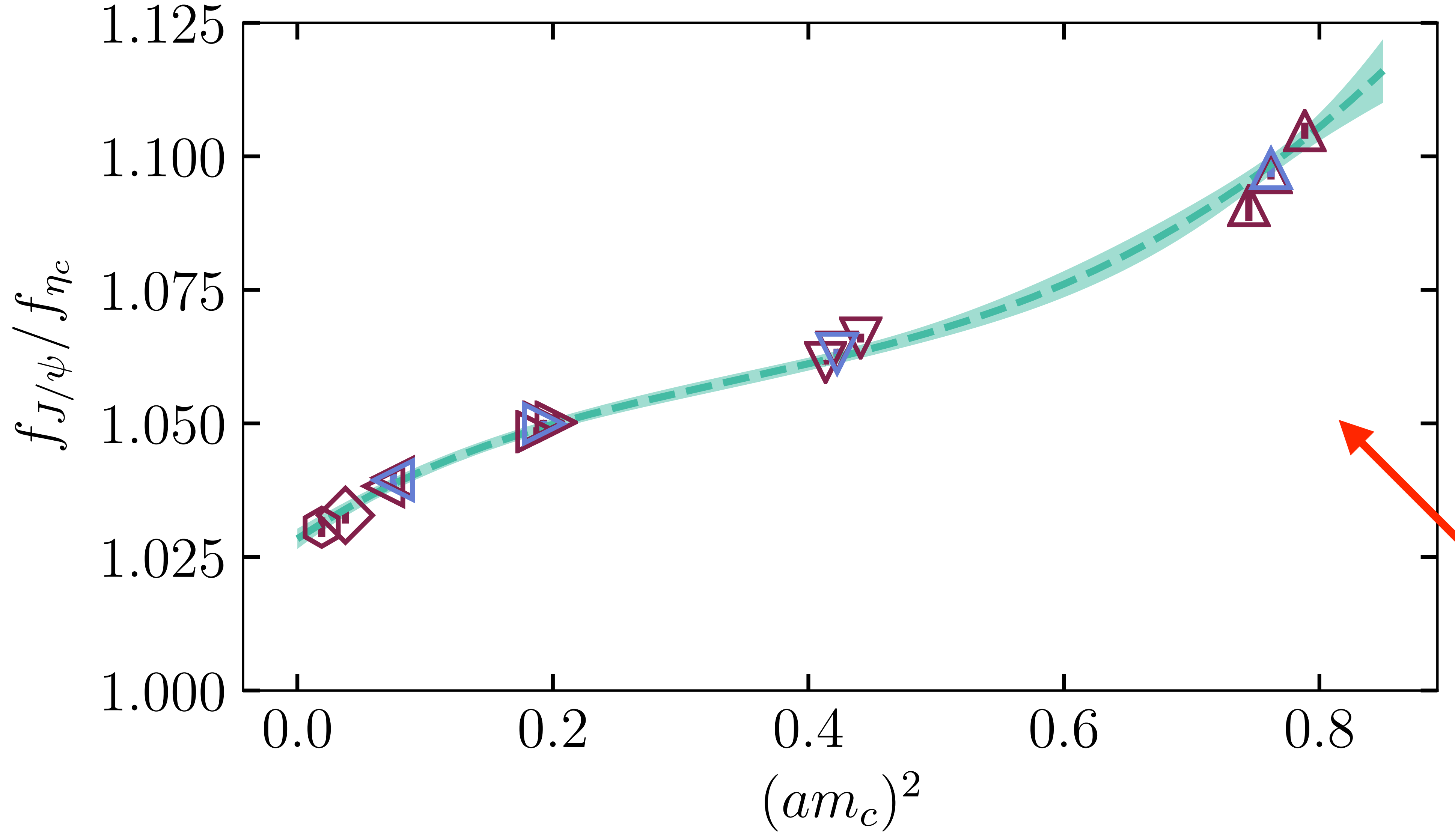
0.4% accurate physical result

Extrapolation to a=0

$$\Gamma(J/\psi \to e^+e^-) = 5.637(49)\ \text{keV}$$



Agrees with, but more accurate than, experiment



Systematics cancel in decay constant ratio

$$\frac{f_{J/\psi}}{f_{\eta_c}} = 1.0284(19)$$

0.2% accurate! and > 1

D. Hatton et al, HPQCD, 2005.01845

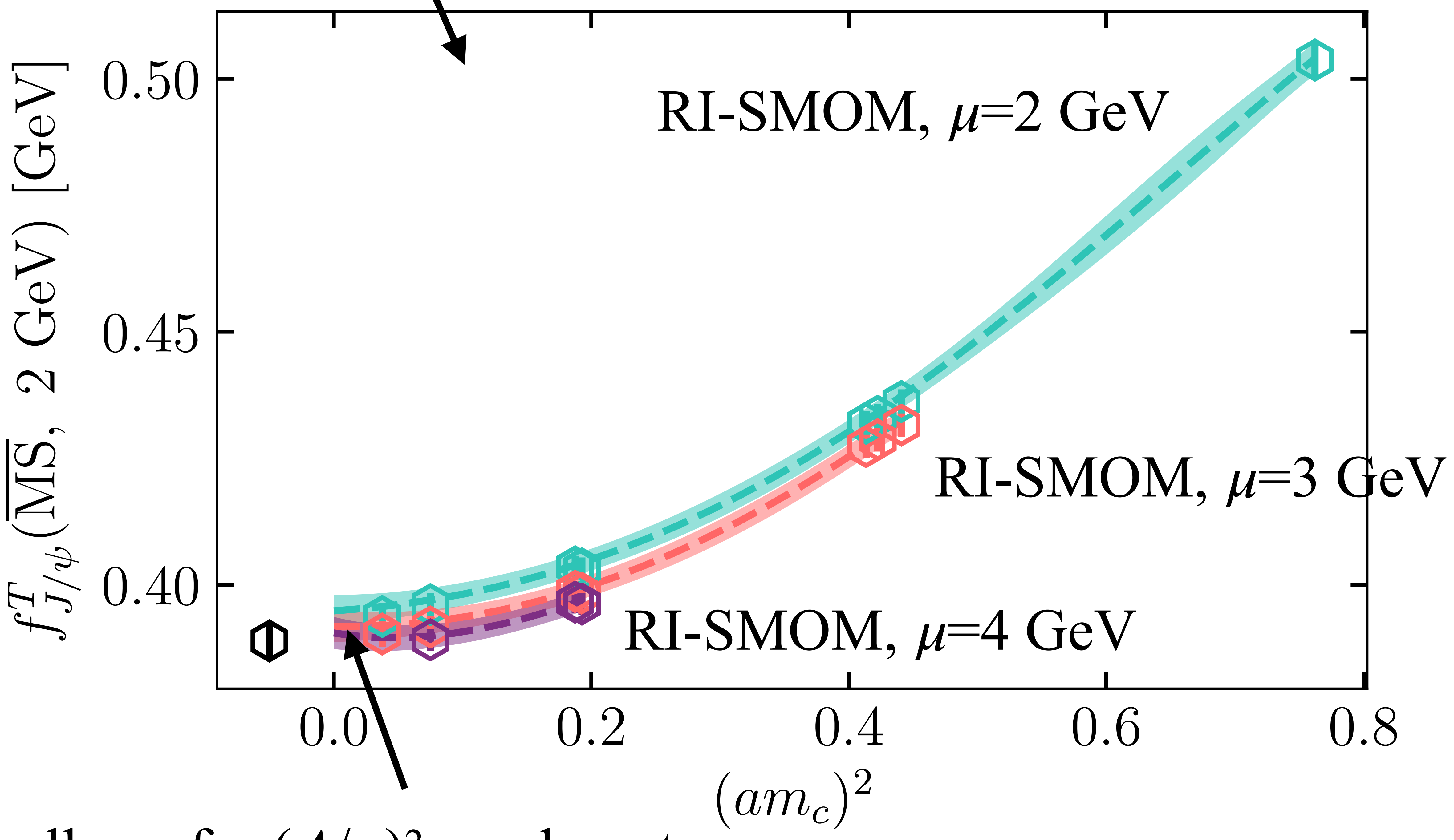
Tensor decay constant of the J/ψ

D. Hatton et al, HPQCD, 2008.02024

$$\langle 0|\overline{\psi}\sigma_{\alpha\beta}\psi|J/\psi\rangle = if^T_{J/\psi}(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha)$$

scheme and scale-dependent

Must renormalise tensor current in lattice QCD - use intermediate momentum-subtraction scheme, RI-SMOM, on lattice and account for nonperturbative 'condensate' contributions

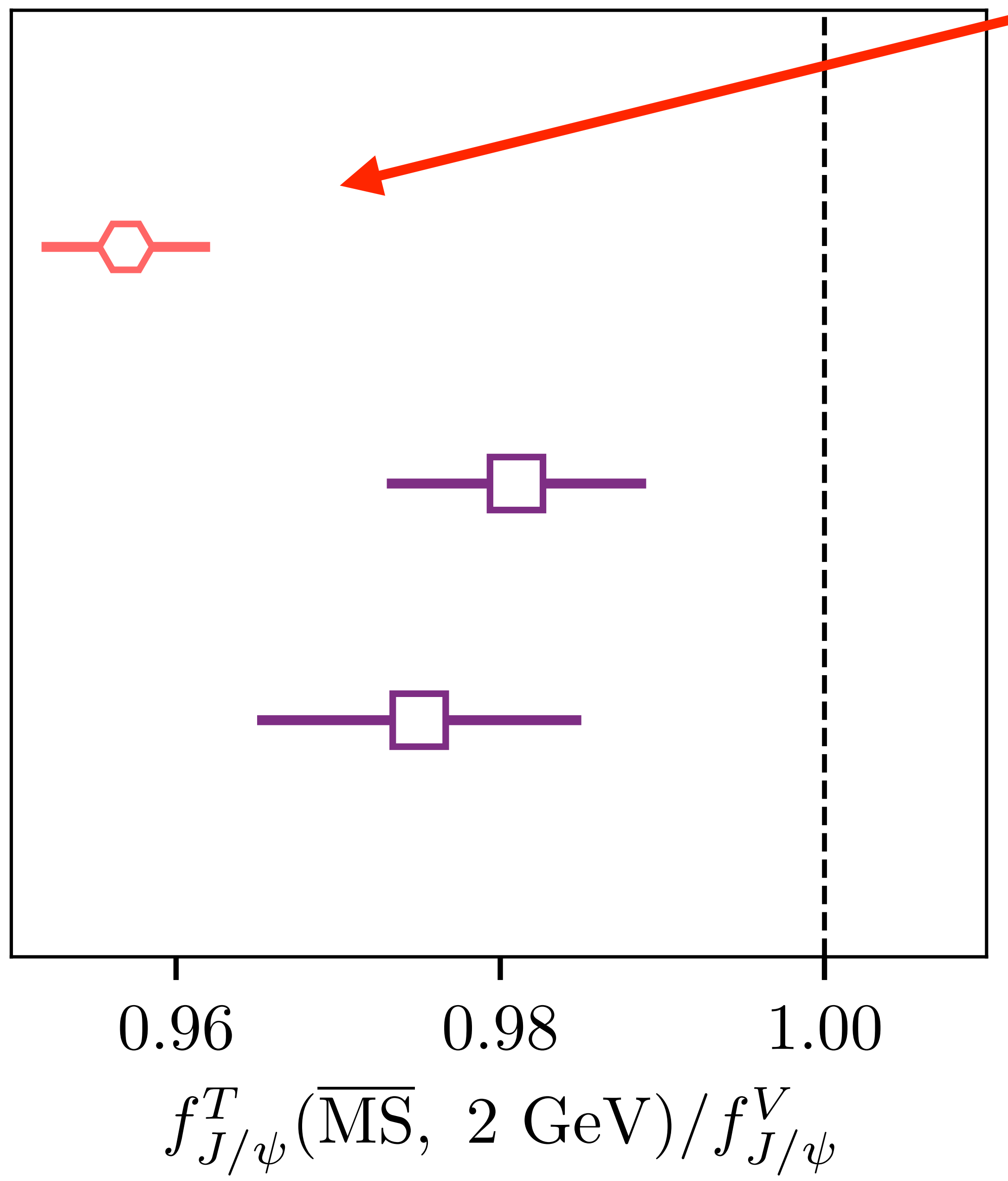


Fit allows for $(\Lambda/\mu)^2$ condensates

Ratio of tensor to vector decay constants

$$\frac{f^T(\overline{\text{MS}}, 2\,\text{GeV})}{f^V} = 0.957(5)$$

Clearly <1



Use to set bounds on BSM decay rates.

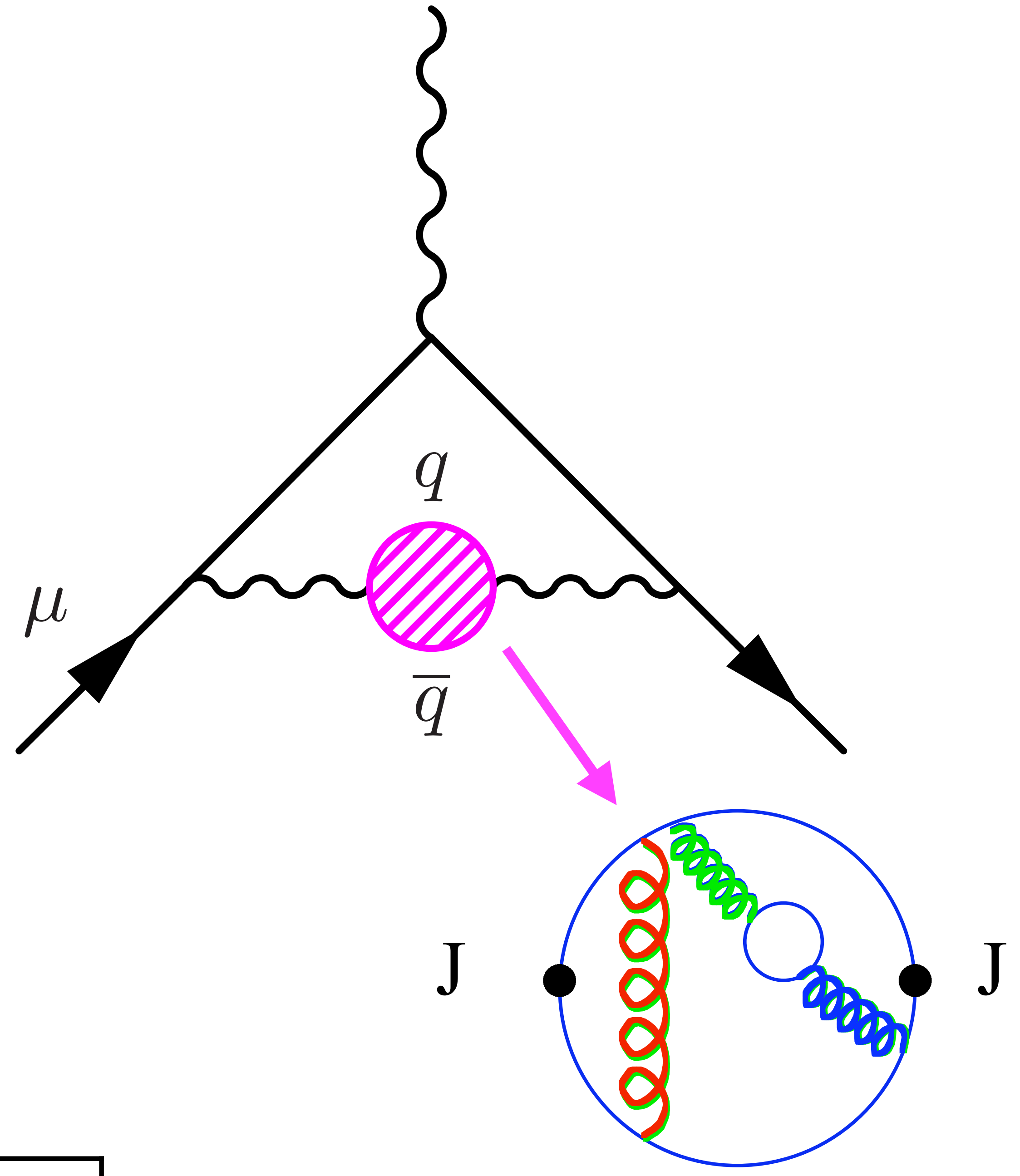
Charm quark contribution to anomalous magnetic moment of the muon, a^c_μ

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$

$$a_\mu = \frac{g-2}{2}$$

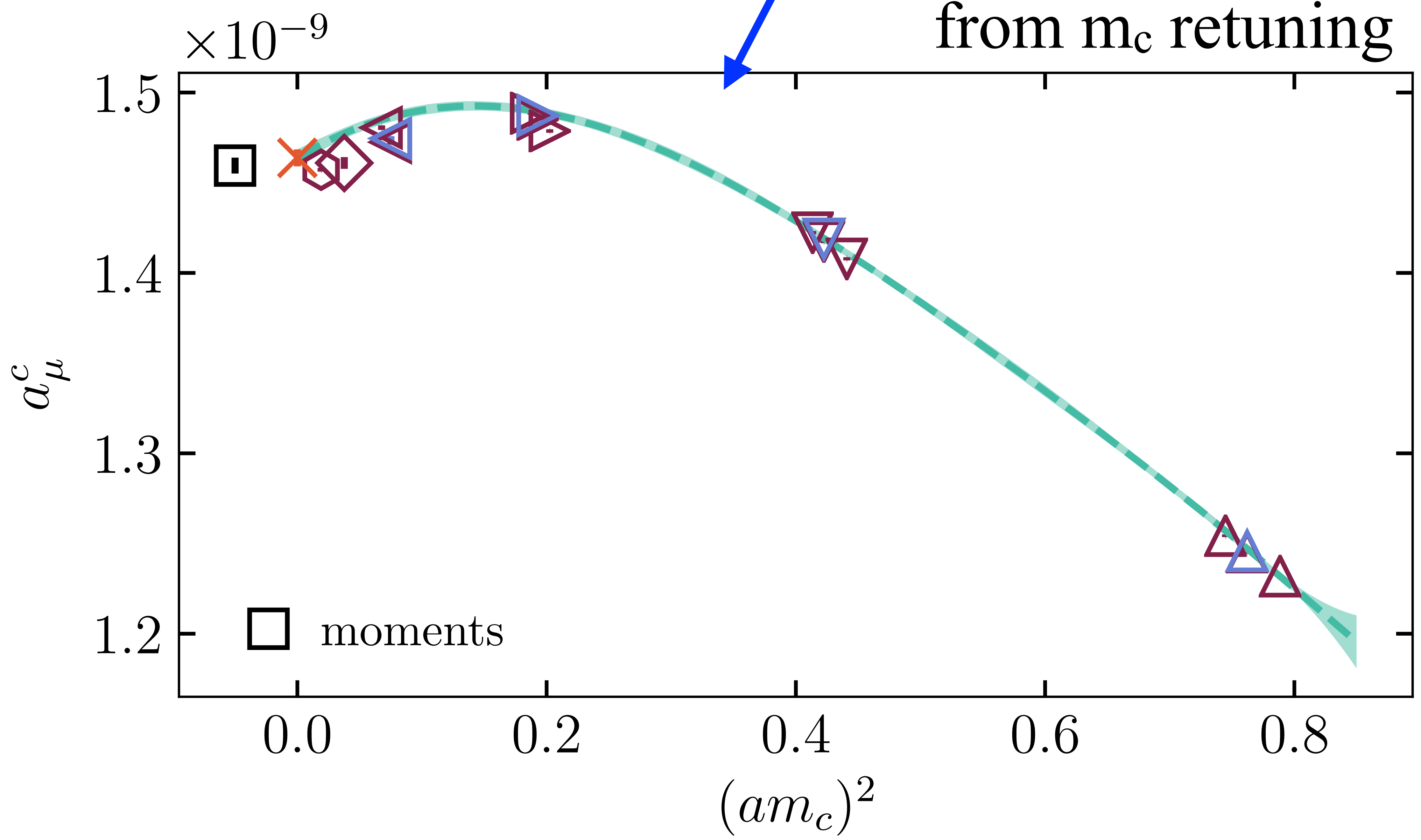
Least well-known contribution is that of 'hadronic vacuum polarisation' - quark bubble for each flavour attached to photon.

Calculate in lattice QCD+QED using vector correlators, either via time-moments or by direct integration



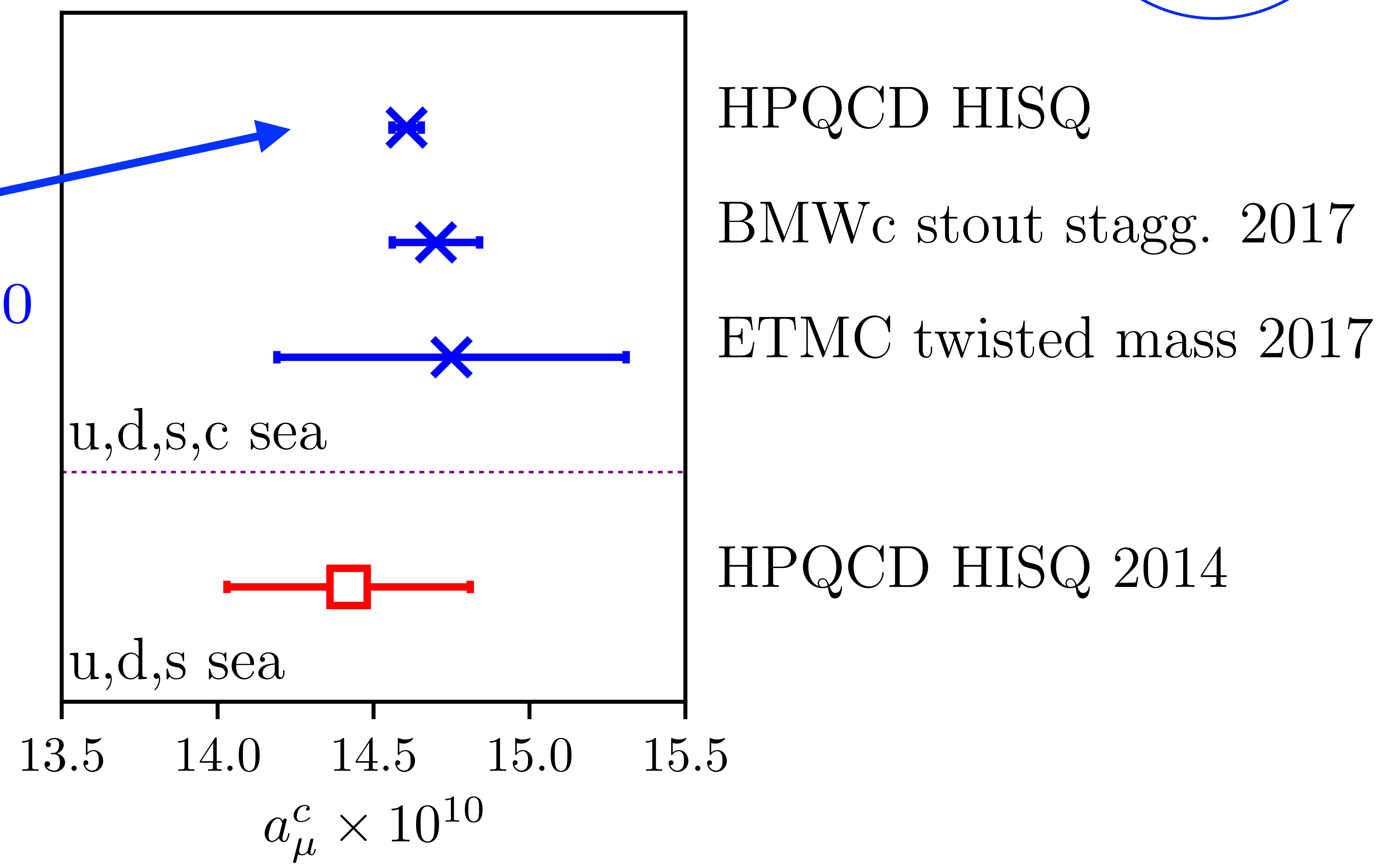
Extrapolation to a=0

Impact of QED = +0.2% from m_c retuning



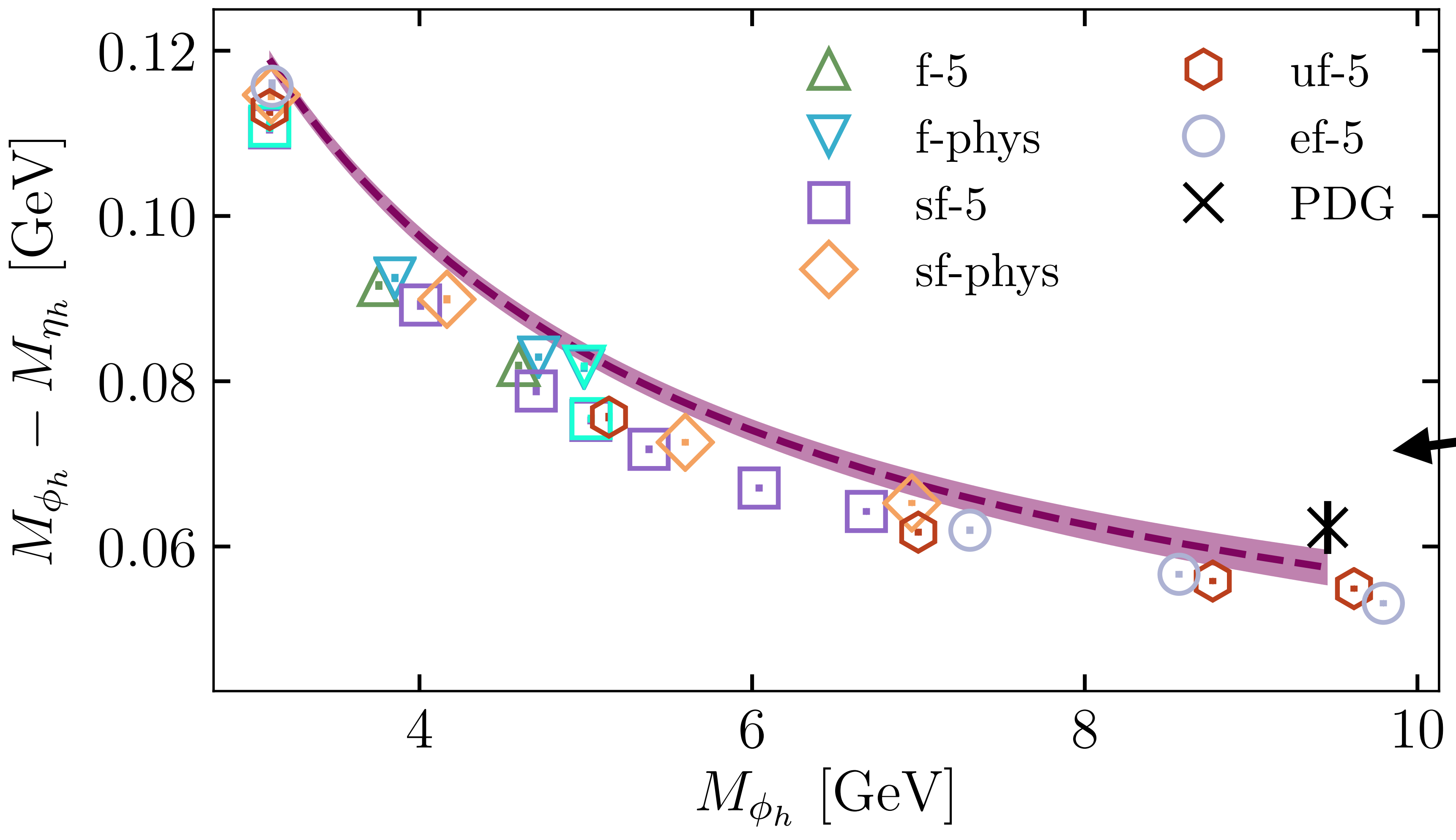
$$a^c_\mu = 14.638(47) \times 10^{-10}$$

Small part of total HVP, but still needed for accurate SM a_μ

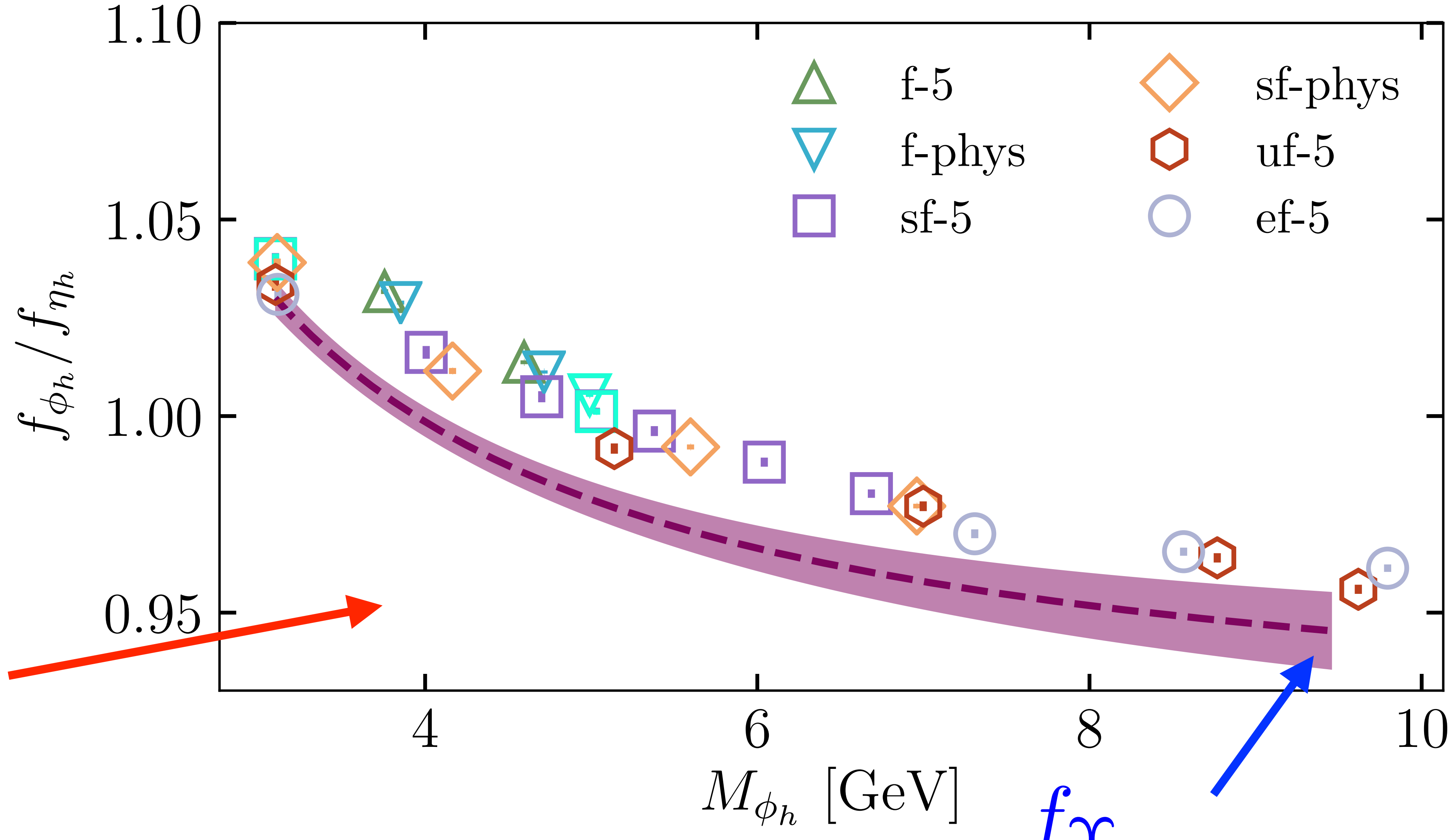


Extend calculations to bottomonium

Use HPQCD's 'heavy-HISQ' approach, fitting results at multiple m_h and multiple a to obtain results at b in continuum.



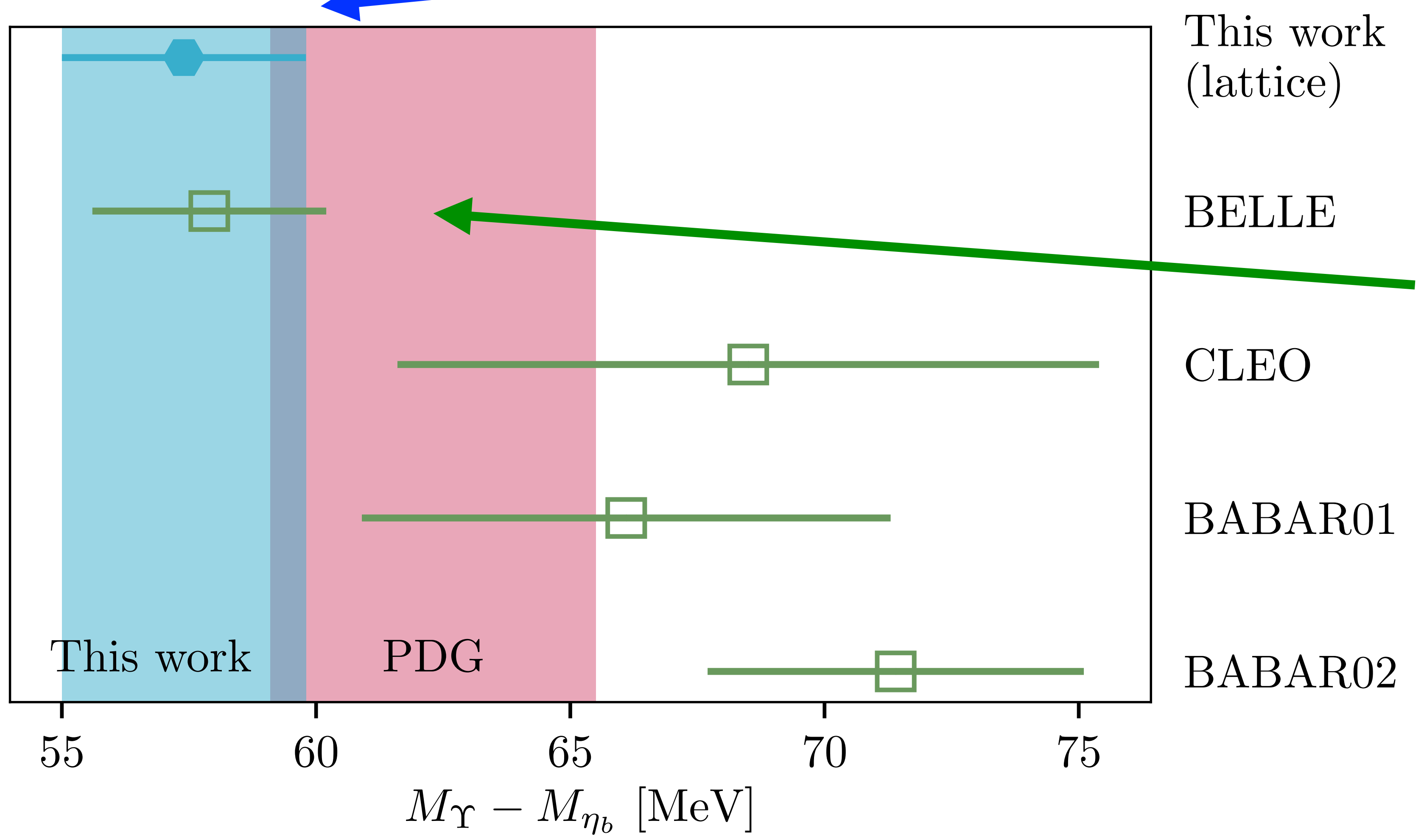
Hyperfine splitting



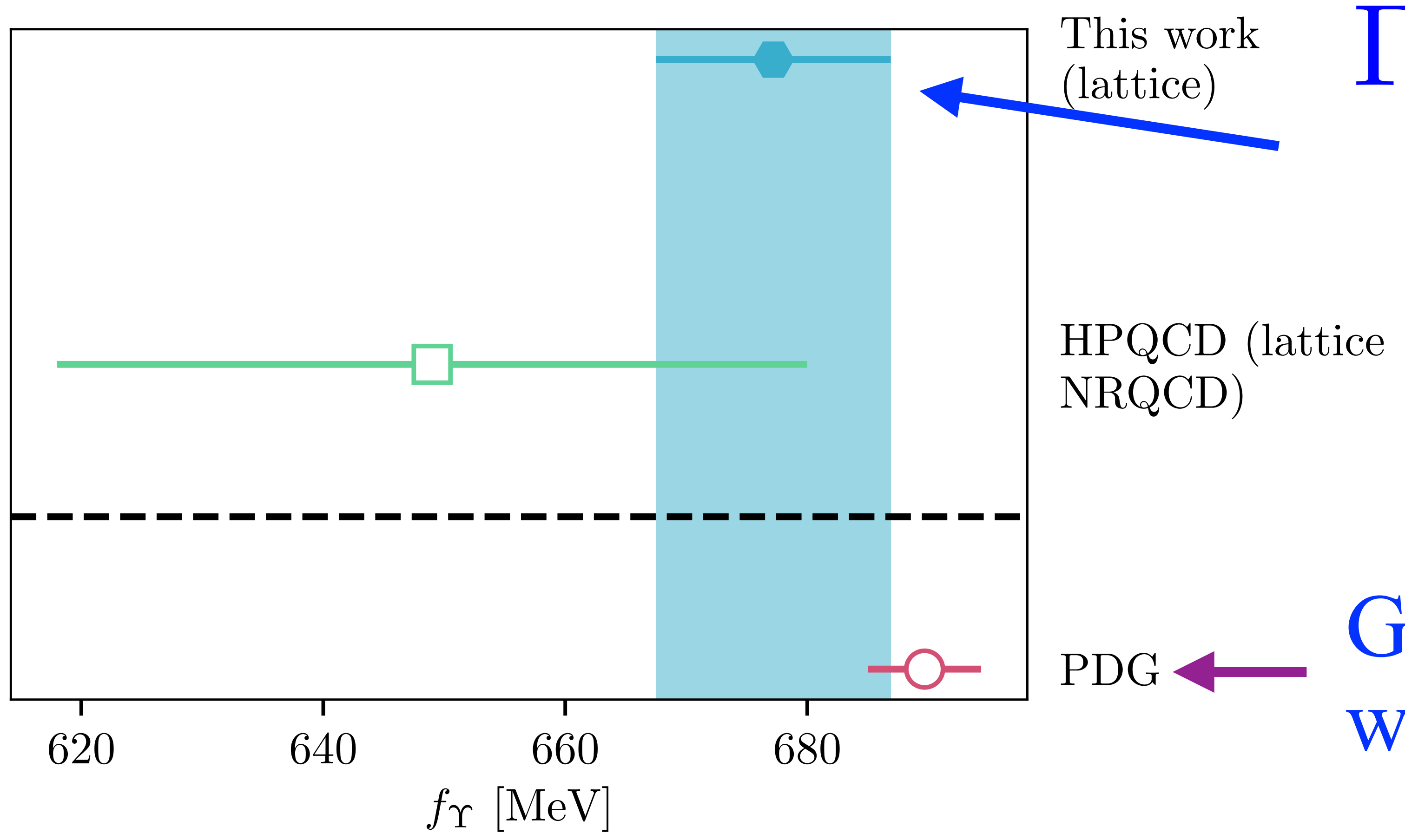
Decay constant ratio >1 for c; <1 for b

$$\frac{f_\Upsilon}{f_{\eta_b}} = 0.945(10)$$

$$M_\Upsilon - M_{\eta_b} = 57.5(2.5)\ \text{MeV}$$



Good agreement with Belle



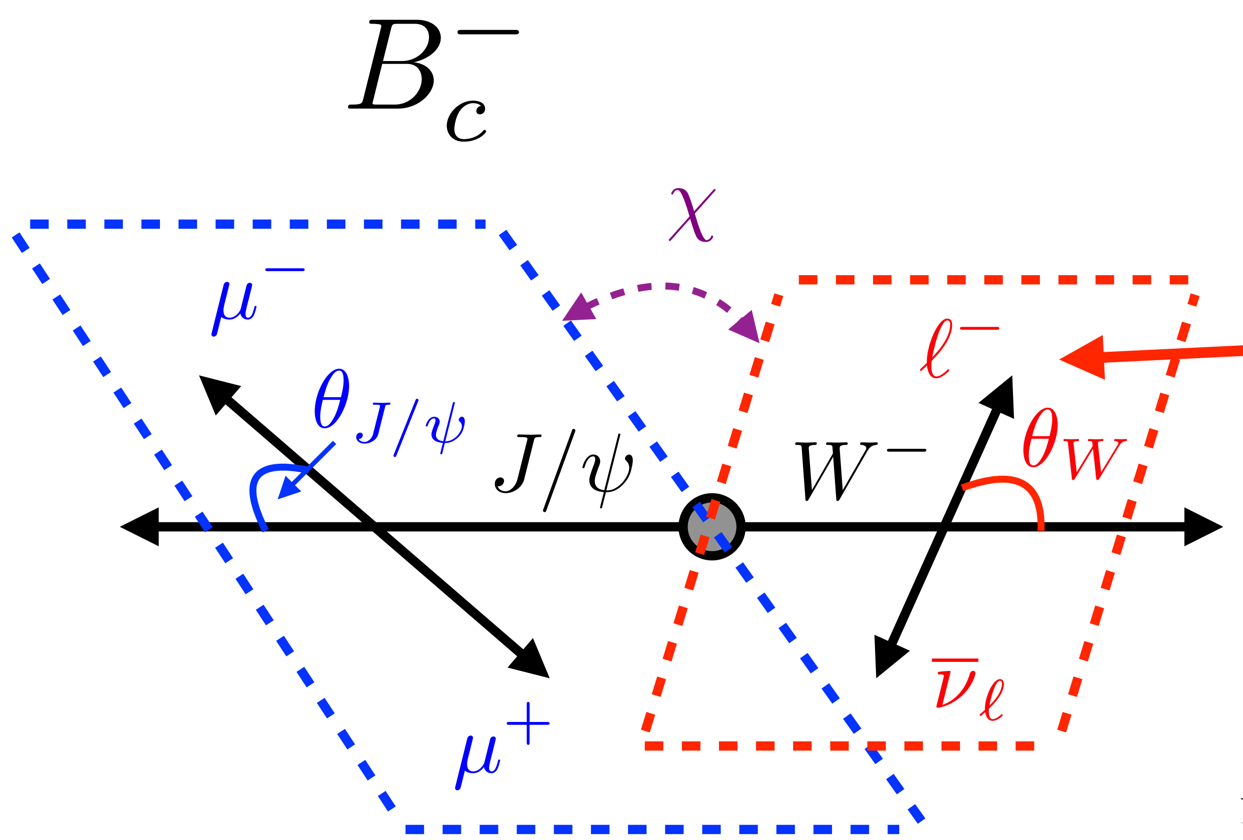
$$\Gamma(\Upsilon \to e^+e^-) = 1.292(37)\ \text{keV}$$

Good agreement with experiment

$B_c \to J/\psi \ell \overline{\nu}$

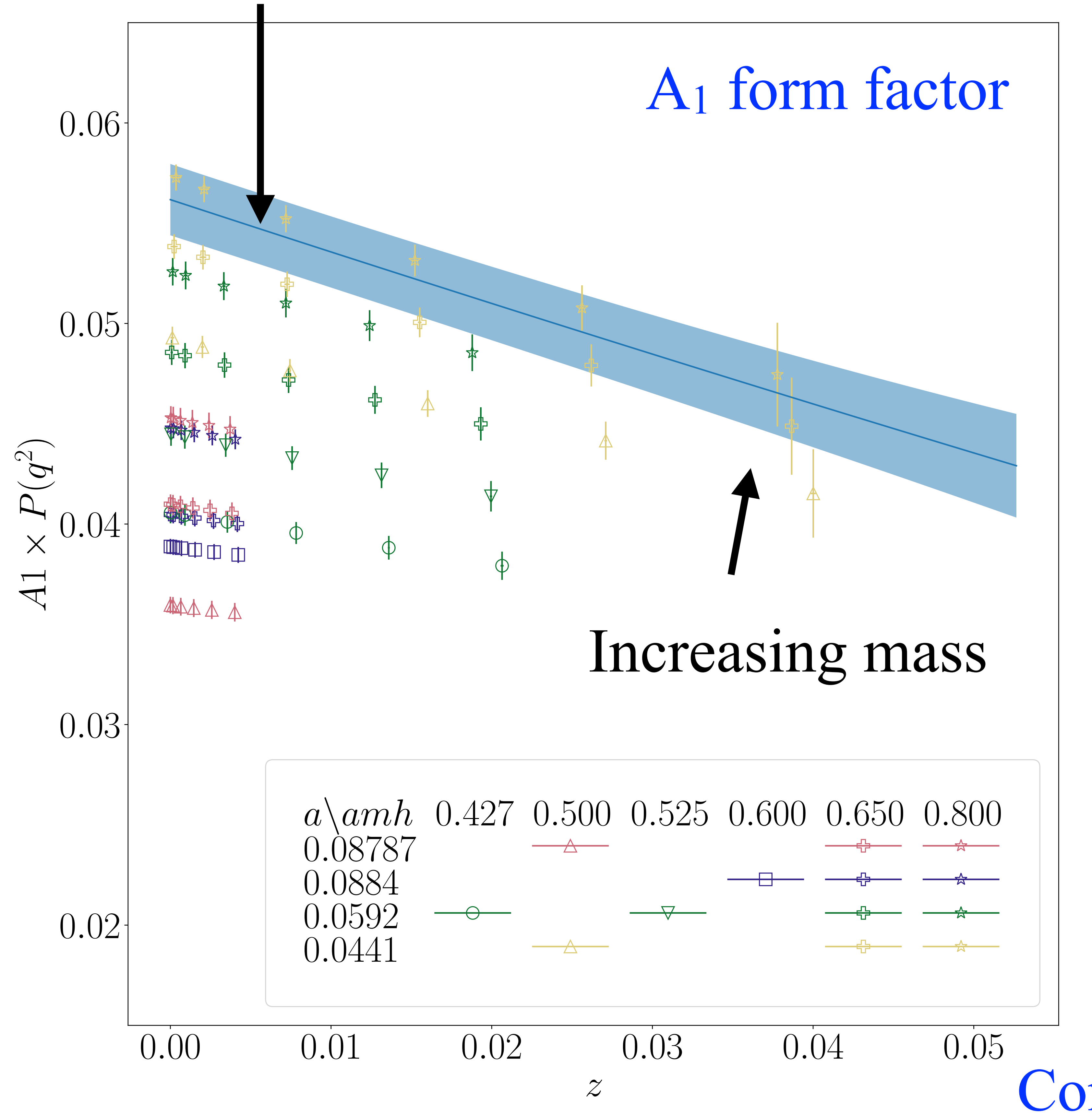
FIRST lattice QCD calculation

J. Harrison et al, HPQCD, 2007.06956, 2007.06957



e, μ, or τ

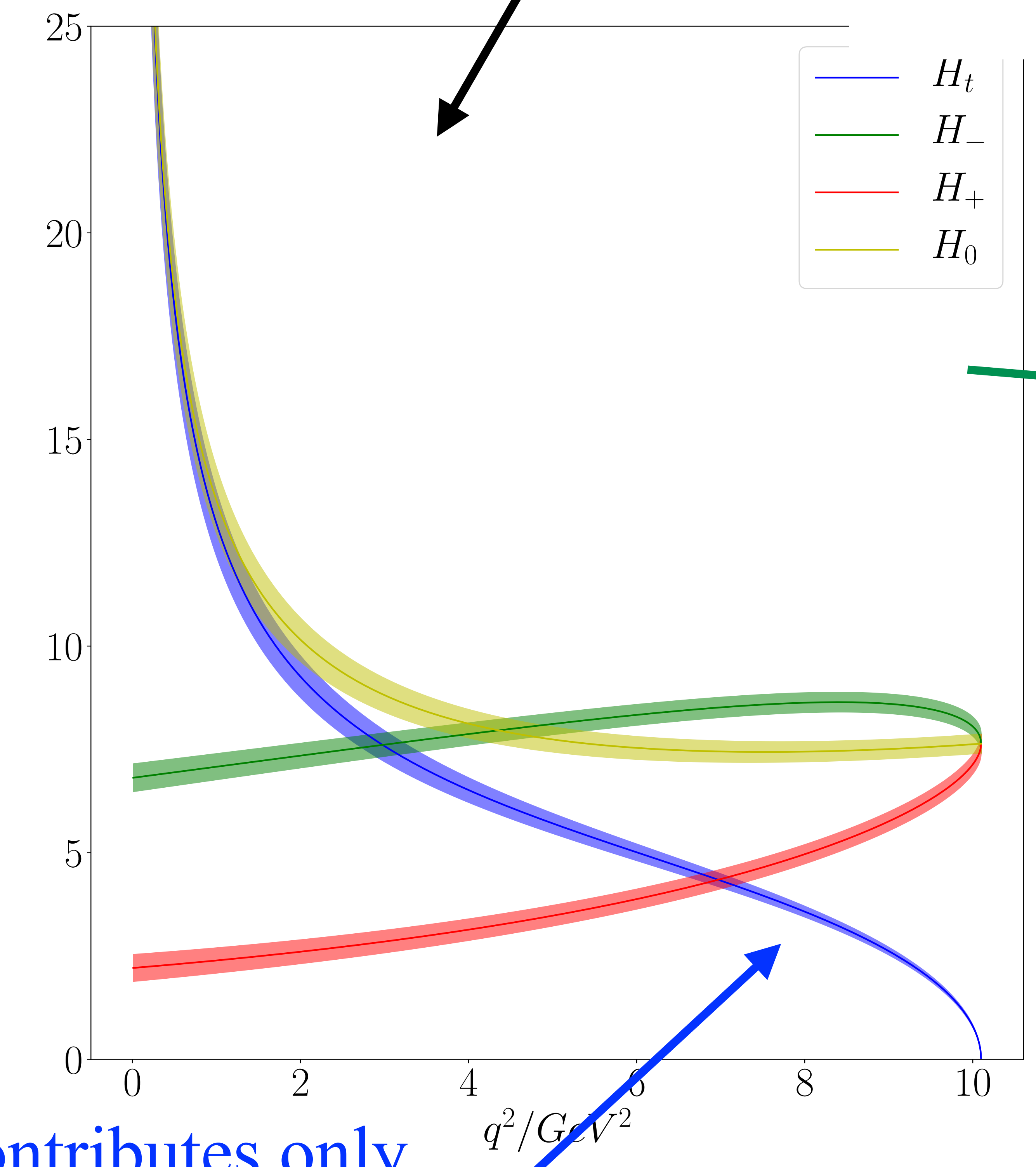
Physical result at b mass



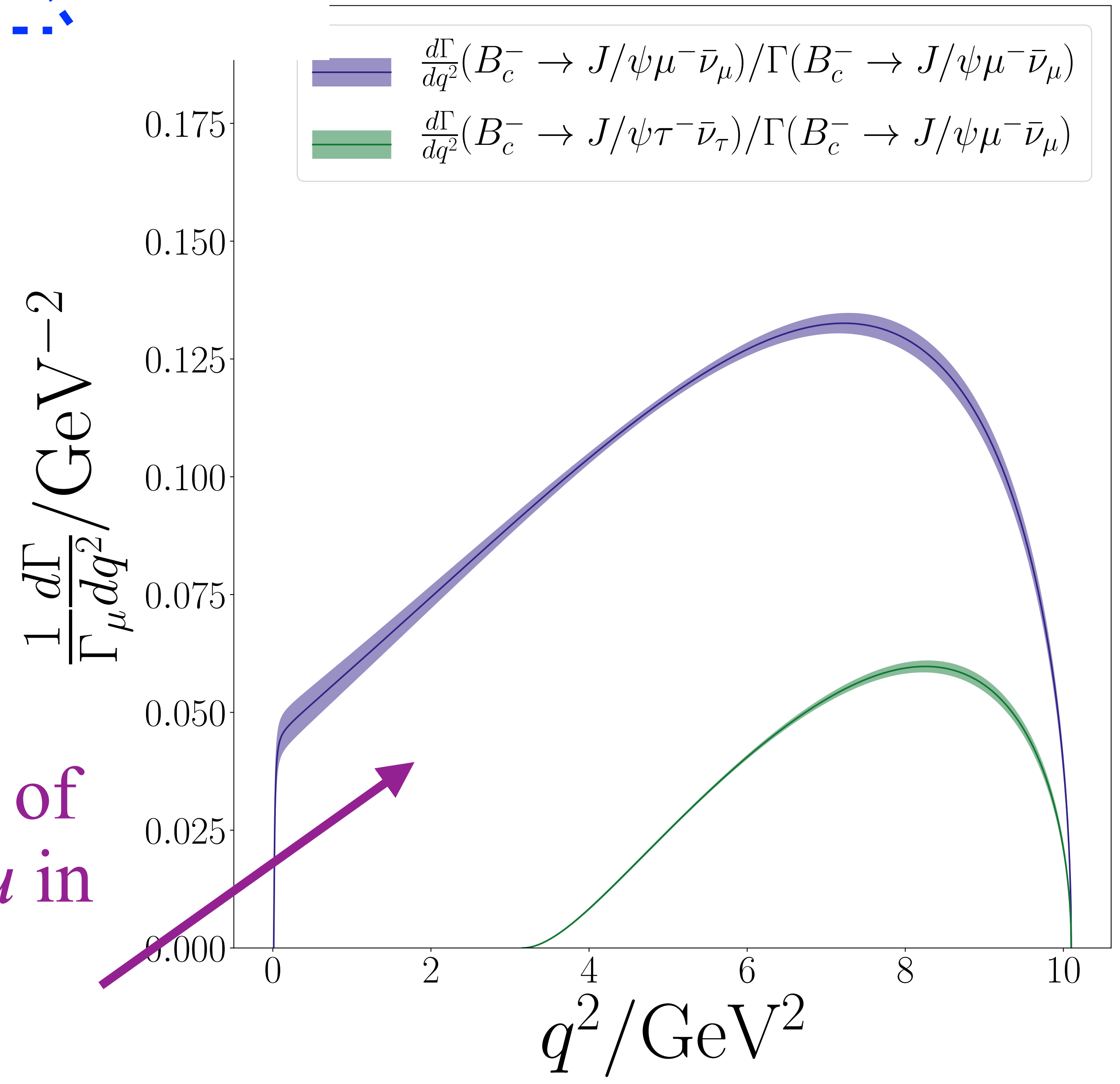
Increasing mass

Use HISQ with multiple heavy quark masses ($am_h < 1$) for a range of fine lattice spacings down to 0.045 fm and fit

Combine the 4 ffs into W helicity amplitudes



Contributes only for heavy leptons i.e. τ, because it appears multiplied by lepton mass in the rate.



Calculate ratio of rates to τ and μ in SM :

$$R(J/\psi) = 0.2582(38)$$

1.5% uncty

First LHCb result:

$$R(J/\psi) = 0.71(25)$$

LHCb, 1711.05623

A discrepancy would imply non-SM lepton flavour universality violation. Hints of this being seen elsewhere ..

Conclusions

• Lattice QCD results for η_c and J/ψ have reached high precision, and now include the effects of the c quark electric charge (QCD+quenched QED).

• The hyperfine splitting result shows that the impact of η_c annihilation on its mass is +7(1) MeV

• The calculation of the J/ψ leptonic width is now more accurate (0.9%) than experiment and agrees well with it.

• Extension of the calculation to bottomonium gives a hyperfine splitting that agrees well with recent experiment, and an Υ leptonic width that also agrees with experiment. The ratio of vector to pseudoscalar decay constants flips from >1 at c to <1 at b.

Future

• Extension of vector and tensor decay constant calculations to heavy-light mesons is underway.

• Form factor calculations using 'heavy-HISQ' also being extended to further processes.

J. Harrison + CD, HPQCD, 2105.11433, Bs to Ds* semileptonic decay