#### Study of initial state of causal dissipative relativistic fluid expansion in *p-p* and *p-Pb* collisions at LHC energies with percolation color sources approach

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# Outline

- Motivation
- Model
  - Thermodynamics
  - Fluid-dynamics
- Results & Conclusions



#### Motivation

One can see the phase transition in QCD as a change in the number of degrees of freedom

$$\varepsilon_{\rm HG} = \frac{\pi^2}{30} 3T^4 \simeq T^4 \rightarrow \varepsilon_{\rm QGP} = \frac{\pi^2}{30} \left[ 2 \times 8 + \frac{7}{8} \times 2(3) \times 2 \times 2 \times 3 \right] T^4.$$
Lattice QCD suggest that it could have had a first-order transition for high densities of baryons and low temperature, while at a high temperature and low baryon density, it is a "crossover".

1.0

1.5

2.0

2.5

3.0

3.5

F. Karsch, Lect. Notes Phys. 583 (2002) 209; J. Phys. Conf. Ser. 46 (2006) 122.

4.0

#### Motivation



• The phase transition in QCD can be described from percolation theory by using critical orden parameters.

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- In SPM we use the 2-dimensional percolation theory over the overlapping area of a collision, S, considering the chromodynamic interaction as color flux tubes stretched among the colliding partons of the proyectiles or targets.

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The number of initial strings, N<sup>s</sup>, depends on the energy of the collision, on the number of participants and on the centrality of the event:

$$N^s = 2 + 4 \frac{S_0}{S} \left(\frac{\sqrt{s}}{m_p}\right)^{2\lambda},$$

where  $m_p = 938.3$ MeV is the mass of the proton and the power lambda (=0.186) describes the multiplicity with the energy in *p*-*p* collisions. The transverse area of a string is a circle,  $S_0$ , with radius  $r_0 = 0.25$ fm.



I. Bautista, J. G. Milhano, C. Pajares and J. Dias de Deus, Phys. Lett. B 715 (2012) 230

As the multiplicity increases the string density will increase to and the strings will start to overlap forming macroscopic clusters, thus marking a phase transition defined by the percolation threshold, the critical string density, to classify the events the string density is defined as



The average multiplicity at central rapidity region for each energy is related to the average number of initial strings through a specific geometrical scaling function of the string density

$$\frac{dN}{d\eta} = \kappa F(\xi^t) N^s, \qquad F(\xi^t) = \sqrt{\frac{1 - e^{-\xi^t}}{\xi^t}},$$

The transverse momentum distribution behaves as the following power law

$$\frac{1}{N} \frac{d^2 N_{ch}}{d\eta dp_T} \bigg|_{\eta=0} = a \frac{(p_0 b)^{\alpha-2}}{(p_T + p_0 b)^{\alpha-1}},$$

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where

$$a = \frac{\alpha - 2}{2\pi} \left. \frac{\left\langle \sum_{i=1}^{M} \sqrt{\frac{n_i S_i}{S_1}} \right\rangle}{\left\langle \sum_{i=1}^{M} \sqrt{\frac{n_i S_i}{S_1}} \right\rangle_{pp}} \frac{dN}{d\eta} \right|_{\eta=0}^{pp}, \qquad b = \sqrt{\left\langle \frac{N}{\sum_{i=1}^{M} \sqrt{\frac{n_i S_i}{S_1}}} \right\rangle / \left\langle \frac{N}{\sum_{i=1}^{M} \sqrt{\frac{n_i S_i}{S_1}}} \right\rangle_{pp}},$$

I. Bautista, J. G. Milhano, C. Pajares and J. Dias de Deus, Phys. Lett. B 715 (2012) 230

• To obtain p<sub>0</sub> and alpha, which are energy parameters, it is necessary to make a fit over the minimum bias transverse momentum distributions from data below, with b=1, as shown in the figure 1, the values of these parameters are shown in table 1.

$$\frac{1}{N} \frac{d^2 N_{ch}}{d\eta dp_T} \bigg|_{\eta=0} = a \frac{(p_0 b)^{\alpha-2}}{(p_T + p_0 b)^{\alpha-1}}, \qquad (1f)$$

S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 72 (2012) 2164;
A. M. Sirunyan et al. [CMS Collaboration], Phys. Rev. D 96 (2017) no.11, 112003;
S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 74 (2014) no.6, 2847.

I. Bautista, J. G. Milhano, C. Pajares and J. Dias de Deus, Phys. Lett. B 715 (2012) 230

#### Minimum bias transverse momentum distributions

$\sqrt{s}(\text{TeV})$	$p_0(\text{GeV})$	$\alpha$
5.02	$2.780{\pm}0.171$	$9.937 {\pm} 1.716$
13	$2.478 \pm 1.862$	$9.980{\pm}0.297$
7	$2.305{\pm}0.079$	$9.752{\pm}0.140$
2.76	$2.032{\pm}0.074$	$9.448 {\pm} 0.147$
0.9	$1.785{\pm}0.071$	$9.287 {\pm} 0.165$

Table 1: Energy dependent parameters from the fit.





Figure 1: Fit over *pPb* and *pp* transverse momentum distributions of charged pions at LHC energies.

#### High multiplicity events

Near the thermodynamic limit

$$\left\langle \frac{N}{\sum_{i=1}^{M} \sqrt{\frac{n_i S_i}{S_0}}} \right\rangle \to \frac{1}{F(\xi^t)}, \text{ so } b \to \sqrt{\frac{F(\xi_0^t)}{F(\xi_{HM}^t)}}$$

that measures the deviation between high multiplicity and minimum bias events. We use the relation above in eq 1f to make a new fit over the high multiplicity events, from the same data references, obtaining the corresponding Color Reduction Factors.

S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 72 (2012) 2164;

A. M. Sirunyan et al. [CMS Collaboration], Phys. Rev. D 96 (2017) no.11, 112003;

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#### High multiplicity events



#### on data

S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 72 (2012) 2164;
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one obtains...

pp 
$$\sqrt{s} = 0.9 \text{ TeV}$$



$$pp \sqrt{s} = 2.76 \text{ TeV}$$



$$pp \ \sqrt{s} = 7 \ TeV$$



pp  $\sqrt{s} = 13 \text{ TeV}$ 



pPb  $\sqrt{s} = 5.02 \text{ TeV}$ 



Color Reduction Factor



## Thermodynamics

## Local temperature

The stress of the macroscopic clusters in SMP fluctuates around their mean value due to chrome-electric field fluctuations from the nature of the quantum vacuum in QCD. These fluctuations determine a Gaussian distribution in terms of the color reduction factor that is related to a thermal distribution. The average temperature of the system is proportional to the average moment of the produced particles, in this way a local temperature is defined, even in small systems, that is expressed as:

$$T(\xi^t) = \frac{\langle p_T \rangle}{\sqrt{2F(\xi^t)}},$$
$$\langle p_T \rangle = 190.25 \pm 11.12 \text{MeV}.$$

where

J. S. Schwinger, Phys. Rev. 128 (1962) 2425. doi:10.1103/PhysRev.128.2425; I. Bautista, A. Fernadez, P. Ghosh, Phys. Re D 92 (2015) 7; A. Bazavov et al., Phys. Rev. D 85 (2012) 054503; A. Bazavov et al., Phys. Rev. D 80 (2009) 014504

Temperature



### Energy Density

The string density is the local order parameter which determines the geometric phase transition of the system. In the case of the phase transition in QCD, the local order parameter is the energy density,  $\varepsilon$ , for which there is also a critical value, the proposal is that the relationship between both order parameters is directly proportional:

![](_page_26_Figure_2.jpeg)

### Energy Density

The string density is the local order parameter which determines the geometric phase transition of the system. In the case of the phase transition in QCD, the local order parameter is the energy density,  $\varepsilon$ , for which there is also a critical value, the proposal is that the relationship between both order parameters is directly proportional:

$$\frac{\varepsilon}{\varepsilon_c} = \frac{\xi^t}{\xi_c^t},$$

where

$$\varepsilon_c / \xi_c^t = 0.56010039 \text{GeV} / \text{fm}^3.$$

Energy Density

![](_page_28_Figure_1.jpeg)

The figure shows the behavior of the energy density with temperature compared to Lattice QCD predictions for 2+1 flavors (two light and one heavy) using 8 lattices with p4 action in blue and asqtad action in red, the curve of the model is represented by the dashed black line.

A. Bazavov et al., Phys. Rev. D 85 (2012) 054503; A. Bazavov et al., Phys. Rev. D 80 (2009) 014504

# Fluid-dynamics

The transformation:

$$x \to \lambda x, \quad \psi_j^{(f)} \to \lambda^{3/2} \psi_j^{(f)}, \quad \mathcal{A}^a_\mu \to \lambda \mathcal{A}^a_\mu,$$

is a symmetry at classical level, which at quantum level is broken, so it is called an anomaly. The Noether current associated with this symmetry is:

$$\partial_{\mu}s^{\mu} = T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s}G^a_{\mu\nu}G^{\mu\nu}_a + [1+\gamma(g_s)]m_{ij}\bar{\psi}_i\psi_j$$

therefore the non-zero trace of the tensor of energy-moment is

$$T^{\mu}_{\mu} = \varepsilon - 3P,$$

the non-zero expected value of the trace anomaly implies the existence of friction coefficients, that is, that the medium must have a certain viscosity.

#### Shear Viscosity

The behavior of the elliptic flow in nuclear collisions suggests that the matter created in these systems behaves like an almost perfect fluid, with a small contribution of viscosity, the indirect measurement of the Shear Viscosity over entropy density  $\eta_s$ /s was proposed as a measure of the fluidity of the medium.

#### Shear Viscosity

In the kinetic-relativistic theory the shear viscosity is related to the mean free path that is proportional to the inverse of the effective number of sources per unit volume

$$\eta_s/s \simeq T/5n\sigma_s$$

and the transverse area that is the inverse of  $(1 - e^{-\xi^{\tau}})/L$ , where *L* is the longitudinal extension of a string (1fm), so:

$$\frac{\eta_s}{s} = \frac{TL}{5(1 - e^{-\xi^t})}$$

I. Bautista, A. Fernadez, P. Ghosh, Phys. Re D 92 (2015) 7

Shear Viscosity

![](_page_33_Figure_1.jpeg)

P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94 (2005) 111601.

#### Trace anomaly

The trace anomaly is the expected value of the trace of the energymomentum tensor in QCD,  $T^{\mu}_{\mu} = \varepsilon - 3P$ . This observable measures the deviation with respect to the conformal behavior and identifies the residual interactions in the medium. It is expected that this observable is related to the viscosity properties of the medium, qualitatively it has been verified that the behavior of this observable is inversely proportional to the  $\eta_s$ /s ratio:

$$\Delta \equiv \frac{\varepsilon - 3P}{T^4} \simeq \frac{s}{\eta_s}.$$

Trace Anomaly

![](_page_35_Figure_1.jpeg)

#### Speed of sound

The adiabatic speed of sound is a well defined quantity for a medium in thermal equilibrium, using thermodynamic identities can be written as follows:

$$c_s^2 = \left(\frac{\partial P}{\partial \varepsilon}\right)_s = \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial \varepsilon}\right)_V = s \left(\frac{\partial T}{\partial \varepsilon}\right)_V,$$

with a fundamental relation  $Ts = \varepsilon + P$ , we can obtain in terms of the model:

$$c_s^2 = \frac{sT}{4\varepsilon} \left( 1 - \frac{e^{-\xi^t}}{F(\xi^t)^2} \right)$$

Adiabatic Speed of Sound

![](_page_37_Figure_1.jpeg)

S. Borsanyi, G. Endrodi, Z. Fodor, A.~Jakovac, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP 1011, 077 (2010)

# Bulk viscosity

The formulation of relativistic hydrodynamics is absolutely not trivial, for the calculation of bulk viscosity, the projection operator's approach was considered to derive the microscopic formulas for the transport coefficients in CDRF that can be seen as a generalization of the Navier-Stokes equation.

# Bulk viscosity

The bulk viscosity characterizes the internal properties of the fluid, the microscopic formula of the bulk viscosity over the corresponding relaxation time of causal dissipative relativistic fluid-dynamics are obtained at finite temperature and chemical potential by using the projection operator method

$$\frac{\eta_b}{\tau_{\Pi}} = \left(\frac{1}{3} - c_s^2\right)(\varepsilon + P) - \frac{2}{9}(\varepsilon - 3P).$$

- To implement the bulk viscosity formula, it is necessary that the speed of sound does not reach saturation until the value of the trace disappears.
- Then we must consider a new definition of speed of sound for dissipative media.

# Results

The increasing entropy is considered with a cylindrical expansion with one of the longitudinal dimensions fixed (L).

• By making the derivative we have

$$c_{sL}^2 = \left(\frac{\partial P}{\partial \varepsilon}\right)_L = \frac{P}{\varepsilon} + \frac{T^3 \Delta}{3s} c_s^2.$$

Speed of Sound

The upper graph shows the approximation of the behavior of the speed of sound in a dissipative medium, whose curve is shown with green dotted line, the adiabatic speed of sound is shown with the black dotted line.

The lower graph shows the comparison between the two calculated speeds, noting that equality occurs at very high temperatures.

![](_page_43_Figure_3.jpeg)

44

Bulk Viscosity

![](_page_44_Figure_1.jpeg)

Bulk-Shear viscosity ratio

![](_page_45_Figure_1.jpeg)

#### CONCLUSIONS

The final result shows the behavior of the shear viscosity over the bulk viscosity, which is well carried out causally using the modification for the speed of sound, the quotient between the two viscosities shows a change in the steep slope that suggests a second-order phase transition for these systems, the quotient is below of the theoretical curve, represented by the dotted green line, based on dual holography, where it is speculated that  $\eta_{\rm b}/\eta_{\rm s} \ge 2(1/3 - c_{\rm s}^{-2})$ .

#### CONCLUSIONS

The signals observed in small collision systems show that perhaps these systems do not reach thermalization, which implies the bulk properties for these systems.

The fact of considering equations for dissipative media allows us to obtain new physics results that are relevant, since the QGP that is formed in collisions of small systems may be different to that found in nuclear collisions.

#### **THANKS!**