ELECTROMAGNETIC COUPLINGS OF HEAVY BARYONS AND PENTAQUARKS

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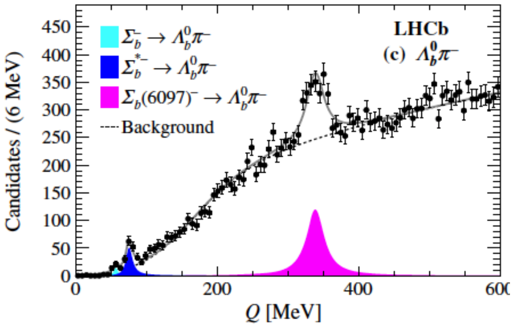
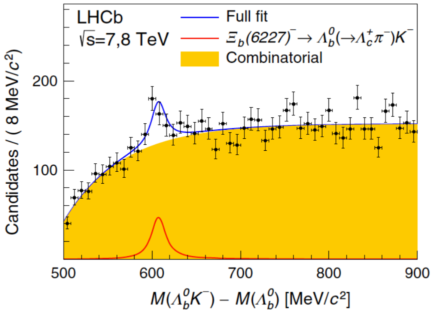
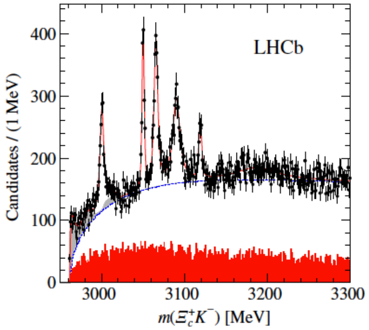
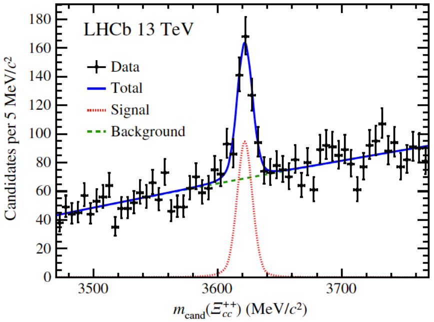
Motivation

- The LHCb collaboration recently announce the observation of two new resonances $\Sigma_b(6097)^{\pm}$ (2019) [1] , $\Xi_b(6227)^{-}$ (2018) [2], **five** new **states** Ω_c^0 (2017) [3] and in the same year the baryon with **double c** Ξ_{cc}^{++} [4].



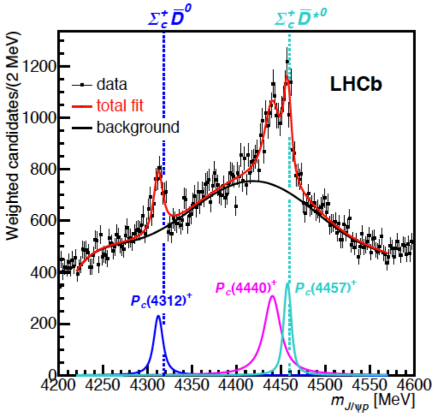






- On the other hand, since 2015 [5] LHCb has announced the observation of **pentaquark** states with quark content $uudc\bar{c}$. Now in 2019 [6] with more statistics they obtain the following signals

State	M [MeV]	Γ [MeV]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$



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[1] R. Aaij et al. (LHCb), Phys. Rev. Lett. **121**, 072002.

[2] R. Aaij et al. (LHCb), (2019),Phys. Rev Lett **122,** 012001.

[3] R. Aaij et al. (LHCb), Phys. Rev. Lett. **118**, 182001 (2017).

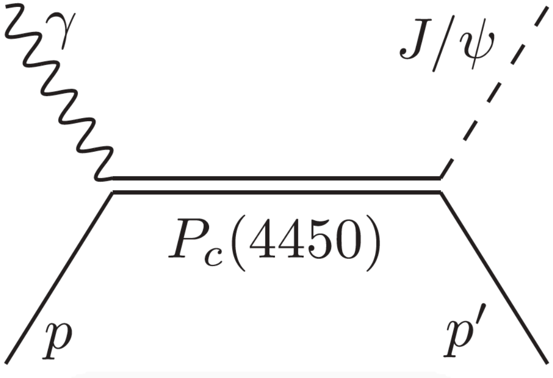
[4] R. Aaij et al. (LHCb) Phys. Rev. Lett. **119**, 112001 (2017).

[5] R. Aaij et al. (LHCb), Phys. Rev. Lett. **115, 072001** (2015).

[6] R. Aaij et al. (LHCb), Phys. Rev. Lett. **Accepted** (2019).

Photoproduction of Pentaquark at Jefferson Lab

- One of the ways to confirm the nature of the pentaquark signal is to photoproduce the state $P_c(4450)$ using an electromagnetic probe $\gamma p \to P_c \to J/\psi p$.



- This experiment has already been approved and carried out by the Jefferson Lab Hall C in February of this year [7].

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[7] Mezziani ZE et al (Hall C collaboration) 2016 A Search for the LHCb Charmed "Pentaquark" using Photoproduction of J/ψ at Threshold in Hall C at Jefferson Lab, Preprint 1609.00676 [hep-ex]

Electromagnetic Couplings

The interaction Hamiltonian for electromagnetic couplings is [8]

$$H = e \int d^3x J^{\mu}_{em}(\vec{x}) A_{\mu}(\vec{x})$$

Helicity Amplitudes $B \rightarrow B' + \gamma$

Since the final state is the base state $J^P = 1/2^+$ the possible values of the helicity are $\nu = 1/2,\ 3/2$. The helicy amplitudes are

$$\mathcal{A}_\nu = \langle \psi_{B'}; 1/2, \nu - 1 | H | \psi_B; J, \nu \rangle,$$

Radiative decay width [9]

$$\Gamma(B \rightarrow B' + \gamma) = 2\pi\rho \frac{1}{(2\pi)^3} \frac{2}{2J+1} \sum_{\nu>0} |\mathcal{A}_\nu(k)|^2 \ ,$$

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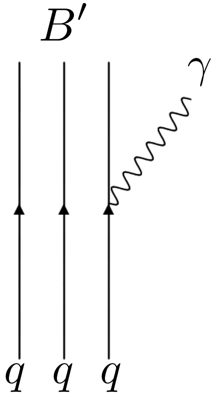
[8] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Hadron Transitions in the Quark Model (Gordon and Breach, NY, USA, 1988).

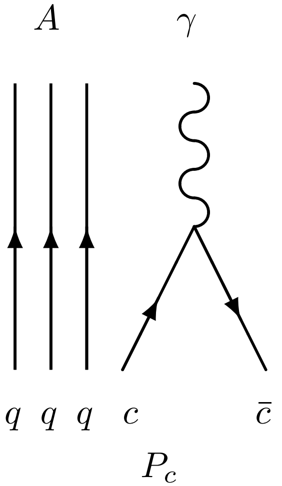
[9] R. Bijker, F. Iachello, and A. Leviatan, Annals of Physics **284**, 89 (2000) .

Processes of interest

$B \to B' + \gamma$



$P_c \to A + \gamma$



Non-relativistic Limit:

$$\bar{u}_{s'}(\vec{p'})\gamma^0 u_s(\vec{p}) \quad \to \quad \chi^\dagger_{s'}\chi_s$$

$$\bar{u}_{s'}(\vec{p'})\gamma^k u_s(\vec{p}) \quad \to \quad \frac{1}{2m}\chi^\dagger_{s'}[(\vec{p}+\vec{p'}) + i\vec{\sigma}\times(\vec{p'}-\vec{p})]\chi_s$$

$$\bar{v}_{s'}(\vec{p'})\gamma^0 u_s(\vec{p}) \quad \to \quad \chi^\dagger_{s'}\frac{\vec{\sigma}\cdot(\vec{p}+\vec{p'})}{2m}\chi_s$$

$$\bar{v}_{s'}(\vec{p'})\gamma^k u_s(\vec{p}) \quad \to \quad \chi^\dagger_{s'}\sigma^k\chi_s$$

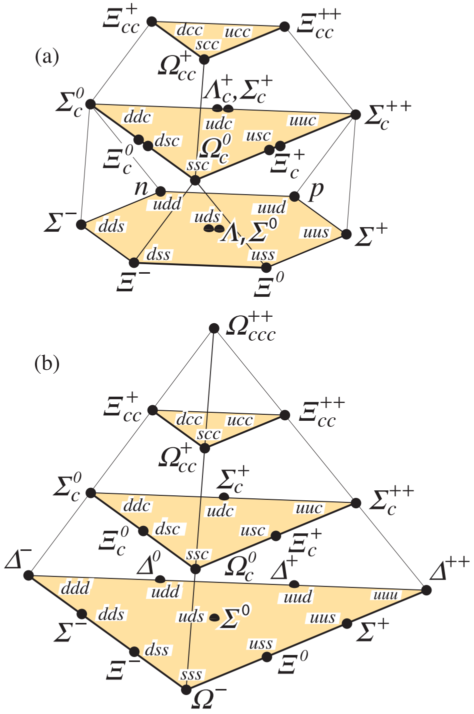
BARYONS: $\mathbf{qqQ}$ y $\mathbf{QQq}$

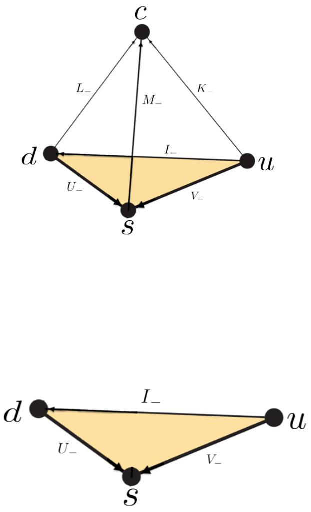
Quark Model

i. The color part of the full-wave function is a **color singlet** $SU(3)$, a completely three-color **antisymmetric** state.

ii. Because quarks are fermions, the total wave function must be completely antisymmetric under any permutation of the three quarks.

$$\psi = \underbrace{\psi^o \phi^f \chi^s}_{Symm} \quad \underbrace{\psi^c}_{Antisymm}$$





Spin WF

The spin wave functions with their maximum projection are given by

$$\chi_\lambda^{1/2} = \frac{1}{\sqrt{6}}(2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle)$$

$$\chi_\rho^{1/2} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle$$

$$\chi_S^{3/2} = |\uparrow\uparrow\uparrow\rangle$$

Orbital WF of Excited States

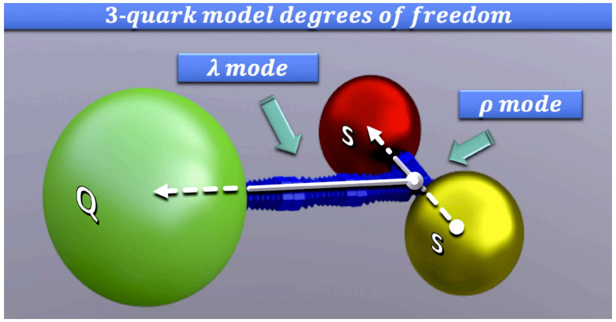
It is obtained by considering the eigenfunctions of the Hamiltonian of the quarks as an harmonic oscillator model. This Hamiltonian is responsible for the confinement of three quarks 1, 2 and 3 with masses $m_1 = m_2 \neq m_3$ [10]

$$H = \frac{{p_1}^2}{2m} + \frac{{p_2}^2}{2m} + \frac{{p_3}^2}{2m'} + \frac{1}{2}C\sum_{i<j}^{3}|\vec{r_i} - \vec{r_j}|^2$$

To obtain the analytical solutions, the relative coordinates are used,

$$\begin{cases} \vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}), \\ \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}), \\ \vec{R} = \frac{m(\vec{r_1}+\vec{r_2})+m'\vec{r_3}}{2m+m'} \end{cases} \rightarrow \begin{cases} \vec{r}_1 = \vec{R} + \frac{1}{\sqrt{2}}\vec{\rho} + \frac{\sqrt{\frac{3}{2}}m'}{2m+m'}\vec{\lambda} \\ \vec{r}_2 = \vec{R} - \frac{1}{\sqrt{2}}\vec{\rho} + \frac{\sqrt{\frac{3}{2}}m'}{2m+m'}\vec{\lambda} \\ \vec{r}_3 = \vec{R} - \frac{\sqrt{6}m}{2m+m'}\vec{\lambda} \end{cases}$$

$$H = \frac{{P_{CM}}^2}{2M} + \frac{{p_\rho}^2}{2m_\rho} + \frac{{p_\lambda}^2}{2m_\lambda} + \frac{3}{2}C\rho^2 + \frac{3}{2}C\lambda^2$$



The Hamiltonian now includes the movement of the center of mass, plus two harmonic oscillators ρ and λ, with the same constant C, but different masses

$$M = 2m + m', \qquad m_\rho \equiv m, \qquad m_\lambda \equiv \frac{3mm'}{2m + m'}.$$

with its harmonic oscillator constant and frequency

$$\alpha_i^2 = (3Cm_i)^{\frac{1}{2}}, \qquad \omega_i = \sqrt{\frac{3C}{m_i}}, \qquad i = \{\rho, \lambda, \}.$$

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[10] A. Hosaka, M. Takayama, and H. Toki, Nucl. Phys. A678, 147 (2000)

The wave function of the baryons of **ground state** are given by

$$\psi_B^o(\vec{r}_1,\vec{r}_2,\vec{r}_3) = \frac{1}{(2\pi)^{3/2}} e^{\vec{P}_{CM}\cdot\vec{R}} \psi_B^{rel}(\vec{\rho},\vec{\lambda})$$

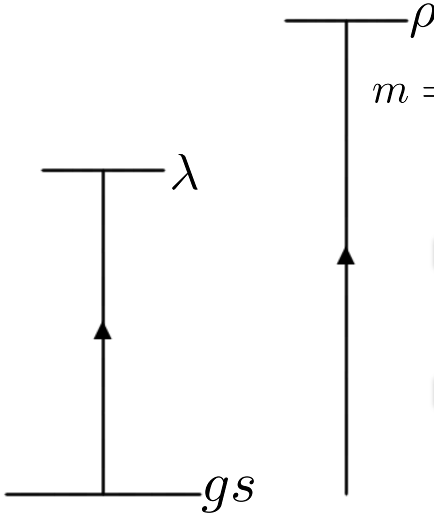
$$\psi_{gs}^{rel}(\vec{\rho},\vec{\lambda}) = \frac{1}{\sqrt{3\sqrt{3}}} \psi_{0,0,0}(\vec{\rho})\psi_{0,0,0}(\vec{\lambda})$$

The wave functions with one **quantum of excitation** in ρ and λ are

$$\psi_{\rho}^{rel}(\vec{\rho},\vec{\lambda}) = \frac{1}{\sqrt{3\sqrt{3}}} \psi_{0,0,0}(\vec{\rho})\psi_{1,1,m_\rho}(\vec{\lambda})$$

$$\psi_{\lambda}^{rel}(\vec{\rho},\vec{\lambda}) = \frac{1}{\sqrt{3\sqrt{3}}} \psi_{0,0,0}(\vec{\rho})\psi_{1,1,m_\lambda}(\vec{\lambda})$$

$a)$ $\boxed{qqQ}$

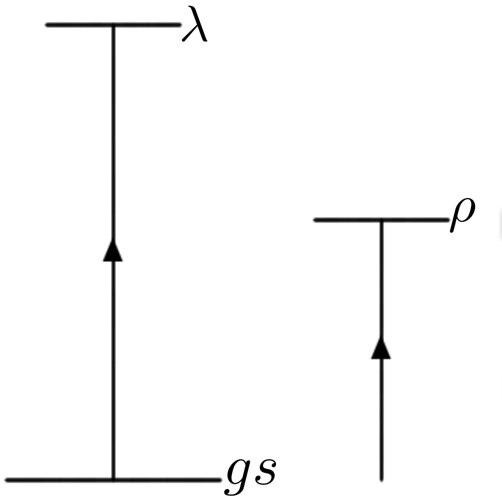


$m = \quad m_1 = m_2 < m_3 \quad = m'$

→ $m_\lambda > m_\rho$

→ $\omega_\lambda < \omega_\rho$

$b)$ $\boxed{QQq}$



$m_1 = m_2 > m_3$

→ $m_\lambda < m_\rho$

→ $\omega_\lambda > \omega_\rho$

Total Wave Functions

The wave functions for the case of different masses can be obtained once the quark b is placed in the third position (as a convention) and a symmetry is made with respect to the remaining two with equal masses in the flavor part.

$$\Lambda \quad : \quad \phi_0 = \frac{1}{\sqrt{2}}(ud - du)b$$

$$\Sigma \quad : \quad \phi_1 = \frac{1}{\sqrt{2}}(ud + du)b$$

Considering only the resonances Λ_b^0 and Σ_b^+

$$\nu = 1/2$$

Λ	ψ_Λ^{osf}	J
${}^4\rho$	$\phi_0\chi_S\psi_\rho$	$\frac{1}{2},\frac{3}{2},\frac{5}{2}$
${}^2\lambda$	$\phi_0\chi_\rho\psi_\lambda$	$\frac{1}{2},\frac{3}{2}$
${}^2\rho$	$\phi_0\chi_\lambda\psi_\rho$	$\frac{1}{2},\frac{3}{2}$
Σ	ψ_Σ^{osf}	J
${}^4\lambda$	$\phi_1\chi_S\psi_\lambda$	$\frac{1}{2},\frac{3}{2},\frac{5}{2}$
${}^2\lambda$	$\phi_1\chi_\lambda\psi_\lambda$	$\frac{1}{2},\frac{3}{2}$
${}^2\rho$	$\phi_1\chi_\rho\psi_\rho$	$\frac{1}{2},\frac{3}{2}$

Ground States →

$$\Lambda_b \quad : \quad \psi_0\frac{1}{\sqrt{2}}(ud - du)b\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow$$

Excited States

$$\left[\begin{array}{lcl} {}^2\lambda(\Lambda_b^*) & : & \psi_\lambda\frac{1}{\sqrt{2}}(ud - du)b\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \\ {}^2\rho(\Lambda_b^*) & : & \psi_\rho\frac{1}{\sqrt{2}}(ud - du)b\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \\ {}^4\rho(\Lambda_b^*) & : & \psi_\rho\frac{1}{\sqrt{2}}(ud - du)b\uparrow\uparrow\uparrow \end{array}\right.$$

Ground States →

$$\Sigma_b^+ : \psi_0 uub\frac{1}{\sqrt{6}}(-2\downarrow\downarrow\uparrow + \uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow)$$

Excited States

$$\left[\begin{array}{lcl} {}^2\rho(\Sigma_b^{*+}) & : & \psi_\rho uub\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow \\ {}^2\lambda(\Sigma_b^{*+}) & : & \psi_\lambda uub\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \\ {}^4\lambda(\Sigma_b^{*+}) & : & \psi_\lambda uub\uparrow\uparrow\uparrow \end{array}\right.$$

Results for Heavy Baryons

Σ_b^{*+}	J	$\mathcal{A}_{\frac{1}{2}}/2\mu\sqrt{\frac{\pi}{k_0}}$	$\mathcal{A}_{\frac{3}{2}}/2\mu\sqrt{\frac{\pi}{k_0}}$
$^2\lambda$	$\frac{1}{2}$	$\frac{k}{3\sqrt{3}}\left(e_b U_{\lambda,3} - 4e_u U_{\lambda,1}\right) - \frac{1}{g}\sqrt{\frac{2}{3}}\left(2e_u T_{\lambda,1} + e_b \lambda, 3\right)$	0
	$\frac{3}{2}$	$\frac{k}{3}\sqrt{\frac{2}{3}}\left(4e_u U_{\lambda,1} - e_b U_{\lambda,3}\right) - \frac{1}{g}\frac{1}{\sqrt{3}}\left(2e_u T_{\lambda,1} + e_b T_{\lambda,3}\right)$	$-\frac{1}{g}\left(2e_u T_{\lambda,1} + e_b T_{\lambda,3}\right)$
$^2\rho$	$\frac{1}{2}$	$\frac{2}{3}k e_u U_{\rho,1}$	0
	$\frac{3}{2}$	$-\frac{2\sqrt{2}}{3}k e_u U_{\rho,1}$	0
$^4\lambda$	$\frac{1}{2}$	$\frac{k}{3}\sqrt{\frac{2}{3}}\left(e_u U_{\lambda,1} - e_b U_{\lambda,3}\right)$	0
	$\frac{3}{2}$	$\frac{k}{3}\sqrt{\frac{2}{15}}\left(e_u U_{\lambda,1} - e_b U_{\lambda,3}\right)$	$\frac{2k}{\sqrt{10}}\left(e_u U_{\lambda,1} - e_b U_{\lambda,3}\right)$
	$\frac{5}{2}$	$k\sqrt{\frac{2}{15}}\left(-e_u U_{\lambda,1} + e_b U_{\lambda,3}\right)$	$\frac{2k}{\sqrt{15}}\left(-e_u U_{\lambda,1} + e_b U_{\lambda,3}\right)$

Λ_b^{*}	J	$\mathcal{A}_{\frac{1}{2}}/2\mu\sqrt{\frac{\pi}{k_0}}$	$\mathcal{A}_{\frac{3}{2}}/2\mu\sqrt{\frac{\pi}{k_0}}$
$^2\lambda$	$\frac{1}{2}$	$-\frac{k}{\sqrt{3}}e_b U_{\lambda,3} - \frac{1}{g}\sqrt{\frac{2}{3}}\left((e_u + e_d)T_{\lambda,1} + e_b T_{\lambda,3}\right)$	0
	$\frac{3}{2}$	$k\sqrt{\frac{2}{3}}e_b U_{\lambda,3} - \frac{1}{g}\frac{1}{\sqrt{3}}\left((e_u + e_d)T_{\lambda,1} + e_b T_{\lambda,3}\right)$	$-\frac{1}{g}\left((e_u + e_d)T_{\lambda,1} + e_b T_{\lambda,3}\right)$
$^2\rho$	$\frac{1}{2}$	$\frac{k}{3}(e_u + e_d)U_{\rho,1}$	0
	$\frac{3}{2}$	$-\frac{\sqrt{2}k}{3}(e_u + e_d)U_{\rho,1}$	0
$^4\rho$	$\frac{1}{2}$	$\frac{k}{\sqrt{3}}\frac{1}{\sqrt{6}}(e_u + e_d)U_{\rho,1}$	0
	$\frac{3}{2}$	$\frac{k}{3\sqrt{10}}(e_u + e_d)U_{\rho,1}$	$\sqrt{\frac{3}{10}}k(e_u + e_d)U_{\rho,1}$
	$\frac{5}{2}$	$-k\frac{1}{\sqrt{10}}(e_u + e_d)U_{\rho,1}$	$-\frac{1}{\sqrt{5}}k(e_u + e_d)U_{\rho,1}$

PENTAQUARKS: $qqqq\bar{q}$

i. *As all physical states, the total wave function should be a* ***color singlet****.*

ii. *The pentaquarks wave function should be antisymmetric under any permutation of the* ***four quarks****.*

The irreducible representation of S_4 , and the isomorphism with the tetrahedral group $\mathcal{T}_d$

S_4		$\mathcal{T}_d$	
$[4]$	$\sim$	A_1	singlet
$[31]$	$\sim$	F_2	triplet
$[22]$	$\sim$	E	doublet
$[211]$	$\sim$	F_1	triplet
$[1111]$	$\sim$	A_2	singlet

$$\psi = \left[\psi^{\mathrm{c}}_{F_1} \times \psi^{\mathrm{osf}}_{F_2}\right]_{A_2}$$ [11]

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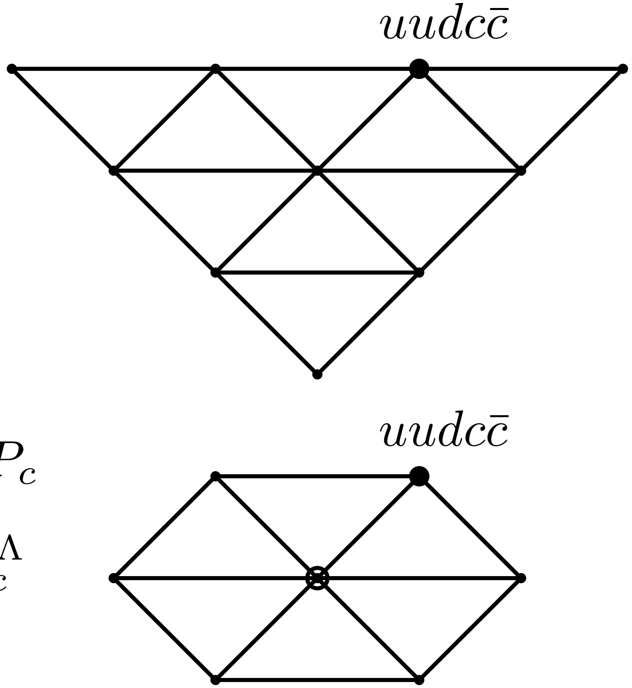
[11] F. Stancu, Oxford Stud. Nucl. Phys. **19** , 1 (1996).

Classification of Pentaquark



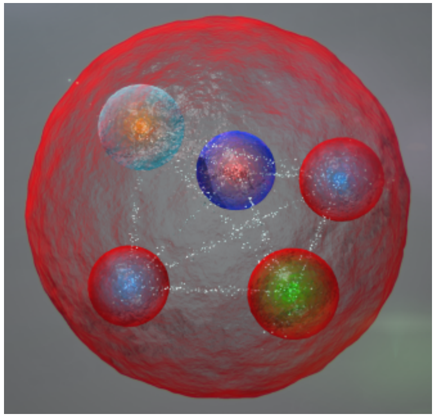
$uudc\bar{c}$

P_c^{Δ}	$nnnc\bar{c}$
$P_c^{\Sigma^*}$	$nnsc\bar{c}$
$P_c^{\Xi^*}$	$nssc\bar{c}$
P_c^{Ω}	$sssc\bar{c}$

$uudc\bar{c}$

$P_c^N \equiv P_c$	$nnnc\bar{c}$
$P_c^{\Sigma}/P_c^{\Lambda}$	$nnsc\bar{c}$
P_c^{Ξ}	$nssc\bar{c}$

Orbital WF for ground states Pentaquarks



The Hamiltonian to describe the confinement of 5 quarks is

$$H = \frac{{p_1}^2}{2m} + \frac{{p_2}^2}{2m} + \frac{{p_3}^2}{2m} + \frac{{p_4}^2}{2m} + \frac{{p_5}^2}{2m} + \frac{1}{2}C\sum_{i<j}^{5}|\vec{r_i} - \vec{r_j}|^2$$

Using relative coordinates,

$$\begin{aligned}
\vec{\rho} &\equiv \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}), \\
\vec{\lambda} &\equiv \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}), \\
\vec{\eta} &\equiv \frac{1}{\sqrt{12}}(\vec{r_1} + \vec{r_2} + \vec{r_3} - 3\vec{r_4}), \\
\vec{\zeta} &\equiv \frac{1}{\sqrt{20}}(\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} - 4\vec{r_5}), \\
\vec{R} &\equiv \frac{1}{5}(\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} + \vec{r_5})
\end{aligned}$$

$$H = \frac{{P_{CM}}^2}{2M} + \frac{{p_\rho}^2}{2m_\rho} + \frac{{p_\lambda}^2}{2m_\lambda} + \frac{{p_\eta}^2}{2m_\eta} + \frac{{p_\zeta}^2}{2m_\zeta} + \frac{5}{2}C\rho^2 + \frac{5}{2}C\lambda^2 + \frac{5}{2}C\eta^2 + \frac{5}{2}C\zeta^2$$

$$\alpha_i^2 = (5Cm_i)^{\frac{1}{2}}, \qquad \omega_i = \sqrt{\frac{5C}{m_i}}, \qquad i = \{\rho, \lambda, \eta, \zeta\}$$

The Hamiltonian now includes the movement of the center of mass, plus four harmonic oscillators ρ, λ, η and ζ, with the same constant C, and equal masses.

$$m_\rho = m_\lambda = m_\eta = m_\zeta = m \qquad M = 5m$$

$$\psi^{o}_{Pc}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5) = \frac{1}{(2\pi)^{3/2}} e^{\vec{P}_{CM}\cdot\vec{R}} \psi^{rel}_{Pc}(\vec{\rho}, \vec{\lambda}, \vec{\eta}, \vec{\zeta})$$

Results for Pentaquarks $J^P = 3/2^-$

Helicity amplitudes for electromagnetic decays of pentaquarks [12] $P_c(uudc\bar{c}) \to p + \gamma$ in terms of $e_c = 2/3$ and the initial momentum of photon k . The results are expresed in the rest frame of the pentaquark where the helicity amplitud is

$$\mathcal{A}_\nu = \sqrt{\frac{5\sqrt{5}}{3\sqrt{3}}}\sqrt{\frac{4\pi\alpha}{k}}e^{-\frac{5}{12\alpha^2}|\vec{k}|^2}\alpha_\nu$$

$$\boxed{k = 2.1GeV}$$

Estado	Nombre	$\alpha_{1/2}$	$\alpha_{3/2}$
$[\phi_{A_1} \times \chi_{F_2}]_{F_2}$	P_c^Δ	0	0
$[\phi_{F_2} \times \chi_{A_1}]_{F_2}$	P_c^Δ	0	0
$[\phi_{F_2} \times \chi_{F_2}]_{F_2}$	P_c^Δ	0	0
$[\phi_{F_2} \times \chi_{A_1}]_{F_2}$	P_c^N	0	0
$[\phi_{F_2} \times \chi_{F_2}]_{F_2}$	P_c^N	$\frac{1}{6\sqrt{2}}e_c$	$\frac{1}{2\sqrt{6}}e_c$
$[\phi_E \times \chi_{F_2}]_{F_2}$	P_c^N	$-\frac{1}{6}e_c$	$-\frac{1}{2\sqrt{3}}e_c$
$[\phi_{F_1} \times \chi_{F_2}]_{F_2}$	P_c^N	$-\frac{1}{2\sqrt{6}}e_c$	$-\frac{1}{2\sqrt{2}}e_c$

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[12] E. Ortiz-Pacheco, R. Bijker, and C. Fernández-Ramírez, J. Phys. G. **46**, 065104 (2019)

Orbital Suppression

$$\mathcal{A}_\nu = \sqrt{\frac{5\sqrt{5}}{3\sqrt{3}}}\sqrt{\frac{4\pi\alpha}{k}}\,e^{-\frac{5}{12\alpha^2}|\vec{k}|^2}\alpha_\nu$$

State	Mass (GeV)	$k = \frac{m_{B'}^2 - m_B^2}{2m_{B'}} (GeV)$
Σ_c^*	2.520	**0.060**
Σ_c	2.455	
Ξ_c	2.645	**0.068**
Ξ_c	2.576	
Ω_c^*	2.766	**0.070**
Ω_c	2.695	

$P_c(uudc\bar{c}) \to p + \gamma$ $\qquad$ $k = 2.1GeV$

Conclusions

HEAVY BARYONS

- For baryon configurations qqQ consistent with the $\Xi_b, \Sigma_b, \Omega_c$ it was obtained that because the frequencies of the modes are such that $\omega_\rho >> \omega_\lambda$. The relevant states for electromagnetic baryon couplings are those with one quantum of excitation in λ .
- In contrast, for configuration QQq, the analysis for baryons as Ξ_{cc}, implies frequencies such that $\omega_\lambda > \omega_\rho$, so in this case the most important contribution is in the ρ mode. The total analysis of electromagnetic coplings is work in progress.

PENTAQUARKS

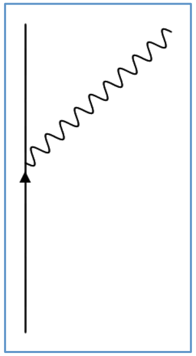
- Due to the symmetry properties of the pentaquark wave function, several configurations can not be excited in photoproduction.
- We find that only a very small number of all possible pentaquark states can be excited in photoproduction experiments $P_c(uudc\bar{c}) \rightarrow p + \gamma$.
- Finally, because the moment associated with the photon is in a very high energy range, there is a very large suppression factor to the photo coupling due to the orbital part.

Thanks for your attention

Electromagnetic Couplings

The Hamiltonian responsible for electromagnetic couplings is given by [5]

$$H = e \int d^3x J^{\mu}_{em}(\vec{x}) A_{\mu}(\vec{x})$$



Helicity Amplitudes $B \to B' + \gamma$

Since the final state is the base state $J^P = 1/2^+$ the possible values of the helicity are $\nu = 1/2,\ 3/2$. The helicy amplitudes are

$$\mathcal{A}_{\nu} = \langle \psi_{B'}; 1/2, \nu - 1 | H | \psi_B; J, \nu \rangle, \qquad \nu = 1/2,\ 3/2.$$

For each J, the only values allowed are

$$\begin{array}{lcll} J & = & 1/2; & \nu = 1/2 \\ J & \neq & 1/2; & \nu = 1/2, 3/2 \end{array}$$

Radiative decay width [6]

$$k = \frac{m^2_{B'} - m^2_B}{2m'_B}$$

$$\Gamma(B \to B' + \gamma) = 2\pi\rho \frac{1}{(2\pi)^3} \frac{2}{2J+1} \sum_{\nu>0} |\mathcal{A}_{\nu}(k)|^2 \ ,$$

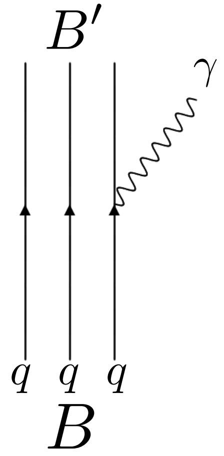
$$\rho = 4\pi \frac{E_B k^2}{m'_B}$$

---

[5] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Hadron Transitions in the Quark Model (Gordon and Breach, NY, USA, 1988).

[6] R. Bijker, F. Iachello, and A. Leviatan, Annals of Physics **284**, 89 (2000) .

Calculating the helicity amplitudes



Non-relativistic Hamiltonian

$$H = 2\sqrt{\frac{\pi}{k_0}}\mu\sum_{j=1}^{3} e_j\left[\underbrace{k s_{j,-} e^{-i\vec{k}\cdot\vec{r}_j}}_{\text{spin-flip}} + \frac{1}{2g}\underbrace{\left(p_{j,-} e^{-i\vec{k}\cdot\vec{r}_j} + e^{-i\vec{k}\cdot\vec{r}_j} p_{j,-}\right)}_{\text{orbit-flip}}\right]$$

Radial contribution of the j-th component:

$$U_{\alpha,j} = \langle\psi_{gs}|e^{-i\vec{k}\cdot\vec{r}_j}|\psi_\alpha\rangle \qquad T_{\alpha,j} = \langle\psi_{gs}|p_{j,-}e^{-i\vec{k}\cdot\vec{r}_j}|\psi_\alpha\rangle$$

$$\begin{aligned}
U_{\lambda,3} &= i\frac{\sqrt{3}m}{2m+m'}\frac{k}{\alpha_\lambda}e^{-\frac{3m^2k^2}{2\alpha_\lambda^2(2m+m')^2}} \\
U_{\lambda,1} &= U_{\lambda,2} = -i\frac{1}{2\alpha_\lambda}\frac{\sqrt{3}m'k}{2m+m'}e^{-\frac{k^2}{8\alpha_\rho^2}}e^{-\frac{3m'^2k^2}{8\alpha_\lambda^2(2m+m')^2}} \\
U_{\rho,1} &= -U_{\rho,2} = -i\frac{k}{2\alpha_\rho}e^{-\frac{k^2}{8\alpha_\rho^2}}e^{-\frac{3}{8}\left(\frac{m'k}{\alpha_\lambda(2m+m')}\right)^2} \\
U_{\rho,3} &= 0
\end{aligned}$$

$$\begin{aligned}
T_{\lambda,3} &= -im'k_0\frac{\sqrt{6}m}{2m+m'}\frac{1}{\alpha_\lambda}e^{-\frac{3}{2}\left(\frac{mk}{\alpha_\lambda(2m+m')}\right)^2} \\
T_{\lambda,1} &= T_{\lambda,2} = imk_0\frac{\sqrt{6}}{2}\frac{1}{\alpha_\lambda}\frac{m'}{2m+m'}e^{-\frac{k^2}{8\alpha_\rho^2}}e^{-\frac{3}{8}\left(\frac{m'k}{\alpha_\lambda(2m+m')}\right)^2} \\
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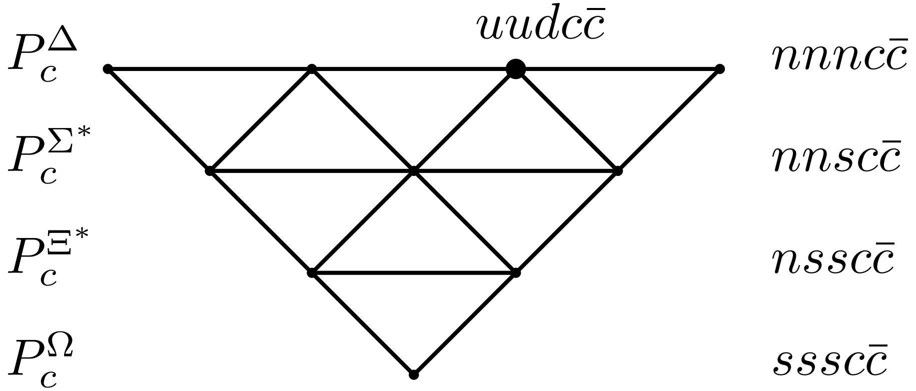
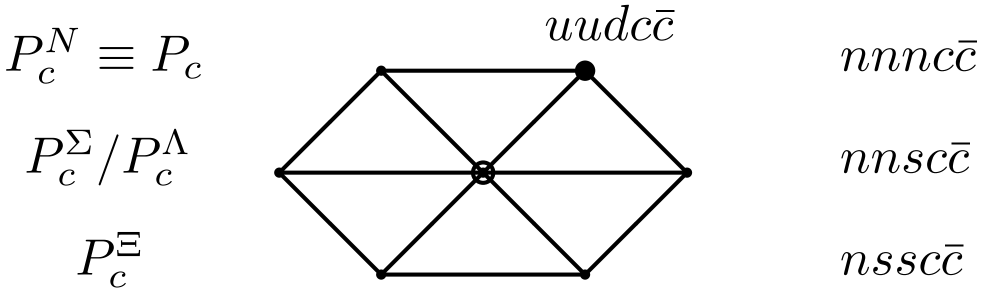
T_{\rho,3} &= 0
\end{aligned}$$

Contribution of **spin-flavor** to the probability amplitude, example: $\nu = 1/2$

$$\begin{aligned}
\langle\Sigma_b^+; 1/2, \nu - 1/2|e_1 s_{1,-}|{}^2\rho(\Sigma_b^{*+}); J, \nu\rangle &= \langle uub\frac{1}{\sqrt{6}}(\downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow - 2\downarrow\downarrow\uparrow)|e_1 s_{1,-}|uub\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow\rangle \\
&= -\frac{1}{\sqrt{3}}e_u
\end{aligned}$$





Spin wave function

$$
\begin{aligned}
|\chi_{A_1}\rangle &= |\frac{1}{2\sqrt{5}}(4\uparrow\uparrow\uparrow\uparrow\downarrow -(\uparrow\uparrow\uparrow\downarrow + \uparrow\uparrow\downarrow\uparrow + \uparrow\downarrow\uparrow\uparrow + \downarrow\uparrow\uparrow\uparrow)\uparrow)\rangle \\
|\chi_{F_{2\rho}}\rangle &= |\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow)\uparrow)\rangle \\
|\chi_{F_{2\lambda}}\rangle &= |\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow)\uparrow)\rangle \\
|\chi_{F_{2\eta}}\rangle &= |\frac{1}{2\sqrt{3}}(3\uparrow\uparrow\uparrow\downarrow - \uparrow\uparrow\downarrow\uparrow - \uparrow\downarrow\uparrow\uparrow - \downarrow\uparrow\uparrow\uparrow)\uparrow)\rangle
\end{aligned}
$$

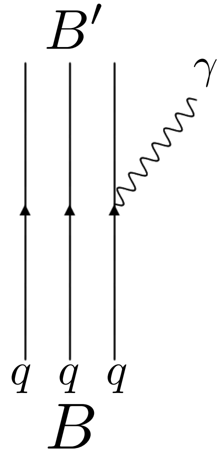
Función de onda de color

$$\begin{aligned}\psi^c_{F_{1\rho}}(q^4\bar{q}) &= \frac{1}{12}\left[|((2gbr-2bgr-rgb+grb+rbg-brg)r+3(rgr-grr)b-3(rbr-brr)g)\bar{r}\rangle\right.\\ &\quad +|((2brg-2rbg-gbr+bgr+grb-rgb)g+3(gbg-bgg)r-3(grg-rgg)b)\bar{g}\rangle\\ &\quad \left.+|((2rgb-2grb-brg+rbg+bgr-gbr)b+3(brb-rbb)g-3(bgb-gbb)r)\bar{b}\rangle\right]\end{aligned}$$

$$\begin{aligned}\psi^c_{F_{1\lambda}}(q^4\bar{q}) &= \frac{1}{4\sqrt{3}}\left[|((2rrg-rgr-grr)b-(2rrb-rbr-brr)g+(rgb+grb-rbg-brg)r)\bar{r}\rangle\right.\\ &\quad +|((2ggb-gbg-bgg)r-(2ggr-grg-rgg)b+(gbr+bgr-grb-rgb)g)\bar{g}\rangle\\ &\quad \left.+|((2bbr-brb-rbb)g-(2bbg-bgb-gbb)r+(brg+rbg-bgr-gbr)b)\bar{b}\rangle\right]\end{aligned}$$

$$\begin{aligned}\psi^c_{F_{1\eta}}(q^4\bar{q}) &= \frac{1}{3\sqrt{2}}\left[|((rgb-grb+brg-rbg+gbr-bgr)r)\bar{r}\rangle\right.\\ &\quad +|((rgb-grb+brg-rbg+gbr-bgr)g)\bar{g}\rangle\\ &\quad \left.+|((rgb-grb+brg-rbg+gbr-bgr)b)\bar{b}\rangle\right]\end{aligned}$$

Calculating the helicity amplitudes



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Contribution of **spin-flavor** to the probability amplitude, example: $\nu = 1/2$

$$\begin{aligned}\langle\Sigma_b^+; 1/2, \nu - 1/2|e_1 s_{1,-}|^2\rho(\Sigma_b^{*+}); J, \nu\rangle &= \langle uub\frac{1}{\sqrt{6}}(\downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow - 2\downarrow\downarrow\uparrow)|e_1 s_{1,-}|uub\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\uparrow\rangle \\ &= -\frac{1}{\sqrt{3}}e_u\end{aligned}$$

Funciones de onda de sabor

Estas funciones de onda de las funciones de onda de espín, simplemente reemplazando $\uparrow\rightarrow\ u$y $\downarrow\rightarrow\ d$ ya que al igual que el espín, los quarks u y d pertenecen a un doblete y Acoplando el quinto estado de sabor $|\phi_t(uudc\bar{c})\rangle = |\phi_t(uudc)\rangle\,|\phi(\bar{c})\rangle$

$$
\begin{aligned}
|\phi_{F_{2\rho}}\rangle &= \left|-\tfrac{1}{4\sqrt{3}|}\,(3(udcu+uduc-ducu-duuc)+2(cduu-dcuu)+ucdu+ucud-cudu-cuud)\,\bar{c}\right\rangle \\
|\phi_{F_{2\lambda}}\rangle &= \left|-\tfrac{1}{12}\,(6uudc+2uucd+5(cudu+ucdu)-4(dcuu+cduu)-3(duuc+uduc)-ducu-udcu-cuud-ucud)\,\bar{c}\right\rangle \\
|\phi_{F_{2\eta}}\rangle &= \left|-\tfrac{1}{3\sqrt{2}}\,(2(cuud+ucud+uucd)-dcuu-ducu-cduu-udcu-cudu-ucdu)\,\bar{c}\right\rangle \\
|\phi_{E_{\rho}}\rangle &= \left|-\tfrac{1}{2\sqrt{2}}\,(ucud+uduc-cuud-duuc-ucdu-udcu+cudu+ducu)\,\bar{c}\right\rangle \\
|\phi_{E_{\lambda}}\rangle &= \left|-\tfrac{1}{2\sqrt{6}}\,(2uucd+2uudc-ucud-uduc-cuud-duuc+2cduu+2dcuu-ucdu-udcu-cudu-ducu)\,\bar{c}\right\rangle \\
|\phi_{F_{1\rho}}\rangle &= \left|-\tfrac{1}{4\sqrt{3}}\,(2dcuu-2cduu-udcu+ducu+ucdu-cudu+3uduc-3duuc-3ucud+3cuud)\,\bar{c}\right\rangle \\
|\phi_{F_{1\lambda}}\rangle &= \left|-\tfrac{1}{4}\,(2uudc-uduc-duuc-2uucd+ucud+cuud+udcu+ducu-ucdu-cudu)\,\bar{c}\right\rangle \\
|\phi_{F_{1\eta}}\rangle &= \left|-\tfrac{1}{\sqrt{6}}\,(dcuu-cduu-ducu+udcu+cudu-ucdu)\,\bar{c}\right\rangle
\end{aligned}
$$

Funcione de onda de color

$$
\begin{aligned}
\psi^{c}_{F_{1\rho}}(q^4\bar{q}) \quad = \quad \frac{1}{12}\,[&|((2gbr-2bgr-rgb+grb+rbg-brg)r+3(rgr-grr)b-3(rbr-brr)g)\bar{r}\rangle \\
&+|((2brg-2rbg-gbr+bgr+grb-rgb)g+3(gbg-bgg)r-3(grg-rgg)b)\bar{g}\rangle \\
&+|((2rgb-2grb-brg+rbg+bgr-gbr)b+3(brb-rbb)g-3(bgb-gbb)r)\bar{b}\rangle]
\end{aligned}
$$

$$\mathbf{qqqQ\bar{Q}}$$

PENTAQUARKS: $\mathrm{qqqQ\bar{Q}}$

*i. Todos los estados físicos deben ser un **singlete de color.***

*ii. La **función de onda total** debe ser completamente **antisimétrica** bajo cualquier permutación de los **cuatro quarks**.*

Representación irreducible de S_4 , e isomorfismo con el grupo tetraedral $\mathcal{T}_d$

S_4		$\mathcal{T}_d$	
$[4]$	$\sim$	A_1	singlet
$[31]$	$\sim$	F_2	triplet
$[22]$	$\sim$	E	doublet
$[211]$	$\sim$	F_1	triplet
$[1111]$	$\sim$	A_2	singlet

$SU_{\mathrm{sf}}(8)$	$\supset$	$SU_{\mathrm{f}}(4)$	$\otimes$	$SU_{\mathrm{s}}(2)$	
$[f]$	$\supset$	$[g]$	$\otimes$	$[g']$	$\psi^{\mathrm{sf}}_{F_2}$
$[31]_{630}$		$[4]_{35}$	$\otimes$	$[31]_3$	$[\phi_{A_1} \times \chi_{F_2}]_{F_2}$
		$[31]_{45}$	$\otimes$	$[4]_5$	$[\phi_{F_2} \times \chi_{A_1}]_{F_2}$
				$[31]_3$	$[\phi_{F_2} \times \chi_{F_2}]_{F_2}$
				$[22]_1$	
		$[22]_{20}$	$\otimes$	$[31]_3$	$[\phi_{E} \times \chi_{F_2}]_{F_2}$
		$[211]_{15}$	$\otimes$	$[31]_3$	$[\phi_{F_1} \times \chi_{F_2}]_{F_2}$
				$[22]_1$	

$$\psi \quad = \quad \left[\psi^{\mathrm{c}}_{F_1} \times \psi^{\mathrm{osf}}_{F_2}\right]_{A_2}$$ [11]

$$\psi = \psi^{\mathrm{o}}_{A_1}\psi^{\mathrm{csf}}_{A_2} = \frac{1}{\sqrt{3}}\left[\psi^{\mathrm{o}}_{A_1}\left(\psi^{\mathrm{c}}_{F_{1\lambda}}\psi^{\mathrm{sf}}_{F_{2\rho}} - \psi^{\mathrm{c}}_{F_{1\rho}}\psi^{\mathrm{sf}}_{F_{2\lambda}} + \psi^{\mathrm{c}}_{F_{1\eta}}\psi^{\mathrm{sf}}_{F_{2\eta}}\right)\right]$$

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[11] F. Stancu, Oxford Stud. Nucl. Phys. **19** , 1 (1996).

Conclusions

HEAVY BARYONS

- For baryon configurations qqQ consistent with the $\Xi_b, \Sigma_b, \Omega_c$ it was obtained that because the frequencies of the modes are such that $\omega_\rho >> \omega_\lambda$. The relevant states for electromagnetic baryon couplings are those with one quantum of excitation in λ.
- In contrast, for configuration QQq, the analysis for baryons as Ξ_{cc}, implies frequencies such that $\omega_\lambda > \omega_\rho$, so in this case the most important contribution is in the ρ mode. The total analysis of electromagnetic coplings is work in progress.

PENTAQUARKS

- Debido a las propiedades de simetría de la función de onda de pentaquark, varias configuraciones no pueden ser excitadas en foto-producción tales como perteneciente al decuplete de SU(3).
- We find that only a very small number of all possible pentaquark states can be excited in photoproduction experiments.
- Finalmente, debido ya que el momento asociado al fotón esta en un rango de energías muy alto, se tiene un factor de supresion muy grande al fotoacoplamiento debido a la parte orbital. Distinguiss between heavy and light quarks is work in progress.