Extension of the Lindhard model, to calculate the "quenching factor" at low energies

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- Importance of Quenching Factor for Dark Matter and Neutrino experiments.
- Summary of Lindhard model and limitations.
- Improvements to the Lindhard model.
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Quenchig Factor

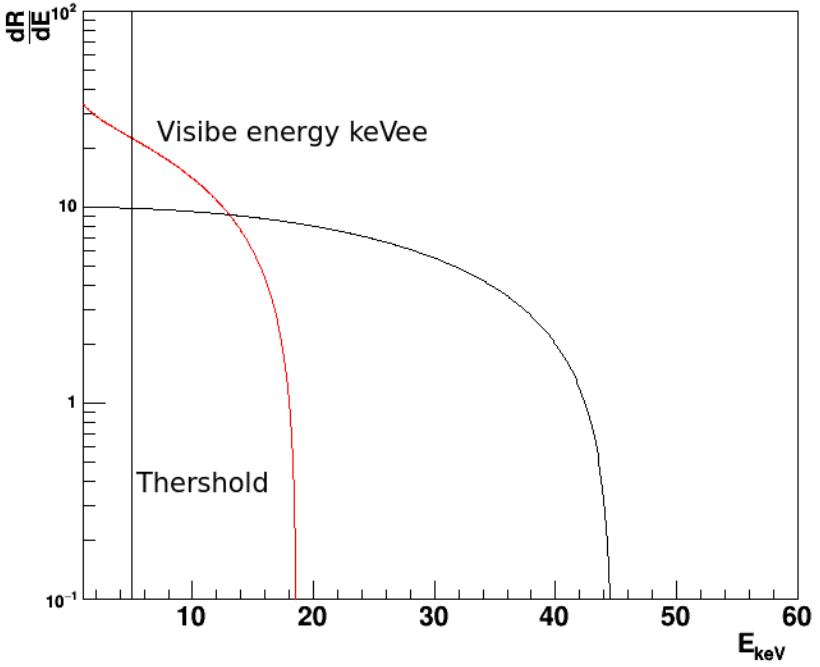
- The amount of electronic excitation produced by a recoiling atom is smaller than that produced by a recoiling electron of the same energy.

- The Q.F is the fraction of energy that is transformed into ionization energy in a crystal.

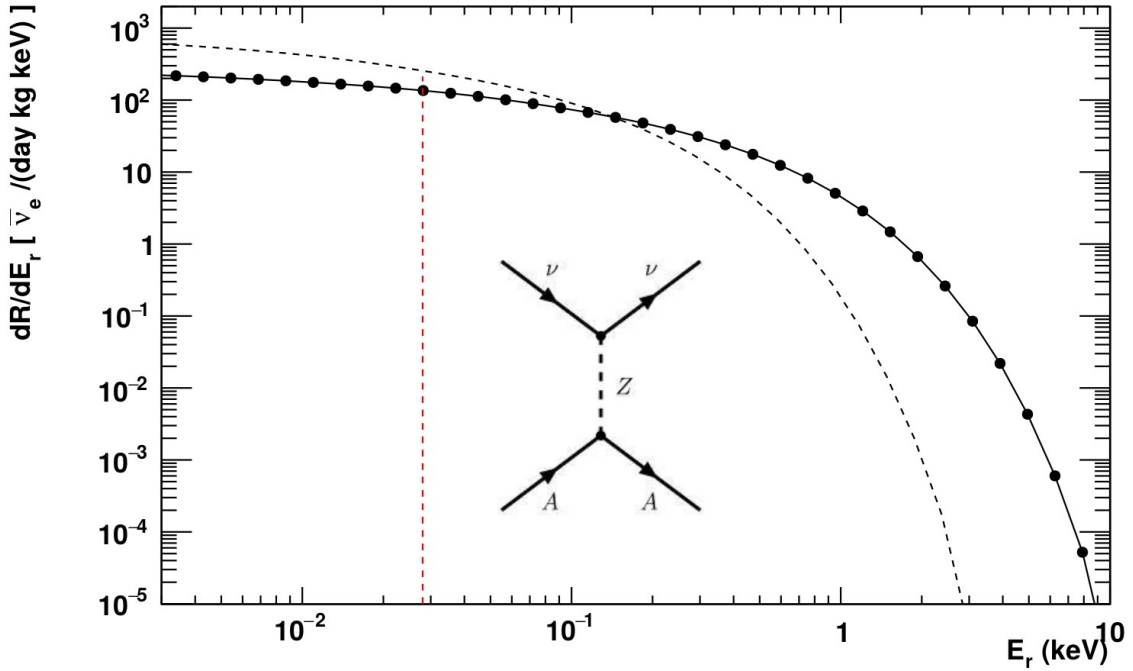
$$f_n = \frac{\eta}{\varepsilon}$$

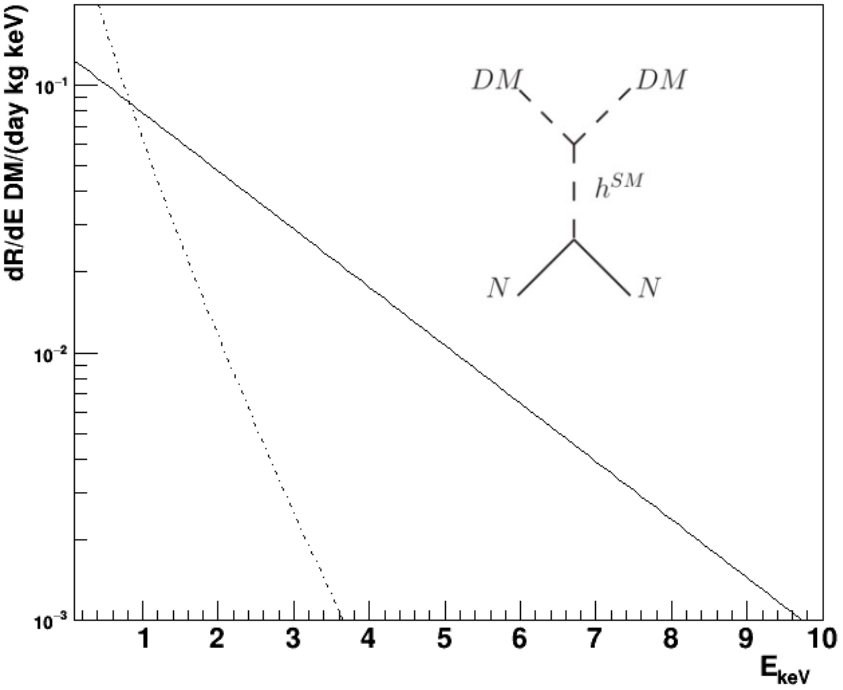
The Q.F shift the spectrum to lower energies.

$$\frac{dR}{dE_v} = \frac{dR}{dE}\frac{dE}{dE_v} = \frac{dR}{dE}\left(f_n + E\frac{df_n}{dE}\right)^{-1}$$



- In the direct dark matter searches experiments or in the measurements of the coherent elastic neutrino-nucleus scattering, the signal usually consists in the detection of the low energy (~ 1k eV) of the recoiling target atoms.
- As the experiments have reduced their detection thresholds well below 1 ~ keV, the understanding of the quenching at low energies has become increasingly important.





φ can be η or ν

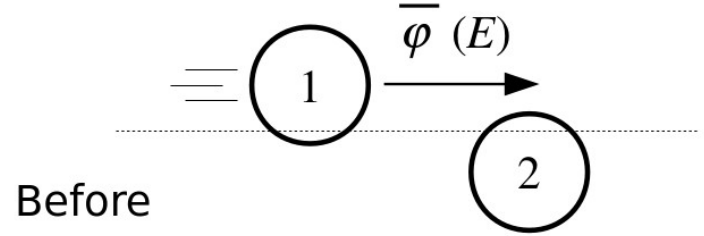
Lindhard Model

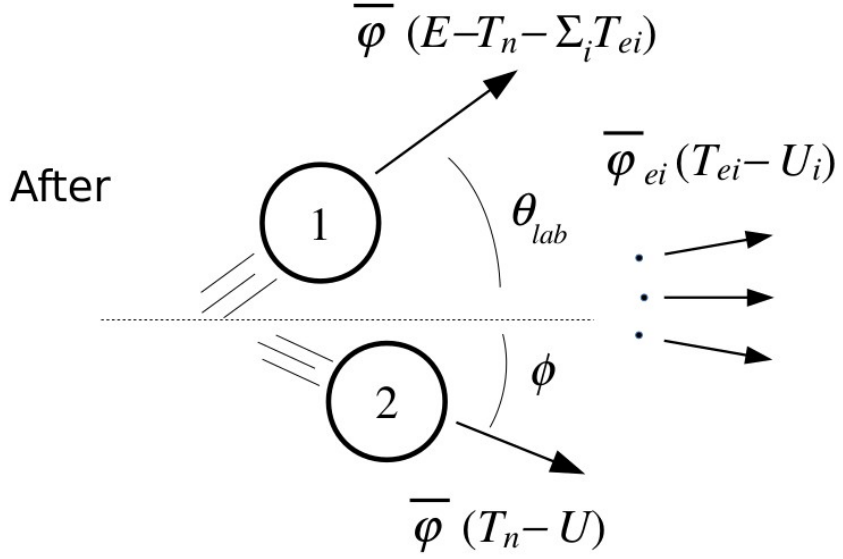
Tarjet atom after collision — projectile atom after collision — projectile atom before collision

$$\int d\sigma_{n,e}\left[\bar{\varphi}(E - T_n - \Sigma T_{ei}) + \bar{\varphi}(T_n - U) - \varphi(E)\right] = 0$$

Energy due to electronic stripping process

Energy due to recoil process

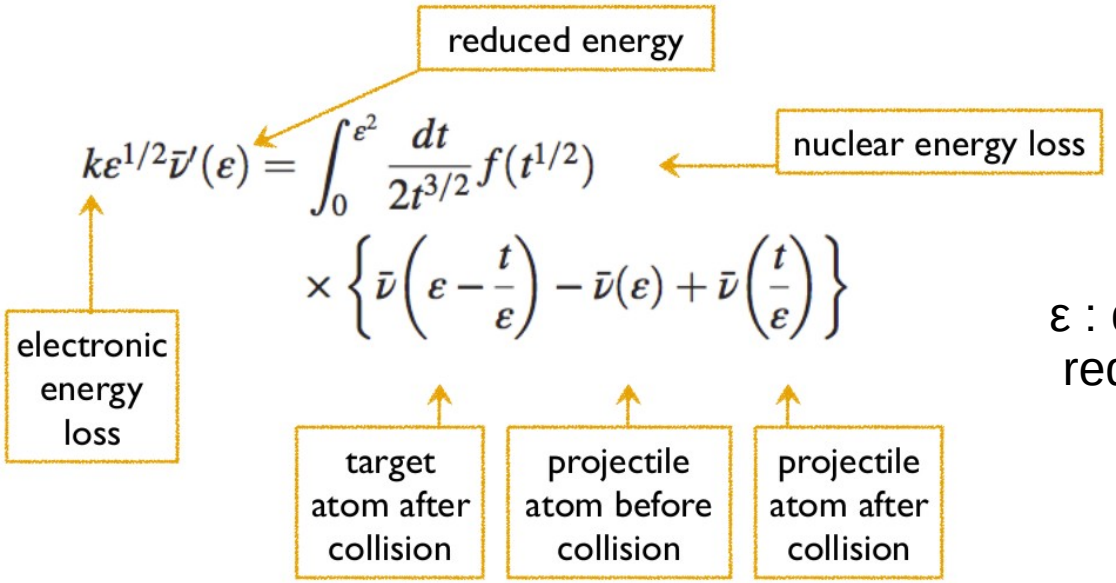




Energy is divided in nuclear and electronic average losses (ν, atomic movement, η ionization losse).

$$E = \bar{\nu}(E) + \bar{\eta}(E).$$

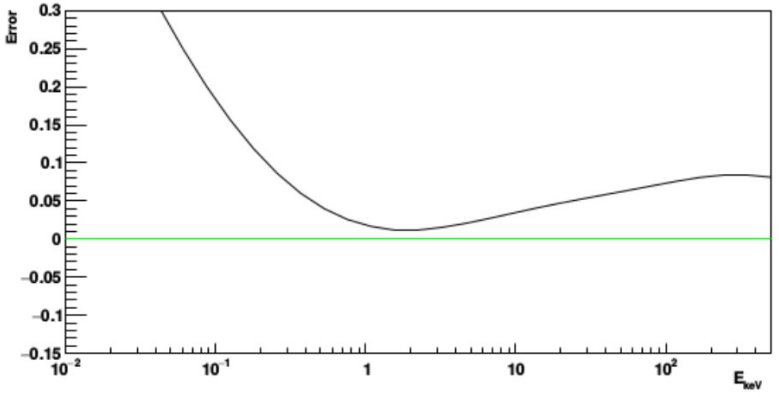
- The physical effect (phi) of the incident particle equals the sum of the physical effect of the scattered particle and that of the struck atom
- When writing the above equation Lindhard took into account the energy conservation in the scattering of two ions. To deal with the equation he made four basic approximations: i) electrons do not contribute to the atomic movement process, ii) the atomic-binding energy U can be neglected, iii) the electronic stripping process is expanded up to first order, iv) nuclear and electronic effects are separable.



ε : dimensionless reduced energy.

- Lindhard after these approximations can deduce this equation .
- Where $t=\varepsilon^2\sin^2(\theta/2)$, f(t) is related to the nuclear cross section scattering.
- Lindhard equation has a problem: the boundary condition at $E \rightarrow 0$, is $\nu(E) \rightarrow E$, so $\nu' \rightarrow 1$ in this limit. But the Integral equation predict $\nu' \rightarrow 0$.
- So the Integral equation can only be solve for energies far from zero.

- Threfore, the equation can be solved making to the Integral equation more raw approximations.
- He found a “reasonable” parametrization from this equation for energies below ε < 1.
- This allowed Lindhard to solve numerically his equation for energies ε >1.
- So this approach doesn’t allow us to give a good prediction in low energy regions.



The parametrization doesn’t solve the Equation at low energies.

$$\bar{\nu}_l(\epsilon) = \frac{\epsilon}{1 + kg(\epsilon)}$$

$$g(\epsilon) = 3\epsilon^{0.15} + 0.7\epsilon^{0.6} + \epsilon$$

Standar high energy parametrization

DASK computer
45 kW
3.5 Tons

Extension of Lindhard model

- We introduce the atomic-binding energy, considering the changing the term

$$\bar{\nu}(t/\varepsilon) \rightarrow \bar{\nu}(t/\varepsilon - u)$$

- Expand the 1st term in the integral equation $\varphi(\varepsilon\text{-}t/\varepsilon\text{-}\Sigma T_{ei})$ to 2nd order in $\Sigma T_{ei,}$ (Probability of electronic stripping process term).
- The lower limit of integration (over dt) can no longer be zero. The min t is now εu. (predicted also by model of scattering of two semi-hard spheres).
- In this model we area going to considere u constant.

- So with this modification we can deduce a new Integral modified equation with atomic-binding energy.

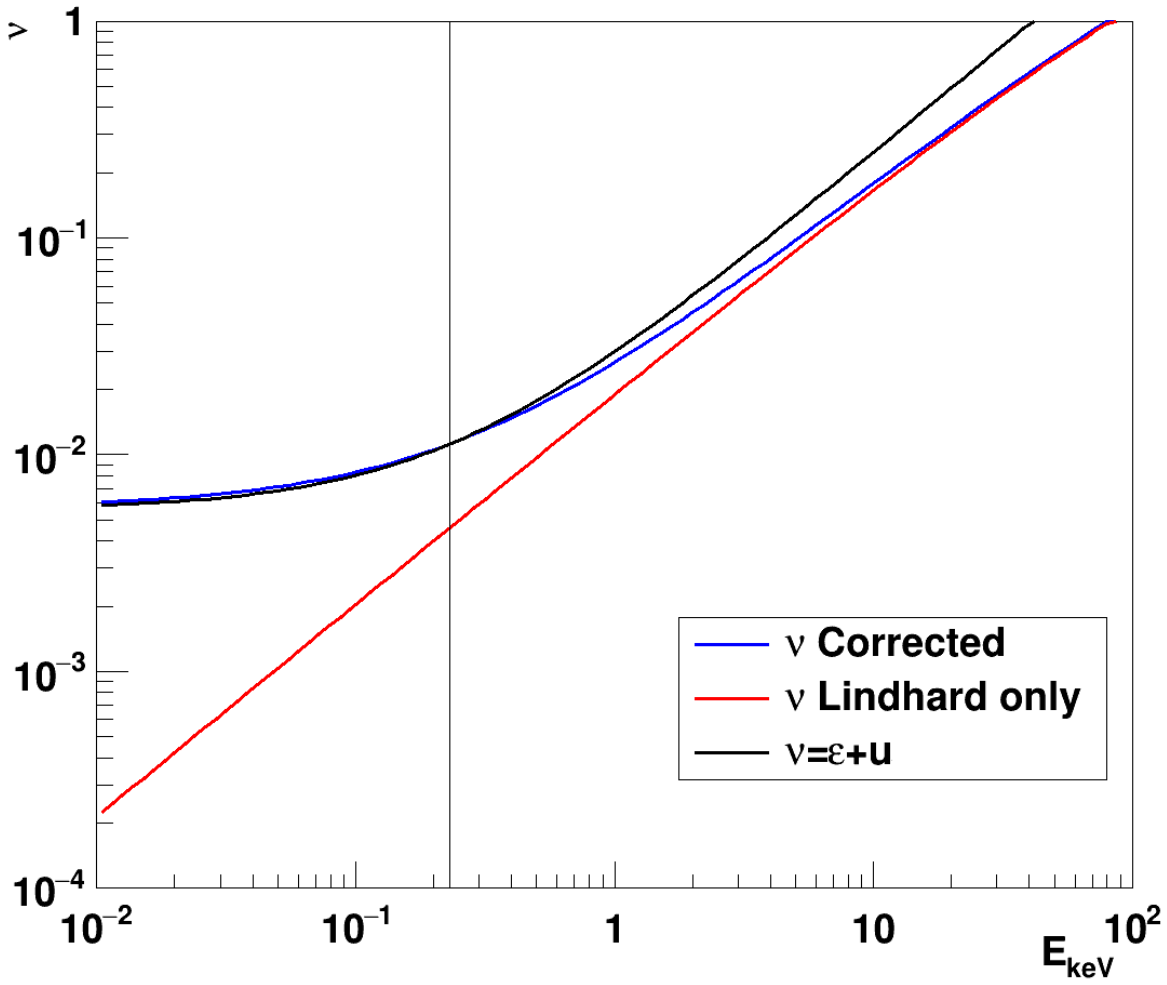
$$-\frac{1}{2}k\varepsilon^{3/2}\bar{\nu}''(\varepsilon)+k\varepsilon^{1/2}\bar{\nu}'(\varepsilon)=\int_{\varepsilon u}^{\varepsilon^2}\frac{dtf(t^{1/2})}{2t^{3/2}}\left[\bar{\nu}(\varepsilon-t/\varepsilon)+\bar{\nu}(t/\varepsilon-u)-\bar{\nu}(\varepsilon)\right]$$

- This equation ***can be solve numericaly from ε≥u***. So the equation predict a threshold energy of u.
- The equation requires the following behavior of the solution.

$$\bar{\nu}_u(\varepsilon)=\begin{cases}\varepsilon+u & \varepsilon<u\\ \varepsilon+u\ -g(\varepsilon) & \varepsilon\geq u\end{cases}\qquad f_n=\frac{\bar{\eta}}{\varepsilon+\bar{u}}=1-\frac{\bar{\nu}}{\varepsilon+\bar{u}}.$$

- The equation admits a solution featuring a "kink" at ε=u (discontinuous 1st derivative).
- We solve the equation by the shooting method pursuing the boundary condition at high energies that ν'' → 0 and for ε=u

Alternative good parametrization



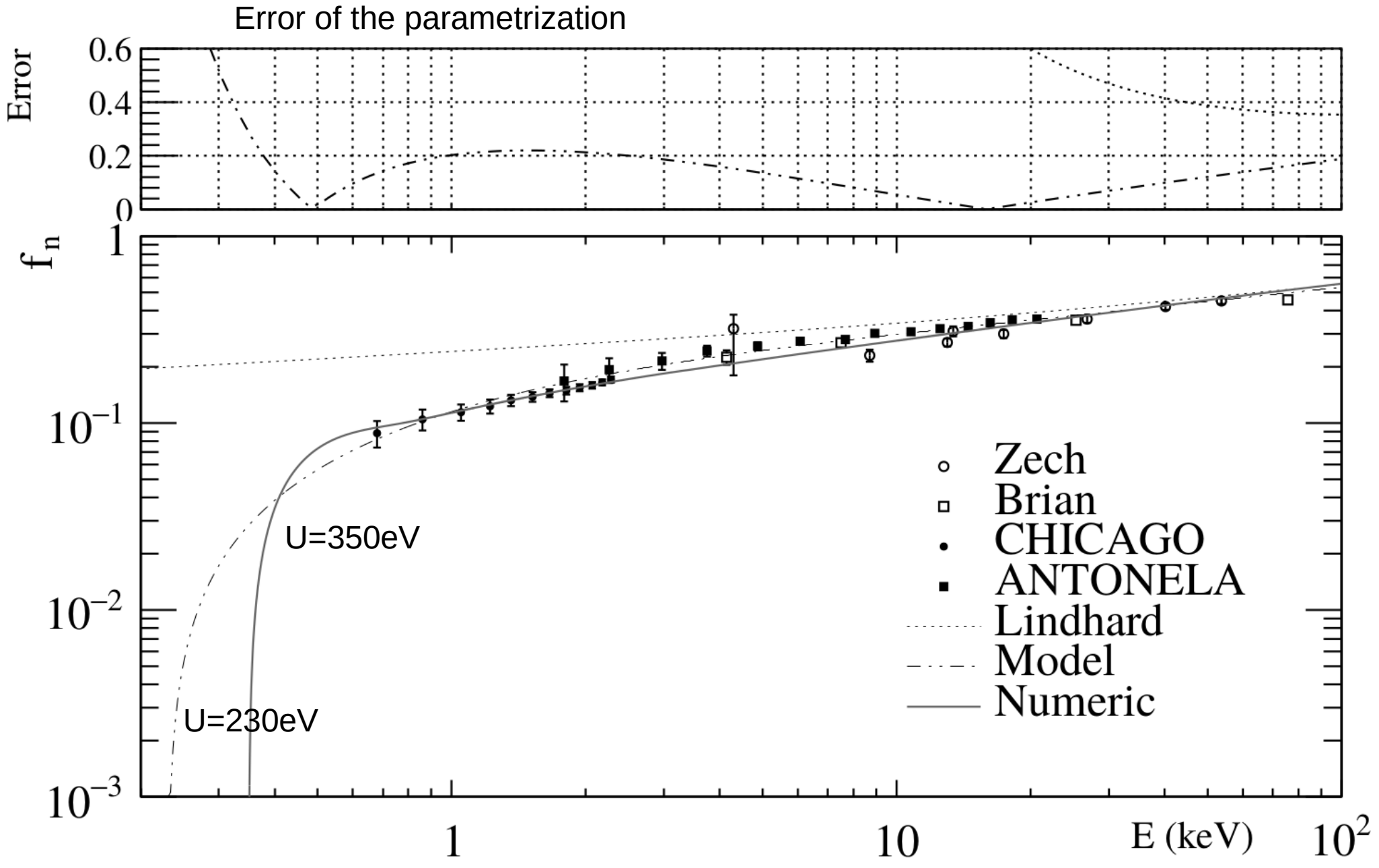
- We can implement a good analitical aproximation to solve the Integral equation.
- The idea is to lessen the ionization contribution, substracting a fraccion of the electronic stopping power.
- $\eta \rightarrow \eta - C' - C\varepsilon^{1/2}$.

  Where C, C' and u are estimated from a fit to the available data.

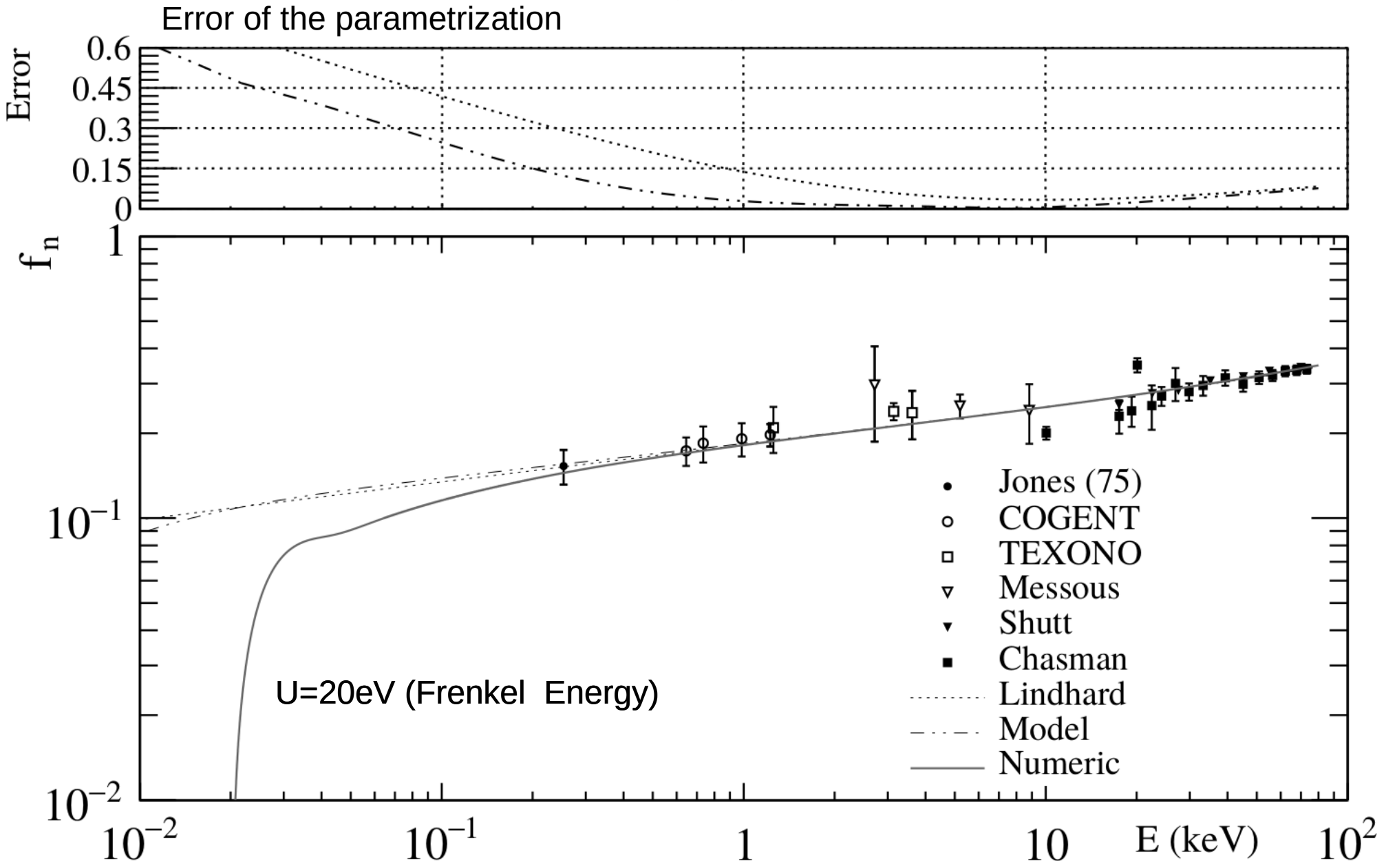
Results for Si, Ge and Xe.

- We assumed as a first simple model a constant atomic-binding energy (average).
- And also we assume that the k factor, of the electronic stopping power is constant too.

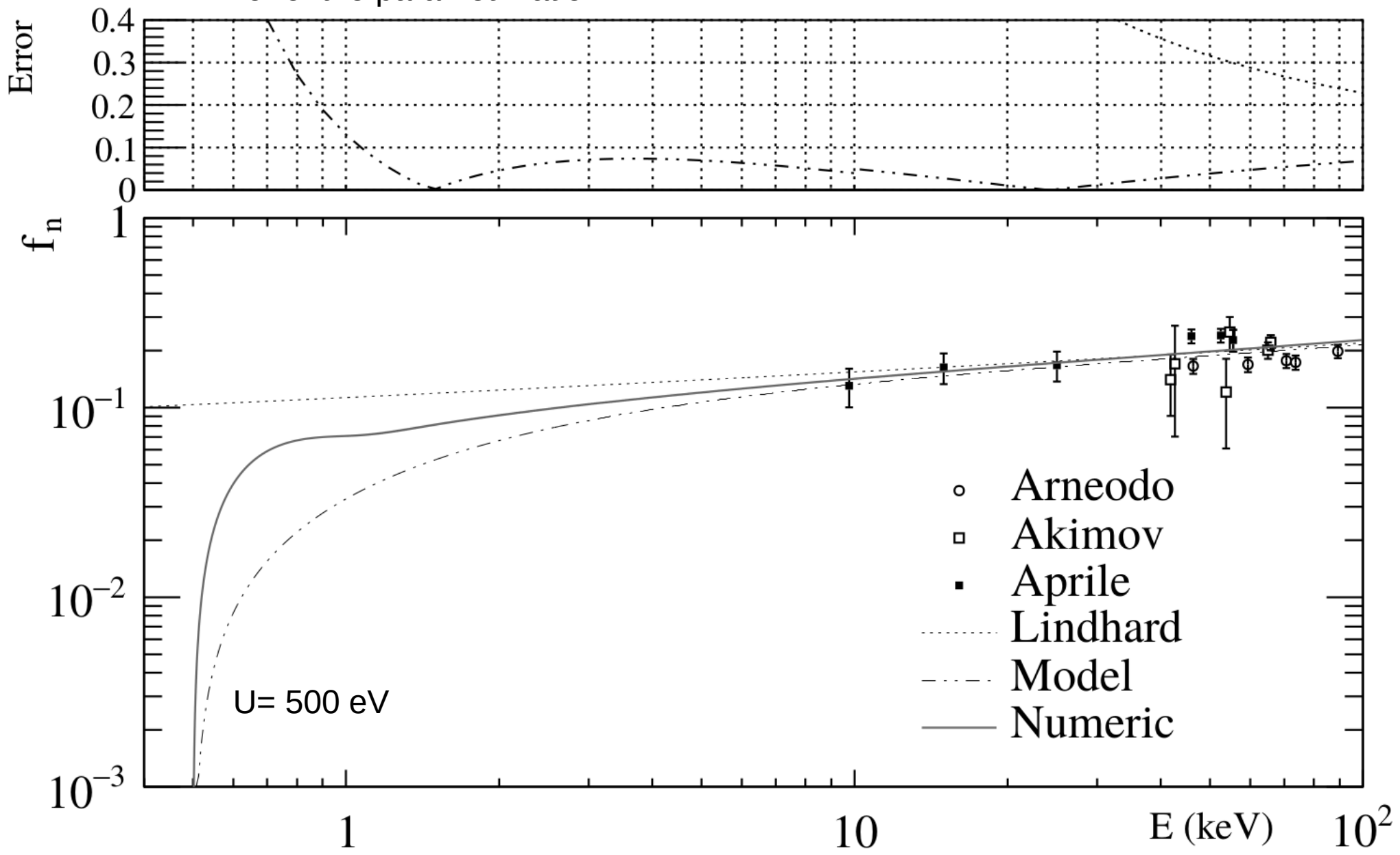
Si


Error of the parametrization
Error
0.6
0.4
0.2
0
f_n
1
10^{-1}
10^{-2}
10^{-3}
U=350eV
U=230eV
Zech
Brian
CHICAGO
ANTONELA
Lindhard
Model
Numeric
1
10
E (keV)
10^2

Ge


Error of the parametrization
Error
0.6
0.45
0.3
0.15
0
f_n
1
10^{-1}
10^{-2}
10^{-2}
10^{-1}
1
10
E (keV)
10^{2}
Jones (75)
COGENT
TEXONO
Messous
Shutt
Chasman
Lindhard
Model
Numeric
U=20eV (Frenkel Energy)

Xe


Error of the parametrization
Error
0.4
0.3
0.2
0.1
0
f_n
1
10^{-1}
10^{-2}
10^{-3}
U= 500 eV
Arneodo
Akimov
Aprile
Lindhard
Model
Numeric
1
10
E (keV)
10^2

Conclusions

- We have implemented an extension to Lindhard model which predicts a cut off at the binding energy.
- We study this new equation and deduce a numeric algorithm that solve the problem with no need of simplifications.
- Also the model now only required the specifications of k and u to have a QF.
- We think that this QF model can have an important impact in the new upcoming results from dark matter and neutrino experiments.
- Other approach (Sorensen) tried to solve this, but in an incomplete way, that doesn’t describe the data available.
- We are going to publish this work very soon.

Agradecimientos

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