ALTERNATIVE METHODS FOR CALCULATIONS IN PQCD

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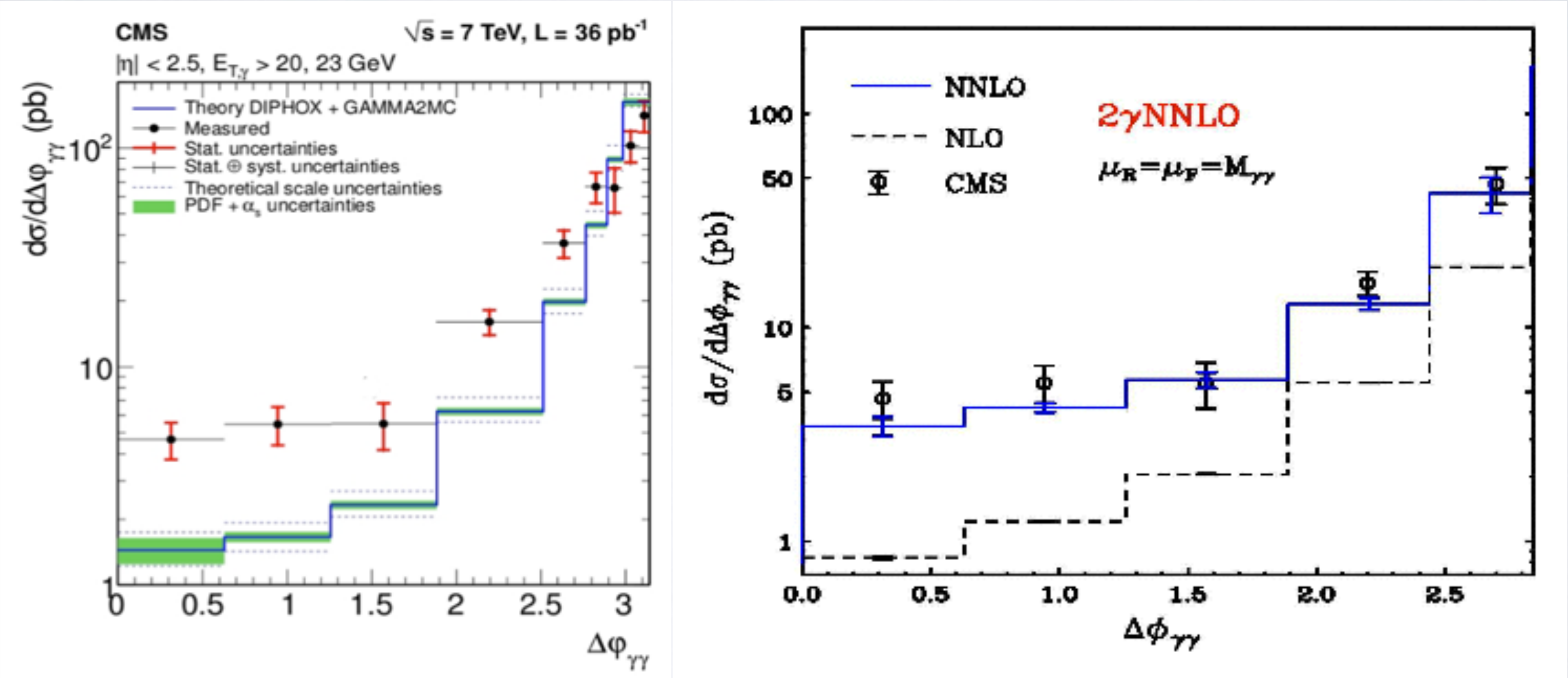
HIGH PRECISION @ LHC

- LHC experiments are delivering more and more data to the HEP community.
- More data sets improve the accuracy of all observables.
- And, when the physics is hidden in small effects, accuracy is crucial to claim discovery.
- And more data from CERN is coming

THEORY MEETS EXPERIMENT

- Precision measurements use accurate theoretical predictions.



- Experimental results needs Monte Carlo simulations in order to compare with nature.

THEORETICAL PREDICTIONS

- The total cross section is computed as,

$$\sigma = \sigma^{(LO)} + \alpha_S(\mu)\sigma^{(NLO)} + \alpha_S^2(\mu)\sigma^{(NNLO)} + \cdots$$

- where

$$\sigma^{(NLO)} = \int_{\Omega} d\sigma^{V} + \int_{\Omega+1} d\sigma^{R}$$

$$\sigma^{(NNLO)} = \int_{\Omega} d\sigma^{VV} + \int_{\Omega+1} d\sigma^{RV} + \int_{\Omega+2} d\sigma^{RR}$$

THEORETICAL ISSUES

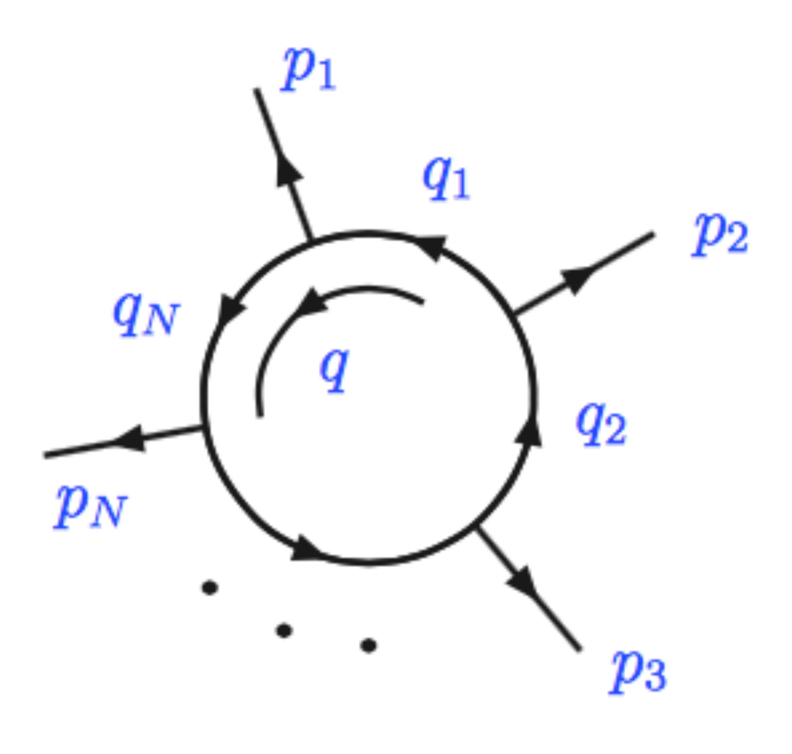
Huge number of diagrams !

	NNLO	N3LO
# of diagrams	~10,000	~100,000
# of integrals	~50,000	~500,000,000
# of master integrals	27	1,028
# of boundary conditions	5	78

Divergences everywhere !

LOOP-TREE DUALITY

- Massive one-loop scalar integrals are,



$$= -\imath \int \frac{d^d q}{(2\pi)^d} \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + \imath 0}$$

- where the +i0 prescription establishes that particles are going forward in time.

- LTD at one loop establishes then

$$L^{(1)}(p_1, \cdots, p_N) = -\sum \int_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} G_D(q_i; q_j)$$

- where Feynman propagators are transformed to dual propagators.

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$

- $\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 - m_i^2)$ and sets internal lines on-shell and in the positive energy mode.

- LTD modify the +i0 prescription, instead of having multiple cuts like in the Feynman Tree Theorem.

- η^μ is a future-like vector, for simplicity we take $\eta^\mu = (1, \mathbf{0})$. In fact, the only relevance is the sign in the prescription.

NUMERICAL IMPLEMENTATION

- Faster computations are needed for the Montecarlo simulations for the LHC observables.
- Using LTD the standard methods become time consuming
- (S. Buchta, et al. , arXiv:1510.00187)

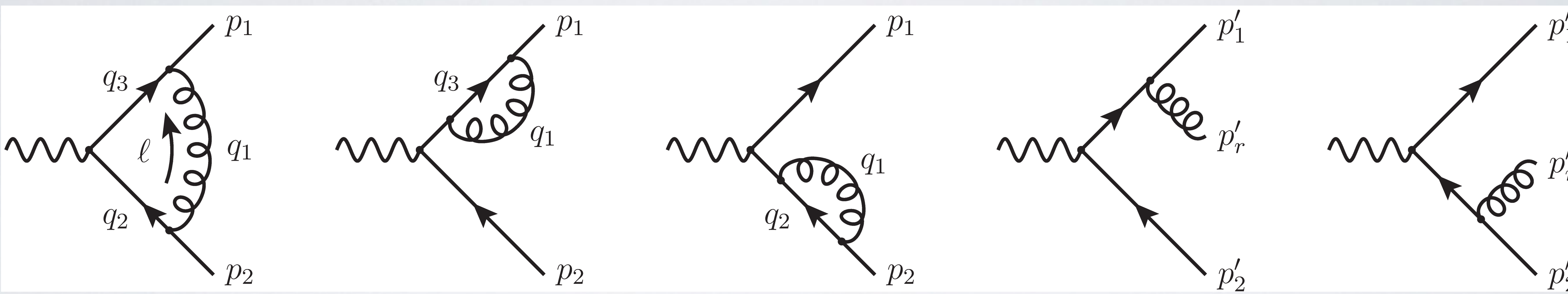
	Rank	Tensor Pentagon	Real Part	Imaginary Part	Time [s]
P16	2	LoopTools	-1.86472×10^{-8}		
		SecDec	$-1.86471(2) \times 10^{-8}$		45
		LTD	$-1.86462(26) \times 10^{-8}$		1
P17	3	LoopTools	1.74828×10^{-3}		
		SecDec	$1.74828(17) \times 10^{-3}$		550
		LTD	$1.74808(283) \times 10^{-3}$		1
P18	2	LoopTools	-1.68298×10^{-6}	$+i\ 1.98303 \times 10^{-6}$	
		SecDec	$-1.68307(56) \times 10^{-6}$	$+i\ 1.98279(90) \times 10^{-6}$	66
		LTD	$-1.68298(74) \times 10^{-6}$	$+i\ 1.98299(74) \times 10^{-6}$	36
P19	3	LoopTools	-8.34718×10^{-2}	$+i\ 1.10217 \times 10^{-2}$	
		SecDec	$-8.33284(829) \times 10^{-2}$	$+i\ 1.10232(107) \times 10^{-2}$	1501
		LTD	$-8.34829(757) \times 10^{-2}$	$+i\ 1.10119(757) \times 10^{-2}$	38

	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P20	1	SecDec	$-1.21585(12) \times 10^{-15}$		36
		LTD	$-1.21552(354) \times 10^{-15}$		6
P21	3	SecDec	$4.46117(37) \times 10^{-9}$		5498
		LTD	$4.461369(3) \times 10^{-9}$		11
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i\ 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) \times 10^{-15}$	$+i\ 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) \times 10^{-12}$	$-i\ 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) \times 10^{-12}$	$-i\ 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i\ 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) \times 10^{-6}$	$+i\ 6.97192(8) \times 10^{-7}$	85

- This results shows have been implemented for several data points for tensor pentagons and hexagons.
- Integrals considering massive internal lines were computed numerically.
- The results using LTD are, in some cases, four order of magnitudes faster than SecDec.
- What about in a physical process ?

$\gamma^* \to q\bar{q}$ AT NLO IN QCD

- In this well known process, the Feynman diagrams are



- Using the LTD we find,

$$\tilde{\sigma}_1^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(19 - 32 \log(2)\right),$$

$$\tilde{\sigma}_2^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(-\frac{11}{2} + 8 \log(2) - \frac{\pi^2}{3}\right),$$

$$\bar{\sigma}_V^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(-\frac{21}{2} + 24 \log(2) + \frac{\pi^2}{3}\right).$$

The sum coincides with the result in DREG

UNIVERSAL DUAL AMPLITUDES

- The one-loop decay amplitudes for the processes for massless vector bosons $H \to VV$, can be written as,

$$|\mathcal{M}^{(1)}_{H\to VV}\rangle = \imath\, e \left(\sum_{f=\phi,t,W} e_f^2 \mathcal{A}^{(1,f)}_{\mu\nu} \right) (\varepsilon^{\mu}(p_1))^* (\varepsilon^{\nu}(p_2))^*$$

- Decomposing $\mathcal{A}^{(1,f)}_{\mu\nu}$,as a function of a tensor basis,

$$\mathcal{A}^{(1,f)}_{\mu\nu} = \sum_{i=1}^{5} A_i^{(1,f)} T^i_{\mu\nu}$$

- Computing the coefficients lead us to,

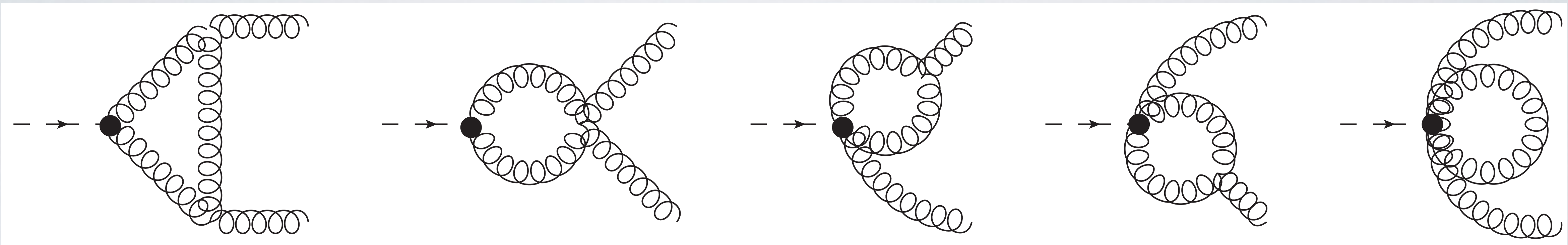
$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12} M_f^2}{(2\ell \cdot p_1)(2\ell \cdot p_2)} c_1^{(f)} + c_2^{(f)} \right) + \frac{2 s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} c_3^{(f)} \right]$$

- and

$$\mathcal{A}_2^{(1,f)} = g_f \frac{c_3^{(f)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right)$$

- All the other coefficients are meaningless.
- A1 have the complete physics information
- A2 integrates to zero.

- The previous calculation can be extended for a gluon loop in the EFT. The Feynman diagrams to compute are,



- Finally, we find

$$\mathcal{A}_1^{(1,g)} = g_g \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12} M_f^2}{(2\ell\cdot p_1)(2\ell\cdot p_2)} c_1^{(g)} + c_2^{(g)} \right) + \frac{2 s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} c_3^{(g)} \right] + \mathcal{S}^{(1,g)}$$

$$\mathcal{A}_2^{(1,g)} = g_g \frac{c_3^{(g)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right)$$

S1 is zero in DREG

The same !

CONCLUSIONS

- New methods for computing higher order corrections are needed for upcoming LHC observables.
- Mapping of momenta between real and virtual corrections permits to cancel soft and final-state collinear singularities.
- Fully local cancellation of IR and UV divergences through the LTD.
- Spanning the amplitudes under a tensor basis allow us tu extract the information in a single coefficient.

THANKS