Conjecture about the 2-Flavour QCD Phase Diagram

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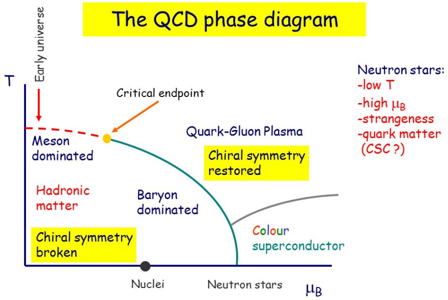
September 27, 2018

Motivation



Conjecture of the QCD phase diagram.

- The QCD phase diagram is one of the most prominent outstanding mysteries within the Standard Model of particle physics.
- Quarks do have masses, but two flavours are very light compared to the intrinsic scale $\Lambda_{QCD} = 341(12)$MeV. (Bruno et al., '16)

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3d O(4) non-linear σ-model

- In QCD with 2 massless quark flavors, the chiral symmetry breaks spontaneously as $SU(2)_L \times SU(2)_R \to SU(2)_{L=R}$. This is isomorphic to $O(4) \to O(3)$, and high T induces dimensional reduction. Therefore, 2-flavor QCD at the chiral phase transition is assumed to belong to the universality class of the 3d O(4) model ("Rajagopal/Wilczek '93"), whose Hamilton function is

$$\mathcal{H} = -\sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j , \qquad \vec{S}_i \in S^3 ,$$

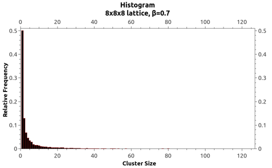
where $\langle i, j \rangle$ means the nearest-neighbor sites on a 3d cubic lattice.

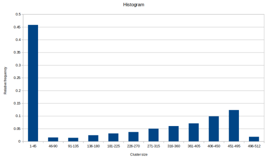
3d O(4) model on an $L \times L \times L$ lattice

Goal: generate field configurations $[\vec{S}] \propto \exp(-\beta \mathcal{H}[\vec{S}])$ for the importance sampling of the functional integral.

Wolff Single Cluster Algorithm. Choose randomly a reflection 3d subspace. Choose randomly a site i in the lattice. Visit all neighbour sites j of i. These sites join to the cluster with certain probability. Repeat until the cluster does not grow anymore. Apply the reflection to all spins in the cluster.

Cluster size distributions on a $8 \times 8 \times 8$ lattice at $\beta = 0.7$ and $\beta = 1.5$; the latter leads to larger clusters.





Computing time

Computing time for one core on an $8 \times 8 \times 8$ lattice.

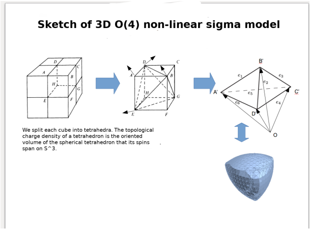
Lattice volume	8x8x8	8x8x8
β	0.7	1.0
Thermalization(s)	2.4	36
Separation of lattices(s)	0.025	0.6
Measuring of Q, M,etc. (s)	0.00076	0.00096
Statistics(s)	0.00057	0.00049
Total time (s)	250	6040

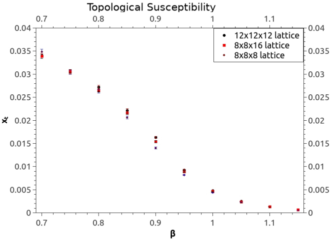
Topology.

The winding number of a field configuration on the sphere S^3 is its topological charge $Q \in Z$. In our low energy effective theory, it represents the baryon number. The definition of topological susceptibility is

$$\chi_t = \frac{\langle Q^2 \rangle}{V},$$

where V is the volume. At large β, there are only few top. windings, which are exponentially suppressed, as one expects for a dilute instanton gas. Below β_c the dependence ceases to be exponential - here the top. windings are quite dense, i.e. not dilute anymore.



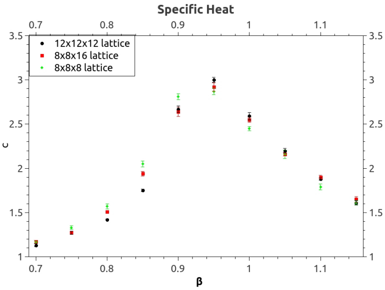


Specific heat

The definition of the specific heat is

$$c = \frac{\beta^2}{V}(\langle \mathcal{H}^2 \rangle - \langle \mathcal{H} \rangle^2)$$

Notice the expected peak between $\beta = 0.9$ and 0.95, where simulations indicate a phase transition of second order at $\beta_c = 0.93590$ (Engels et al., '03).



Magnetic susceptibility

The definition of magnetization is

$$\vec{M} = \sum_x \vec{S_x}$$

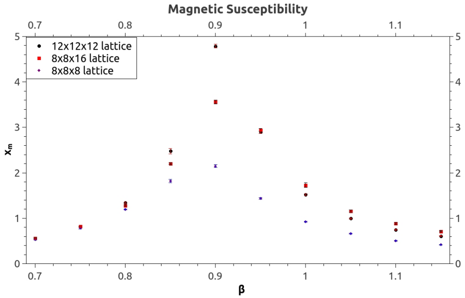
from where we proceed to define the magnetic susceptibility

$$\chi_m = \frac{\beta}{V}(\langle M^2\rangle - \langle M\rangle^2).$$

We can see the expected divergence in χ_m due to the phase transition between $\beta = 0.9$ and 0.95.

The peak is enhanced when the volume increases.

Thus as one approaches to the limit $V \to \infty$, this peak turns into a divergence.



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Correlation lengths

The correlation function $C(r)$ is given by

$$C(r) = \langle \vec{S}_{x3} \cdot \vec{S}_{x3+r} \rangle \propto \cosh\left(\frac{r - L/2}{\xi}\right),$$

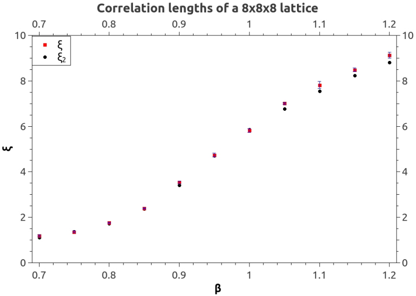
where $\vec{S}_{x3} = \frac{1}{L^2} \sum_{x1,x2} \vec{S}_x$.
The second moment correlation length is

$$\xi_{2nd} = \left(\frac{(\chi/\mathcal{F}) - 1}{4\sin^2(\pi/L)}\right)^{1/2}.$$

ξ is the correlation length and

$$\mathcal{F} = \frac{1}{V} \sum_{x,y} \langle \vec{e}_x \cdot \vec{e}_y \rangle \cos\left(\frac{2\pi(x_1 - y_1)}{L}\right)$$

The physical size is $L\xi$. Therefore these data points are affected by strong finite-size effects, except for the data at the lowest β-values. This is why the divergence of ξ at β_c is not yet visible - it will show up in larger volumes.



Conclusions and Outlook

The 3d O(4) mode has been simulated successfully; the results agree with the expected 2nd order phase transition, and the topological charge was measured.
Next the baryonic chemical potential μ will be included,

$$\mathcal{H} = -\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \mu Q[\vec{S}]$$

which will reveal the phase diagram of the effective theory at finite baryon density.

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Acknolewdgments

MN thanks to Edgar López Contreras, José Antonio García Hernández, Ilya Orson Sandoval Cárdenas, Héctor Miguel Mejía Díaz, Dr. Mario Rodríguez Cahuantzi and Dr. Pedro González Zamora for their help during the realization of this work.

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THANK YOU