Lecture Plan:

1) Cosmic Ray acceleration- accelerated spectrum, efficient accelerators, nuclei friendly

PROBLEMS

2) Cosmic Ray proton + nuclei interaction rates in extragalactic radiation fields

PROBLEMS

3) Cosmic Ray propagation through Galactic and extragalactic magnetic fields

Andrew Taylor
When $E^{-2}$ and when not?
Strong shock wave propagating at supersonic velocity (sound speed depends on temperature)

\[ E_2 = E_1 \left( \frac{1 + \beta \mu_1}{1 + \beta \mu_2} \right) \]

\[ E_2 = E_1, \mu_1 \]

\[ E_2, \mu_2 \]

\[ E'_1, \mu'_1 \]

\[ E'_1, \mu'_2 \]

Andrew Taylor
### Fermi Acceleration (more)

#### Energy

\[
\frac{\Delta E}{E} = \frac{4v}{3c} = \frac{4}{3} \beta \text{(energy gain)}
\]

\[
E_1 = \left(1 + \frac{4}{3} \beta \right) E_0
\]

\[
E_n = \left(1 + \frac{4}{3} \beta \right)^n E_0
\]

#### Number

\[
\frac{\Delta N}{N} = -\frac{4v}{3c} = -\frac{4}{3} \beta \text{ (advection downstream)}
\]

\[
N_1 = \left(1 - \frac{4}{3} \beta \right) N_0
\]

\[
N_n = \left(1 + \frac{4}{3} \beta \right)^n N_0
\]

So \( n \sim 1/\beta \) crossings are needed before the particle population is significantly altered.

\[\rightarrow\] SNRs have \( v_{sh} \sim 10^3 \text{ km s}^{-1} \)

so \( \beta \sim 10^{-2} \)
Fermi Acceleration (more)

Energy

Number

$\beta \sim 10^{-2}$
Fermi Acceleration (more)

So,

\[
\frac{\Delta N}{\Delta E} = \frac{N_0}{E_0} \left( \frac{1 - 4\beta/3}{1 + 4\beta/3} \right)^n
\]

\[
\approx \frac{N_0}{E_0} \left( 1 + 4\beta/3 \right)^{-2n}
\]

\[
\approx N_0 E_0 E^{-2}
\]
Stochastic Acceleration/Propagation

\[ D_{xx} D_{pp} \approx \beta_{\text{scat}}^2 p^2 \]
Random Walks

\[ \gamma(t + 1) = t! \]

\[ \gamma(t + 1) = \int_0^\infty x^t e^{-x} dx \]

\[ f(x, t) = \frac{\gamma(t + 1)}{[\gamma([t - x]/2 + 1)\gamma([x + t]/2 + 1)]/(2^t)} \]

\[ f(x, t) \approx \frac{e^{-x^2}/(2t)}{[\pi/(t/2)]^{1/2}} \]

Andrew Taylor
Random Walks

Spatial spread:
\[ \frac{dN}{dx} \propto e^{-x^2/4D_{xx}t} \]
\[ \frac{dN}{dx} \propto e^{-x^2/4c^2t_{scat}t} \]

Momentum spread:
\[ \frac{\Delta E}{E} \propto \beta \]
\[ \frac{dN}{dp} \propto e^{-(\ln p)^2/4(D_{pp}/p^2)t} \]
\[ \frac{dN}{dp} \propto e^{-(\ln p)^2/4(t/t_{acc})} \]
Gamma-Ray Probes of Particle Acceleration – Flaring Blazars (Mrk 501)

\[ \frac{\partial}{\partial t} n(p, t) = \nabla_p \cdot D_p \nabla_p n(p, t) - \frac{n(p, t)}{\tau_{\text{esc.}}(p)} + Q(p, t) \]

No energy losses


Andrew Taylor
Fermi (Second Order) Acceleration

\[
\frac{\partial f}{\partial t} = \nabla_p \cdot \left[ \left( D_{pp} \nabla_p f \right) - \frac{p}{\tau_{\text{loss}}(p)} f \right] - \frac{f}{\tau_{\text{esc}}(p)} + \frac{Q}{p^2}
\]

- Acceleration
- Radiative Losses
- Escape
- Source term

Andrew Taylor
Stochastic Particle Acceleration - Random Walk Result (Spatial)

\[ \nabla \cdot (D_{xx} \nabla f) = \delta(r) \]

\[ \frac{\partial^2 f}{\partial r^2} + \frac{2}{r} \frac{\partial f}{\partial r} = \delta(r) \]

\[ f = r^{-\alpha} \]

\[ -\alpha(-\alpha - 1) - 2\alpha = 0 \]

\[ \alpha(\alpha - 1) = 0 \]
Stochastic Particle Acceleration - Random Walk Result (Momentum)

\[
\frac{\partial f}{\partial t} = \nabla_p \cdot \left[ \left( D_{pp} \nabla_p f \right) - \frac{p}{\tau_{\text{loss}}(p)} f \right] - \frac{f}{\tau_{\text{esc}}(p)} + \frac{Q}{p^2}
\]

- Steady state
- No losses
- Delta injection

\[ D_{pp} \propto p^q \]

\[
\frac{\partial^2 f}{\partial p^2} + \left( 2 + q \right) \frac{\partial f}{p} - \frac{4\tau_{\text{acc}}}{\tau_{\text{esc}}} \frac{f}{p^2} = \delta(p)
\]

For \( f = p^{-\alpha} \) and \( q = 2 \)

\[
\alpha^2 - 3\alpha - \frac{4\tau_{\text{acc}}}{\tau_{\text{esc}}} = 0
\]

Andrew Taylor
Stochastic Particle Acceleration-
Random Walk Result (Momentum)

\[ \alpha^2 - 3\alpha - \frac{4\tau_{\text{acc}}}{\tau_{\text{esc}}} = 0 \]

\[ \alpha = \frac{3}{2} \pm \left( \frac{4\tau_{\text{acc}}}{\tau_{\text{esc}}} + \frac{9}{4} \right)^{1/2} \]

\[ \frac{\tau_{\text{acc}}}{\tau_{\text{esc}}} = 1 \]

\[ f = \frac{dN}{d^3p} = p^{-4} \]

Andrew Taylor
Fermi (First Order) Acceleration Time

\[ t_{\text{acc}} = E \frac{\Delta t_{\text{cycle}}}{\Delta E_{\text{cycle}}} \]

Transport of particles in each region is dictated by competition between diffusion and advection downstream and upstream.

\[ t_{\text{diff}} = \frac{R^2}{D_{xx}} \quad t_{\text{adv}} = \frac{R}{v_{\text{adv}}} \]

Balancing these timescales

\[ t_{\text{resid}} = \frac{D_{xx}}{(c\beta_{sh})^2} \]
Fermi (First Order)
Acceleration Time

\[ t_{\text{acc}} = E \frac{\Delta t_{\text{cycle}}}{\Delta E_{\text{cycle}}} \]

\[ t_{\text{resid}} = \frac{D_{xx}}{(c \beta_{sh})^2} \]

However, during the time it takes advection to dominate over diffusion, the particle will have crossed the shock \( \frac{1}{\beta} \) times

\[ \Delta t_{\text{cycle}} = \frac{D_{xx}}{(c^2 \beta_{sh})} \]
Fermi (First Order)
Acceleration Time

\[ t_{\text{acc}} = E \frac{\Delta t_{\text{cycle}}}{\Delta E_{\text{cycle}}} \]

\[ \Delta t_{\text{cycle}} = \frac{D_{xx}}{(c^2 \beta_{sh})} \]

\[ \Delta E_{\text{cycle}} = E \beta_{sh} \]

\[ t_{\text{acc}} = \frac{D_{xx}}{(c \beta_{sh})^2} = \frac{t_{\text{scat}}}{\beta_{sh}^2} \]

\[ E_2 = E_1 \left( \frac{1 + \beta \mu_1}{1 + \beta \mu_2} \right) \]
Fermi (Second Order) Acceleration Time

\[ t_{acc} = E \frac{\Delta t_{scat}}{\Delta E_{scat}} \]

\[ \Delta E_{scat} = E \beta_{scat}^2 \]

\[ t_{acc} = \frac{t_{scat}}{\beta_{scat}^2} \]
Efficient Accelerators....what means efficient?
Particle Acceleration in AGN

\[ t_{\text{acc}} = \eta \frac{R_{\text{lar}}}{c \beta^2} \]

\[ t_{\text{esc.}} = \frac{R^2}{\eta c R_{\text{lar}}} \]

Maximum energy (Hillas criterion)

\[ R_{\text{lar}} = \frac{\beta}{\eta} R \]

AM Hillas (1984)

\[ R_{\text{lar}}(E, B) = \left( \frac{E}{10 \text{ EeV}} \right) \left( \frac{1 \text{ mG}}{B} \right) 10 \text{ pc} \]
Compactness of UHECR Sources: Proton/Nuclei Synchrotron Losses

AM Hillas (1984)

\[ \eta \approx 1 \quad \text{assumed in above plot} \]
Particle Acceleration with Cooling

\[ t_{\text{acc}} = \eta \frac{R_{\text{lar}}}{c \beta^2} \]

\[ t_{\text{cool}} = \frac{9}{8\pi \alpha} \left( \frac{m_e}{E_{\text{sync}}} \right) t_{\text{lar}} \]

\[ E_{\text{sync}} \approx \frac{9}{4} \eta^{-1} \beta^2 \frac{m_e}{\alpha} \]

Maximum synchrotron energy tells us how efficient accelerator is!

\[ \eta < 10^3 \]

Andrew Taylor
Emission Site?

Where are the misaligned (X)HBLs?

Hardcastle et al. (1103.1744)

\[ \eta < 10^3 \]
Future Probes- Cutoff Region

\[ E_{\gamma} \frac{dN}{dE_{\gamma}} = \int \left( \frac{E_{\gamma}}{E_e^2} \right) \frac{dN}{dE_{\gamma}} \left( \frac{E_{\gamma}}{E_e^2} \right) E_e \frac{dN}{dE_e} dE_e \]

Possibility to probe cutoff region

\[ E_{\text{sync}} = \Gamma_e^2 \left( \frac{B}{B_{\text{crit}}} \right) m_e \]

\[ B_{\text{crit}} = 4 \times 10^{13} \text{ G} \]
Nuclei Friendly Accelerators
UHECR Air Showers

$N_{ch}$ [total charge set to 1]

Shower Height [km]

grammage [g cm$^{-2}$]

Andrew Taylor
UHECR Air Showers

\[ \langle X_{\text{max}} \rangle = \frac{1}{N} \sum_{n=1}^{N} X_{\text{max},n} \]

\[ \text{RMS}(X_{\text{max}}) = \frac{1}{N} \sqrt{\sum_{n=1}^{N} (X_{\text{max},n} - \langle X_{\text{max}} \rangle)^2} \]
Composition Measurements by the PAO

\[ \langle X_{\text{max}} \rangle \]

\[ \langle X_{\text{max}} \rangle = \sum_i f_i X_{i,\text{max}} \]

**RMS** (\( X_{\text{max}} \))

\[ \text{RMS}^2 (X_{\text{max}}) = \sum_i f_i \text{RMS}^2_{X_{i,\text{max}}} + \sum_i f_i (X_{i,\text{max}} - \langle X_{\text{max}} \rangle)^2 \]

DESY.
Nuclei Transmutation Within their Source

![Graph showing the surviving fraction of nuclei over time. The x-axis represents time in Myr (百万年), ranging from 0 to 4500, and the y-axis represents the surviving fraction, ranging from 0 to 1. The legend indicates different isotopes (A=10 to A=56) with various line styles and colors.](image-url)
IMPLICATIONS for UHECR Sources

\[ f = \frac{t_{\text{trap}}}{t_{\text{int.}}} \]

\[ t_{\text{int.}}^{\text{CR}\gamma} \approx \frac{1}{n_\gamma \sigma_{\text{CR}\gamma} c} \]

\[ n_\gamma = \frac{L_\gamma}{c4\pi R^2 \epsilon_\gamma} \]

\[ t_{\text{trap}} \approx \frac{R^2}{2D} = \frac{3R^2}{2R_{\text{Larmor}}} \]

\[ f^{\text{CR}\gamma} = \frac{3L_\gamma \sigma_{\text{CR}\gamma} Z B}{8\pi \epsilon_\gamma E_{\text{CR}}} \]
IMPLICATIONS for UHECR Sources

\[ f_{\text{CR} \gamma} = \frac{3L_\gamma \sigma_{\text{CR} \gamma} ZB}{8\pi \epsilon_\gamma E_{\text{CR}}} = \frac{s_1}{s_2} \]

Photo-disintegration threshold:

\[ 2E_{\text{CR} \epsilon_\gamma} > A m_p c^2 E_{\text{bind.}} , \text{ where } m_p c^2 E_{\text{bind.}} = 10^{16} \text{ eV}^2 \]

Since,

\[
L_\gamma [10^{44} \text{ erg s}^{-1}] = 2 \times 10^{45} \text{ eV cm}^{-1} \\
\sigma_{\text{CR} \gamma} [A \text{ mb}] = A \times 10^{-27} \text{ cm}^2 \\
B [10^{-4} \text{ G}] = 3 \times 10^{-2} \text{ eV cm}^{-1}
\]

\[ \frac{L_\gamma \sigma_{\text{CR} \gamma} B}{A} = 6 \times 10^{16} \text{ eV}^2 , \text{ ergo... } f_{\text{CR} \gamma} = 50 \frac{Z}{26} \]

A similar expression holds for TeV photon transparency
Since,
\[
\frac{L_\gamma^{\text{Edd.}} \sigma_{\text{CR}_{\gamma}} B^{\text{Edd.}}}{A} = 4 \times 10^{23} \left( \frac{M}{M_\odot} \right)^{1/2} \text{ eV}^2
\]

Only heavily sub-Eddington power objects need apply!

If magnetic + photon luminosity are in equipartition:
\[
L_\gamma \approx \beta R^2 B^2
\]

Requiring, \( B < 4 \times 10^{-5} \text{ G} \) to ensure safe passage.

OVERALL MESSAGE: Compact Sources Disfavoured.
Are there Any Candidate Sources Left?

Accelerators of $10^{20}$ eV Iron nuclei

**Hillas Diagram**

- **AD (small)**
- **AD (large)**
- **GRB**
- **LHC**
- **JET**
- **SFR**
- **CC**

- Lines indicate $R_{\text{Larmor}} = 10^n R_{\text{Source}}$ for $n = 3, 6$.

**DESY.**
Are there Any Candidate Sources Left?
Example Candidate UHECR Source  
(a Nuclei Friendly Environment)

Stochastic Acceleration in Radio Lobes:

\[ B_{\text{source}} \sim 10^{-4} \text{ G} \]

\[ R_{\text{Larmor}}(10^{20} \text{ eV Fe}) \sim 0.04 \text{ kpc} \]

\[ t_{\text{acc}} < 10^6 \text{ yrs} \quad \text{for} \quad \beta_{\text{scat.}} > 10^{-2} \]

Diagram taken from Ferrari -1998

General PROBLEM for Large Accelerators-ACCELERATION TIME  
Andrew Taylor
Can Centaurus A's Radio Lobes Accelerate UHECR?

Yes, but requires:

\[ \beta_A > 0.1 \]

where

\[ \beta_A = \frac{1}{c} \frac{B}{\sqrt{4\pi m_p n_p}} \]

**Diffusion Coefficient**

- From resonant scattering between particles and magnetic field perturbations with Larmor radius $R_L$:

\[
B_0 + \delta B(k) \quad \text{resonance for } k \sim R_L^{-1}
\]

\[
P(k) \propto k^{-q}
\]

Probability to scatter off resonant mode within Larmor period.

\[
\frac{D_{xx}}{\beta} = \left\langle \frac{B_0^2}{(\delta B(k))^2} \right\rangle R_L = \frac{R_L}{kP(k)}
\]

\[
\propto p^{2-q}
\]

Since

\[
\frac{p^2}{D_{pp}} \sim \frac{D_{xx}}{\beta_{scat}^2}
\]

\[
D_{pp} \propto p^q
\]

- Bohm $\rightarrow q=1$
- Kolmogorov $\rightarrow q=5/3$
- Kraichnan $\rightarrow q=3/2$
- Hard-sphere $\rightarrow q=2$

Andrew Taylor
Particle Transport Equation

- Cut-offs arise naturally in the general solution of the transport equation for particles

\[
\frac{\partial f}{\partial t} = \nabla_p \cdot \left[ (D_{pp} \nabla_p f) - \frac{p}{\tau_{\text{loss}}(p)} f \right] - \frac{f}{\tau_{\text{esc}}(p)} + \frac{Q}{p^2}
\]

- Acceleration
- Radiative Losses
- Escape
- Source term

Andrew Taylor
Cut-off Shape

- Interplay of acceleration and cooling defines the value of the cut-off of the primary particles:

\[
\frac{dN}{dE_e} \propto E_e^{-\Gamma} e^{-(E_e/E_{max})^{\beta_e}} \quad \beta_e = 2 - q - r
\]

- In the following, demonstrations for this result will be shown for the case of stochastic acceleration scenarios. However, in reality, this result is more general, holding also for shock acceleration scenarios.

[see Schlickeisser et al. 1985, Zirakashvili et al. 2007, Stawarz et al. 2008]
A Simple Case- q=1, only escape

• Bohm diffusion (q=1) + only escape results in simple exponential cutoff.

• Some simplifications to the transport equation:

\[
\frac{\partial f}{\partial t} = \nabla_p \cdot \left[ (D_{pp} \nabla_p f) - \frac{p}{\tau_{\text{loss}}(p)} f \right] - \frac{f}{\tau_{\text{esc}}(p)} + \frac{Q}{p^2}
\]

- Steady state
- No losses
- Delta injection

Andrew Taylor
A Simple Case (II)- q=1, only escape

Rearranging the terms (and explicitly stating the dependences from p of the parameters):

\[
\frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D_0 \frac{p}{p_0} \frac{\partial f}{\partial p} \right) - \frac{f}{\tau_{\text{esc}}(p)} = \delta(p), \quad \tau_{\text{esc}}(p) \propto p^{-1}
\]

\[
\frac{\partial^2 f}{\partial p^2} + \frac{3}{p} \frac{\partial f}{\partial p} - \left( \frac{1}{D_0 \tau_0} \right) f = \delta(p)
\]

Cutoff comes from balancing 1\textsuperscript{st} and 3\textsuperscript{rd} term

Recall generally, \( \beta_e = 2 - q - r \)

\( q = 1, \ r = 0, \ \rightarrow \ \beta_e = 1 \)

(Note- energy losses for the \( r = 0 \) case will not alter this result)
Intuitive Insights into Cut-off Shape Origin

Consider the steady-state case of diffusion (constant diffusion coefficient) of particles into an absorbing medium

$$ \nabla \cdot (D_{xx} \nabla f) - \frac{f}{\tau(x)} = \delta(r) $$

For $\tau(x) = \tau_*(x/x_*)^2$ \hspace{1cm} f \propto \text{const.}$

For $\tau(x) = \tau_*$ \hspace{1cm} f \propto e^{-x/x_\tau}$

For $\tau(x) = \tau_*(x/x_*)^{-2}$ \hspace{1cm} f \propto e^{-(x/x_\tau)^2}$

Andrew Taylor
End of First Lecture
Shock Acceleration

\[ E_2 = E_1 \left( \frac{1 + \beta \mu_1}{1 + \beta \mu_2} \right) \]

\[ E_2 = \Gamma^2 E_1 (1 - \beta \mu_1) (1 + \beta \mu'_2) \]

\[ \mu' = \frac{\mu - \beta}{1 - \beta \mu} \]

\[ E_2 = \Gamma^2 E_1 (1 - \beta \mu_1) \left( 1 + \beta \left( \frac{\mu_2 - \beta}{1 - \beta \mu_2} \right) \right) \]
Random Walks

$$f(x, t) = \gamma(t + 1)/[\gamma([t - x]/2 + 1)\gamma([x + t]/2 + 1)]/(2^t)$$

From Stirling’s formula

$$\gamma(x) \approx \frac{(x/e)^x}{\pi^{1/2}}$$

$$\gamma(x + 1) \approx (2\pi x)^{1/2}(x/e)^x$$

$$f(x, t) \approx \frac{t^t e^{-t}}{[(t - x)/2]^{(t-x)/2}[(t + x)/2]^{(t+x)/2}e^{-t}}$$

$$\log[f(x, t)] \approx \frac{t}{[\frac{1}{2}(t - x)(\log t/2 - x/2t)] + [\frac{1}{2}(t + x)(\log t/2 + x/2t)]}$$

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DESY.
Particle Acceleration with Cooling

\[
\frac{\text{d}E_e}{\text{d}t} = \frac{4}{3} \Gamma_e^2 \sigma_T U_B
\]

\[
t_{\text{cool}} = \frac{9}{8 \pi \alpha} \left( \frac{m_e}{E_{\gamma}^{\text{sync}}} \right) t_{\text{lar}}
\]

\[
t_{\text{cool}} = E_e \frac{\text{d}t}{\text{d}E_e}
\]

\[
\sigma_T U_{B_{\text{crit}}} \frac{hc}{(m_e c^2)^2} = \left( \frac{2 \pi}{3} \right) \alpha
\]

\[
t_{\text{cool}} = \frac{9}{8 \pi \alpha} \frac{h}{E_e} \frac{U_{B_{\text{crit.}}}}{U_B}
\]

Andrew Taylor
Particle Acceleration with Cooling

\[ t_{\text{cool}} = \frac{9}{8\pi\alpha} \left( \frac{E_e}{E_{\text{sync}}} \right) \frac{h}{m_e} U_{B_{\text{crit}}} \]

\[ t_{\text{lar}} = \frac{2\pi E_e}{eBc} = \Gamma_e \left( \frac{B_{\text{crit}}}{B} \right) \frac{h}{m_e} \]

\[ E_{\text{sync}}^\gamma = \Gamma_e^2 \left( \frac{B}{B_{\text{crit}}} \right) m_e \]

Andrew Taylor
Consider the steady-state case of diffusion (constant diffusion coefficient) of particles into an absorbing medium

\[ \nabla \cdot (D_{xx} \nabla f) - \frac{f}{\tau(x)} = \delta(r) \]

For \( \tau(x) = \tau_*(x/x_*)^2 \) \( f \propto \text{const.} \)

For \( \tau(x) = \tau_* \) \( f \propto e^{-x/x_\tau} \)

For \( \tau(x) = \tau_*(x/x_*)^{-2} \) \( f \propto e^{-(x/x_\tau)^2} \)

Andrew Taylor
Intuitive Insights into Cut-off Shape Origin

\[ \begin{align*}
D_{xx} \frac{\partial^2 f}{\partial x^2} + D_{xx} \frac{2}{x} \frac{\partial f}{\partial x} - \frac{f}{\tau(x)} &= 0 \\
\text{For } &\quad \tau(x) = \tau_* \\
&\quad f \propto e^{-x/x_\tau}
\end{align*} \]
Cut-off Shape: Electrons & Photons

\[ dN_{ee} / dE_e \propto E_e^{-1} e^{-\left( E_e / E_{cut} \right) \beta_e} \]

\[ E_\gamma = \gamma_e^2 \left( \frac{B}{B_{crit}} \right) m_e \]

\[ f(x) = \int_0^{\infty} e^{-\left( x / y^2 \right)} e^{-y^{\beta_e}} dy \]
\[ y^2 \left( y^{\beta_e} - \frac{1}{\beta_e} \right) = \frac{2x}{\beta_e} \]

\[ y^2 \approx \left( \frac{2x}{\beta_e} \right)^{\frac{2}{\beta_e+2}} \]

\[ \frac{x}{y^2} \approx x^{\frac{\beta_e}{\beta_e+2}} \quad \rightarrow \quad \beta_\gamma = \frac{\beta_e}{\beta_e + 2} \]
Cut-off Shape- Emission Dependence

\[ \frac{dN}{dE_e} \propto E_e^{-\Gamma} e^{-\left(E_e/E_{\text{max}}\right)^{\beta_e}} \]

\[ \frac{dN}{dE_\gamma} \propto E_\gamma^{-\Gamma} e^{-\left(E_\gamma/E_{\text{max}}\right)^{\beta_\gamma}} \]

- Different emission processes dictate different relation between electrons and gamma rays

\[ dN/dE_e \propto E_e^{-\Gamma} e^{-\left(E_e/E_{\text{max}}\right)^{\beta_e}} \]

\[ dN/dE_\gamma \propto E_\gamma^{-\Gamma} e^{-\left(E_\gamma/E_{\text{max}}\right)^{\beta_\gamma}} \]

• Synchrotron/IC Thomson: \( \beta_\gamma = \frac{\beta_e}{\beta_e + 4} \)
• SSC: \( \beta_\gamma = \beta_e \)
• IC (Klein Nishina) \( \beta_\gamma = \frac{\beta_e}{\beta_e + 2} \)

Good measurement of gamma ray cut-off can give insight on the cut-off region of primary electrons

Andrew Taylor

DESY.
Observation of Cut-offs in Gamma-ray Spectra

- Test case- Vela Pulsar (brightest source)

\[
\frac{dN}{dE_\gamma} \propto E_\gamma^{-\Gamma} e^{-(E_\gamma/E_{max})^{\beta_\gamma}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) [ph/cm(^2)/s/GeV]</td>
<td>((1.39^{+0.12}_{-0.10}) \times 10^{-5})</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>(1.019 \pm 0.011)</td>
</tr>
<tr>
<td>(E_c) [GeV]</td>
<td>(0.238 \pm 0.016)</td>
</tr>
<tr>
<td>(\beta_\gamma)</td>
<td>(0.464 \pm 0.009)</td>
</tr>
<tr>
<td>(E_s) (fixed) [GeV]</td>
<td>(0.83255)</td>
</tr>
</tbody>
</table>

- Note- MCMC method used to explore ‘good-fit’ region. This has the benefit of being stable on the landscape being explored


Andrew Taylor
MCMC Parameter Constraints

\[ \frac{dN}{dE_\gamma} \propto E_\gamma^{-\Gamma} e^{-\left( \frac{E_\gamma}{E_{\text{max}}} \right)^\beta_\gamma} \]

False minima

DESY.
Observation of Cut-offs in Gamma-ray Spectra

- Brightest AGN Flare: 3C 454 Nov 2010

![Graph showing energy spectrum of 3C 454.3 with fit parameters and cut-off around 1 GeV]

- Parameters:
  - $N$ [ph/cm$^2$/s/GeV] = $(4.7^{+3.9}_{-1.2}) \times 10^{-5}$
  - $\Gamma = 1.87^{+0.08}_{-0.12}$
  - $E_c$ [GeV] = $1.1^{+1.6}_{-0.9}$
  - $\beta_\gamma = 0.4 \pm 0.1$
  - $E_s$ (fixed) [GeV] = 0.41275

- Caveats:
  - Values obtained on a 7 days integration (for statistics)
  - Spectrum variable during the flare -> superposition effects?

Observation of Cut-offs in Gamma-ray Spectra

- 2nd Brightest AGN Flare - 3C 279 June 2015


Values obtained on a 3 days integration
Note- X-ray observations during flare indicated that $\Gamma = 1.17 \pm 0.06$

$\beta_{\gamma} = \frac{\beta_e}{\beta_e + 2}$  
Andrew Taylor
3C 279 June 2015 Flare-Temporal Evolution
Prospects for CTA (South)

- Study using the expected CTA performance
- Fermi data integrated over 3 days
- Constraint on $\beta\gamma$ parameter at 10% level obtained during only 0.5 hr flare!
H.E.S.S. Phase I: 2002-2012
▪ 4 telescopes of 12m
▪ 100 GeV - 100 TeV

H.E.S.S. Phase II: 2012-++
▪ Addition of CT5 to the array: 28m
▪ ~30 GeV - 100 TeV

CT5 allows $E < 100$ GeV measurements
— best for:
● High redshift AGN + GRBs
● EBL studies at large $z$
Can We Do Better Already?
Fermi + H.E.S.S.II Fit

- Joint fit of Fermi-LAT data (9 hours centred on HESSII obs.) taken on night 2
  
  \[ \beta_\gamma = 0.34^{+0.32}_{-0.14} \]

(HESSII data taken from ICRC2017 Presentation)
The pp Cross-Section
Cut-Offs for Primary and Secondaries

\[ \Phi_\gamma(E_\gamma) = 4\pi n_H \int \frac{d\sigma}{dE_\gamma}(p_p, E_\gamma) J(p_p) dp_p \]

For spectra of the form,

\[ J_p(p_p) = \frac{A}{p_p^\alpha} \exp \left[ - \left( \frac{p_p}{p_p^{\text{max}}} \right)^\beta \right] \]

and the cutoff regions may be fit with a function of the form

\[ \Phi_\gamma(E_\gamma) = \frac{A'}{E_\gamma^{\alpha'}} \exp \left[ - \left( \frac{E_\gamma}{E_\gamma^{\text{max}}} \right)^{\beta'} \right] \]

where \[ \beta' = \frac{a\beta}{\beta+b} \]

Kafexhiju et al., *Phys.Rev. D90* 12, 123014 (2014)
\[ \sigma_\pi = \sigma_{\text{inel}} \langle n_{\pi_0} \rangle \]


Note- Kamae description has artificially high threshold (~0.5 GeV) at DESY.
Constraining the Particle Spectra in Solar flares

- Optimal level of statistics (bright low energy transients, plenty of photons)
- Retrieve the primary particle spectrum (using the most up-to-date cross sections)

\[ \frac{dN}{dT} = \frac{dN}{dT} \bigg|_{T=T_0} \left( \frac{T}{T_0} \right)^{-\alpha} \]

\[ \alpha = 2.5, \quad T_p^{\text{cut}} \approx \frac{1}{\text{GeV/mic}} \]

If we try to fit high energy cut-off, strong degeneracy exists with the spectral index.
Constraining the Particle Spectra in Solar flares

- Optimal level of statistics (bright low energy transients, plenty of photons)
- Retrieve the primary particle spectrum (using the most up-to-date cross sections)

\[ \frac{dN}{dT} = \frac{dN}{dT}|_{T=T_0} \left( \frac{T}{T_0} \right)^{-\alpha} \]

\[ \alpha = 2.5, \quad T_p^{cut} \approx 1 \text{GeV} \]

This degeneracy can be broken by the lower energy emission detected by GBM, which nuclear de-excitation is expected to contribute/dominate.
Future Sources to be Probed.....GRB CTA (South)

- Evolution of spectra during flare
- Detection of as yet undetected VHE transients (e.g., GRB)
- Detection of unexpected new VHE transient phenomena
Recent HESSII GRB Upper Limits

From Ackermann et al. 2011

- Time-integrated photon spectrum (3.3 s – 21.6 s)

Upper limits for GRB140818B

- Center of the H.E.S.S. FoV
- 1.5 deg off the center of the FoV
- BAT best fit model in T0 < t < T0 + 21.6s

HESS PRELIMINARY
\[
y^2 \left( y^{\beta_e} - \frac{1}{\beta_e} \right) = \frac{2x}{\beta_e}
\]

\[
y^2 \approx \left( \frac{2x}{\beta_e} \right)^{\frac{2}{\beta_e+2}}
\]

\[
\frac{x}{y^2} \approx x^{\frac{\beta_e}{\beta_e+2}}
\]
\[
\beta_e = 1
\]

\[
y^{\frac{2\beta_e}{\beta_e + 2}} \left( y^{\beta_e} - \frac{1}{\beta_e} \right) = \left( \frac{2}{2 + \beta_e} \right) x^{\frac{\beta_e}{\beta_e + 2}}
\]

\[
y^2 \approx x^{\frac{2}{\beta_e + 4}}
\]

\[
\frac{x}{y^2} \approx x^{\frac{\beta_e + 2}{\beta_e + 4}}
\]

\[
\left( \frac{x}{y^2} \right)^{\frac{\beta_e}{\beta_e + 2}} \approx x^{\frac{\beta_e}{\beta_e + 4}}
\]

Andrew Taylor
\[ \beta_e = 1 \]
\[ \beta_\gamma = 1/3 \]

\[ E_\gamma \text{d}N/\text{d}E = 1.2 \cdot \exp\left(-\frac{x}{0.64}\right)^{0.37} \]

\[ \beta_e = 2 \]
\[ \beta_\gamma = 1/2 \]

\[ E_\gamma \text{d}N/\text{d}E = 1.2 \cdot \exp\left(-\frac{x}{0.8}\right)^{0.58} \]
pN Interactions
There are also multiple channels by which multi-MeV gamma-ray emission can be produced from non-thermal electrons:

- Secondary Bremstrahlung
- Secondary Annihilation in flight
- Primary Bremstrahlung
There are multiple channels by which multi-MeV gamma-ray emission can be produced from non-thermal protons:

- **Nuclear Line Emission**
  - \( a+B \rightarrow B^* \)

- **Nuclear Line Continuum**:
  - statistical photons
  - direct photons
  - pre-equilibrium processes

- **Hard Photon Emission**
  (nuclear Bremstrahlung)
Other Bright Sources Seen By Fermi

- Gamma ray emission during flaring events
- Most probable scenario, magnetic reconnection in Solar Corona

Emission of gamma rays!
Most important channels:
- De-excitation of atomic nuclei (low energy)
- Decay of neutral pions $\pi_0 - \gamma\gamma$ (high energy)
π Spectra for $T_p^{\text{th}} < T_p < 1$ GeV


Note - Kamae description has artificially high threshold (~0.5 GeV)
γ-ray Spectra for $T_p^{th} < T_p < 1$ GeV

Stochastic Particle Acceleration - Random Walk Result (Spatial)

\[ \nabla \cdot (D_{xx} \nabla f) = \delta(r) \]

Spherically symmetric case:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} f \right) = \delta(r) \]

\[ u = rf \]

\[ \frac{1}{r} \frac{\partial^2 u}{\partial r^2} = \delta(r) \]

Andrew Taylor
Stochastic Particle Acceleration-
Random Walk Result (Spatial)

\[
\frac{1}{r} \frac{\partial^2 u}{\partial r^2} = \delta(r)
\]

\[
u = Ar + B
\]

\[
f = A + \frac{B}{r}
\]
• Relativistic particle will lose its energy on a timescale that depends on the different processes

\[ \tau_{\text{cool}}(E) \propto E^r \]

- Synchrotron: \( r = -1 \)
- Inverse Compton (Thomson): \( r = -1 \)
- Inverse Compton (K.N): \( r = 1 \)
Radiative Loss Timescale

\[ \tau_{\text{cool}}(E) \propto E^r \]

Synchrotron: 
\[ r = -1 \]

Inverse Compton (Thomson):
\[ r = -1 \]

Inverse Compton (K.N):
\[ r = 1 \]

\[
E_\gamma \approx \gamma_e^2 \left( \frac{B}{B_{\text{crit}}} \right) m_e \\
= bE_e
\]

\[
E_\gamma = \left( \frac{b}{1 + b} \right) E_e
\]