Lecture Plan:

1) Cosmic Ray acceleration - accelerated spectrum, efficient accelerators, nuclei friendly

PROBLEMS

2) Cosmic Ray proton + nuclei interaction rates in extragalactic radiation fields

PROBLEMS

3) Cosmic Ray propagation through Galactic and extragalactic magnetic fields
COSMIC RAYS: High Energy Proton and Nuclei Interactions During Propagation
Cosmic Ray Proton Energy Losses
The Interaction Rate

\[ R = \int_0^\infty d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma} \int_{-1}^1 \frac{1}{2} d(\cos \theta) \frac{d\sigma}{d\cos \theta} (1 - \beta \cos \theta) \]

All values above in lab frame
The Interaction Rate

\[ R = \int_{0}^{\infty} d\epsilon_{\gamma} \frac{dn}{d\epsilon_{\gamma}} \int_{-1}^{1} \frac{1}{2} d(\cos \theta) \frac{d\sigma}{d\cos \theta} (1 - \beta \cos \theta) \]

Since,

\[ \epsilon_{\gamma} E_{P} = \epsilon' E_{P} (1 + \beta \cos \theta) \]

\[ (1 + \beta \cos \theta) d \cos \theta = \frac{\epsilon_{\gamma} E_{P}}{\epsilon' E_{P}} \frac{d(\epsilon' E_{P})}{\epsilon' E_{P}} \]

\[ R = \int_{0}^{\infty} d\epsilon_{\gamma} \frac{dn}{d\epsilon_{\gamma}} \int_{0}^{2\epsilon_{\gamma} E_{P}} d(\epsilon_{\gamma} E_{P}) \frac{\epsilon_{\gamma} E_{P}}{\epsilon'_{\gamma} E_{P}^2} \frac{d\sigma}{d(\epsilon_{\gamma} E_{P})} \]

\[ = \frac{m_{p}^2}{2E_{P}^2} \int_{0}^{\infty} d\epsilon'_{\gamma} \frac{1}{\epsilon'_{\gamma}^2} \frac{dn}{d\epsilon'_{\gamma}} \int_{0}^{2\epsilon'_{\gamma} E_{P}/m_{p}} d\epsilon_{\gamma} \frac{d\sigma}{d\epsilon_{\gamma}} \]
Cosmic Ray Proton Interactions

For $E_{\text{proton}} < 10^{19.6}$ eV

$\gamma \rightarrow p \rightarrow e^- \rightarrow e^+$

For $E_{\text{proton}} > 10^{19.6}$ eV

$\gamma \rightarrow p \rightarrow \pi^+ \rightarrow n$
Cosmic Ray Proton Interactions

For $E_{\text{proton}} < 10^{19.6}$ eV

For $E_{\text{proton}} > 10^{19.6}$ eV

$E_{\gamma}^{\text{th}} \sim 1\text{MeV}$

$E_{\gamma}^{\text{th}} \sim 140\text{ MeV}$
Cosmic Radiation Fields - Energy Density

CMB

Dust

Stellar

\[ E_{\gamma}^2 \frac{dN_{\gamma}}{dE_{\gamma}} \text{ [eV cm}^{-3}\text{]} \]

\[ nW m^{-2} sr^{-1} \]

\[ E_{\gamma} \text{ [eV]} \]

\[ \nu \text{ [Hz]} \]

DIRBE
ISO
HST
IRTS
Cosmic Radiation Fields - Number Density

\( E_{\gamma} dN_{\gamma}/dE_{\gamma} [\text{cm}^{-3}] \)

\( \nu [\text{Hz}] \)

\( E_{\gamma} [\text{eV}] \)

CMB

CIB
CMB- Total Number Density

\[ \frac{dn}{d\epsilon} = \frac{8\pi}{(hc)^3} \frac{\epsilon^2}{e^{\epsilon/kT} - 1} \]

\[ n_{\gamma}^{BB} = \frac{8\pi (kT)^3}{(hc)^3} \int_0^\infty \frac{x^2}{e^x - 1} dx \]

\[ \frac{8\pi (kT_{CMB})^3}{(hc)^3} \approx 170 \text{ cm}^{-3} \]

\[ n_{\gamma}^{CMB} = 8\pi \frac{(kT_{CMB})^3}{(hc)^3} \gamma(3) \zeta(3) \approx 400 \text{ cm}^{-3} \]

\[ \zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x} \]
Energy Loss Rates due to Proton Interactions

\[ R = \frac{m_p^2 c^4}{2E^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{dn}{d\epsilon_\gamma} \int_0^{2E\epsilon_\gamma/(m_p c^2)} d\epsilon'_\gamma \epsilon'_\gamma \sigma_{p\gamma}(\epsilon'_\gamma) K_p \]

where \( R \) is the energy loss rate

where \( K_p \) is the inelasticity
Energy Loss Rates due to Proton Interactions

\[ R = \frac{m_p^2 c^4}{2E^2} \int_0^\infty \int_0 \frac{d\varepsilon_\gamma}{\varepsilon_\gamma^2} \frac{1}{d\varepsilon_\gamma} \int_0^{2E\varepsilon_\gamma/(m_p c^2)} d\varepsilon'_\gamma \varepsilon'_\gamma \sigma_{\gamma\gamma}(\varepsilon'_\gamma) K_p \]

where \( R \) is the energy loss rate

where \( K_p \) is the inelasticity
Energy Loss Rates due to Proton Interactions

\[ R = \frac{m_p^2 c^4}{2E^2} \int_0^\infty \frac{1}{\epsilon^2} \frac{dn}{d\epsilon} \int_0^{2E\epsilon / (m_pc^2)} d\epsilon' \epsilon' \sigma_{p\gamma}(\epsilon') K_p \]

where \( R \) is the energy loss rate

where \( K_p \) is the inelasticity
....with Different IR Backgrounds
Interactions of Cosmic Ray Protons with CMB:

Pair Creation-

\[ p + \gamma \rightarrow p + e^+ + e^- \]

\[ E_\gamma \sim 1 \text{ MeV} \]

Photo-Meson Production-

\[ p + \gamma \rightarrow n + \pi^+/p + \pi^0 \]

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

\[ E_{\gamma}^{\text{th}} \sim 140 \text{ MeV} \]
Threshold Energy- Proton Pair Production

\[
(E_p + E_\gamma)^2 - (p_p - E_\gamma)^2 = (m_p + 2m_e)^2
\]

\[
m_p^2 + 2E_pE_\gamma + 2p_pE_\gamma \approx m_p^2 + 4m_p m_e
\]

\[
E_p \approx \frac{m_e}{E_\gamma} m_p \approx \left( \frac{0.5 \times 10^6}{6 \times 10^{-4}} \right) 0.9 \times 10^9 = 8 \times 10^{17} \text{ eV}
\]

Repeat this calculation for pion production
Photo-Pion Production Rate

\[ R = \frac{m_p c^4}{2E^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{1}{d\epsilon_\gamma} \int_0^{2E\epsilon_\gamma / (m_p c^2)} d\epsilon_\gamma' \epsilon_\gamma' \sigma_{p\gamma}(\epsilon_\gamma') K_p \]

Assuming the cross-section is approximately:

\[ \sigma_{p\gamma}(\epsilon_\gamma) = 0 \quad \epsilon_\gamma < E - \Delta \]

\[ \sigma_{p\gamma}(\epsilon_\gamma) = \sigma_{p\gamma} \quad \epsilon_\gamma > E + \Delta \]

Where \( \sigma_{p\gamma} = 0.5 \text{ mb}, \quad E = 300 \text{ MeV}, \quad \Delta = 100 \text{ MeV} \)
Photo-Pion Production Rate

\[ R(\Gamma) \approx \sigma_0 \int_{(E_0-\Delta_0)/2\Gamma}^{(E_0+\Delta_0)/2\Gamma} \left( \frac{\epsilon^2 - \left[ \frac{(E_0 - \Delta_0)/2\Gamma}{\epsilon^2} \right]^2}{\epsilon^2} \right) \frac{dn}{d\epsilon} \, d\epsilon + \cdots \]

\[ \sigma_0 \int_{\infty}^{(E_0+\Delta_0)/2\Gamma} \left( \frac{\left[ \frac{(E_0 + \Delta_0)/2\Gamma}{2\Gamma} \right]^2 - \left[ \frac{(E_0 - \Delta_0)/2\Gamma}{2\Gamma} \right]^2}{\epsilon^2} \right) \frac{dn}{d\epsilon} \, d\epsilon \]

- \( E = 10^{18} \text{ eV} \)
- \( E = 10^{18.5} \text{ eV} \)
- \( E = 10^{19} \text{ eV} \)
- \( E = 10^{19.5} \text{ eV} \)
- \( E = 10^{20} \text{ eV} \)
- \( E = 10^{20.5} \text{ eV} \)
- \( E = 10^{21} \text{ eV} \)
Photo-Pion Production Rate

\[ R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{(x^2 - x_1(\Gamma)^2)}{e^x - 1} \, dx + \]

\[ n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{(x_2^2(\Gamma) - x_1^2(\Gamma))}{e^x - 1} \, dx \]

Where,

\[ x_1 = \frac{(E - \Delta)m_p}{2kT_{\text{CMB}}E_p} \]
Photo-Pion Production Rate

\[ R = \frac{m_p^2 c^4}{2E^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{d\gamma}{d\epsilon_\gamma} \int_0^{2E\epsilon_\gamma/(m_p c^2)} \epsilon_\gamma' \sigma_{p\gamma}(\epsilon_\gamma') K_p \]

\[ \approx 0.2 \sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{d\gamma}{d\epsilon_\gamma} \]

Since,

\[ \epsilon_\gamma \frac{d\gamma}{d\epsilon_\gamma} = 170 \left( \frac{\epsilon_\gamma}{kT_{\text{CMB}}} \right)^3 \frac{e^{\epsilon_\gamma/kT_{\text{CMB}}} - 1}{e^{\epsilon_\gamma/kT_{\text{CMB}}}} \text{ cm}^{-3} \]

Where \( \Gamma = \frac{E_p}{m_p c^2} \) is the proton Lorentz factor.

\[ E_\gamma \frac{dN_\gamma}{dE_\gamma} = \sigma_{p\gamma}(E_\gamma) \frac{dkT_{\text{CMB}}}{E_\gamma} \]

\[ \sigma_{p\gamma}(E_\gamma) \propto E_\gamma^2 \]

\[ E_\gamma \approx 2.7kT_{\text{CMB}} \]
Photo-Pion Production Rate

With, \( kT_{\text{CMB}} \approx 2 \times 10^{-4} \text{ eV} \)

\[
R \approx 0.2\sigma_{p\gamma} \int \frac{E + \Delta}{2\Gamma} \frac{dn}{d\epsilon_{\gamma}} d\epsilon_{\gamma} \\
\approx \left( \frac{l_0}{e^{-x_1} (1 - e^{-x_1})} \right)^{-1}
\]

Where \( l_0 \) is 5 Mpc and

\[
x_1 = \frac{(E - \Delta)m_p}{2kT_{\text{CMB}}E_p} = \frac{10^{20.7} \text{ eV}}{E_p} \]

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Photo-Pion Production Rate

With, \[ kT_{\text{CMB}} \approx 2 \times 10^{-4} \text{ eV} \]

\[
R \approx 0.2\sigma_{p\gamma} \int \frac{E + \Delta}{2\Gamma} d\varepsilon_\gamma \frac{dn}{d\varepsilon_\gamma} \approx \left( \frac{l_0}{e^{-x_1} (1 - e^{-x_1})} \right)
\]

Where \( l_0 \) is 5 Mpc and \( x_1 = \frac{10^{20.5} \text{ eV}}{E_p} \)
Cosmic Ray Nuclei Energy Losses
Cosmic Ray Nuclei Interactions

For $10^{19.7} < E_{(A,Z)} < 10^{20.2}$ eV

For $E_{(A,Z)} < 10^{19.7}$ and $E_{(A,Z)} < 10^{20.2}$ eV
Cosmic Ray Nuclei Interactions

Photo-disintegration:

\[ N_{(A,Z)} + \gamma \rightarrow N'_{(A',Z')} + (Z-Z')p + (A-A'+Z'-Z)n, \quad E_\gamma \sim 30\text{MeV} \]

\[ n \rightarrow p + e^- + \nu_e \]
Energy Loss Rates due to Nuclei Interactions

\[ R = \frac{A^2 m_p^2 c^4}{2E^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{dn}{d\epsilon_\gamma} \int_0^{2E\epsilon_\gamma/(Am_p c^2)} d\epsilon'_\gamma \epsilon'_\gamma \sigma_{N\gamma}(\epsilon'_\gamma) K_p \]

where \( R \) is the energy loss rate
Cosmic Radiation Fields

E^2 dN_γ/dE_γ [eV cm^{-3}]

ν [Hz]

10^{11} 10^{12} 10^{13} 10^{14} 10^{15}

CMB

Dust

Stellar

ν_γ [Hz]

E_γ [eV]

DIRBE
ISO
HST
IRTS

nW m^{-2} sr^{-1}

10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}

10^{0} 10^{1} 10^{2} 10^{3}

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Cosmic Ray Disintegration During Propagation
Cosmic Ray Spectra
Assumptions on Source Population

Spatial Distribution

\[
\frac{dN}{dV_C} \propto (1 + z)^3 \quad z < 1.9
\]

\[
\frac{dN}{dV_C} \propto (1 + 1.9)^3 \quad 1.9 < z < 2.7
\]

\[
\frac{dN}{dV_C} \propto (1 + 1.9)^3 e^{-z/1.7} \quad z > 2.7
\]

Energy Distribution

\[
\frac{dN}{dE} \propto E^{-\alpha} \exp\left[-E/E_{Z,\text{max}}\right]
\]

\[
E_{Z,\text{max}} = (Z/26) \times E_{Fe,\text{max}}
\]

Note- magnetic field horizon effects are neglected in the following. This amounts to assuming: \( d_s < (ct_H \lambda_{\text{scat}})^{1/2} \)

ie. the source distribution may be approximated to be spatially continuous (also note, presence of \( t_H \) term comes from temporally continuous assumption)
A Cosmological Distribution of Sources

Distribution of sources in a comoving volume

\[ dV_c = 4\pi \chi^2 d\chi \]

\[ d\chi = \frac{dz}{H} \approx \frac{dz}{H_0 (\Omega_M (1 + z)^3 + \Omega_\Lambda)^{1/2}} \]
Do Protons or Nuclei Fit the Data?
Or
**Assumptions on Source Population**

### Spatial Distribution

\[
\frac{dN}{dV_C} \propto (1 + z)^n \quad z < z_{\text{max}}
\]

\(n = -6, -3, 0, 3\)

### Energy Distribution

\[
\frac{dN}{dE} \propto E^{-\alpha} \exp\left[-\frac{E}{E_{Z,\text{max}}}\right]
\]

\(E_{Z,\text{max}} = \left(\frac{Z}{26}\right) \times E_{\text{Fe, max}}\)

Note- magnetic field horizon effects are neglected in the following. This amounts to assuming:

\[d_s < (ct_H \lambda_{\text{scat}})^{1/2}\]

ie. the source distribution may be approximated to be spatially continuous (also note, presence of \(t_H\) term comes from temporally continuous assumption)
MCMC Likelihood Scan: Spectral + Composition Fits

\[ L(f_p, f_{He}, f_N, f_{Si}, E_{\text{max}}, \alpha) \propto \exp\left(-\chi^2/2\right) \]

n=3 evolution result

![Graph showing energy spectra and fits with various compositions](image)

- \( E_{Fe, \text{max}} = 10^{20.2} \text{ eV} \)
- \( \alpha = 0.6 \)


- Graphs showing energy spectra with Auger 2014 points
- RMS(X^max) [g cm\(^{-2}\)] vs. log\(_{10}\) Energy [eV]
- Protons and Iron data points
MCMC Likelihood Scan: “Soft” Spectra Solutions

\[ L(f_p, f_{He}, f_N, f_{Si}, E_{max}, \alpha) \propto \exp(-\chi^2/2) \]

n=-6 evolution result

\[
E^2 \frac{dN}{dE} \text{ [eV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}] = \begin{cases} 
E_{Fe, max}=10^{20.5} \text{ eV} \\
\alpha=1.8
\end{cases}
\]
Similar conclusion arrives to by others (eg. ADD REF. TO KAMPERT ET AL.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$n = -6$</th>
<th>$n = -3$</th>
<th>$n = 0$</th>
<th>$n = 3$</th>
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<tbody>
<tr>
<td></td>
<td>Best-fit Value</td>
<td>Posterior Mean &amp; Standard Deviation</td>
<td>Best-fit Value</td>
<td>Posterior Mean &amp; Standard Deviation</td>
</tr>
<tr>
<td>$f_p$</td>
<td>0.03</td>
<td>0.14 ± 0.12</td>
<td>0.08</td>
<td>0.15 ± 0.13</td>
</tr>
<tr>
<td>$f_{He}$</td>
<td>0.50</td>
<td>0.21 ± 0.17</td>
<td>0.42</td>
<td>0.17 ± 0.16</td>
</tr>
<tr>
<td>$f_N$</td>
<td>0.40</td>
<td>0.50 ± 0.18</td>
<td>0.42</td>
<td>0.51 ± 0.19</td>
</tr>
<tr>
<td>$f_{Si}$</td>
<td>0.06</td>
<td>0.11 ± 0.12</td>
<td>0.08</td>
<td>0.12 ± 0.13</td>
</tr>
<tr>
<td>$f_{Fe}$</td>
<td>0.01</td>
<td>0.052 ± 0.039</td>
<td>0.0</td>
<td>0.053 ± 0.042</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.8</td>
<td>1.83 ± 0.31</td>
<td>1.6</td>
<td>1.67 ± 0.36</td>
</tr>
<tr>
<td>$\log_{10}(E_{Fe,\text{max}}/\text{eV})$</td>
<td>20.5</td>
<td>20.55 ± 0.26</td>
<td>20.5</td>
<td>20.52 ± 0.27</td>
</tr>
</tbody>
</table>

Flatter spectra preferred for negative source evolution

Hard spectra preferred for source evolution following that of the SFR
An Analytic Description of these Results
Differential Equation Describing System State

\[
\frac{d}{dt} \begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix} = \Lambda \begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix}
\]

\[
\Lambda = \begin{pmatrix}
- \left( \frac{1}{\tau_{56 \rightarrow 55}} + \frac{1}{\tau_{56 \rightarrow 54}} + \cdots \right) & 0 & 0 \\
- \left( \frac{1}{\tau_{55 \rightarrow 54}} + \frac{1}{\tau_{55 \rightarrow 53}} + \cdots \right) & 0 & 0 \\
- \left( \frac{1}{\tau_{54 \rightarrow 53}} + \frac{1}{\tau_{54 \rightarrow 52}} + \cdots \right) & 0 & 0 
\end{pmatrix}
\]

by

\[
f_q(t) = \sum_{n=q}^{56} A_n f_n(t)
\]

then

\[
f_q(t) = \sum_{n=q}^{56} A_n e^{-\lambda_n t} f_n(0)
\]

(where \(A_n\) values are set by the initial conditions)
Only Considering Single Nucleon Losses

\[ \Lambda = \begin{pmatrix} - \frac{1}{\tau_{56 \to 55}} & 0 & 0 \\ 0 & - \frac{1}{\tau_{55 \to 54}} & 0 \\ 0 & 0 & - \frac{1}{\tau_{54 \to 53}} \end{pmatrix} \]

and

\[ f_q(t) = \sum_{n=q}^{56} f_{56}(0) \frac{\tau_q \tau_{56-q-1}}{\prod_{p=q}^{56} (\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}} \]
Consider

\[
\frac{df_q}{dt} + \frac{f_q}{\tau_q} = \frac{f_{q+1}}{\tau_{q+1}}
\]

\[
e^{-\frac{t}{\tau_q}} \frac{d}{dt} \left[ e^{\frac{t}{\tau_q}} f_q \right] = \frac{f_{q+1}}{\tau_{q+1}}
\]

\[
f_q = e^{\frac{-t}{\tau_q}} \int e^{\frac{t}{\tau_q}} \frac{f_{q+1}}{\tau_{q+1}} dt
\]

Assume solution is true for q, apply to q+1

\[
\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1} \tau_{n}^{56-q-2}}{\prod_{p=q+1}^{56} (\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}
\]
Nuclear Cascade Description

Assume solution is true

\[
\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1} \tau_{n}^{56-q-2}}{\prod_{p=q+1}^{56} (\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}
\]

\[
f_q = e \left( \frac{-t}{\tau_q} \right) \int e \left( \frac{t}{\tau_q} \right) \frac{f_{q+1}}{\tau_{q+1}} dt
\]

\[
\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{n}^{56-q-2}}{\prod_{p=q+1}^{56} (\tau_n - \tau_p)} \left[ \left( \frac{1}{\tau_q} - \frac{1}{\tau_n} \right)^{-1} e^{-\frac{t}{\tau_n}} \right] - ce^{\frac{-t}{\tau_q}}
\]

Since \( f_q(0) = 0 \)

\[
c = \sum_{n=q+1}^{56} \frac{\tau_{q} \tau_{n}^{56-q-1}}{\prod_{p=q}^{56} (\tau_n - \tau_p)}
\]
These are equivalent if:

\[
\sum_{n=q+1}^{56} \frac{\tau_n^{56-q-2}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}} = \sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_q}}
\]

Consider:

\[
\frac{w^2}{(w-x)(w-y)(w-z)} + \frac{x^2}{(x-w)(x-y)(x-z)} + \frac{y^2}{(y-w)(y-x)(y-z)} = -\frac{z^2}{(z-w)(z-x)(z-y)}
\]
End of Second Lecture
CMB- Total Number Density

\[ n^\text{BB}_\gamma = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3) \zeta(3) \]

\[ n^\text{BB}_\gamma = \frac{8\pi (kT)^3}{(hc)^3} \int_0^\infty \frac{x^2}{e^x - 1} \, dx \]

\[ \int_0^\infty x^2 e^{-x} \, dx = \gamma(3) \]

\[ \frac{x^n}{e^x - 1} = \frac{e^{-x}x^n}{1 - e^{-x}} \]
CMB- Total Number Density

\[ n_{BB}^\gamma = 8\pi \frac{(kT)^3}{(hc)^3} \Gamma(3)\zeta(3) \]

\[ \frac{x^n}{e^x - 1} = \frac{e^{-x}x^n}{1 - e^{-x}} \]

\[ = \sum_{m=0}^{\infty} e^{-mx}e^{-x}x^n \]

\[ = \sum_{m=1}^{\infty} e^{-mx}x^n \]
CMB- Total Number Density

\[ n_{\gamma}^{BB} = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3) \zeta(3) \]

\[ \int \frac{x^n}{e^x - 1} \, dx = \sum_{m=1}^{\infty} \int e^{-mx} x^n \, dx \]

Let \( y = mx \)

\[ \int \frac{x^n}{e^x - 1} \, dx = \sum_{m=1}^{\infty} \int e^{-y} \left( \frac{y}{m} \right)^n \, d\left( \frac{y}{m} \right) \]

\[ \int \frac{x^n}{e^x - 1} \, dx = \sum_{m=1}^{\infty} \frac{1}{m^{n+1}} \int y^n e^{-y} \, dy = \gamma(n + 1) \zeta(n + 1) \]
Threshold Energy- Proton Pion Production

\[
(E_p + E_\gamma)^2 - (p_p - E_\gamma)^2 = (m_p + m_\pi)^2
\]

\[
m_p^2 + 2E_p E_\gamma + 2p_p E_\gamma \approx m_p^2 + 2m_p m_\pi
\]

\[
E_p \approx \frac{m_\pi}{2E_\gamma} m_p \approx \left(\frac{135 \times 10^6}{2 \times 6 \times 10^{-4}}\right) 0.9 \times 10^9 = 10^{20} \text{ eV}
\]
Photo-Pion Production Rate

\[ R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{(x^2 - x_1(\Gamma)^2)}{e^x - 1} \, dx + \]

\[ n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{(x_2^2(\Gamma) - x_1^2(\Gamma))}{e^x - 1} \]

\[ R(\Gamma) \approx \frac{1}{l_0} \left[ (\gamma_i(3, x_2(\Gamma)) - \gamma_i(3, x_1(\Gamma))) - x_1(\Gamma)^2 (\gamma_i(1, x_2(\Gamma)) - \gamma_i(1, x_1(\Gamma))) + x_2(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) - x_1(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) \right] \]

\[ \gamma_i(3, x) = 2 - (2 + 2x + x^2) \exp(-x) \quad \gamma_i(1, x) = 1 - \exp(-x) \]

\[ R(\Gamma) \approx \frac{2}{l_0} \left[ e^{-x_1} (1 - e^{-x_1} + x_1 (1 - 2e^{-x_1})) \right] \]
Consider the case

\[
\begin{align*}
\frac{w^2}{(w-x)(w-y)(w-z)} + \frac{x^2}{(x-w)(x-y)(x-z)} + \frac{y^2}{(y-w)(y-x)(y-z)} &= -\frac{z^2}{(z-w)(z-x)(z-y)} \\
\begin{vmatrix}
1 & w & w^2 & w^2 \\
1 & x & x^2 & x^2 \\
1 & y & y^2 & y^2 \\
1 & z & z^2 & z^2 \\
\end{vmatrix} &= 0
\end{align*}
\]
Integrating Out the Time Variable of the Green’s Function

\[
\frac{dN(t)}{d^3r} = \frac{e^{-\frac{r^2}{4Dt}}}{(4\pi Dt)^{3/2}}
\]

\[
\frac{dn}{dr} = \int_0^\infty \frac{dN(t)}{d^3r} dt
\]

Let \[x = \frac{r^2}{4Dt}\]

\[
(4Dt)^{3/2} = r^3x^{-3/2} \quad \quad dt = -\frac{r^2}{4Dx^2} dx
\]

\[
\frac{dn}{dr} = \frac{1}{(\pi)^{3/2} 4Dr} \int_0^\infty x^{-1/2}e^{-x} dx
\]
INTERACTION OF ULTRA-HIGH ENERGY COSMIC RAYS WITH MICROWAVE BACKGROUND RADIATION

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Abstract. The formation of the "bump" and the "black-body cutoff" in the cosmic-ray (CR) spectrum arising from the annihilation photoproduction reaction in collision of UHE CR with the microwave background radiation (MBR) is studied. A kinetic equation which describes CR proton propagation in the MBR with account for the catastrophic nature of the annihilation photoproduction process is derived. The equilibrium CR proton spectrum obtained from the solution of this kinetic kinetic equation is in general agreement with the spectrum obtained under assumption of the constant energy loss approximation. However, the spectrum from point sources noticeably differ from those obtained in the constant loss approximation. Both, the equilibrium and the point source spectra are modified when taking into account the possible deviation of the MBR spectrum from the Planckian one in the Wien region. Thus, for the recently measured MBR spectrum, which reveals an essential "bump" in the submillimeter region, the "black-body cutoff" and the preceding "bump" shift towards lower energies.

1. Introduction

The ultra-high energy cosmic-ray (CR) interaction in the intergalactic space with the microwave background radiation (MBR) gives rise to a "black-body cutoff" of the CR spectrum predicted more than 20 years ago (Greisen, 1966; Zatsepin and Kuzmin, 1966). Unfortunately, the available experimental data do not allow us to draw an unambiguous conclusion concerning the presence or absence of such a spectral peculiarity (see, e.g., Watson, 1985). At the same time, in the energy range $E = 10^{19} \text{ eV}$ the Fly's Eye has detected some excess (a "bump") in the spectrum (Baltz et al., 1988), which agrees with the evidence obtained by Haverah Park (Cunningham et al., 1987), Volcano Ranch (Lincoln, 1985), and Alcomo (Teshima et al., 1987) groups to a tendency of spectrum flattening in this energy region. With a better confidence this peculiarity is also revealed in the data of Yakovlev (Kriestensen, 1985) and Sydney (Wynn et al., 1985) extensive air shower (EAS) arrays.

Still and Scharaun (1985), examining the UHE proton transfer in the MBR field, arrived at a rather important conclusion that due to the pion photoproduction process, besides the "black-body cutoff", there is also formed a "bump" (preceding the cutoff). The latter spectral peculiarity is apparently due to a sharp (exponential) energy dependence of the proton-free path (owing to the threshold nature of the $\pi^+ \rightarrow \pi^0\gamma$ process). Protons with energy $E < 10^{18} \text{ eV}$ interact only with the Wien "tail" of the MBR spectrum. Protons with energy $E \geq 5 \times 10^{19} \text{ eV}$ effectively interact with the MBR, deposit energy

of this equation we present in the form of an iterative series

$$F(E, t) = g(E) e^{-E/T} \int_0^t dt' e^{-t'/T} \left[ \frac{d}{dt} F(E, t) \right],$$ (A2-2)

where $F_0(E, t) = g(E) e^{-E/T}$ is the initial approximation for the spectrum. For numerical calculations it is convenient to pass to a new function $f(E, t)$ using the replacement

$$f(E, t) = g(E) f(E, t).$$ (A2-3)

Then for $f(E, t)$ we obtain a solution in the form

$$f(E, 0) = g(E) + \int_0^t dt' e^{-t'/T} \left[ \frac{d}{dt} F_0(E, t) \right],$$ (A2-4)

where the integral term is

$$A_t f = \frac{e^{tT}}{2 \pi (e^{tT})^2} \int_0^{e^{tT}} \left[ c(t, a) \right] \left[ \prod_{a=1}^{m} \left( 1 - \exp \left( -c(t, a) \right) \right) \right] \left[ \prod_{a=1}^{n} \left( 1 - \exp \left( -c(t, a) \right) \right) \right] \left[ \prod_{a=1}^{m} \left( 1 - \exp \left( -c(t, a) \right) \right) \right],$$ (A2-5)

where $c_{1/2}$ and $c_{1/4}$ are determined by the expressions (11).

In the energy region $E \leq 10^{19} \text{ eV}$ the integral term may be approximately presented as

$$A_t f = f(E, 0) e^{-tT} \left[ \frac{d}{dt} f(E, 0) \right],$$ (A2-6)

where

$z_0 = f(E, 0).$ (A2-7)

The solution for the function $f(E, t)$ can be presented as

$$f(E, t) = \sum_{n=0}^{\infty} \frac{(y - 1)^n}{n!} \int_0^t dt' e^{-t'/T} \left[ \prod_{a=1}^{m} \left( 1 - \exp \left( -c(t, a) \right) \right) \right],$$ (A2-8)

where

$$y = \frac{E}{(E, 0)}, \quad v_n = \left( \frac{E}{(E, 0)} \right)^n.$$ (A2-9)

The MBR deviation from the Planckian spectrum (in case of its approximation by the compounded black-body radiation spectrum (14)) for the proton spectrum from a point source, can be taken into account just like in case of the equilibrium proton spectrum (see Appendix 1).
Injecting a $10^{20}$ eV Fe Nucleus and Tracking the Subsequent Nuclei-
Comparison of Analytic and Monte Carlo Results

\[ \frac{E^2 \text{d}N}{\text{d}E} \text{[eV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}] \]

\[ \log_{10} E \text{[eV]} \]

\[ \langle A \rangle \]

\[ \log_{10} E \text{[eV]} \]
Conclusions

• The Pierre Auger Observatory is able to provide much more than just the cosmic ray flux measurement

• Due to the $\ln E_0$ dependence of $X_{\text{max}}$, excellent energy resolution is required to pull out the composition information

• The $X_{\text{max}}$ and energy spectrum data collectively can provide useful information about the source injection spectrum and cutoff energy

• The propagation of nuclei can be easily understood through the application of an analytic description of the photo-disintegration process
Cascade of Nuclei Through Species- single nucleon loss

Since nuclei Lorentz factor remains \( \sim \) conserved, and cross-section varies mildly with \( A \) (nuclear mass)

\[
\tau_{56 \rightarrow 55} \approx \tau_{55 \rightarrow 54} \ldots
\]

For the case

\[
\tau_{56 \rightarrow 55} = \tau_{55 \rightarrow 54} \ldots
\]

\[
f_q = \frac{t^{(q_{\text{max}}-q)}}{\tau_q (q_{\text{max}}-q)!} e^{-t/\tau_q}
\]

ie. Gaisser-Hillas type function! (used to describe air showers)
Cascade of Nuclei Through Species-Comparison of Approximation

Starting with Fe, \( q_{\text{max}} = 56 \)

\[
f_{50} = \frac{t^6}{6!} e^{-\frac{t}{\tau_{50}}} 
\]

\[
f_{40} = \frac{t^{16}}{16!} e^{-\frac{t}{\tau_{40}}} 
\]

\[
f_{30} = \frac{t^{26}}{26!} e^{-\frac{t}{\tau_{30}}} 
\]
Composition – an Excellent Probe of the Local Source Distribution
(if you know the source composition)
Local Scales Effect Highest Energies
(logarithmic scale)

$E_{\text{max,Fe}} = 10^{21}$ eV
$\alpha = 1.8$
100% Iron

Monte Carlo
Analytic

81-243 Mpc
27-81 Mpc
9-27 Mpc
3-9 Mpc
0-3 Mpc