Lecture Plan:

1) Cosmic Ray acceleration- accelerated spectrum, efficient accelerators, nuclei friendly

PROBLEMS

2) Cosmic Ray proton + nuclei interaction rates in extragalactic radiation fields

PROBLEMS

3) Cosmic Ray propagation through Galactic and extragalactic magnetic fields

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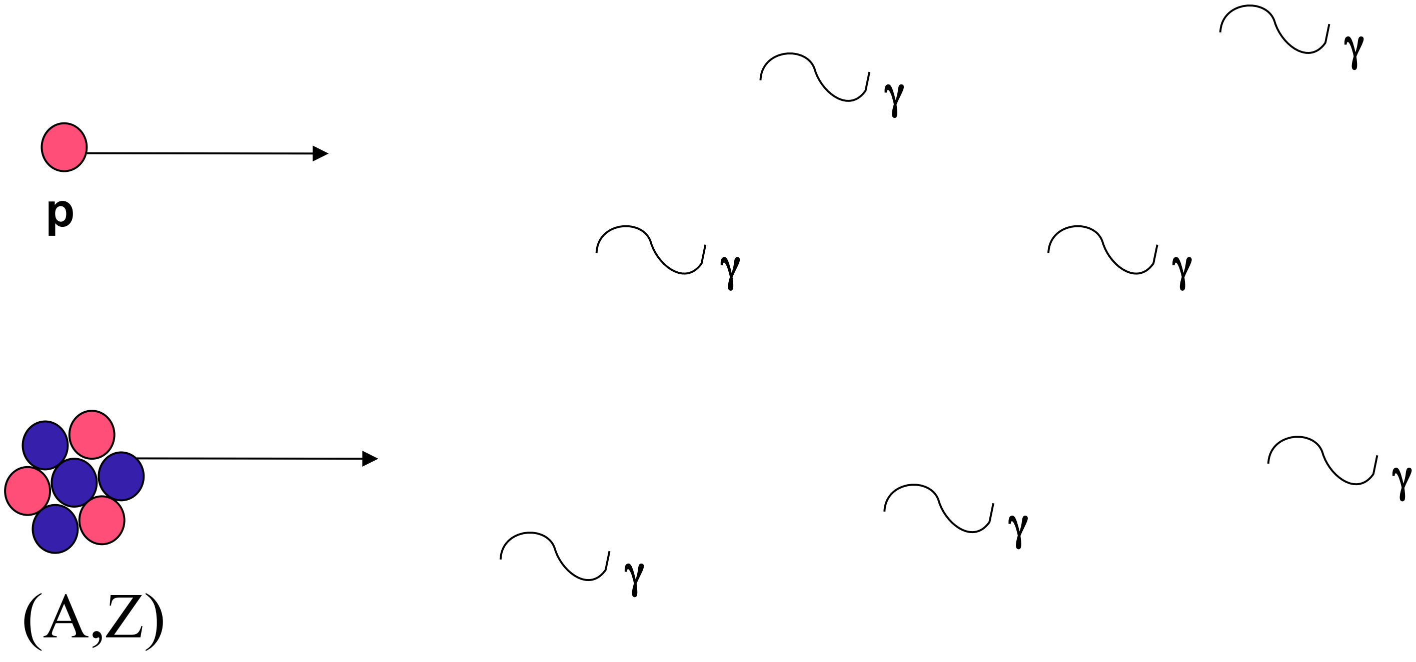
COSMIC RAYS: High Energy Proton and Nuclei Interactions During Propagation



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Cosmic Ray Proton Energy Losses

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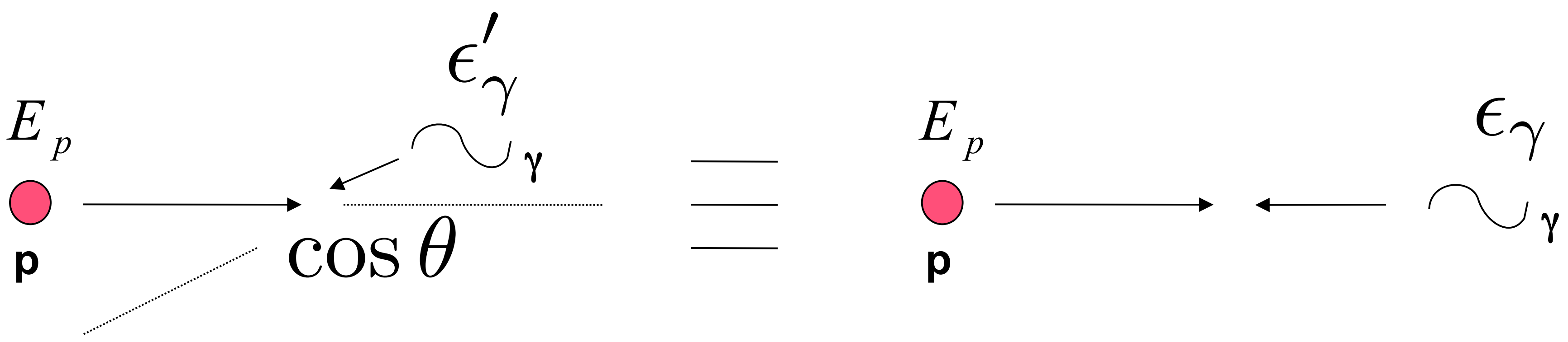
The Interaction Rate

$$\mathbf{R} = \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-\mathbf{1}}^{\mathbf{1}} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta) \frac{\mathbf{d}\sigma}{\mathbf{d}\cos\theta} (\mathbf{1} - \beta\cos\theta)$$

All values above in lab frame



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The Interaction Rate

$$\mathbf{R} = \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-1}^{1} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta) \frac{\mathbf{d}\sigma}{\mathbf{d}\cos\theta} (\mathbf{1} - \beta\cos\theta)$$

Since, $$\epsilon_\gamma \mathbf{E_p} = \epsilon' \mathbf{E_p} (\mathbf{1} + \beta\cos\theta)$$

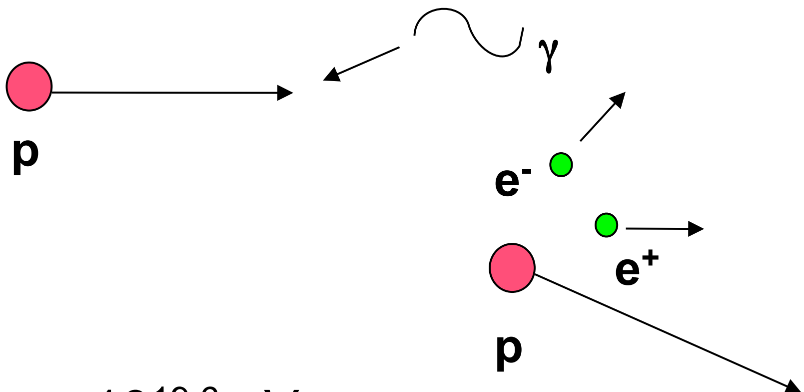
$$(\mathbf{1} + \beta\cos\theta)\mathbf{d}\cos\theta = \frac{\epsilon_\gamma \mathbf{E_p}}{\epsilon' \mathbf{E_p}} \frac{\mathbf{d}(\epsilon' \mathbf{E_p})}{\epsilon' \mathbf{E_p}}$$

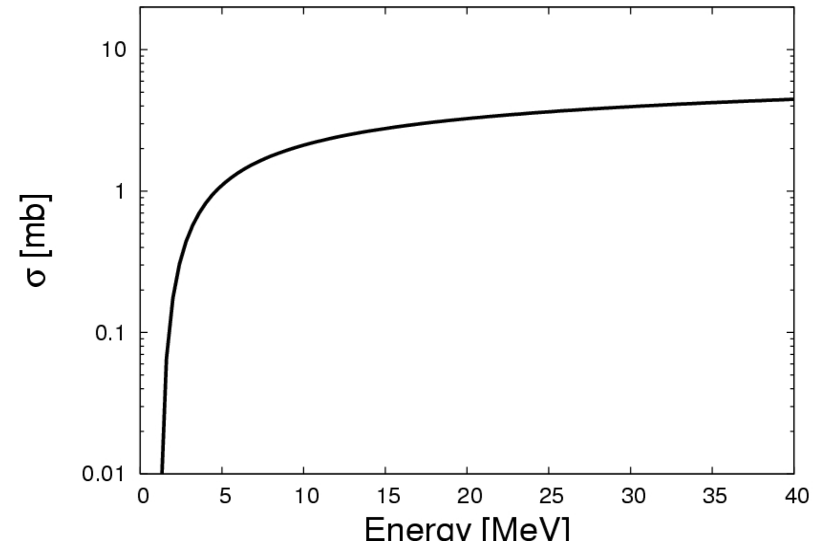
$$\mathbf{R} = \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2}\epsilon_\gamma \mathbf{E_p}} \mathbf{d}(\epsilon_\gamma \mathbf{E_p}) \frac{\epsilon_\gamma \mathbf{E_p}}{\epsilon_\gamma'^{\mathbf{2}} \mathbf{E_p^2}} \frac{\mathbf{d}\sigma}{\mathbf{d}(\epsilon_\gamma \mathbf{E_p})}$$

$$= \frac{\mathbf{m_p^2}}{\mathbf{2E_p^2}} \int_0^\infty \mathbf{d}\epsilon'_\gamma \frac{\mathbf{1}}{\epsilon_\gamma'^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon'_\gamma} \int_0^{\mathbf{2}\epsilon'_\gamma \frac{\mathbf{E_p}}{\mathbf{m_p}}} \mathbf{d}\epsilon_\gamma \epsilon_\gamma \frac{\mathbf{d}\sigma}{\mathbf{d}\epsilon_\gamma}$$

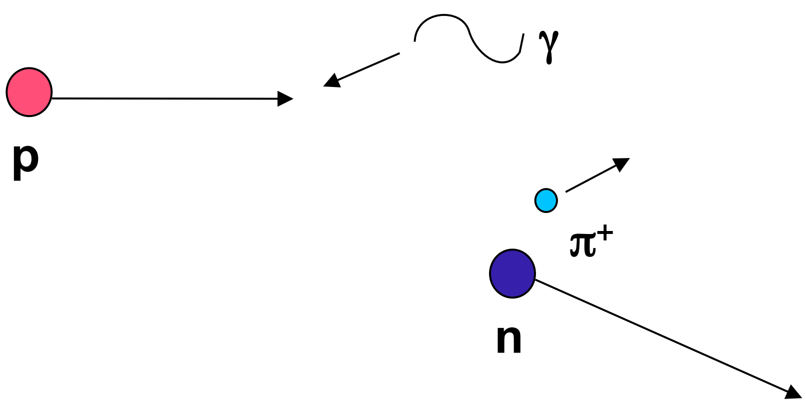
Cosmic Ray Proton Interactions

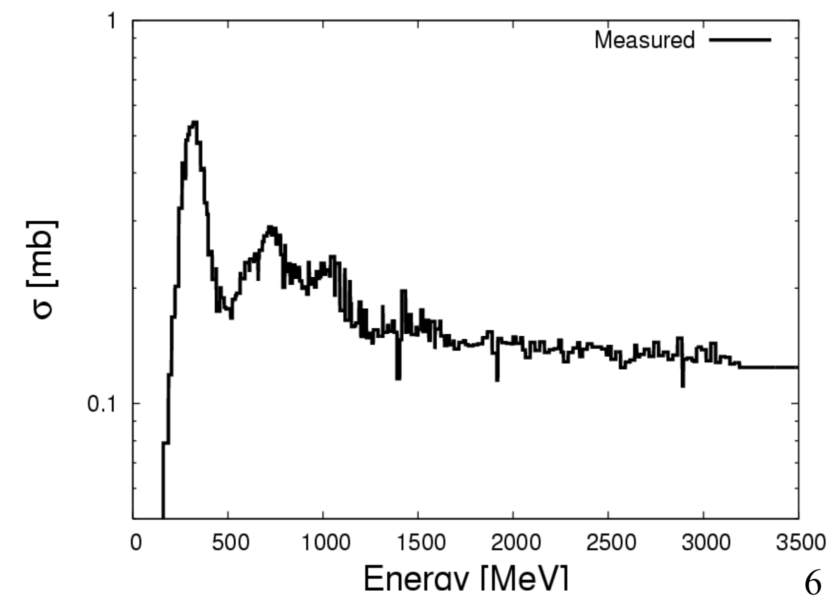
For $E_{proton} < 10^{19.6}$ eV





For $E_{proton} > 10^{19.6}$ eV





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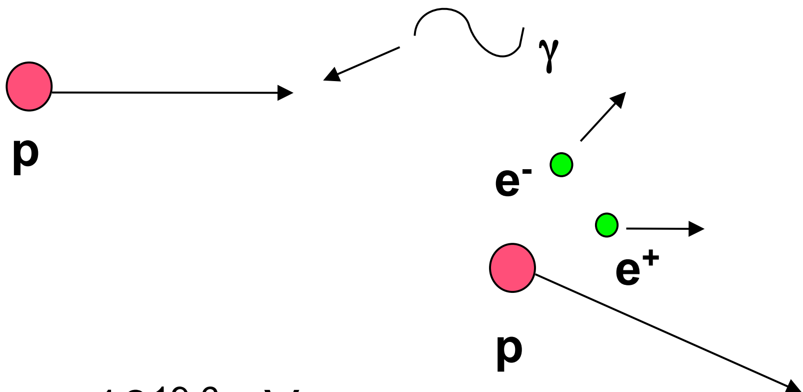
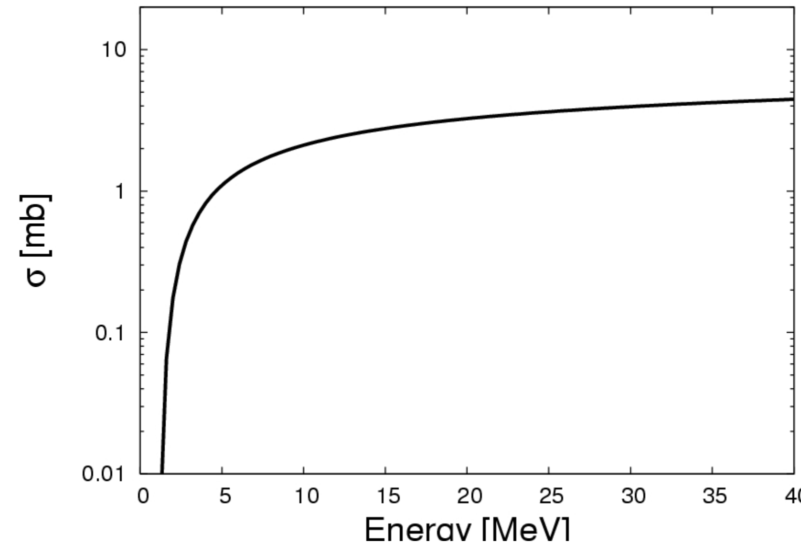
Cosmic Ray Proton Interactions

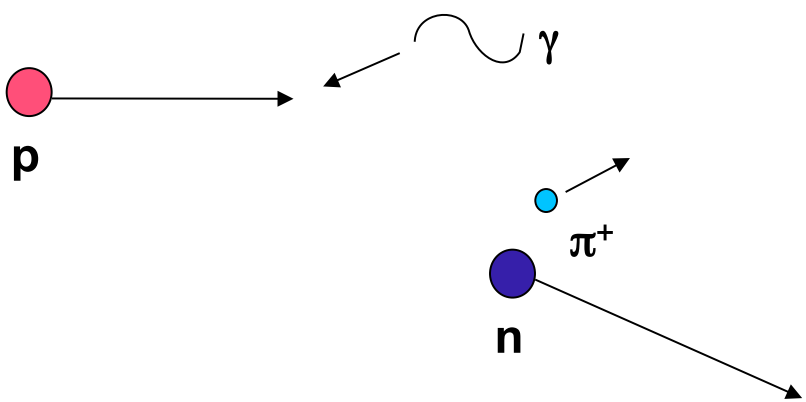
For $E_{proton}<10^{19.6}$ eV

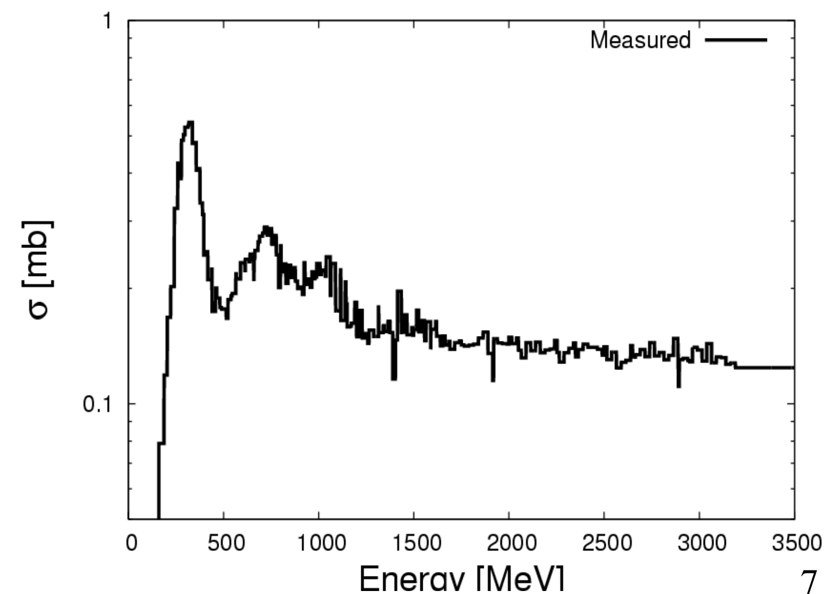




E_{γ}^{th} ~ 1MeV

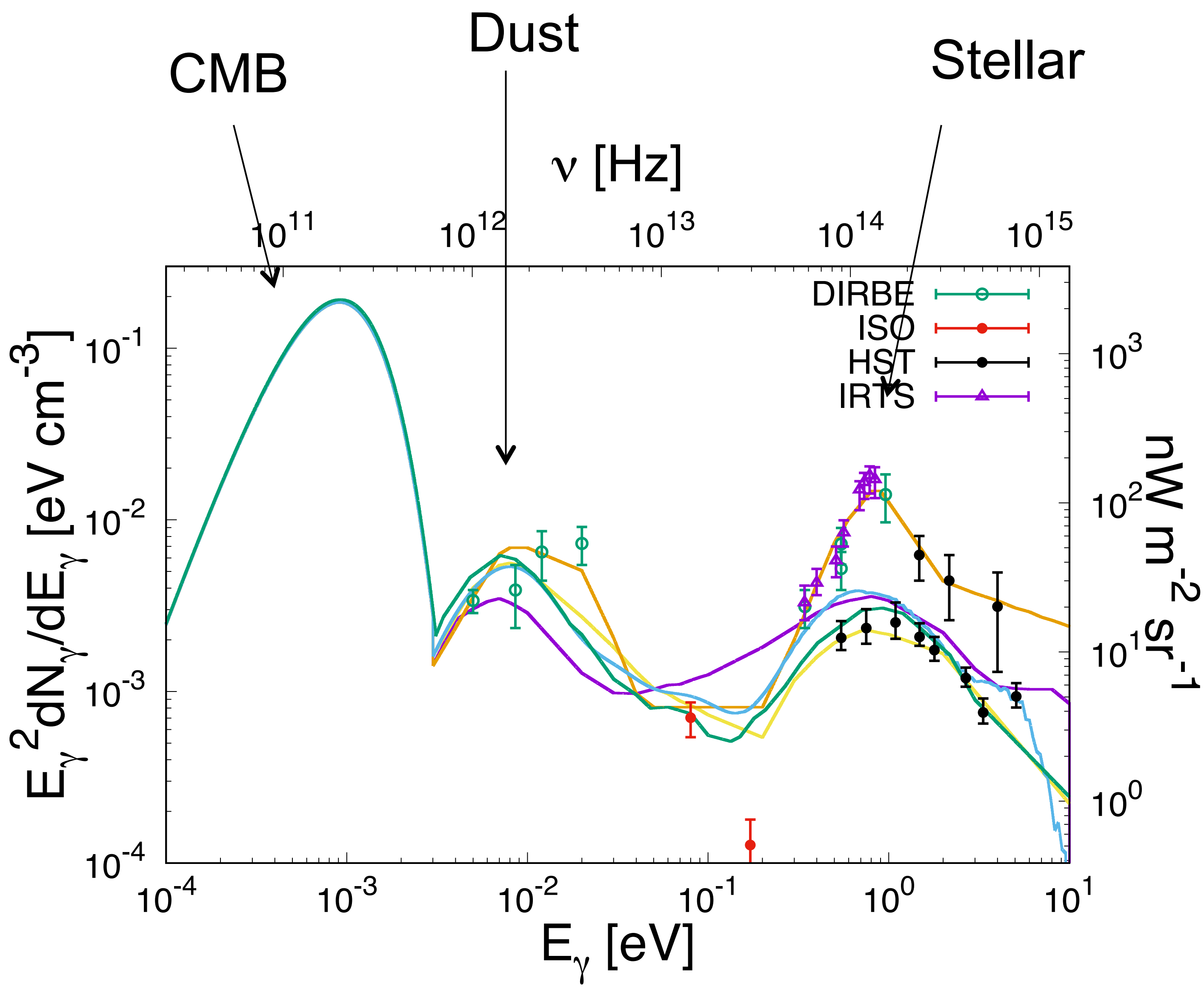
For $E_{proton}>10^{19.6}$ eV



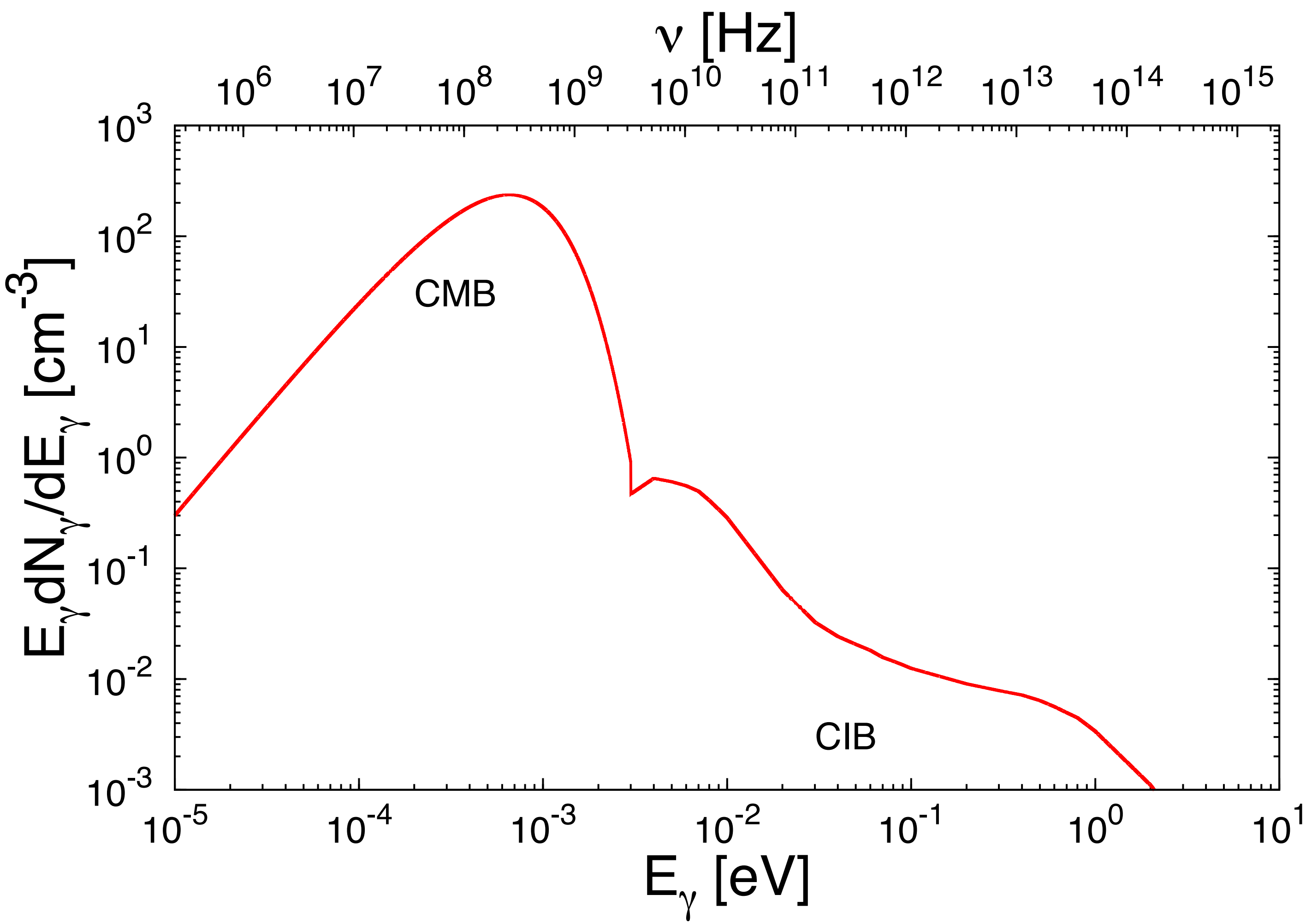


E_{γ}^{th} ~ 140 MeV

Cosmic Radiation Fields- Energy Density



Cosmic Radiation Fields- Number Density



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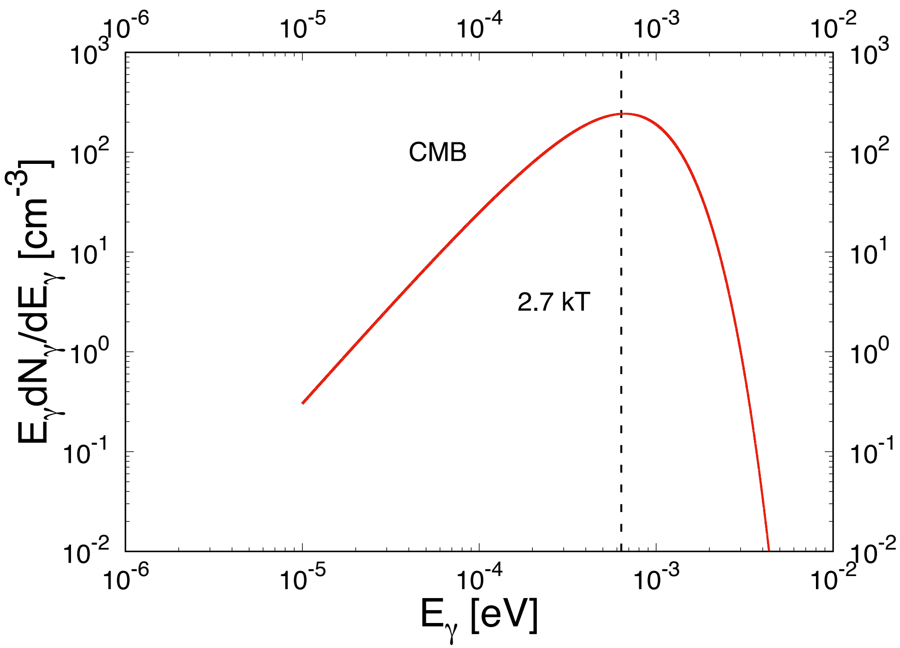
CMB- Total Number Density

$$\frac{dn}{d\epsilon_\gamma} = \frac{8\pi}{(hc)^3}\frac{\epsilon_\gamma^2}{e^{\epsilon_\gamma/kT} - 1}$$



$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

$$\frac{8\pi(kT_{CMB})^3}{(hc)^3} \approx 170 \text{ cm}^{-3}$$

$$\zeta(x) = \sum_{n=1}^{\infty}\frac{1}{n^x}$$

$$n_\gamma^{CMB} = 8\pi\frac{(kT_{CMB})^3}{(hc)^3}\gamma(3)\zeta(3) \approx 400 \text{ cm}^{-3}$$

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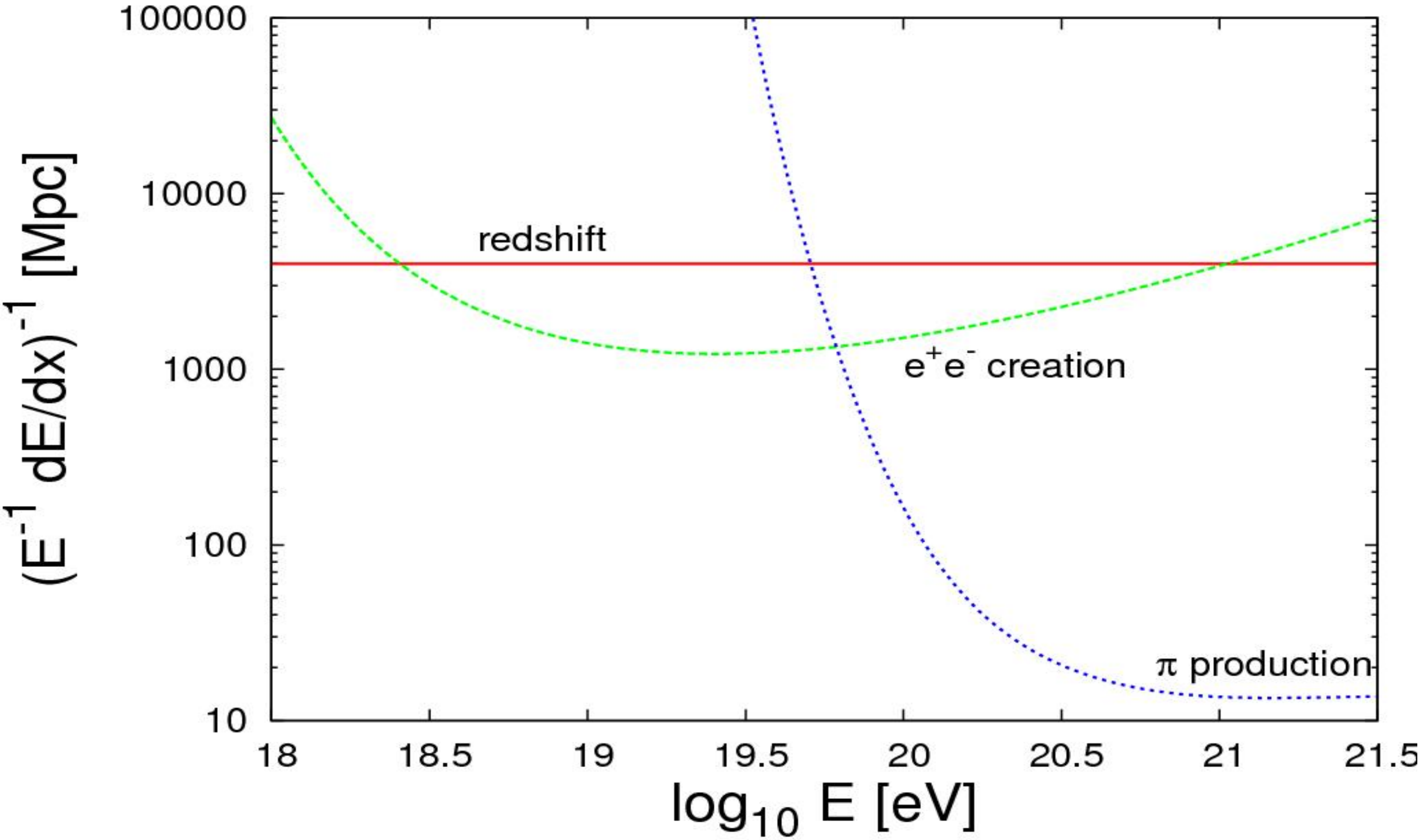
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

where K_p is the inelasticity



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Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

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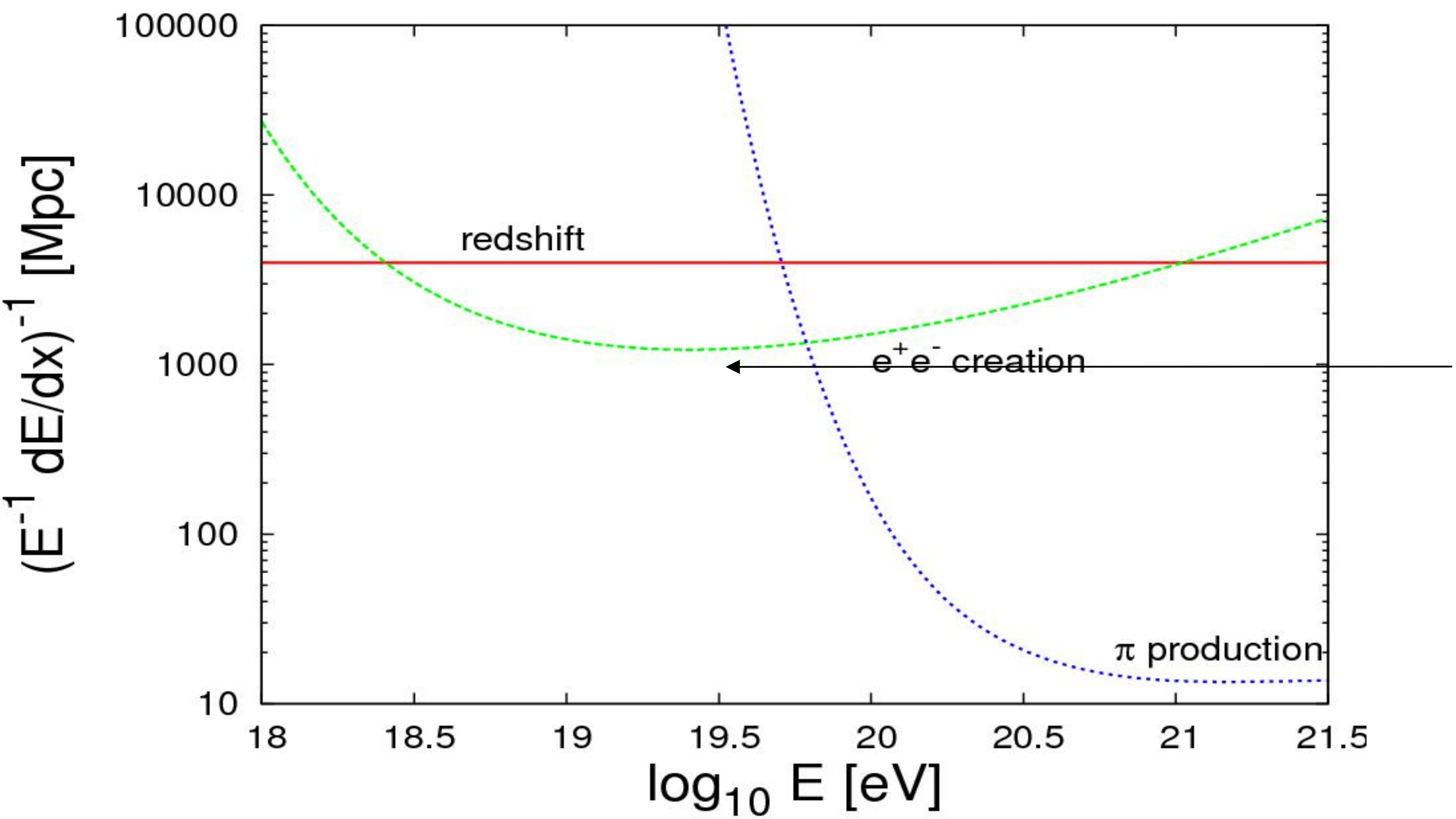

$$\approx \frac{\mathbf{m_p}}{\mathbf{m_e}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

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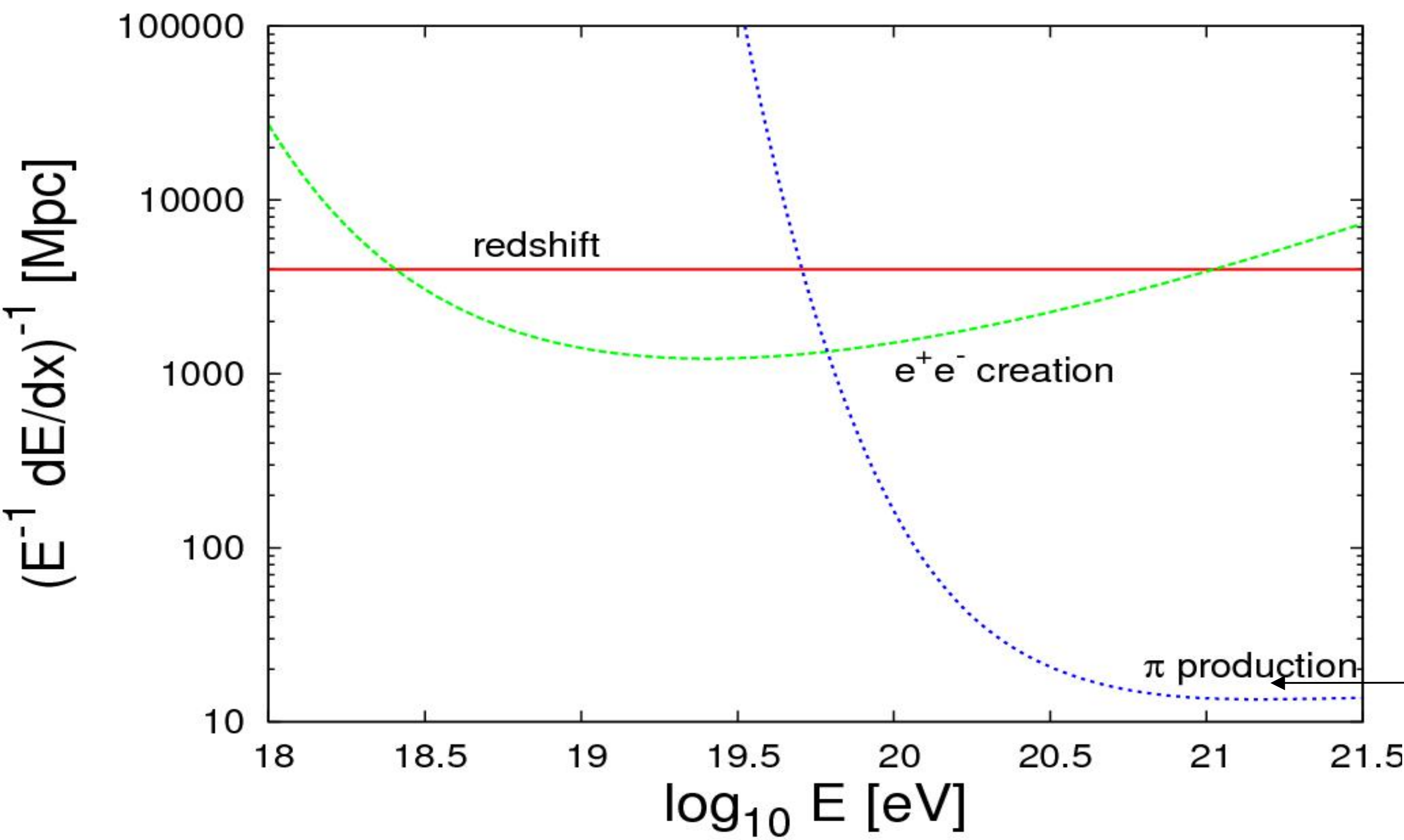
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

where K_p is the inelasticity



$$\approx \frac{\mathbf{m_p}}{\mathbf{m_\pi}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

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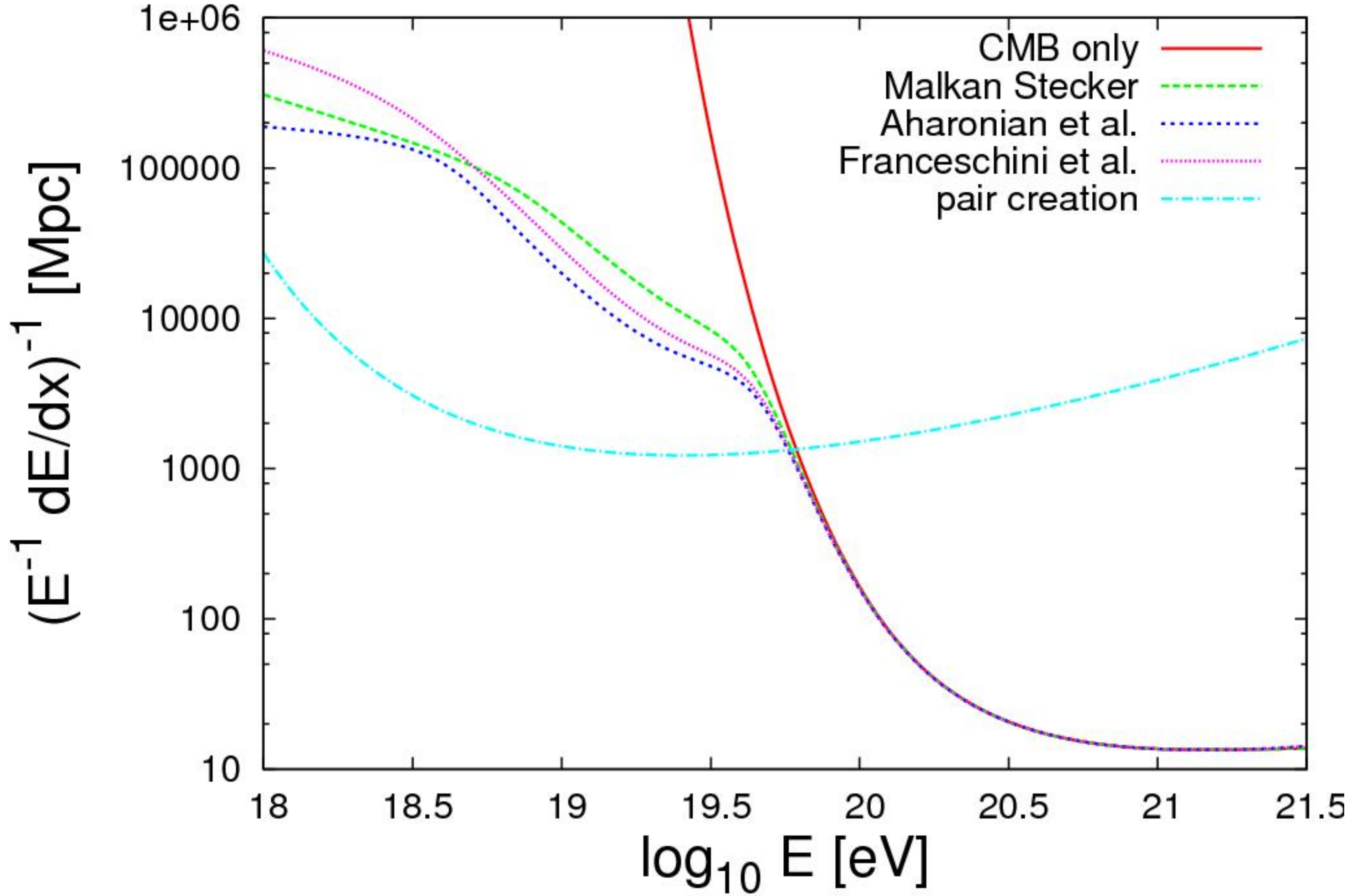
....with Different IR Backgrounds



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Interactions of Cosmic Ray Protons with CMB:

Pair Creation-

$$\mathbf{E}_\gamma \sim \mathbf{1\ MeV}$$

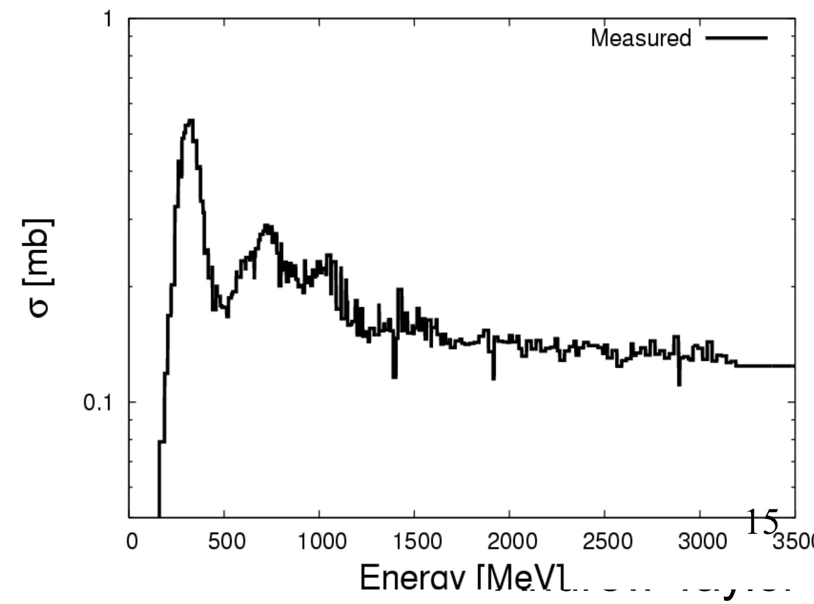
$$\mathbf{p} + \gamma \rightarrow \mathbf{p} + \mathbf{e}^+ + \mathbf{e}^-$$

Photo-Meson Production-

$$\mathbf{E}_\gamma^{\mathbf{th}} \sim \mathbf{140\ MeV}$$

$$\mathbf{p} + \gamma \rightarrow \mathbf{n} + \pi^+ / \mathbf{p} + \pi^\mathbf{0}$$

$$\mathbf{n} \rightarrow \mathbf{p} + \mathbf{e}^- + \bar{\nu}_\mathbf{e}$$



Threshold Energy- Proton Pair Production

$$(\mathbf{E_p} + \mathbf{E}_\gamma)^2 - (\mathbf{p_p} - \mathbf{E}_\gamma)^2 = (\mathbf{m_p} + \mathbf{2m_e})^2$$

$$\mathbf{m_p^2} + \mathbf{2E_pE}_\gamma + \mathbf{2p_pE}_\gamma \approx \mathbf{m_p^2} + \mathbf{4m_pm_e}$$

$$\boxed{\mathbf{E_p} \approx \frac{\mathbf{m_e}}{\mathbf{E}_\gamma}\mathbf{m_p}} \approx \left(\frac{\mathbf{0.5 \times 10^6}}{\mathbf{6 \times 10^{-4}}}\right)\mathbf{0.9 \times 10^9} = \mathbf{8 \times 10^{17}\ eV}$$

Repeat this calculation for pion production

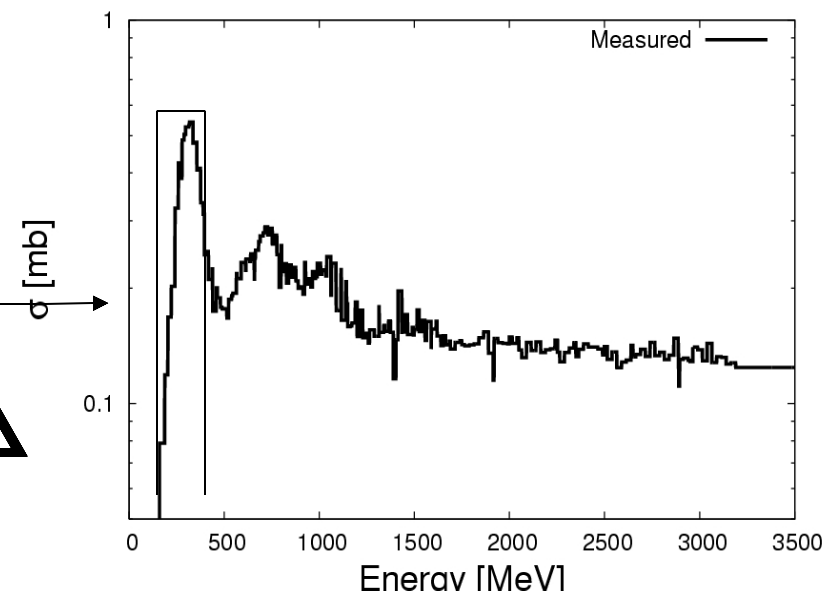
Photo-Pion Production Rate

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{\mathbf{0}}^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

Assuming the cross-section is approximately:

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \mathbf{0} \qquad \begin{matrix} \epsilon_\gamma < \mathbf{E} - \boldsymbol{\Delta} \\ \epsilon_\gamma > \mathbf{E} + \boldsymbol{\Delta} \end{matrix}$$

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \sigma_{\mathbf{p}\gamma} \qquad \mathbf{E} - \boldsymbol{\Delta} < \epsilon_\gamma < \mathbf{E} + \boldsymbol{\Delta}$$

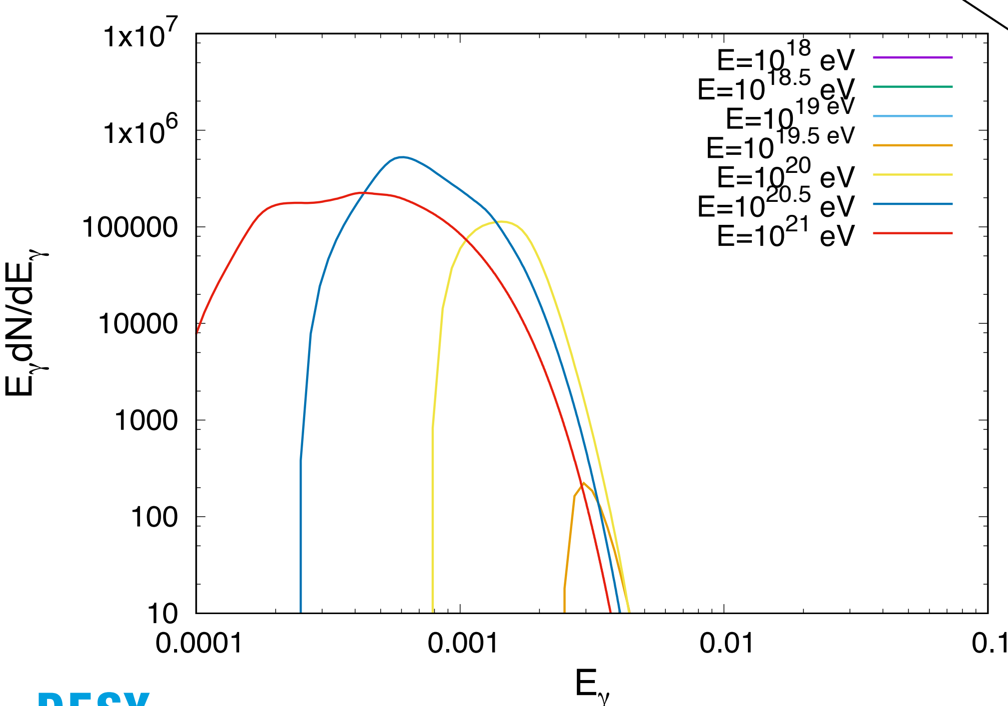


Where $\sigma_{\mathbf{p}\gamma} = \mathbf{0.5\ mb}$, $\mathbf{E} = \mathbf{300\ MeV}$, $\boldsymbol{\Delta} = \mathbf{100\ MeV}$

Photo-Pion Production Rate

$$\mathbf{R}(\boldsymbol{\Gamma}) \approx \sigma_{\mathbf{0}} \int_{(\mathbf{E_0}-\boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}}^{(\mathbf{E_0}+\boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}} \left(\frac{\epsilon^{\mathbf{2}} - \left[(\mathbf{E_0} - \boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}\right]^{\mathbf{2}}}{\epsilon^{\mathbf{2}}} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon +$$

$$\sigma_{\mathbf{0}} \int_{(\mathbf{E_0}+\boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}}^{\infty} \left(\frac{\left[(\mathbf{E_0} + \boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}\right]^{\mathbf{2}} - \left[(\mathbf{E_0} - \boldsymbol{\Delta}_\mathbf{0})/\mathbf{2}\boldsymbol{\Gamma}\right]^{\mathbf{2}}}{\epsilon^{\mathbf{2}}} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon$$



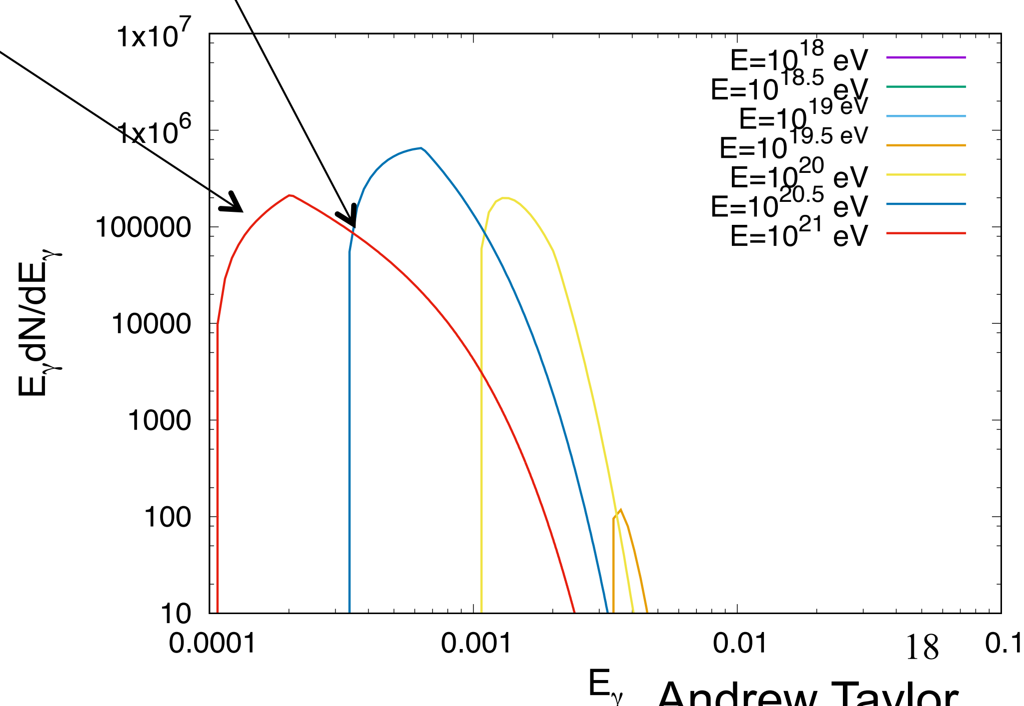


Photo-Pion Production Rate

$$\mathbf{R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1}\left(1 - e^{-x_1} + x_1\left(1 - 2e^{-x_1}\right)\right)\right]}$$

Where, $$\mathbf{x_1 = \frac{(E - \Delta) m_p}{2kT_{CMB} E_p}}$$

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Photo-Pion Production Rate

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma)\mathbf{K_p}$$

$$\approx \mathbf{0.2}\sigma_{\mathbf{p}\gamma} \int_{\frac{\mathbf{E}-\mathbf{\Delta}}{\mathbf{2\Gamma}}}^{\frac{\mathbf{E}+\mathbf{\Delta}}{\mathbf{2\Gamma}}} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma}$$

Where $\mathbf{\Gamma} = \frac{\mathbf{E_p}}{\mathbf{m_p c^2}}$ is the proton Lorentz factor

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Since,

$$\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} = \mathbf{170} \frac{(\epsilon_\gamma/\mathbf{kT_{CMB}})^{\mathbf{3}}}{\mathbf{e}^{\epsilon_\gamma/\mathbf{kT_{CMB}}} - \mathbf{1}} \mathbf{cm^{-3}}$$

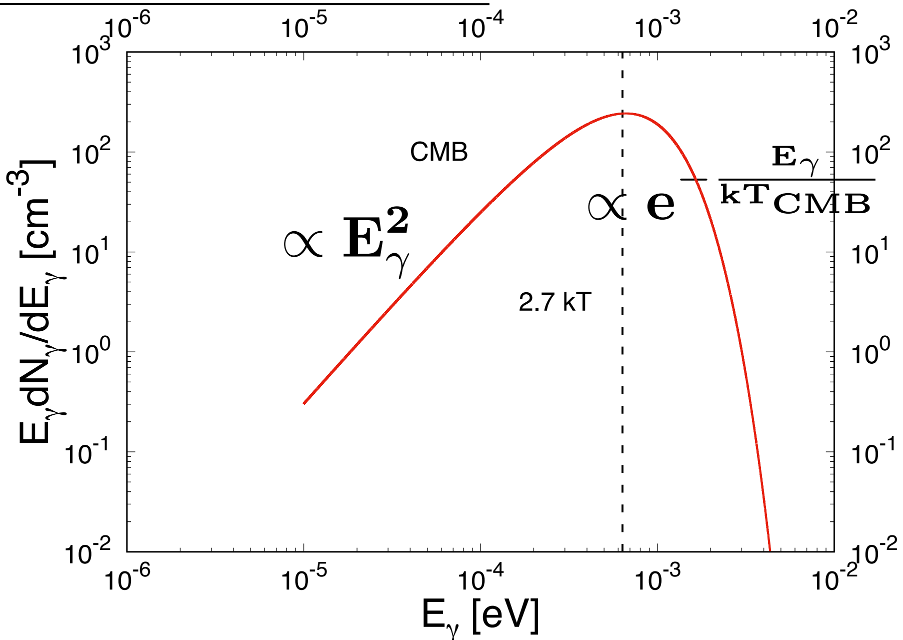

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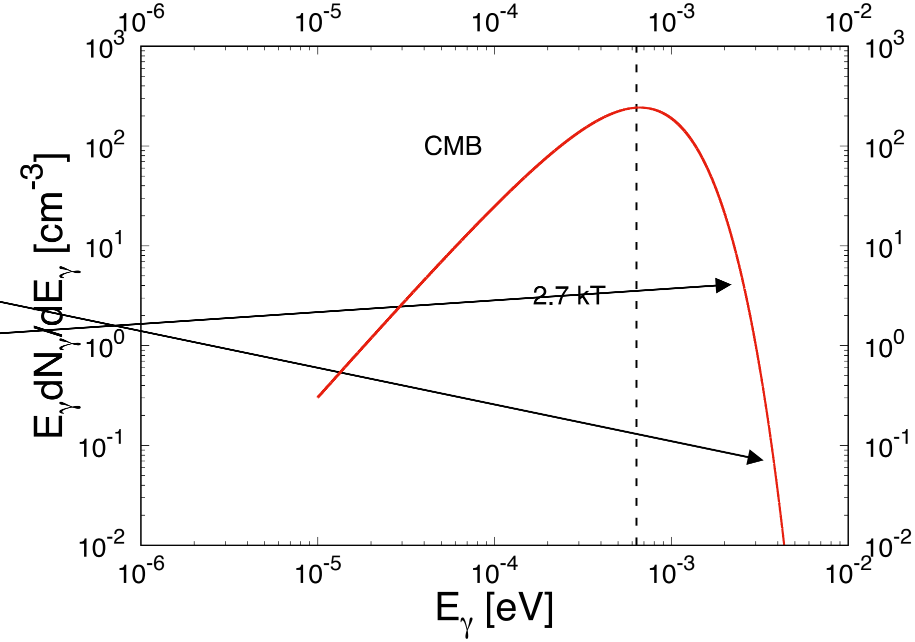
Photo-Pion Production Rate

With, $\mathbf{kT_{CMB} \approx 2 \times 10^{-4}\ eV}$

$$\mathbf{R \approx 0.2\sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma}}$$



$$\mathbf{\approx \left(\frac{l_0}{e^{-x_1}(1-e^{-x_1})}\right)^{-1}}$$

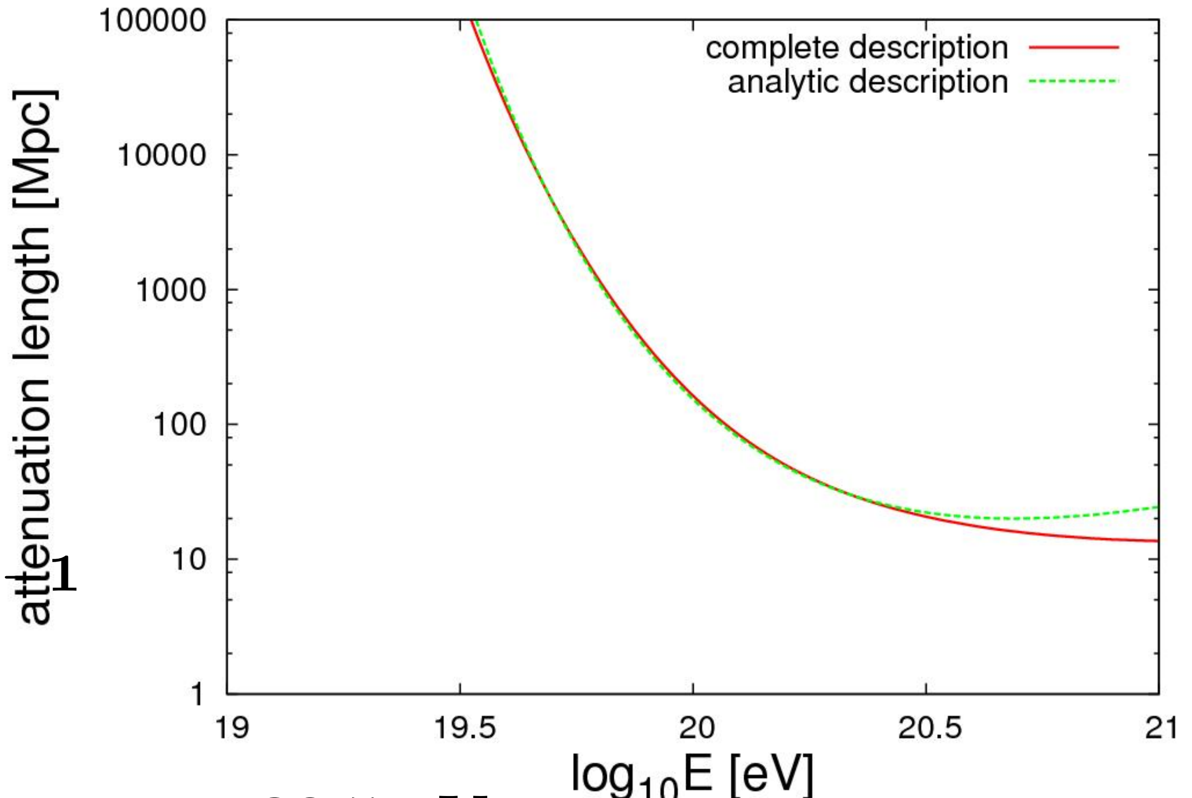
Where $\mathbf{l_0}$ is 5 Mpc and $$\mathbf{x_1 = \frac{(E-\Delta)m_p}{2kT_{CMB}E_p} = \frac{10^{20.7}\ eV}{E_p}}$$

Photo-Pion Production Rate

With, $\mathbf{kT_{CMB} \approx 2 \times 10^{-4}\ eV}$

$$\mathbf{R \approx 0.2\sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma}}$$

$$\approx \left(\frac{\mathbf{l_0}}{\mathbf{e^{-x_1}(1 - e^{-x_1})}} \right)^{-1}$$

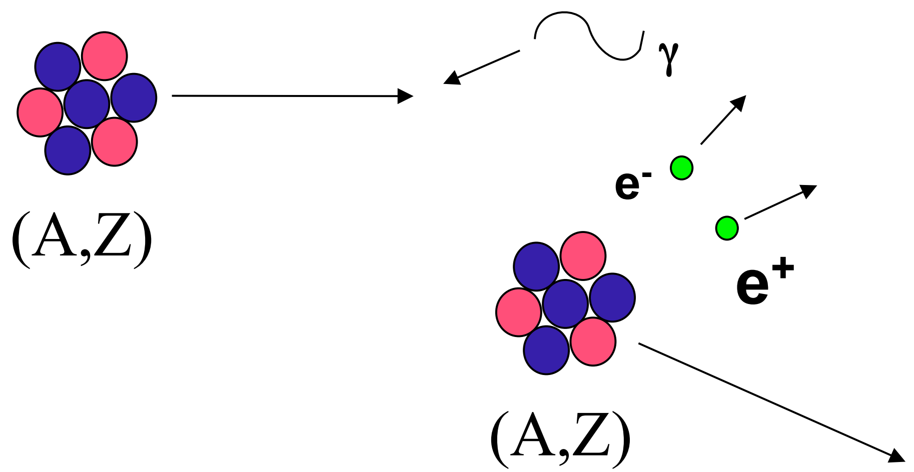


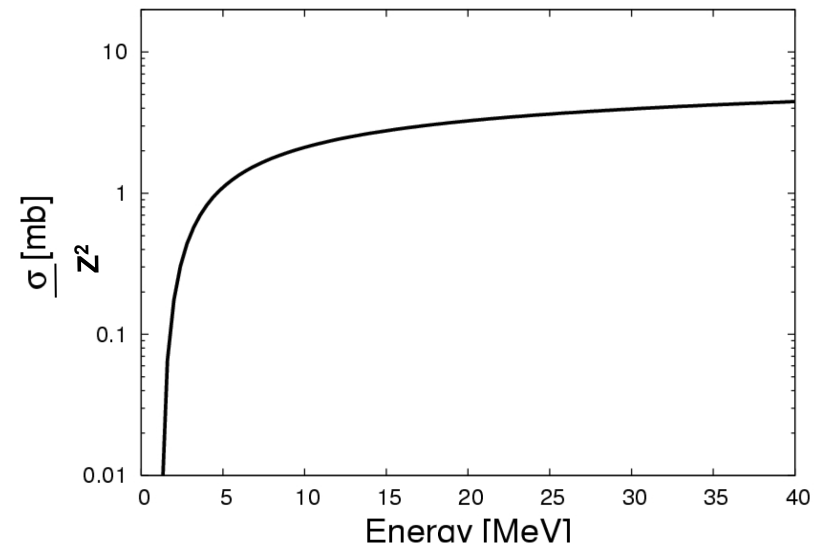
Where $\mathbf{l_0}$ is 5 Mpc and $\mathbf{x_1 = \frac{10^{20.5}\ eV}{E_p}}$

Cosmic Ray Nuclei Energy Losses

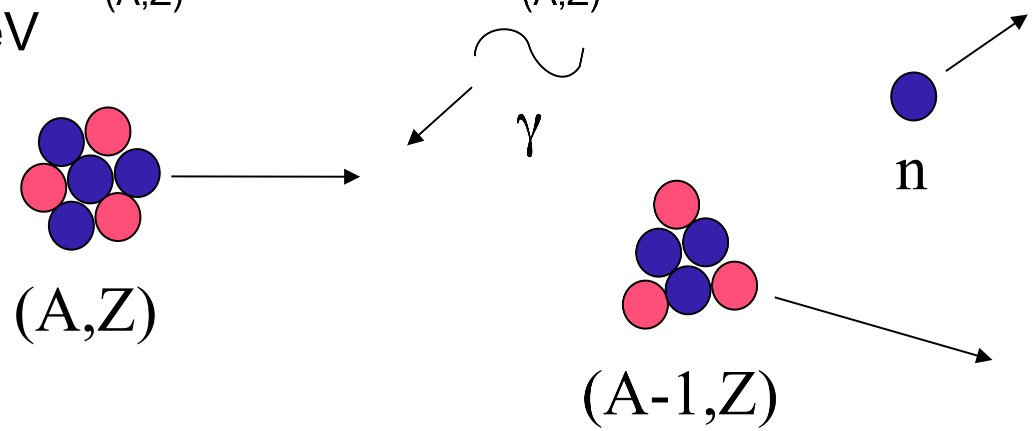
Cosmic Ray Nuclei Interactions

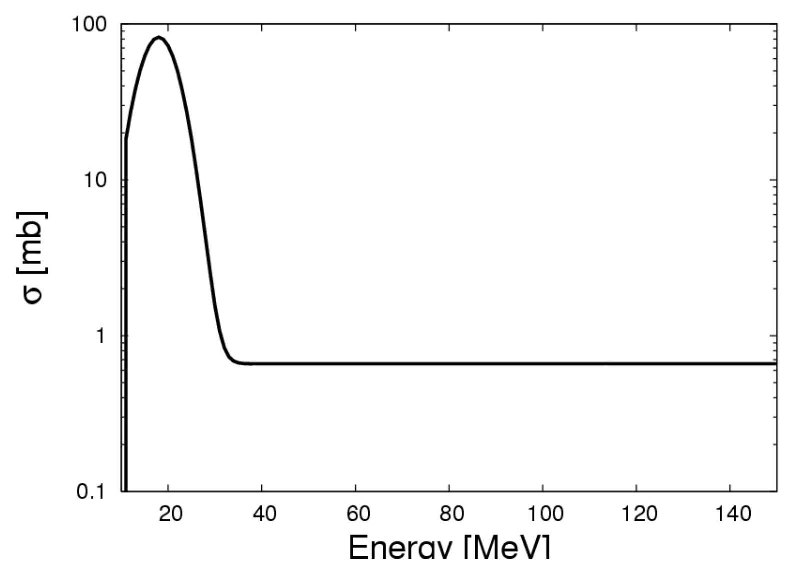
For $10^{19.7} < E_{(A,Z)} < 10^{20.2}$ eV



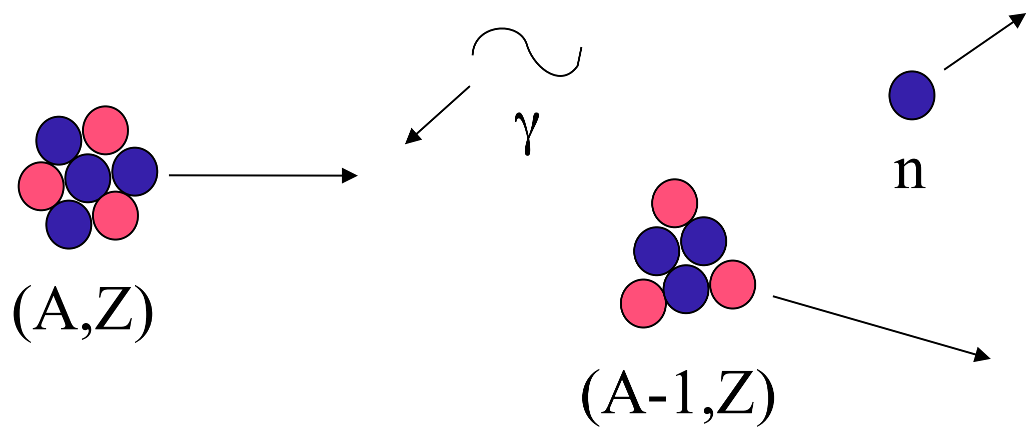


For $E_{(A,Z)} < 10^{19.7}$ and $E_{(A,Z)} < 10^{20.2}$ eV





Cosmic Ray Nuclei Interactions



<u>Photo-disintegration</u>-

$$N_{(A,Z)} + \gamma \longrightarrow N'_{(A',Z')} + (Z-Z')p + (A-A'+Z'-Z)n, \quad E_{\gamma} \sim 30\text{MeV}$$

$$n \longrightarrow p + e^{-} + \bar{\nu}_e$$

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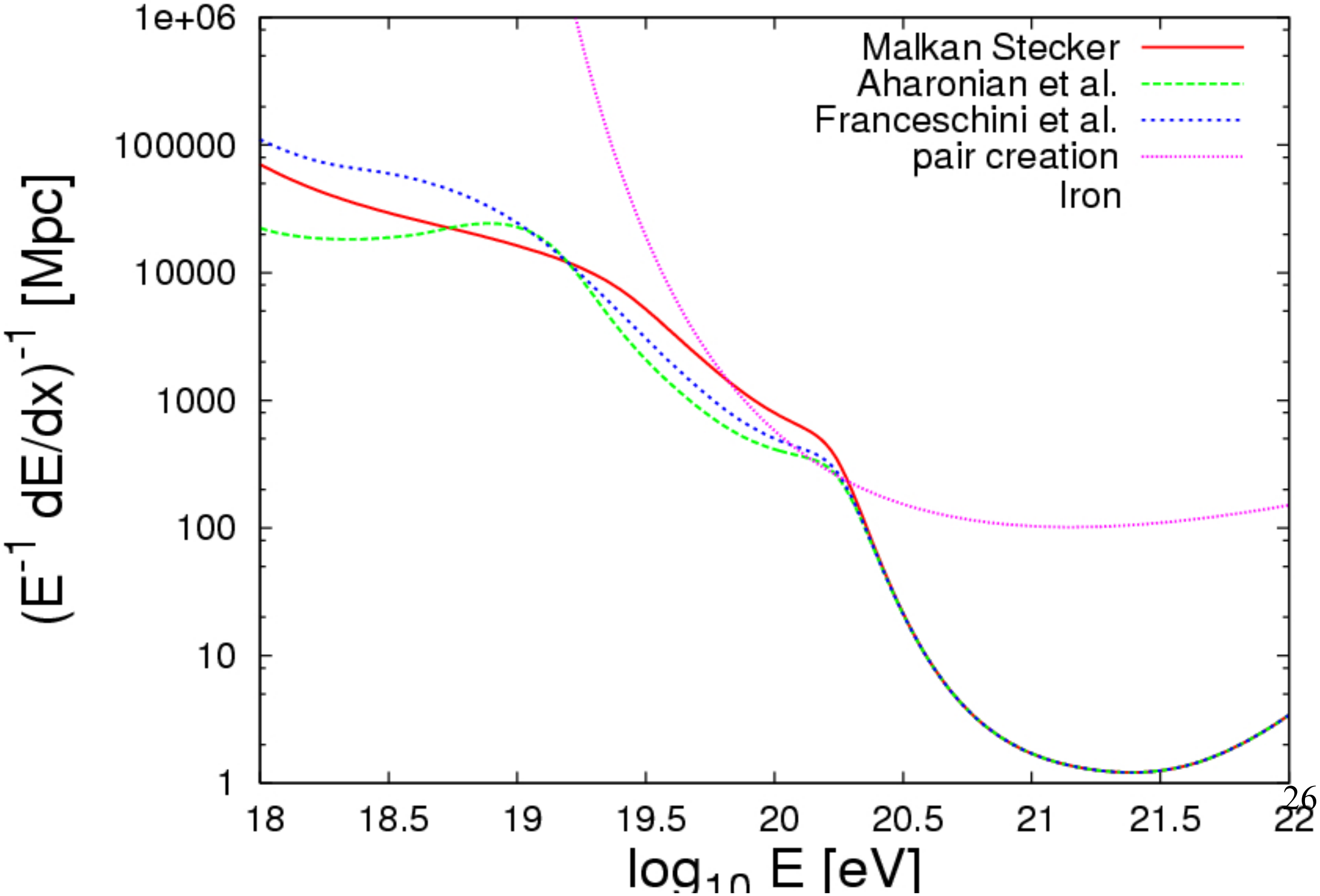
Energy Loss Rates due to Nuclei Interactions

$$\mathbf{R} = \frac{\mathbf{A^2 m_p^2 c^4}}{\mathbf{2E^2}} \int_0^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{Am_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{N}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate



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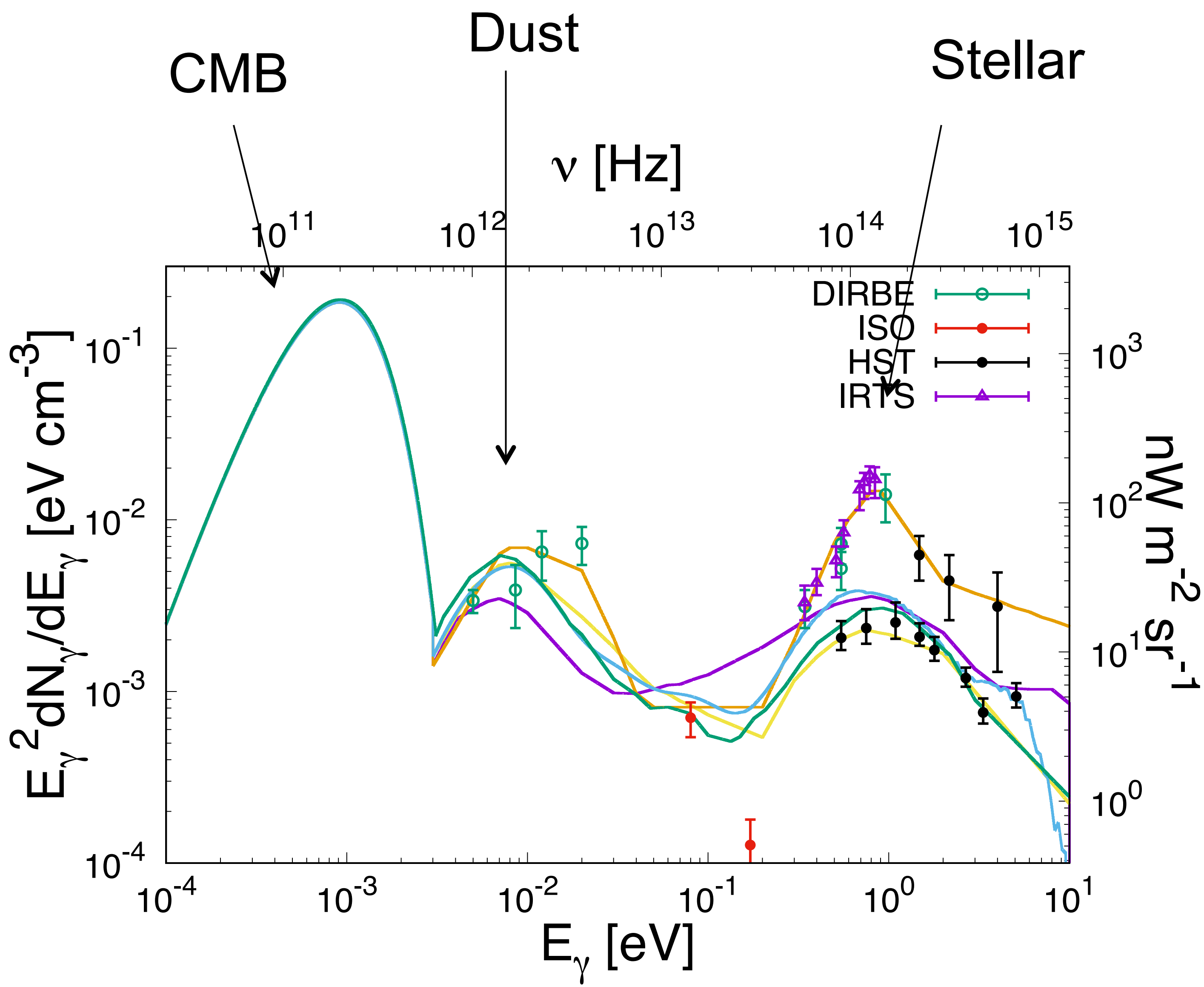
Cosmic Radiation Fields



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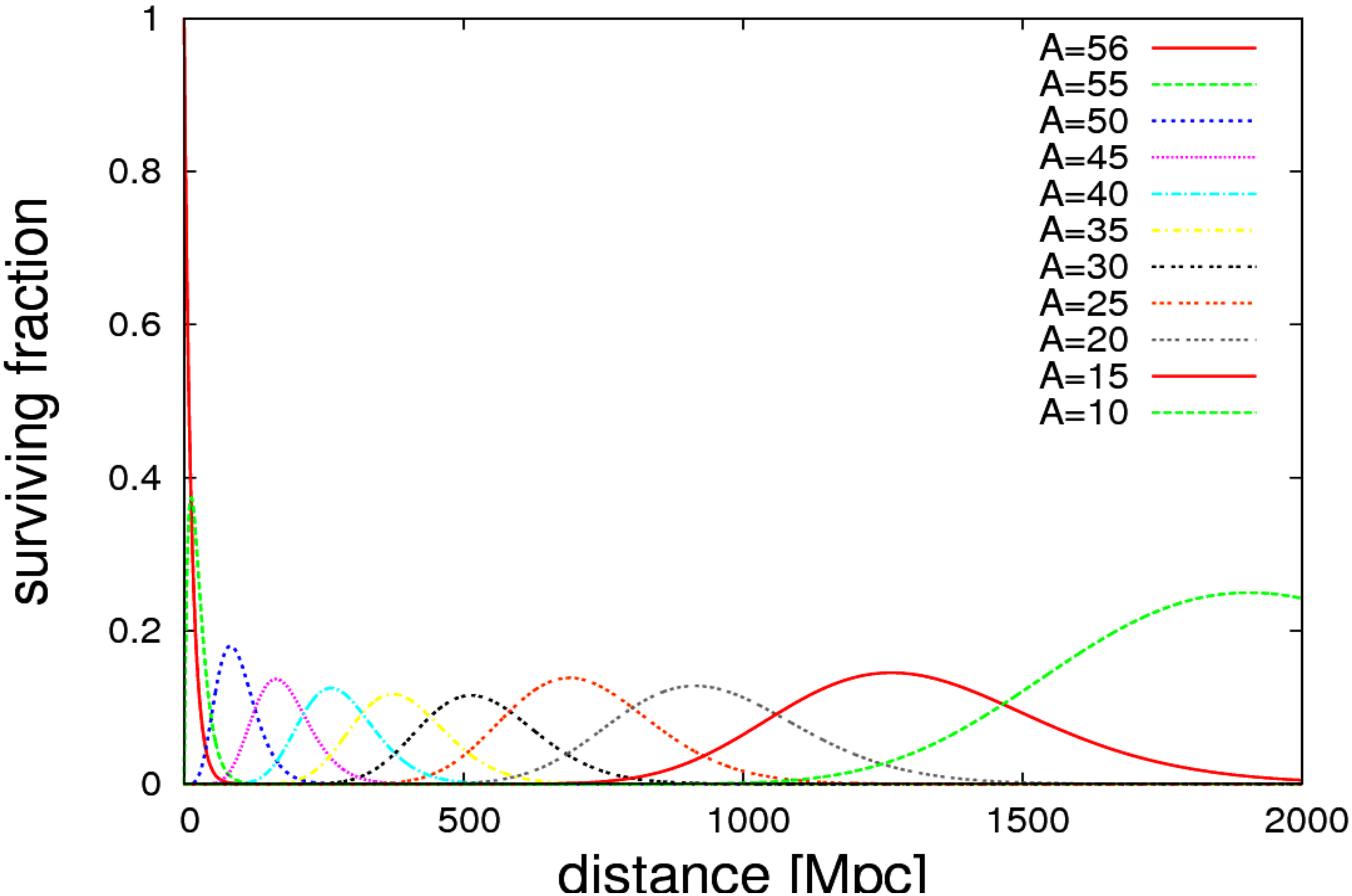
Cosmic Ray Disintegration During Propagation



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Cosmic Ray Spectra

Assumptions on Source Population

Spatial Distribution

motivated by star formation rate evolution

$$\frac{dN}{dV_C} \propto (1+z)^3 \qquad z < 1.9$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 \qquad 1.9 < z < 2.7$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 e^{-z/1.7} \qquad z > 2.7$$

Energy Distribution

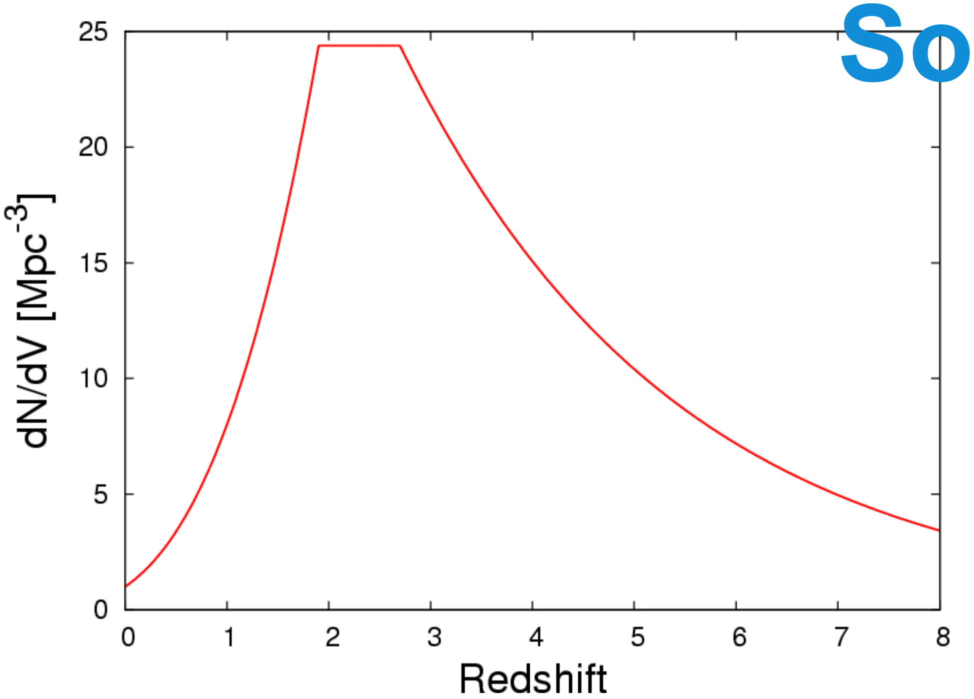
motivated by Fermi acceleration theory

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following. This amounts to assuming: $d_s < (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

A Cosmological Distribution of Sources

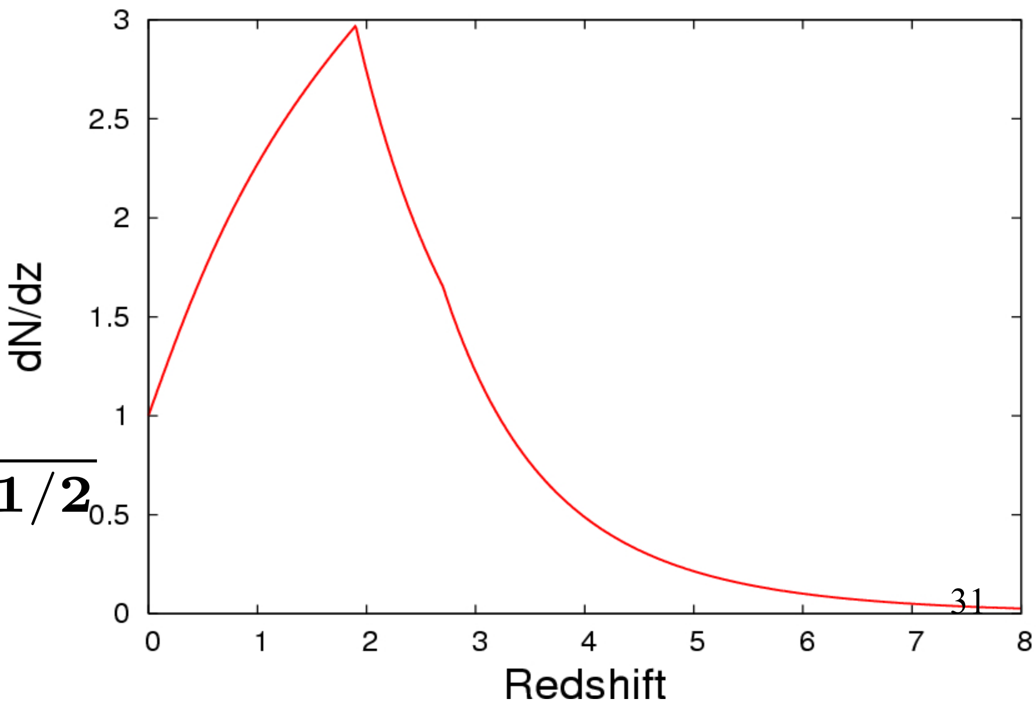


Distribution of sources in a comoving volume

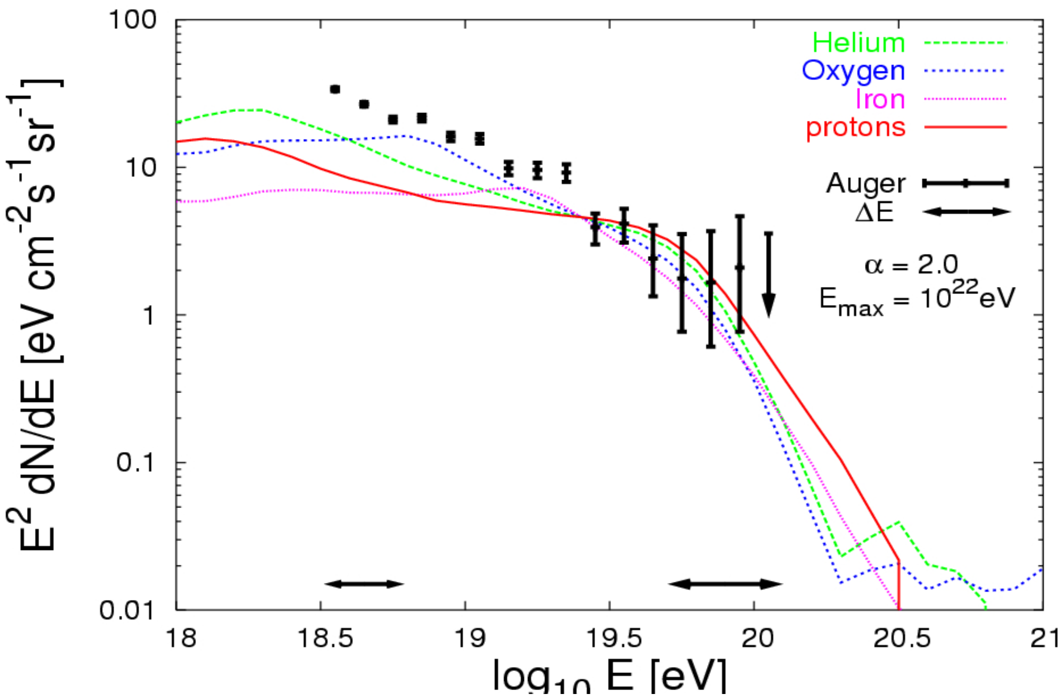
$$\mathbf{dV_c = 4\pi\chi^2 d\chi}$$

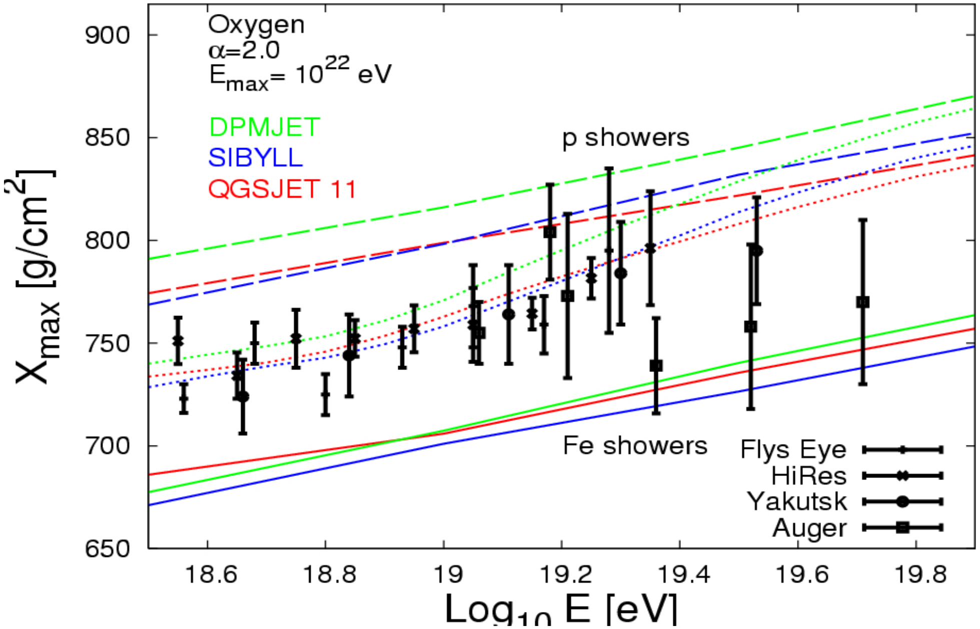
$$\mathbf{d\chi = \frac{dz}{H}}$$

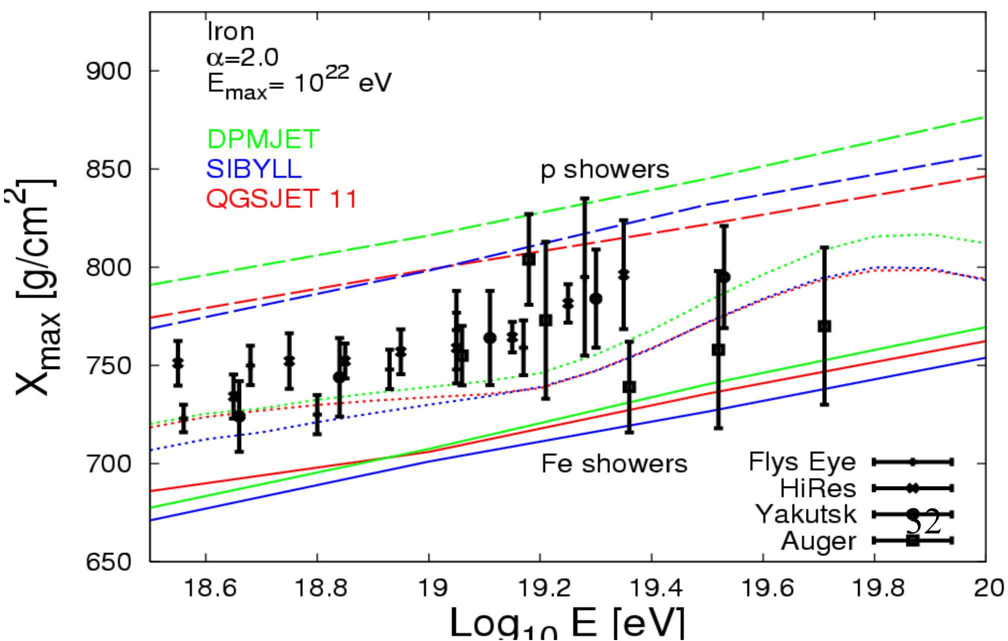
$$\approx \frac{\mathbf{dz}}{\mathbf{H_0(\Omega_M(1+z)^3 + \Omega_\Lambda)^{1/2}}}$$



Do Protons or Nuclei Fit the Data?







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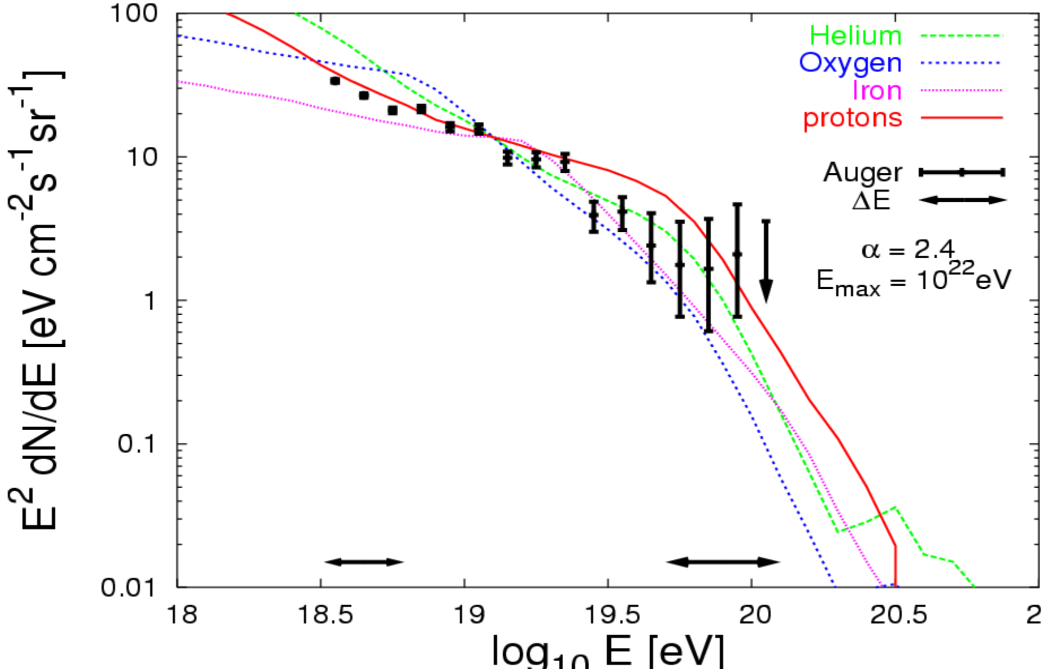
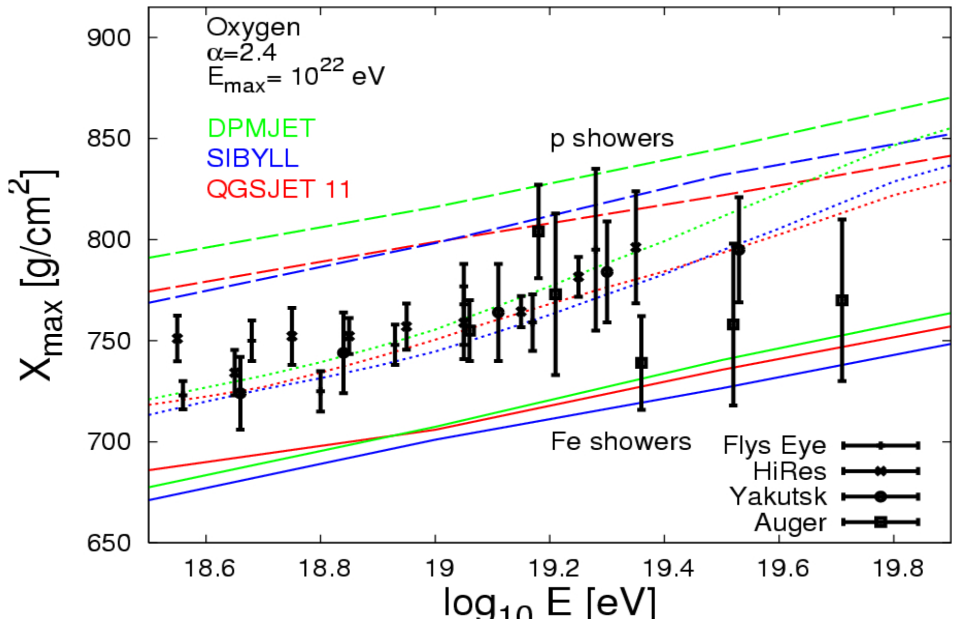
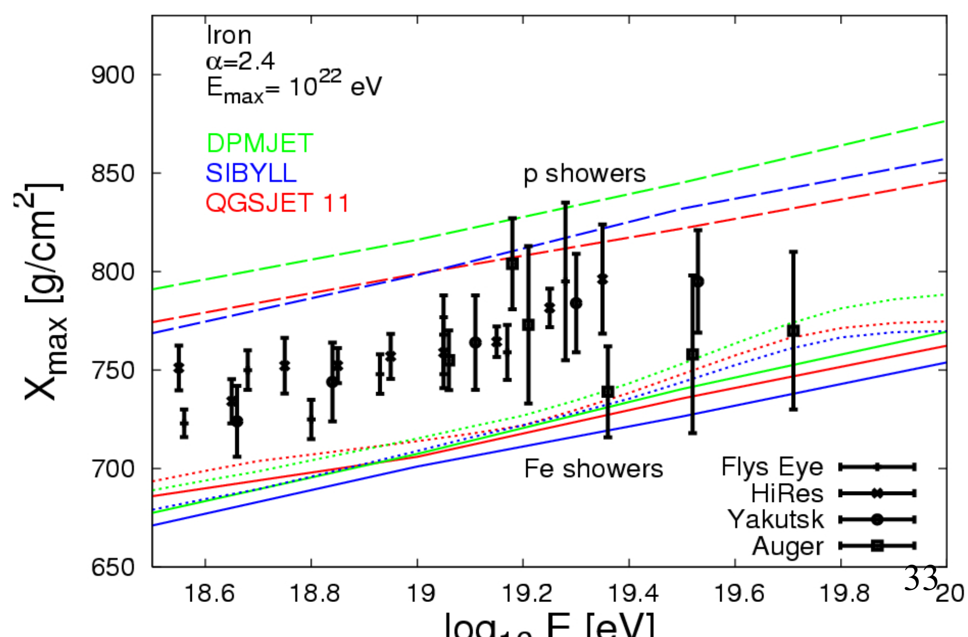
Or


E^2 dN/dE [eV cm^-2 s^-1 sr^-1]
log10 E [eV]
Helium
Oxygen
Iron
protons
Auger
ΔE
α = 2.4
Emax = 10^22 eV


Oxygen
α=2.4
Emax= 10^22 eV
DPMJET
SIBYLL
QGSJET 11
p showers
Fe showers
Flys Eye
HiRes
Yakutsk
Auger
Xmax [g/cm^2]
log10 E [eV]


Iron
α=2.4
Emax= 10^22 eV
DPMJET
SIBYLL
QGSJET 11
p showers
Fe showers
Flys Eye
HiRes
Yakutsk
Auger
Xmax [g/cm^2]
log10 E [eV]

Assumptions on Source Population

Spatial Distribution

$$\frac{dN}{dV_C} \propto (1+z)^n$$

$$z < z_{max}$$

$$n = -6,\ -3,\ 0,\ 3$$

Energy Distribution

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

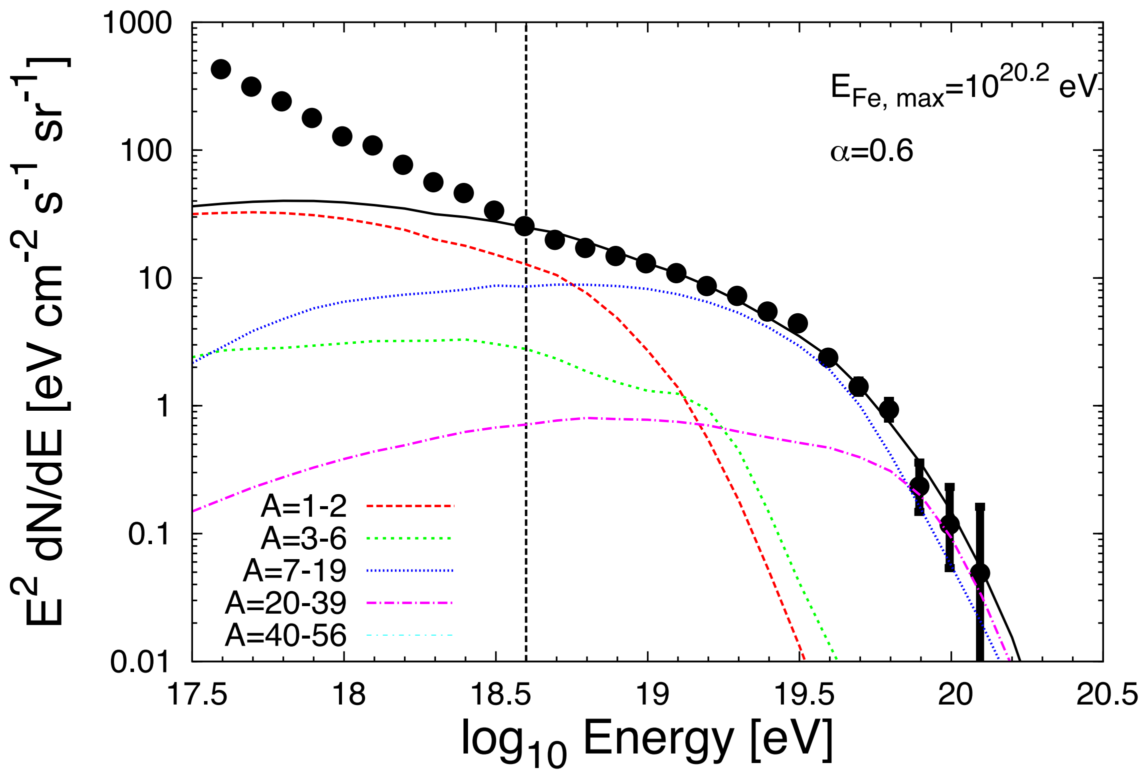
$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

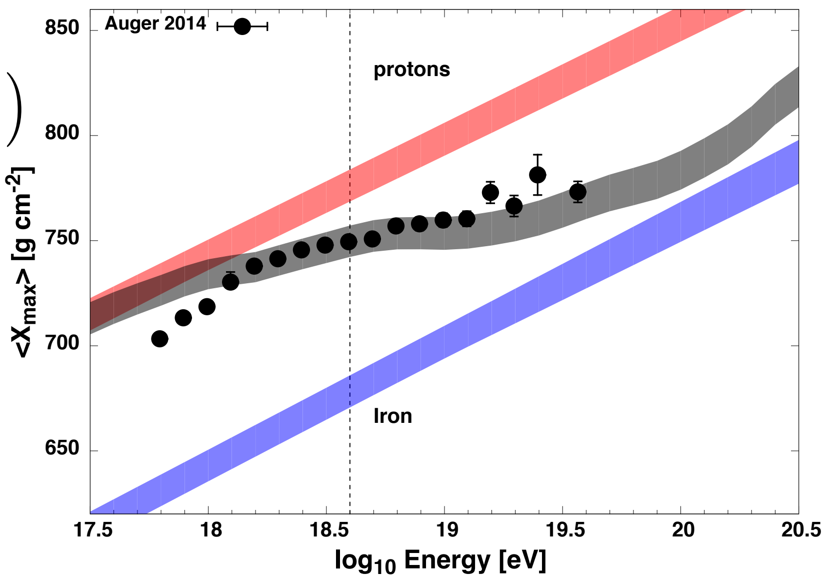
Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s \ll (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

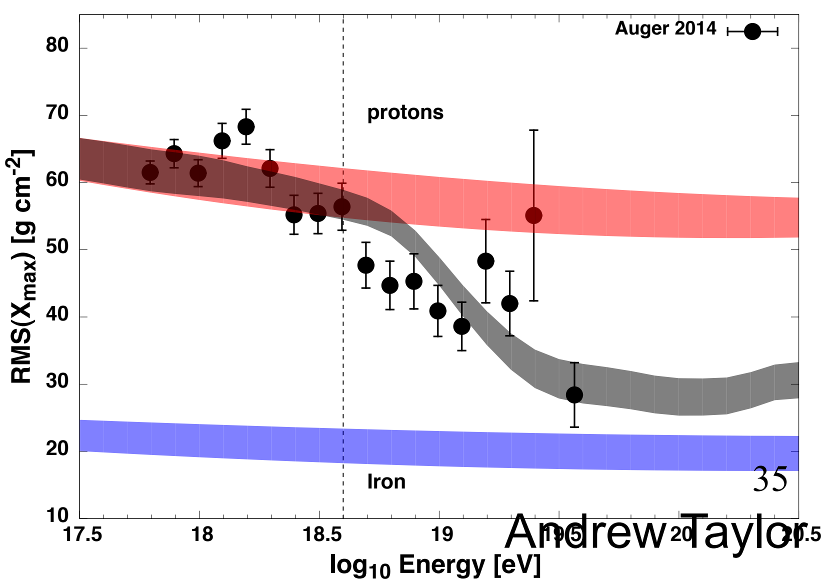
MCMC Likelihood Scan: Spectral + Composition Fits

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=3 evolution result







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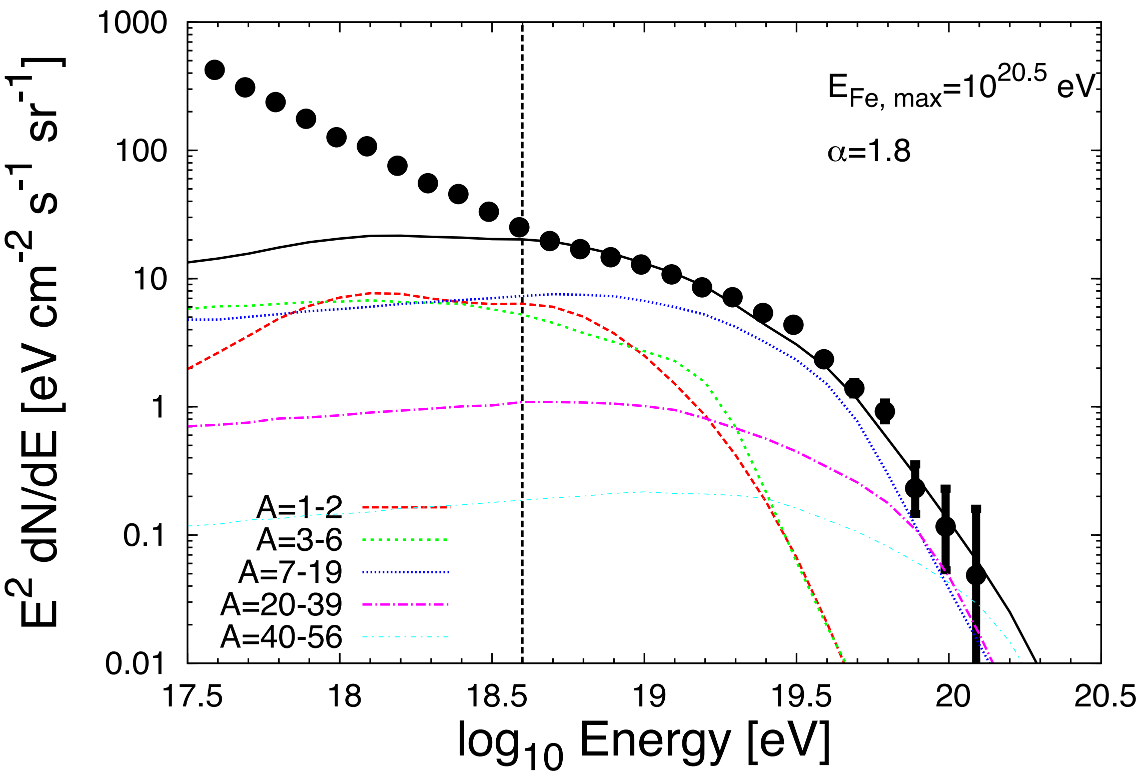
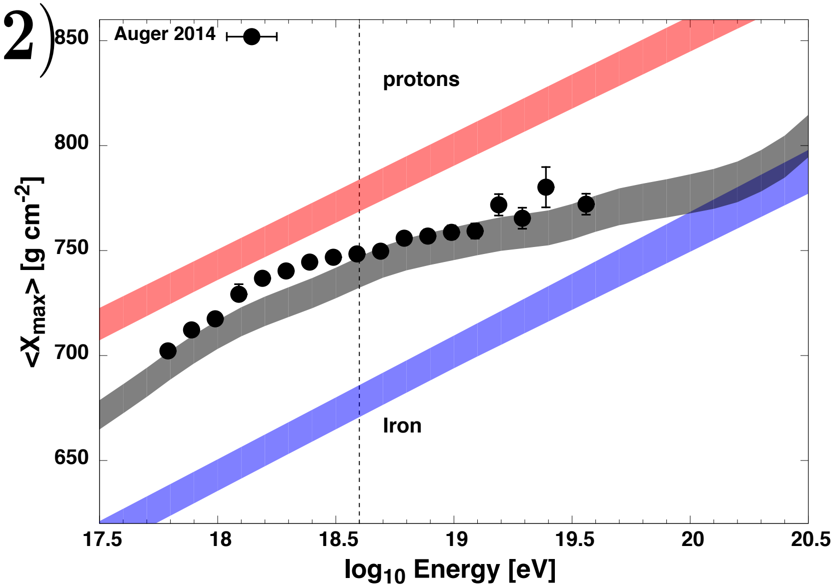
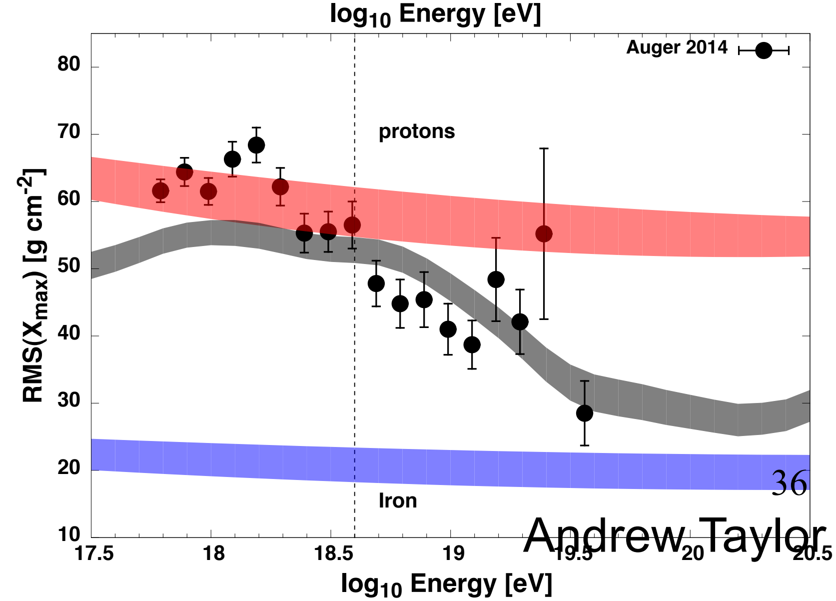
MCMC Likelihood Scan: “Soft” Spectra Solutions

$$L(f_p, f_{He}, f_N, f_{Si}, E_{max}, \alpha) \propto \exp(-\chi^2/2)$$

n=-6 evolution result







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MCMC Results Table

Similar conclusion arrives to by others (eg. ADD REF. TO KAMPERT ET AL.)

Parameter	$n=-6$		$n=-3$		$n=0$		$n=3$	
	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation
f_p	0.03	0.14 ± 0.12	0.08	0.15 ± 0.13	0.17	0.17 ± 0.16	0.19	0.20 ± 0.16
$f_{\rm He}$	0.50	0.21 ± 0.17	0.42	0.17 ± 0.16	0.53	0.20 ± 0.17	0.32	0.23 ± 0.20
$f_{\rm N}$	0.40	0.50 ± 0.18	0.42	0.51 ± 0.19	0.29	0.47 ± 0.19	0.43	0.45 ± 0.21
$f_{\rm Si}$	0.06	0.11 ± 0.12	0.08	0.12 ± 0.13	0.0	0.11 ± 0.12	0.06	0.078 ± 0.086
$f_{\rm Fe}$	0.01	0.052 ± 0.039	0.0	0.053 ± 0.042	0.01	0.050 ± 0.038	0.0	0.044 ± 0.034
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\rm Fe,max}}{\rm eV}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

Flatter spectra preferred for negative source evolution

Hard spectra preferred for source evolution following that of the SFR

An Analytic Description of these Results

Differential Equation Describing System State

$$\frac{\mathbf{d}}{\mathbf{dt}}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix} = \mathbf{\Lambda}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix}$$

$$\mathbf{\Lambda} = \begin{pmatrix} -\left(\frac{1}{\tau_{56\to55}} + \frac{1}{\tau_{56\to54}} + \ldots\right) & 0 & 0 \\ \frac{1}{\tau_{56\to55}} & -\left(\frac{1}{\tau_{55\to54}} + \frac{1}{\tau_{55\to53}} + \ldots\right) & 0 \\ \frac{1}{\tau_{56\to54}} & \frac{1}{\tau_{55\to54}} & -\left(\frac{1}{\tau_{54\to53}} + \frac{1}{\tau_{54\to52}} + \ldots\right) \end{pmatrix}$$

by $$\mathbf{f_q(t)} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \mathbf{A_n f_n(t)}$$

then $$\mathbf{f_q(t)} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \mathbf{A_n e^{-\lambda_n t} f_n(0)}$$

(where A_n values are set by the initial conditions)

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Only Considering Single Nucleon Losses

$$\mathbf{\Lambda} = \begin{pmatrix} -\frac{1}{\tau_{56\to55}} & 0 & 0 \\ \frac{1}{\tau_{56\to55}} & -\frac{1}{\tau_{55\to54}} & 0 \\ 0 & \frac{1}{\tau_{55\to54}} & -\frac{1}{\tau_{54\to53}} \end{pmatrix}$$

and

$$\mathbf{f_q(t) = \sum_{n=q}^{56} f_{56}(0) \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}}$$

Nuclear Cascade Description

Consider

$$\frac{df_q}{dt} + \frac{f_q}{\tau_q} = \frac{f_{q+1}}{\tau_{q+1}}$$

$$e^{\left(\frac{-t}{\tau_q}\right)} \frac{d}{dt}\left[e^{\left(\frac{t}{\tau_q}\right)} f_q\right] = \frac{f_{q+1}}{\tau_{q+1}}$$

$$f_q = e^{\left(\frac{-t}{\tau_q}\right)} \int e^{\left(\frac{t}{\tau_q}\right)} \frac{f_{q+1}}{\tau_{q+1}} dt$$

Assume solution is true for q, apply to q+1

$$\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1}\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

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Nuclear Cascade Description

Assume solution is true

$$\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1}\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

$$f_q = e^{\left(\frac{-t}{\tau_q}\right)} \int e^{\left(\frac{t}{\tau_q}\right)} \frac{f_{q+1}}{\tau_{q+1}} dt$$

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} \left[\left(\frac{1}{\tau_q} - \frac{1}{\tau_n}\right)^{-1} e^{\frac{-t}{\tau_n}}\right] - ce^{\frac{-t}{\tau_q}}$$

Since $\quad f_q(0) = 0$

$$c = \sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)}$$

Nuclear Cascade Description

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_n^{56-q-2}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{\frac{-t}{\tau_n}} - \sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{\frac{-t}{\tau_q}}$$

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

These are equivalent if:

$$\sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} = \frac{\tau_q \tau_q^{56-q-1}}{\prod_{p=q}^{56}(\tau_q - \tau_p)}$$

Consider:

$$\frac{w^2}{(w-x)(w-y)(w-z)} + \frac{x^2}{(x-w)(x-y)(x-z)} + \frac{y^2}{(y-w)(y-x)(y-z)} = -\frac{z^2}{(z-w)(z-x)(z-y)}$$

End of Second Lecture

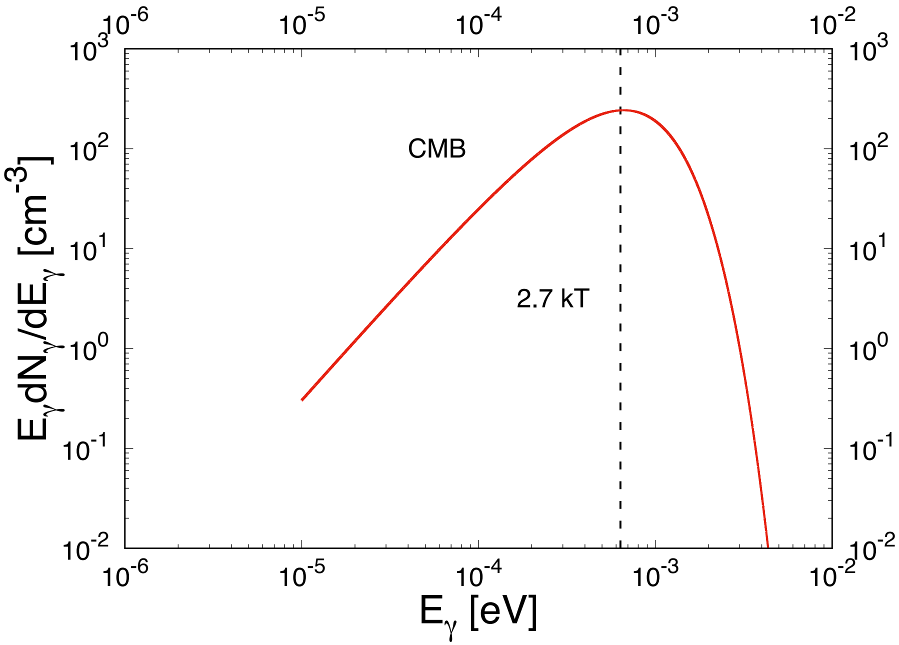
CMB- Total Number Density

$$n_\gamma^{BB} = 8\pi \frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3)$$

$$n_\gamma^{BB} = \frac{8\pi(kT)^3}{(hc)^3}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

$$\int_0^\infty x^2 e^{-x} dx = \gamma(3)$$

$$\frac{x^n}{e^x - 1} = \frac{e^{-x}x^n}{1 - e^{-x}}$$



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CMB- Total Number Density

$$\mathbf{n_\gamma^{BB}} = \mathbf{8}\pi\frac{\mathbf{(kT)^3}}{\mathbf{(hc)^3}}\gamma(\mathbf{3})\zeta(\mathbf{3})$$

$$\frac{\mathbf{x^n}}{\mathbf{e^x - 1}} = \frac{\mathbf{e^{-x}x^n}}{\mathbf{1 - e^{-x}}}$$

$$= \sum_{\mathbf{m=0}}^{\infty} \mathbf{e^{-mx}e^{-x}x^n}$$

$$= \sum_{\mathbf{m=1}}^{\infty} \mathbf{e^{-mx}x^n}$$

CMB- Total Number Density

$$n_\gamma^{BB} = 8\pi \frac{(kT)^3}{(hc)^3}\gamma(3)\zeta(3)$$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty}\int e^{-mx}x^n dx$$

Let $y = mx$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty}\int e^{-y}\left(\frac{y}{m}\right)^n d\left(\frac{y}{m}\right)$$

$$\int \frac{x^n}{e^x - 1}dx = \sum_{m=1}^{\infty}\frac{1}{m^{n+1}}\int y^n e^{-y}dy = \gamma(n+1)\zeta(n+1)$$

Threshold Energy- Proton Pion Production

$$(\mathbf{E_p + E_\gamma})^2 - (\mathbf{p_p - E_\gamma})^2 = (\mathbf{m_p + m_\pi})^2$$

$$\mathbf{m_p^2 + 2E_pE_\gamma + 2p_pE_\gamma \approx m_p^2 + 2m_pm_\pi}$$

$$\mathbf{E_p \approx \frac{m_\pi}{2E_\gamma} m_p \approx \left(\frac{135 \times 10^6}{2 \times 6 \times 10^{-4}}\right) 0.9 \times 10^9 = 10^{20}\ eV}$$

Photo-Pion Production Rate

$$\mathbf{R(\Gamma) \approx n_0\sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0\sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{1}{l_0} [\ (\gamma_i(3, x_2(\Gamma)) - \gamma_i(3, x_1(\Gamma))) - x_1(\Gamma)^2(\gamma_i(1, x_2(\Gamma)) - \gamma_i(1, x_1(\Gamma))) +}$$

$$\mathbf{x_2(\Gamma)^2(1 - \gamma_i(1, x_2(\Gamma))) - x_1(\Gamma)^2(1 - \gamma_i(1, x_2(\Gamma)))\ ]}$$

$$\mathbf{\gamma_i(3, x) = 2 - (2 + 2x + x^2)\exp(-x) \qquad \gamma_i(1, x) = 1 - \exp(-x)}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0}\left[e^{-x_1}(1 - e^{-x_1} + x_1(1 - 2e^{-x_1}))\right]}$$

Nuclear Cascade Description

$$\sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}}-\tau_{\mathbf{p}})} = \frac{\tau_{\mathbf{q}}\tau_{\mathbf{q}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{q}}-\tau_{\mathbf{p}})}$$

Consider the case

$$\frac{\mathbf{w^2}}{(\mathbf{w}-\mathbf{x})(\mathbf{w}-\mathbf{y})(\mathbf{w}-\mathbf{z})} + \frac{\mathbf{x^2}}{(\mathbf{x}-\mathbf{w})(\mathbf{x}-\mathbf{y})(\mathbf{x}-\mathbf{z})} + \frac{\mathbf{y^2}}{(\mathbf{y}-\mathbf{w})(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{z})} = -\frac{\mathbf{z^2}}{(\mathbf{z}-\mathbf{w})(\mathbf{z}-\mathbf{x})(\mathbf{z}-\mathbf{y})}$$

$$\begin{vmatrix} 1 & w & w^2 & w^2 \\ 1 & x & x^2 & x^2 \\ 1 & y & y^2 & y^2 \\ 1 & z & z^2 & z^2 \end{vmatrix} = \mathbf{0}$$

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Integrating Out the Time Variable of the Green's Function

$$\frac{dN(t)}{d^3r} = \frac{e^{-\frac{r^2}{4Dt}}}{(4\pi Dt)^{3/2}}$$

$$\frac{dn}{dr} = \int_0^\infty \frac{dN(t)}{d^3r} dt$$

Let $$x = \frac{r^2}{4Dt}$$

$$(4Dt)^{3/2} = r^3 x^{-3/2}$$

$$dt = -\frac{r^2}{4Dx^2} dx$$

$$\frac{dn}{dr} = \frac{1}{(\pi)^{3/2} 4Dr} \int_0^\infty x^{-1/2} e^{-x} dx$$

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INTERACTION OF ULTRA-HIGH ENERGY COSMIC RAYS WITH MICROWAVE BACKGROUND RADIATION

F. A. AHARONIAN[1], B. L. KANEVSKY[2], and V. V. VARDANIAN[1*]

(Received 18 October, 1989)

Abstract. The formation of the 'bump' and the 'black-body cutoff' in the cosmic-ray (CR) spectrum arising from the π-meson photoproduction reaction in collisions of UHE CR protons with the microwave background radiation (MBR) is studied. A kinetic equation which describes CR proton propagation in the MBR with account of the catastrophic nature of the π-meson photoproduction process is derived. The equilibrium CR proton spectrum obtained from the solution of the stationary kinetic equation is in general agreement with the spectrum obtained under assumption of the continuous energy loss approximation. However, the spectra from point sources noticeably differ from those obtained in the continuous loss approximation. Both, the equilibrium and the point source spectra are modified when taking into account the possible deviation of the MBR spectrum from the Planckian one in the Wien region. Thus, for the recently measured MBR spectrum, which reveals an essential 'excess' in the submillimeter region, the 'black-body cutoff' and the preceding 'bump' shift towards lower energies.

1. Introduction

The ultra-high energy cosmic-ray (CR) interaction in the intergalactic space with the microwave background radiation (MBR) gives rise to a 'black-body cutoff' of the CR spectrum predicted more than 20 years ago (Greisen, 1966; Zatsepin and Kuzmin, 1966). Unfortunately, the available experimental data do not allow us to draw an unambiguous conclusion concerning the presence or absence of such a spectral peculiarity (see, e.g., Watson, 1985). At the same time, in the energy range $E > 10^{19}$ eV the Fly's Eye has detected some excess (a 'bump') in the spectrum (Baltrusaitis *et al.*, 1985), which agrees with the evidence obtained by Haverah Park (Cunningham *et al.*, 1980), Volcano Ranch (Linsley, 1985), and Akeno (Teshima *et al.*, 1987) groups to a tendency of spectrum flattening in this energy region. With a lesser confidence this peculiarity is also revealed in the data of Yakutsk (Khristiansen, 1985) and Sydney (Winn *et al.*, 1985) extensive air shower (EAS) arrays.

Hill and Schramm (1985), examining the UHE proton transfer in the MBR field, arrived at a rather important conclusion that due to the pion photoproduction process, besides the 'black-body cutoff', there is also formed a 'bump' (preceding the cutoff). The latter spectral peculiarity is apparently due to a sharp (exponential) energy dependence of the proton-free path (owing to the threshold nature of the $\gamma p \to \pi N$ process, protons with energy $E < 10^{20}$ eV interact only with the Wien 'tail' of the MBR spectrum). Protons with energy $E \geq 5 \times 10^{19}$ eV effectively interact with the MBR, deposit energy

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of this equation we present in the form of an iterative series

$$F_n(E, t) = q(E)\, e^{-t/\tau} + q(E) \int_0^t \mathrm{d}t'\, e^{-(t-t')/\tau} \hat{A} F_{n-1}(E, t), \qquad \text{(A2–2)}$$

where $F_0(E, t) = q(E)\, e^{-t/\tau}$ is the initial approximation for the spectrum. For numerical calculations it is convenient to pass to a new function $f(E, t)$ using the replacement

$$F(E, t) = q(E) f(E, t). \qquad \text{(A2–3)}$$

Then for $f(E, t)$ we obtain a solution in the form

$$f_n(E, t) = e^{-t/\tau} + \int_0^t \mathrm{d}t'\, e^{-(t-t')/\tau} \hat{A}_1 f_{n-1}(E, t), \qquad \text{(A2–4)}$$

where the integral term is

$$\hat{A}_1 f = \frac{ckT}{2\pi^2 (c\hbar)^3 \Gamma^2} \int_{\varepsilon_0}^{\infty} \mathrm{d}\omega_r\, \sigma(\omega_r) \varphi(\omega_r) \omega_r \times$$

$$\times \int_{z_-(\omega_r)}^{z_+(\omega_r)} \mathrm{d}z\, z^{\gamma+1} f(E/z, t) \left[-\ln\left(1 - \exp\left(-\frac{\omega_r z}{2\Gamma kT}\right)\right)\right], \qquad \text{(A2–5)}$$

where $z_\pm$ and φ are determined by the expressions (11).

In the energy region $E \leq 3 \times 10^{20}$ eV the integral term may be approximately presented as

$$\hat{A}_1 f = f(E/z_0, t) z_0^{\gamma-1} / \tau(E/z_0), \qquad \text{(A2–6)}$$

where

$$z_0 = 1 - f(\varepsilon_0). \qquad \text{(A2–7)}$$

The the solution for the function $f(E, t)$ can be presented as

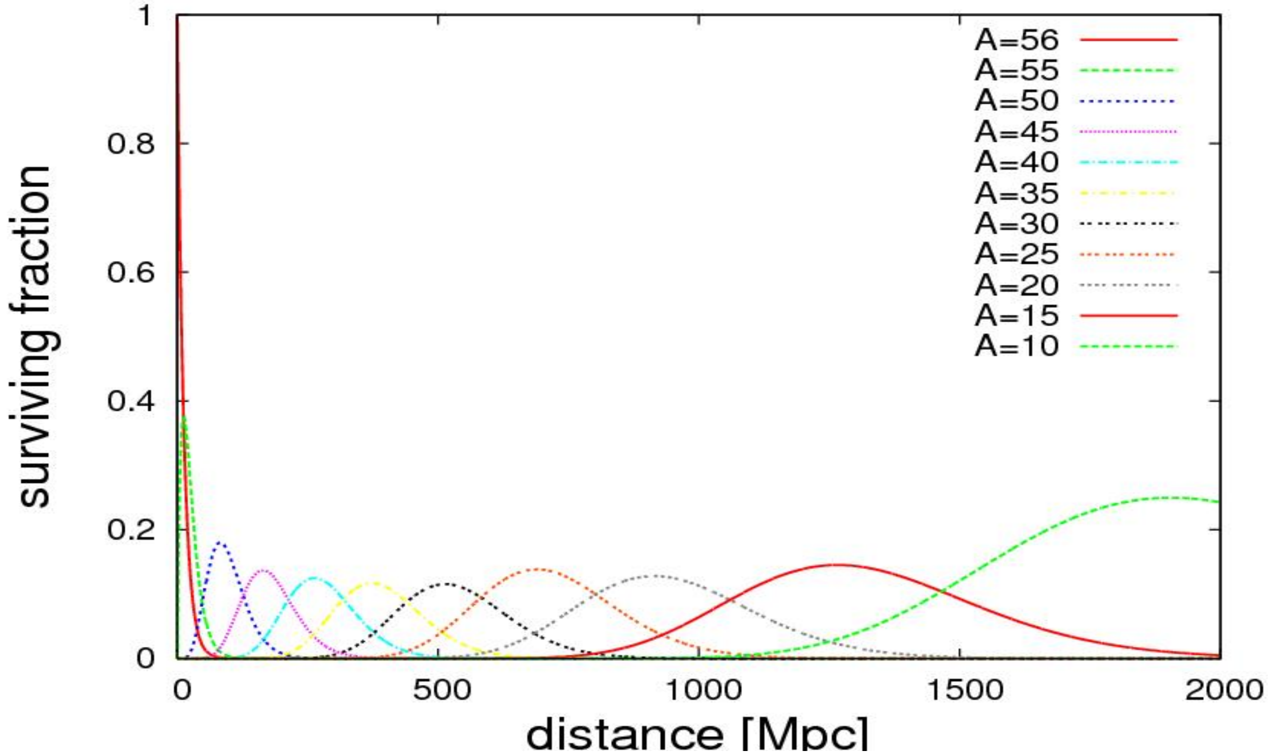
$$f(E, t) = \sum_{n=0}^{\infty} z_0^{n(\gamma-1)} \sum_{j=0}^{n} \frac{\exp(-t/\tau_j) \tau_0/\tau_j}{\prod_{k\neq j}^{n} [1 - \tau_k/\tau_j]}, \qquad \text{(A2 – 8)}$$

where

$$\tau_j = \tau(E/z_0^j); \quad \tau_0 = \tau(E). \qquad \text{(A2 – 9)}$$

The MBR deviation from the Planckian spectrum (in case of its approximation by the comptonized black-body radiation spectrum (14)) for the proton spectrum from a point source, can be taken into account just like in case of the equilibrium proton spectrum (see Appendix 1).

Injecting a 10^{20}eV Fe Nucleus and Tracking the Subsequent Nuclei-



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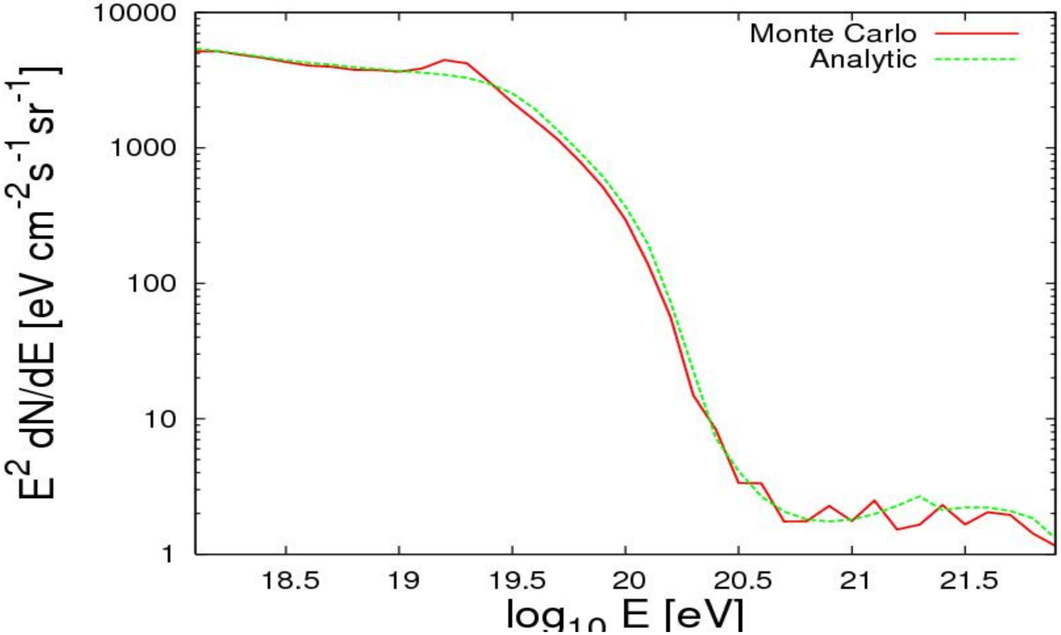
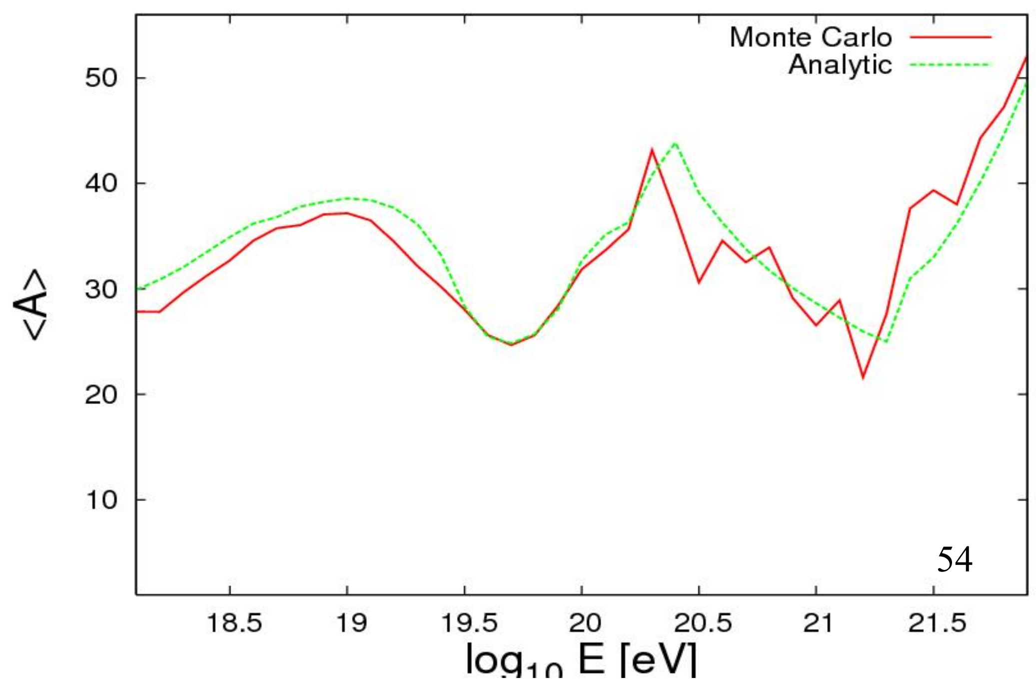
Comparison of Analytic and Monte Carlo Results





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Conclusions

- The Pierre Auger Observatory is able to provide much more than just the cosmic ray flux measurement
- Due to the ln E_0 dependence of X_{max}, excellent energy resolution is required to pull out the composition information
- The X_{max} and energy spectrum data collectively can provide useful information about the source injection spectrum and cutoff energy
- The propagation of nuclei can be easily understood through the application of an analytic description of the photo-disintegration process

Cascade of Nuclei Through Species- single nucleon loss

Since nuclei Lorentz factor remains ~conserved, and cross-section varies mildly with A (nuclear mass)

$$\tau_{56\rightarrow 55} \approx \tau_{55\rightarrow 54}...$$

For the case $\tau_{56\rightarrow 55} = \tau_{55\rightarrow 54}...$

$$f_q = \frac{t^{(q_{max}-q)}}{\tau_q (q_{max}-q)!} e^{-t/\tau_q}$$

ie. Gaisser-Hillas type function!
(used to describe air showers)

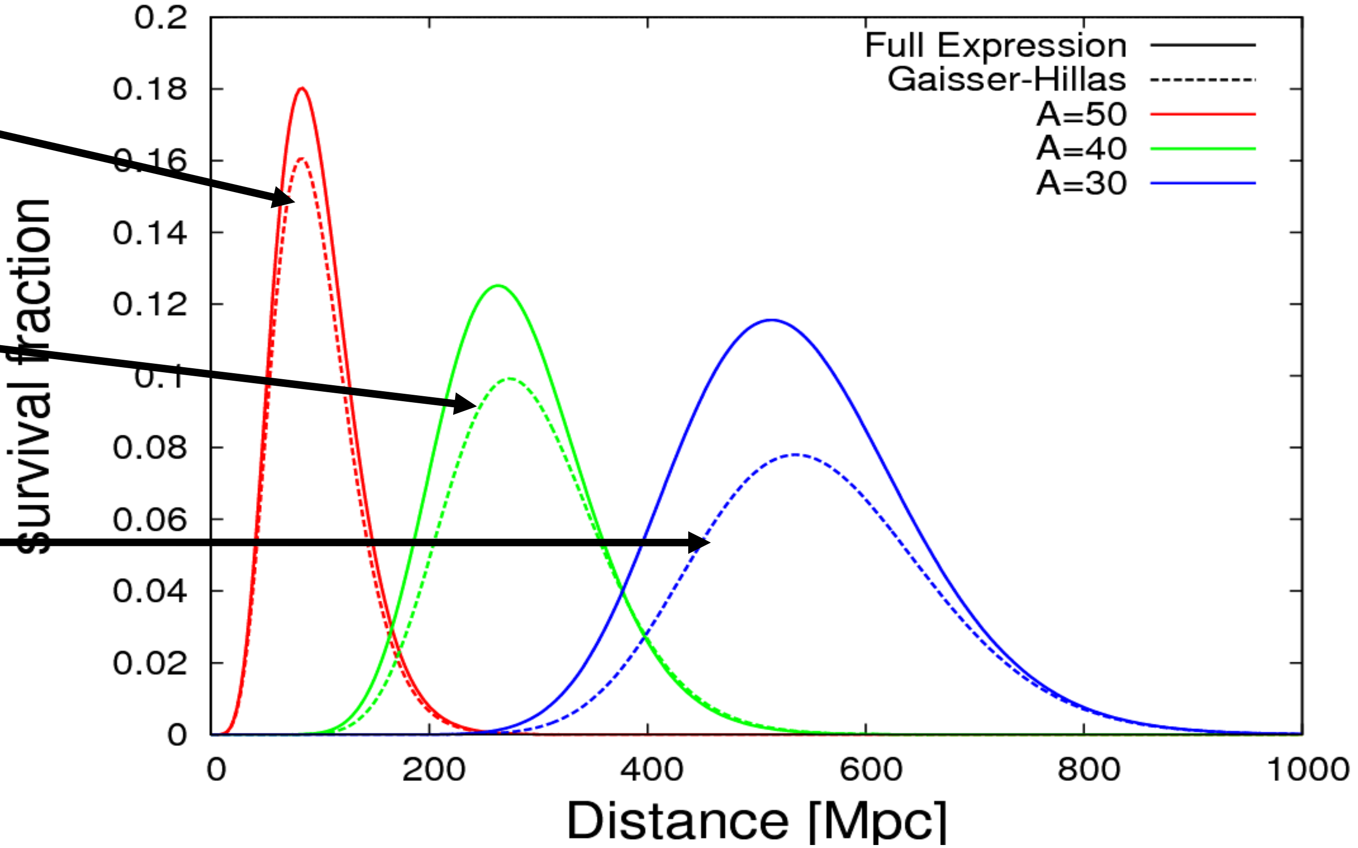
Cascade of Nuclei Through Species- Comparison of Approximation

Starting with Fe, $q_{\mathbf{max}} = \mathbf{56}$

$$f_{50} = \frac{t^6}{6!}e^{-\frac{t}{\tau_{50}}}$$

$$f_{40} = \frac{t^{16}}{16!}e^{-\frac{t}{\tau_{40}}}$$

$$f_{30} = \frac{t^{26}}{26!}e^{-\frac{t}{\tau_{30}}}$$



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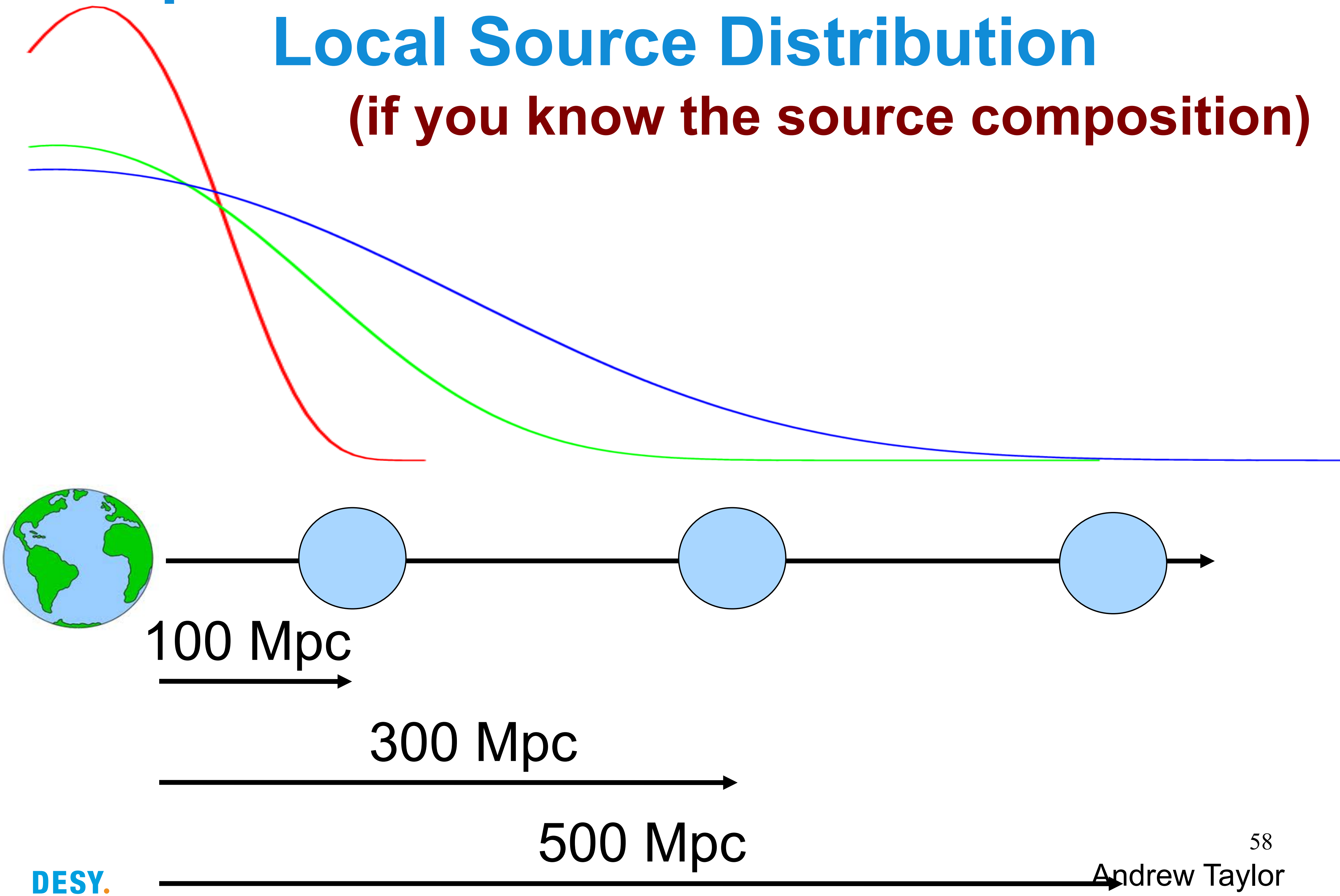
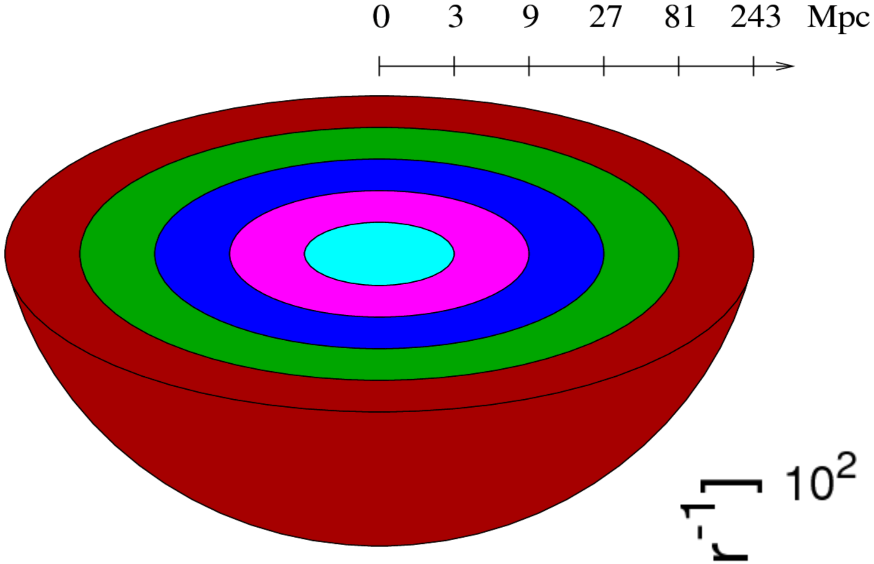
Composition – an Excellent Probe of the Local Source Distribution

(if you know the source composition)



Local Scales Effect Highest Energies



(logarithmic scale)