Flavor Physics: Introduction to C, P and T symmetries

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General Aspects of CP Violation

When we look for CP violation ($\not{CP}$), we search for situations where probability for one process differs from its CP conjugate process,

$$P(A \to B) \neq P(\bar{A} \to \bar{B})$$

with

$$A, B \xrightarrow{CP} \bar{A}, \bar{B}.$$

On the other hand, T and CPT violation are related to

$$\begin{aligned} \not{T}: &\quad P(A \to B) \neq P(B \to A) \\ \not{CPT}: &\quad P(A \to B) \neq P(\bar{B} \to \bar{A}). \end{aligned}$$

- So far $\not{CP}$ has only been unambiguously observed in K_L, B_0, and $B^{\pm}$ decays.
- $\not{CP}$ is typically measured through asymmetries. These are ratios of branching ratios of the form

$$\mathcal{A} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}.$$

- There are basically two processes sensitive to CP: decay and oscillation.

In order to see $\not{CP}$ we need two amplitudes to interfere. Thus there are three options:

- Direct $\not{CP}$: CP violation in decay, interference between decay amplitudes.
- Indirect $\not{CP}$: CP violation in mixing.
- $\not{CP}$ in interference of mixing and decay.

Direct $\not{CP}$

- It is conceptually simple: For a CP asymmetry in the decay $X \to f$ we have

$$\mathcal{A}_{CP} = \frac{|\langle f|X\rangle|^2 - |\langle \bar{f}|\bar{X}\rangle|^2}{|\langle f|X\rangle|^2 + |\langle \bar{f}|\bar{X}\rangle|^2},$$

  where the $\overline{X}$ and $\bar{f}$ are the CP conjugates of X and f respectively.
- ...but the price we pay for this simplicity is that they are hard to compute from first principles.

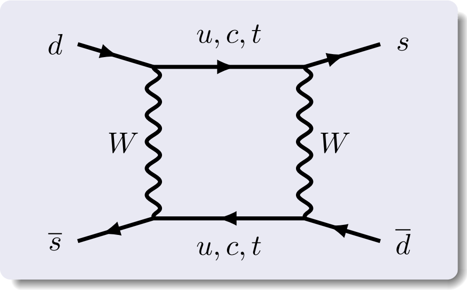
Meson Mixing and Oscillation

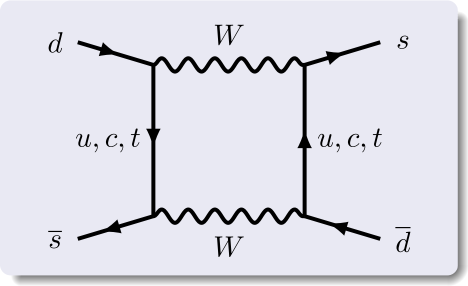
- Mixing: Theoretical concept. Flavor eigenstates are different from the mass eigenstates.
- Oscillation: Time evolution of a flavor state transforms it into another flavor state due to the mismatch of energy eigenstates.

The most famous example is kaon mixing between

$$K^0 = \bar{s}d \quad \text{and} \quad \overline{K}^0 = \bar{d}s.$$

The flavor-changing $K^0 - \overline{K}^0$ mixing occurs only at loop level due to box diagrams of the form





Weisskopf-Wigner mixing formalism

- Generic neutral meson-antimeson system $X^0 - \overline{X}^0$.
- $X^0 = K^0$, D^0, B^0 or B_s.
- Transformation properties under P and C

$$P|X^0\rangle = -|X^0\rangle \qquad P|\overline{X}^0\rangle = -|\overline{X}^0\rangle$$
$$C|X^0\rangle = |\overline{X}^0\rangle \qquad C|\overline{X}^0\rangle = |X^0\rangle$$

- CP transformation

$$CP|\bar{X}^0\rangle = -|X^0\rangle \qquad \text{and} \qquad CP|\bar{X}^0\rangle = -|X^0\rangle\,.$$

- Time evolution and mixing: Two state Hamiltonian
- Finite life-time effects: Non-Hermitian Hamiltonian (Open system, one particle states may evolve into states that are not accounted for in the two state Hamiltonian)
- $\Rightarrow$ Effective Hamiltonian

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

where

$$\mathbf{M}^\dagger = \mathbf{M}, \qquad \mathbf{\Gamma}^\dagger = \mathbf{\Gamma},$$
$$|1\rangle = |X^0\rangle, \qquad |2\rangle = |\overline{X}^0\rangle.$$

- CPT invariance of $\mathbf{H}$ requires

$$M_{11} = M_{22} = M, \qquad \Gamma_{11} = \Gamma_{22} = \Gamma.$$

$$\Rightarrow \mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}.$$

- Diagonalizing $\mathbf{H}$ gives the two mass eigenvalues and the two widths.

Diagonalizing 2×2 matrices

Let's focus first in the limit of vanshing $\mathbf{\Gamma}$:

- Given a Hermitian 2×2 matrix,

$$\begin{pmatrix} a & b \\ b^* & c \end{pmatrix} \tag{1}$$

  one should remember that the mixing angle θ is given by

$$\tan 2\theta = \frac{2|b|}{a-c}. \tag{2}$$

- In the case where $a = c$, we get a maximal mixing $\theta = \pi/4$ even when $|b|$ is very small!

- Taking for example

$$\mathbf{H} = M \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix},$$

in the Schrdinger equation

$$i\frac{d}{dt}\begin{pmatrix} X^0(t) \\ \overline{X}^0(t) \end{pmatrix} = M \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} X^0(t) \\ \overline{X}^0(t) \end{pmatrix},$$

gives

$$X^0(t) = e^{-iMt}\left[\cos(\epsilon Mt) X^0(0) - i\sin(\epsilon Mt)\overline{X}^0(0)\right],$$
$$\bar{X}^0(t) = e^{-iMt}\left[\cos(\epsilon Mt)\overline{X}^0(0) - i\sin(\epsilon Mt) X^0(0)\right].$$

- This is the magic of meson-mixing: small perturbation $\Rightarrow$ large effect (full mixing).
- A small ϵ implies slow oscillation, but the oscillation turns the initial X^0 into 100% $\overline{X}^0$ in half a period.

General case $\Gamma \neq 0$

The physical eigenstates are labeled conventionally as Heavy (H) and Light (L)

$$|X_H\rangle = p|X^0\rangle + q|\overline{X}^0\rangle, \qquad |X_L\rangle = p|X^0\rangle - q|\overline{X}^0\rangle$$

and the corresponding eigenvalues are defined as

$$M_{X_{\substack{H \\ L}}} - \frac{i}{2}\Gamma_{X_{\substack{H \\ L}}} = M - \frac{i}{2}\Gamma \pm \frac{1}{2}(\Delta M - \frac{i}{2}\Delta\Gamma).$$

Note that for $q = p$ these are CP-eigenstates: $CP|X_{\substack{H \\ L}}\rangle = \mp|X_{\substack{H \\ L}}\rangle$.

In terms of the **H** parameters,

$$\frac{p}{q} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}},$$

while ΔM and $\Delta\Gamma$ are the solutions of

$$\begin{aligned}(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2 \\ \Delta M \Delta\Gamma &= 4\mathrm{Re}(M_{12}\Gamma_{12}^*).\end{aligned}$$

$K^0 - \overline{K}^0$ System

For Kaons we label the states differently: Long (L) and Short (S) . The eigenvalues of the 2×2 Hamiltonian are

$$M_{K_{\substack{L\\S}}} - \frac{i}{2}\Gamma_{K_{\substack{L\\S}}} = M - \frac{i}{2}\Gamma \pm \frac{1}{2}(\Delta M - \frac{i}{2}\Delta\Gamma)$$

and the corresponding eigenvectors are

$$|K_{\substack{L\\S}}\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\overline{K}^0\rangle\right]$$

with

$$\frac{p}{q} = \frac{1+\epsilon}{1-\epsilon}.$$

In the limit $\epsilon \to 0$ these are CP-eigenstates:

$$CP|K_L\rangle = -|K_L\rangle, \qquad CP|K_S\rangle = |K_S\rangle.$$

Since

$$CP|\pi\pi\rangle_{\ell=0} = |\pi\pi\rangle_{\ell=0}, \qquad CP|\pi\pi\pi\rangle_{\ell=0} = -|\pi\pi\pi\rangle_{\ell=0}.$$

if CP were a good symmetry:

- $K_L \to \pi\pi\pi$ and $K_S \to \pi\pi$ are allowed,
- $K_L \to \pi\pi$ and $K_S \to \pi\pi\pi$ are forbidden.

CP-violation in mixing occurs when $\epsilon \neq 0$, allowing the observation of $K_L \to \pi\pi$ and $K_S \to \pi\pi\pi$.

This is very close to what is observed:

$$\begin{aligned} \mathrm{Br}(K_S \to \pi\pi) &= 100.00 \pm 0.24\% \\ \mathrm{Br}(K_L \to \pi\pi) &= 0.297 \pm 0.023\% \\ \mathrm{Br}(K_L \to \pi\pi\pi) &= 33.9 \pm 1.2\% \end{aligned}$$

Hence, we conclude

- ϵ is small
- CP is not a symmetry
- The longer life-time of K_L

$$\begin{aligned} \tau_{K_S} &= 0.59 \times 10^{-10}\ \mathrm{s} \\ \tau_{K_L} &= 5.18 \times 10^{-8}\ \mathrm{s} \end{aligned}$$

  is accidental ($m_K - 3m_\pi$ leaves less phase space for the decay in comparison with $m_K - 2m_\pi$).

CPV in Mixing

For the $K^0 - \overline{K}^0$ system we define the K_L semileptonic decay charge-asymmetry, which is a measure of CP violation:

$$\delta = \frac{\Gamma(K_L \to \pi^- e^+ \nu) - \Gamma(K_L \to \pi^+ e^- \overline{\nu})}{\Gamma(K_L \to \pi^- e^+ \nu) + \Gamma(K_L \to \pi^+ e^- \overline{\nu})}$$

In order to compute this we use the expansion of K_L in terms of flavor eigenstates K^0 and $\overline{K}^0$ and we assume

- The underlying process is $s \to u e^- \bar{\nu}$ (or $\bar{s} \to \bar{u} e^+ \nu$), so that

$$\langle \pi^- e^+ \nu | H_W | \overline{K}^0(t) \rangle = 0 = \langle \pi^+ e^- \nu | H_W | K^0(t) \rangle$$

- $\not{C}\!\!\not{P}$ is in the mixing only (through the parameter ϵ)
- CP is a good symmetry of the decay amplitude: $\langle \pi^- e^+ \nu | H_W | K^0(t) \rangle = \langle \pi^+ e^- \nu | H_W | \overline{K}^0(t) \rangle$.

With these assumptions one can show that

$$\delta = \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{|1+\epsilon|^2 + |1-\epsilon|^2} \approx 2\mathrm{Re}\epsilon$$

Experimental measurement gives $\delta_{\rm exp} = 0.330 \pm 0.012\%$, from which $\mathrm{Re}\epsilon \simeq 1.65 \times 10^{-3}$.

Time Evolution in X^0-$\overline{X}^0$ mixing.

As K_L and K_S are eigenvectors of H their time evolution is quite simple

$$
\begin{aligned}
i\frac{d}{dt}|X_{H,L}\rangle &= (M_{H,L} - \tfrac{i}{2}\Gamma_{H,L})|X_{H,L}\rangle \\
\Rightarrow \qquad |X_{H,L}(t)\rangle &= e^{-iM_{H,L}t}e^{-\frac{1}{2}\Gamma_{H,L}t}|X_{H,L}(0)\rangle.
\end{aligned}
$$

Inverting in favor of the original states:

$$
\begin{aligned}
|X^0\rangle &= \tfrac{1}{2p}\left(|X_H\rangle + |X_L\rangle\right), \\
|\overline{X}^0\rangle &= \tfrac{1}{2q}\left(|X_H\rangle - |X_L\rangle\right).
\end{aligned}
$$

we obtain

$$
|X^0(t)\rangle = \frac{1}{2p}\left[e^{-iM_Ht}e^{-\frac{1}{2}\Gamma_Ht}|X_H(0)\rangle + e^{-iM_Lt}e^{-\frac{1}{2}\Gamma_Lt}|X_L(0)\rangle\right]
$$

and in terms of the states X^0 and $\overline{X}^0$ at $t = 0$

$$|X^0(t)\rangle = f_+(t)|X^0(0)\rangle + \tfrac{q}{p} f_-(t)|\overline{X}^0(0)\rangle$$

where

$$\begin{aligned} f_\pm(t) &= \tfrac{1}{2}\left[e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t} \pm e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t}\right] \\ &= \tfrac{1}{2} e^{-iM_H t} e^{-\frac{1}{2}\Gamma_H t}\left[1 \pm e^{i\Delta M t} e^{\frac{1}{2}\Delta\Gamma t}\right] \\ &= \tfrac{1}{2} e^{-iM_L t} e^{-\frac{1}{2}\Gamma_L t}\left[e^{-i\Delta M t} e^{-\frac{1}{2}\Delta\Gamma t} \pm 1\right] \end{aligned}$$

Similarly,

$$|\overline{X}^0(t)\rangle = \tfrac{p}{q} f_-(t)|X^0(0)\rangle + f_+(t)|\overline{X}^0(0)\rangle.$$

$\not{C}\not{P}$ in Interference of Mixing and Decay

Consider an asymmetry constructed from $\Gamma = \Gamma(X^0 \to f)$ and $\bar{\Gamma} = \Gamma(\overline{X}^0 \to \bar{f})$, where f stands for some final state and $\bar{f}$ for its CP conjugate. If both X^0 and $\overline{X}^0$ can decay to the same common state one can have

$$\begin{tikzcd}
 & X^0 \arrow[dr] & \\
X^0 \arrow[ur] \arrow[dr] & & f \\
 & \overline{X}^0 \arrow[ur] &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
 & X^0 \arrow[dr] & \\
\overline{X}^0 \arrow[ur] \arrow[dr] & & \bar{f} \\
 & \overline{X}^0 \arrow[ur] &
\end{tikzcd}$$

- Specializing in the case of the B^0 meson, the approximations $|p/q| = 1$ and $\Delta\Gamma = 0$ simplify the evolution expressions:

$$f_\pm(t) \approx e^{-iMt} e^{-\frac{1}{2}\Gamma t} \begin{cases} \cos\left(\frac{1}{2}\Delta M t\right) \\ -i\sin\left(\frac{1}{2}\Delta M t\right) \end{cases}$$

- Assuming $\bar{f} = \pm f$ (valid for D^+D^- or, to good approximation, $J/\psi K_S$) we have $A_{\bar{f}} = \pm A_f$ and $\bar{A}_{\bar{f}} = \pm \bar{A}_f$ rendering

$$\mathcal{A}_{f_{CP}} = \frac{|\frac{p}{q} f_-(t) A_f + f_+(t)\bar{A}_f|^2 - |f_+(t)A_f + \frac{q}{p} f_-(t)\bar{A}_f|^2}{|\frac{p}{q} f_-(t) A_f + f_+(t)\bar{A}_f|^2 + |f_+(t)A_f + \frac{q}{p} f_-(t)\bar{A}_f|^2}$$

- Now, dividing by $|A_f|^2$ and defining

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

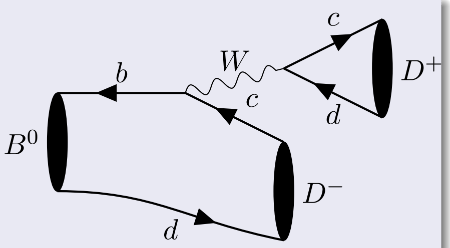
we have

$$\begin{aligned}
\mathcal{A}_{f_{\mathrm{CP}}} &= \frac{|f_-(t) + f_+(t)\lambda_f|^2 - |f_+(t) + f_-(t)\lambda_f|^2}{|f_-(t) + f_+(t)\lambda_f|^2 - |f_+(t) + f_-(t)\lambda_f|^2} \\
&= -\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos(\Delta M t) + \frac{2\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}\sin(\Delta M t) \\
&\equiv -C_f \cos(\Delta M t) + S_f \sin(\Delta M t)
\end{aligned}$$

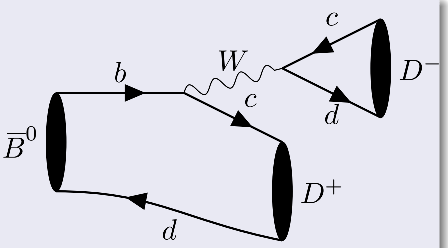
- λ_f is a physical combination of parameters. The first fraction has to do with mixing while the second fraction has to do with decay.

Surprisingly, the coefficients C_f and S_f can be computed in terms of CKM elements only. They are independent of non-computable, non-perturbative matrix elements!

Example: $B^0 \to f$ and $\overline{B}^0 \to f$ with $f = D^+D^-$,



$A_{D^+D^-} \propto V_{cb}^* V_{cd}$



$\bar{A}_{D^+D^-} \propto V_{cb} V_{cd}^*$

- As far as the strong interactions are concerned, the two diagrams are identical, thus

$$\frac{\bar{A}_{D^+D^-}}{A_{D^+D^-}} = \frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}.$$

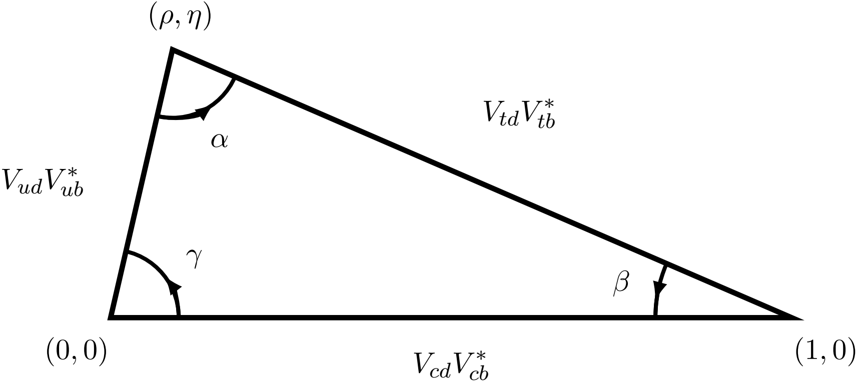
- Since $|\bar{A}_{D^+D^-}/A_{D^+D^-}| = 1$, this is a pure phase, and we see that the phase is given purely in terms of CKM elements.
  To complete the argument we need q/p. For B^0, Γ_{12} is negligible.
  A detailed calculation shows that

$$\frac{p}{q} = \frac{2M_{12}}{\Delta M} = \frac{\Delta M}{2M_{12}^*} = \frac{M_{12}}{|M_{12}|} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}.$$

- Collecting results

$$\mathrm{Im}\left(\lambda_{D^+D^-}\right) = \mathrm{Im}\left(\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right) = \mathrm{Im}(e^{2i\beta}) = \sin(2\beta)$$

The angle β has a simple interpretation: is one of the angles of the unitary triangle defined by one unitarity condition of the CKM:



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Thank you!