Flavor Physics:
Introduction to C, P and T symmetries

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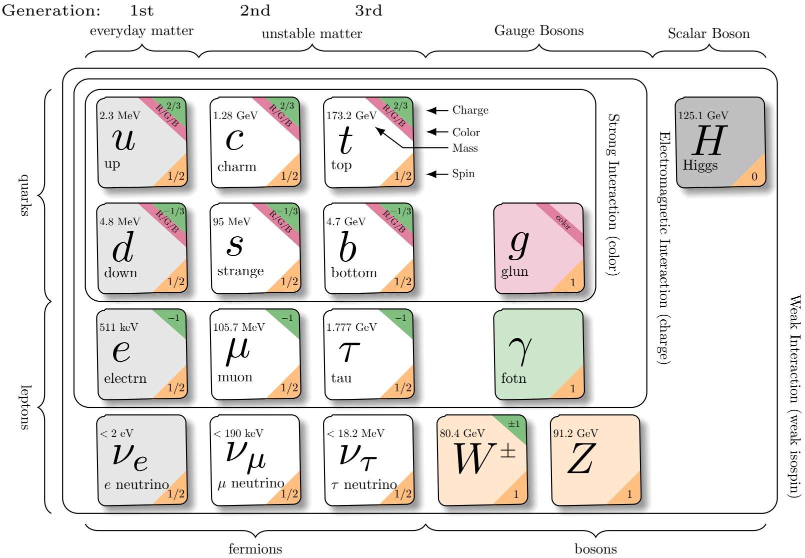
Outline

EW SM ingredients



Flavor Physics

- Flavor physics is the study of different types of quarks and leptons, or flavors, their spectrum and the transmutations among them.
- Flavor physics is very rich. Check out http://pdg.lbl.gov for the many different transition rates among hadrons with different quark content. We aim at understanding this wealth of information in terms of some simple basic principles.

C, P and T

- C, P and T are discrete spacetime transformations
- *A priori*, they have noting to do with flavor physics, as flavor has to do with internal symmetries. However, it turns out that in nature, all observations of CP violation happen to come along with flavor violation.

C, P and T in Quantum Mechanics

- Parity: P performs a spatial inversion through the origin $\mathbf{x} \to -\mathbf{x}$

$$U_P \psi(t, \mathbf{x}) = \eta_P \psi(t, -\mathbf{x})$$

  - Introduced by Wigner in 1927/28
  - Unitary transformation
  - Applying parity twice restores the original state, $U_P^2 = 1$ up to an unobservable phase. From this the parity of the U_P eigenfunctions has to be either even, $\eta_P = +1$, or odd, $\eta_P = -1$.

• Time reversal: T performs reversal of motion in time (sense of time evolution). That is $t \to -t$ with exchange of initial and final states.

$$A_T \psi(t, \mathbf{x}) = \eta_T \psi(-t, \mathbf{x})$$

- Introduced by Wigner in 1932
- Antiunitary transformation $A_T = U_T K$ (Necessary to preserve $[x_i, p_j] = i\hbar\delta_{ij}$).
- Antiunitary: unitary- for conserving probabilities, anti- for complex conjugation (antilinear).

• Charge Conjugation: C reverses the sign of the electric charge, colour charge and magnetic moment of a particle.

- Introduced by Kramers in 1937.
- Requires quantum field theory, as it is better understood as particle-antiparticle interchange

Maxwell Equations, C, P and T

$$\mathcal{L}(A_\mu) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + j_\mu A^\mu; \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Equations of Motion:

$$\partial_\mu F^{\mu\nu} = j^\nu,$$

$$\nabla \cdot \mathbf{E} = \rho, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad \nabla \times \mathbf{B} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t},$$
$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

are invariant under:

- PARITY: $\mathbf{x} \to -\mathbf{x}$
- TIME REVERSAL $t \to -t$
- CHARGE CONJUGATION $\rho \to -\rho$

	P	T	C		P	T	C
t	$+$	-	$+$				
$\mathbf{x}$	-	$+$	$+$	x^μ	x_μ	$-x_\mu$	x^μ
ρ	$+$	$+$	-				
$\mathbf{j}$	-	-	-	j^μ	j_μ	j_μ	$-j^\mu$
ϕ	$+$	$+$	-				
$\mathbf{A}$	-	-	-	A^μ	A_μ	A_μ	$-A^\mu$
$\mathbf{E}$	-	$+$	-				
$\mathbf{B}$	$+$	-	-	$F^{\mu\nu}$	$F_{\mu\nu}$	$-F_{\mu\nu}$	$-F^{\mu\nu}$

Fermion Fields

The fermion fields transformation rules under C, P and T symmetry follow from

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} - iQA_{\mu})\psi - \bar{\psi}m\psi$$

$$\begin{aligned} P\mathcal{L}_{QED}(t,\mathbf{x})P^{-1} &= \mathcal{L}_{QED}(t,-\mathbf{x}) \\ C\mathcal{L}_{QED}(t,\mathbf{x})C^{-1} &= \mathcal{L}_{QED}(t,\mathbf{x}) \\ T\mathcal{L}_{QED}(t,\mathbf{x})T^{-1} &= \mathcal{L}_{QED}(-t,\mathbf{x}) \end{aligned}$$

with $\bar{\psi} = \psi^{\dagger}\gamma^{0}$ and using

$$\begin{aligned} \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} &= 2g^{\mu\nu} \\ \gamma^{0}\gamma^{\mu\dagger}\gamma^{0} &= \gamma^{\mu} \end{aligned}$$

$$
\begin{aligned}
\psi_P(t,\mathbf{x}) = P\psi(t,\mathbf{x})P^{-1} &= \mathcal{P}\psi(t,-\mathbf{x}) \\
\psi_C(t,\mathbf{x}) = C\psi(t,\mathbf{x})C^{-1} &= \mathcal{C}\bar{\psi}^T(t,\mathbf{x}) \\
\psi_T(t,\mathbf{x}) = T\psi(t,\mathbf{x})T^{-1} &= \mathcal{T}\psi(-t,\mathbf{x})
\end{aligned}
$$

where P and C are unitary operators and T is a anti-unitary operator. We obtain (in the chiral representation):

$$
\begin{aligned}
\mathcal{P}\gamma^\mu\mathcal{P}^{-1} = (\gamma^\mu)^\dagger = \gamma_\mu &\rightarrow \mathcal{P} = \gamma^0 \\
\mathcal{C}^{-1}\gamma^\mu\mathcal{C} = -(\gamma^\mu)^T &\rightarrow \mathcal{C} = -i\gamma^0\gamma^2 = -\mathcal{C}^T \\
\mathcal{T}\gamma^\mu\mathcal{T}^{-1} = (\gamma^\mu)^T &\rightarrow \mathcal{T} = i\gamma^1\gamma^3 = -\mathcal{T}^*
\end{aligned}
$$

Using the 16 Dirac matrices which form a complete basis for the Clifford Algebra, one can build the corresponding bilinear forms:

$$\begin{aligned} s_{12}(x) &= :\bar{\psi}_1(x)\psi_2: \\ p_{12}(x) &= :\bar{\psi}_1(x)i\gamma^5\psi_2: \\ v_{12}^{\mu}(x) &= :\bar{\psi}_1(x)\gamma^{\mu}\psi_2: \\ a_{12}^{\mu}(x) &= :\bar{\psi}_1(x)\gamma^{\mu}\gamma^5\psi_2: \\ t_{12}^{\mu\nu}(x) &= :\bar{\psi}_1(x)\sigma^{\mu\nu}\psi_2: \end{aligned}$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

	$s_{12}(x^\rho)$	$p_{12}(x^\rho)$	$v^\mu_{12}(x^\rho)$	$a^\mu_{12}(x^\rho)$	$t^{\mu\nu}_{12}(x^\rho)$
P	$s_{12}(x_\rho)$	$-p_{12}(x_\rho)$	$v_\mu^{12}(x_\rho)$	$-a_\mu^{12}(x_\rho)$	$t_{\mu\nu}^{12}(x_\rho)$
T	$s_{12}(-x_\rho)$	$-p_{12}(-x_\rho)$	$v_\mu^{12}(-x_\rho)$	$a_\mu^{12}(-x_\rho)$	$-t_{\mu\nu}^{12}(-x_\rho)$
C	$s_{21}(x^\rho)$	$p_{21}(x^\rho)$	$-v^\mu_{21}(x^\rho)$	$a^\mu_{21}(x^\rho)$	$-t^{\mu\nu}_{21}(x^\rho)$

CPT Theorem

- Under CPT, any hermitian local Poincaré invariant theory described by $\mathcal{L}$ satisfies

$$\mathcal{L}(x) \to (CPT)\mathcal{L}(x)(CPT)^{-1} = \mathcal{L}^{\dagger}(-x) = \mathcal{L}(-x)$$

  Thus, the action is invariant under CPT.

- $\Rightarrow$ Particles and antiparticles have equal masses, equal total lifetimes and opposite charges

$$|CPT\alpha > \equiv CPT|\alpha > \equiv |\bar{\alpha} >$$

$$m_{\alpha} = m_{\bar{\alpha}}, \quad \tau(\alpha) = \tau(\bar{\alpha}), \quad Q(\alpha) + Q(\bar{\alpha}).$$

Status of discrete symmetries

- No evidence for CPT violation

$$\left|\frac{m_{K^0} - m_{\overline{K}^0}}{m_{K^0}}\right| < 10^{-18},$$

$$\left|\frac{\Gamma(K^0) - \Gamma(\overline{K}^0)}{m_{K^0}}\right| < 10^{-17},$$

$$\left|\frac{Q(p) + Q(\bar{p})}{e}\right| < 10^{-21}.$$

- No evidence for C, P, or T violation in purely electromagnetic or strong interactions.

- P and C maximally broken in weak interactions
- Violation of CP and T has been observed in weak interactions.
- The amount of CP and T observed is small

Standard Model

- Gauge symmetry is $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$
- There are three fermion generations:

$$Q_{Li} \sim (\mathbf{3}, \mathbf{2}, 1/6), \quad u_{Ri} \sim (\mathbf{3}, \mathbf{1}, 2/3), \quad d_{Ri} \sim (\mathbf{3}, \mathbf{1}, -1/3),$$
$$L_{Li} \sim (\mathbf{1}, \mathbf{2}, -1/2), \quad e_{Ri} \sim (\mathbf{1}, \mathbf{1}, -1)$$

- The scalar representation is given by

$$\phi \sim (\mathbf{1}, \mathbf{2}, 1/2)$$

- The pattern of symmetry breaking is given by:

$$G_{SM} \to SU(3)_C \times U(1)_{EM}$$

- The SM lagrangian is the most general renormalizable lagrangian consistent with the gauge symmetry and the given particle content:

$$\begin{aligned} \mathcal{L}_{\rm SM} &= \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa} \\ -\mathcal{L}_{\rm Yukawa}^{\rm Leptons} &= \lambda_E^{ij} \bar{L}_{Li} \phi e_{Rj} + \text{h.c.}, \\ -\mathcal{L}_{\rm Yukawa}^{\rm Quarks} &= \lambda_D^{ij} \bar{Q}_{Li} \phi d_{Rj} + \lambda_U^{ij} \bar{Q}_{Li} \tilde{\phi} u_{Rj} + \text{h.c.}, \end{aligned}$$

where $\tilde{\phi} = i\tau_2 \phi^*$.

- Global accidental symmetry: SM is invariant under the accidental symmetry

$$U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

  Including neutrino masses, the accidental symmetry is reduced to $U(1)_B \times U(1)_L$.
- Taking into account the chiral anomaly and the topological gauge structure of the SM which implies that $(B + L)$ is significantly violated through instantons and sphalerons at early times of the Universe, the accidental symmetry is reduced to $U(1)_{B-L}$.
- P (C) explicitly and maximally broken.
- CP violation not obvious, since both P and C transformations take left- and right-handed fields into one another.

Cabibbo Kobayashi Maskawa

After SSB $\langle\phi\rangle = (0, v/\sqrt{2})^T$ (suppressing flavor indices):

$$-\mathcal{L}_{\rm m} = \frac{v}{\sqrt{2}}\bar{u}_L\lambda_U u_R + \frac{v}{\sqrt{2}}\bar{d}_L\lambda_D d_R + \frac{v}{\sqrt{2}}\bar{e}_L\lambda_E e_R + \text{h.c.}$$

Diagonalization (quark sector):

- Field redefinition (flavor eigenstates → mass eigenstates)

$$u_R \to V_{u_R}u_R, \quad u_L \to V_{u_L}u_L, \quad d_R \to V_{d_R}d_R, \quad d_L \to V_{d_L}d_L.$$

$$V_{u_L}^\dagger\lambda_U V_{u_R} = \lambda'_U, \quad V_{d_L}^\dagger\lambda_D V_{d_R} = \lambda'_D\ .$$

Here the matrices λ'_U and λ'_D, are diagonal, real and positive, and the transformation matrices $V_{u,d_{L,R}}$ are unitary.

Then from

$$-\mathcal{L}_{\mathrm{m}} = \frac{v}{\sqrt{2}}\Big(\bar{u}_L \lambda'_U u_R + \bar{d}_L \lambda'_D d_R + \bar{e}_L \lambda_E e_R + \text{h.c.}\Big)$$
$$= \frac{v}{\sqrt{2}}\Big(\bar{u}\lambda'_U u + \bar{d}\lambda'_D d + \bar{e}\lambda_E e\Big)$$

we read off the diagonal mass matrices, $m_U = v\lambda'_U/\sqrt{2}$, $m_D = v\lambda'_D/\sqrt{2}$ and $m_E = v\lambda_E/\sqrt{2}$.

• However, in general the field redefinitions in are not symmetries of the Lagrangian. We must check the induced Lagrangian dependency on $V_{u,d_{L,R}}$.

- Kinetic Terms are invariant

$$\bar{u}_L i\partial\!\!\!/ u_L \to (\bar{u}_L V_{u_L}^\dagger) i\partial\!\!\!/ (V_{u_L} u_L) = \bar{u}_L (V_{u_L}^\dagger V_{u_L}) i\partial\!\!\!/ u_L = \bar{u}_L i\partial\!\!\!/ u_L$$

- Electromagnetic and weak neutral currents are invariant (GIM mechanism)

$$\bar{u}_L Z\!\!\!/ u_L \to (\bar{u}_L V_{u_L}^\dagger) Z\!\!\!/ (V_{u_L} u_L) = \bar{u}_L (V_{u_L}^\dagger V_{u_L}) Z\!\!\!/ u_L = \bar{u}_L Z\!\!\!/ u_L$$

- Charged currents are not invariant

$$\bar{u}_L W\!\!\!/^+ d_L + \bar{d}_L W\!\!\!/^- u_L \to \bar{u}_L (V_{u_L}^\dagger V_{d_L}) W\!\!\!/^+ d_L + \bar{d}_L (V_{d_L}^\dagger V_{u_L}) W\!\!\!/^- u_L$$

A relic of our field redefinitions has remained in the form of the unitary matrix

$$V = V_{u_L}^{\dagger} V_{d_L}.$$

We call this the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

- This is the place where CP violation and flavor meet. CP can be broken by the terms

$$\bar{u}_L V \not{W}^{+} d_L + \bar{d}_L V^{\dagger} \not{W}^{-} u_L.$$

  To see this, recall that under CP

$$\bar{u}_L \gamma^{\mu} d_L \xrightarrow{CP} -\bar{d}_L \gamma_{\mu} u_L, \qquad W^{+\mu} \xrightarrow{CP} -W_{\mu}^{-}.$$

  Hence CP invariance requires $V^{\dagger} = V^{T}$, or $V^{*} = V$. This condition can be read as

  "physical non-zero phase" = "CP violation".

How does flavor enter the picture? Number of generations: N_G.

- A general $N_G \times N_G$ unitary matrix V is characterized by N_G^2 real parameters: $N_G(N_G-1)/2$ moduli and $N_G(N_G+1)/2$ phases.
- the case of V, many of these parameters are irrelevant because we can always choose arbitrary quark phases.
- Under the phase redefinitions $u_i \to e^{i\phi_i}\, u_i$ and $d_j \to e^{i\theta_j}\, d_j$, the mixing matrix changes as $V_{ij} \to V_{ij}\, e^{i(\theta_j-\phi_i)}$; thus, $2N_G-1$ phases are unobservable.
- The number of physical free parameters in the quark-mixing matrix then gets reduced to $(N_G-1)^2$: $N_G(N_G-1)/2$ moduli and $(N_G-1)(N_G-2)/2$ phases.

• Cabibbo: $N_G = 2$.

In this simple case, V is determined by a single parameter. One then recovers the Cabibbo rotation matrix

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} .$$

This matrix satisfies $V = V^*$, $\Rightarrow$ NO CP violation induced by the field redefinition.

• Kobayashi-Maskawa: $N_G = 3$.

The CKM matrix is described by three angles and one phase.

- It is useful to label the matrix elements by the quarks they connect:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

  Standard CKM parameterization:

$$V = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{bmatrix}.$$

  Here $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, with $c_{ij} \geq 0$, $s_{ij} \geq 0$ and $0 \leq \delta \leq 2\pi$.

Notice that δ is the only complex phase in the SM Lagrangian. Therefore, it is the only possible source of CP-violation phenomena.

In fact, it was for this reason that the third generation was assumed to exist! With two generations, the SM could not explain the observed CP violation in the K system.

Manifestly basis-independent form of the CP violating phase: Jarlskog invariant

$$J = \mathrm{Im}\left(V_{ud} V_{cd}^* V_{cb} V_{ub}^*\right).$$

In the standard parameterization:

$$J = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta.$$

Conditions for CP violation

Let's assume the following form for a physical amplitude:

$$A = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$$

where $A_{1,2}$ are two complex partial weak amplitudes with CP-conserving dynamical phases $\delta_{1,2}$.

$$A \xrightarrow{CP} \bar{A} = A_1^* e^{i\delta_1} + A_2^* e^{i\delta_2} \neq A^*$$

The CP-asymmetry in decay widths is:

$$\mathcal{A}_{CP} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}$$

$$= \frac{-2\mathrm{Im}(A_1 A_2^*) \sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2\mathrm{Re}(A_1 A_2^*) \cos(\delta_1 - \delta_2)}$$

A non-zero CP-asymmetry requires at least two partial amplitudes with

1. a relative CP-violating phase (weak phase)
2. a relative dynamical CP-conserving phase (strong phase)