Flavor symmetries in heterotic orbifolds

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Outline

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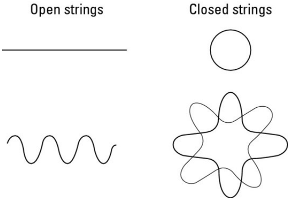
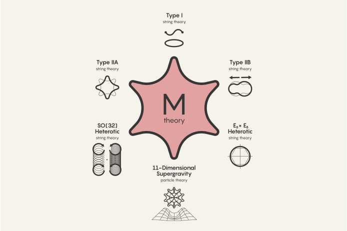
String theory and Heterotic String.

- Possibly the prime candidate for an UV-complete and unified description of the universe.
- Five types: I, IIA, IIB, Heterotic $E_8 \times E_8$, Heterotic $SO(32)$.
- All five string theories are only consistent in a 10 dimensional space-time and they have space-time supersymmetry.
- The theories are all connected by dualities.
- HETEROTIC STRING*:
    - Theory of closed strings.
    - Two types, $E_8 \times E_8$ and $SO(32)$ as gauge groups with $N = 1$ supersymmetry.





*David Gross et. al., The Heterotic String, Phys. Rev. Lett 54 ; 1985.

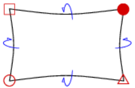
Compactifications and orbifolds.

- To make contact with "real world" compactification of six dimensions on a space X is necessary.
- The $D = 4$ theory depends on space X. Election of X is hence subject to requirements in $D = 4$.
- Many choices of 4 dimensional theories. Expecting $\mathcal{N} = 1$ SUSY at some scale, the six extra dimensions must be compactified on Calabi-Yau manifolds, or orbifolds*

ORBIFOLDS:

- Toroidal orbifolds are simpler than CY.
- $\mathbb{O} = \mathbb{T}^n/G = \mathbb{R}^n/S$. Elements of S, $g = (\theta, v)$
- They are flat except in a finite set of singular points. $X_f = \theta X_f + n_k e_k$. Not equivalent fixed points can be specified by their constructing elements $g_f = (\theta^{(f)}, n_k^{(f)} e_k)$

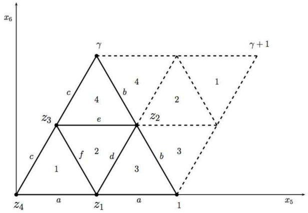
- Roto-translations, $g = (\theta, v) \in S$ with $v \notin \Lambda$
- The set of all θ in S is called the point group P. Cyclic groups $\mathbb{Z}_n$ and $\mathbb{Z}_n \times \mathbb{Z}_m$.
- $S = \{G, \Lambda\}$.
- The product of two space group elements g and $h = (\omega, u)$, is defined as $h \circ g = (\omega\theta, \omega u + v)$.



*Dixon, Harvey, Vafa y Witten; 1985

Flavor symmetries in orbifold compactifications

- In extra dimensional theories, is natural to interpret as flavor symmetries (See Mario's talk) permutation symmetries between fields living in fixed points of an orbifold.
- In 6D, compactification of 2 space dimensions in the orbifold $\mathbb{T}^2/\mathbb{Z}_2$ leads to the A_4 symmetry*.
- In this cases, the symmetry is just due to the orbifold geometry.



*Altarelli et. al.,Tri-bimaximal neutrino mixing from orbifolding, Nucl. Phys. B775, 2007

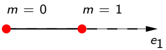
Flavor Symmetries in heterotic orbifolds

1. Flavor symmetries appear also in heterotic orbifolds. The possibility of find matter fields living in the fixed points of the orbifold allow us to consider that as flavor symmetries.Selection rules for couplings add symmetry and the final symmetry is obtained as a combination with the orbifold geometry*.

Selection Rules: space group selection rule. Trivial product of constructing elements:

$$\prod_i^r (\theta_i^{(f)}, \ell_i^f) \sim (\mathbb{I}, (\mathbb{I}-\theta)\Lambda)$$

Example: $\mathbb{S}^1/\mathbb{Z}_2$



- $x \sim -x + e_1$, $\theta = e^{i\pi}$
- Sub-lattice $(\mathbb{I}-\theta)\Lambda = 2e_1$
- Two fixed points (θ, me_1) with $m = 0, 1$

- Space group selection rule $\prod_i^r (\theta_i, m^i e_1) = (\theta^r, e_1(-m^1 + m^2 - m^3 + ...))$, then $\theta^r = \mathbb{I} \Rightarrow \mathbb{Z}_2$ and $\sum_j m^j = 0 \; mod \; 2 \Rightarrow \mathbb{Z}_2$
- In addition, there exist a relabeling symmetry $m \to m+1 \;\; mod \; 2$, because of the degeneracy of fields. $\Rightarrow$ permutation symmetry S_2,
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ are normal subgroups. Final symmetry,

$$S_2 \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2) \sim D_4$$

*Kobayash et. al., Stringy origin of non-Abelian discrete flavor symmetries, Nucl. Phys. B768, 2007

Other examples:

A $\mathbb{T}^6/\mathbb{Z}_4$ Orbifold, Geometry $(3,1)^*$

- In θ sector fixed points are $(\theta, m_3 e_3 + m_6 e_6)$ with $m_3, m_6 = 0, 1, 2, 3$
- We have then the multiplicative closure of
  $$S_4 \times S_4 \cup \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4$$
  or
  $$(S_4 \times S_4) \ltimes (\mathbb{Z}_4^5 \times \mathbb{Z}_2^2)$$
- The other sectors θ^2, θ^3 are not empty and then the symmetry is well defined in all.

With roto-translations.

- Orbifolds with roto-translations do not lead to fixed points in all sectors and then twisted matter fields are not defined for every sector.
- The symmetry, if it exists, will be defined between the fields living in the fixed points of the existing twisted sectors.

We used* to seek for all the symmetries in abelian-toroidal orbifolds. There are 118 geometries for $\mathbb{Z}_n \times \mathbb{Z}_m$ and 19 for $\mathbb{Z}_n$ orbifolds. Among the non-abelian discrete symmetries found are:

$$\Delta(54)^m \times \mathbb{Z}_3^n \times \mathbb{Z}_6^l,\ D_4^m \times \mathbb{Z}_2^l \times \mathbb{Z}_4^n,\ S_4^n \ltimes \mathbb{Z}_4^l \times \mathbb{Z}_2^m \ldots$$

Symmetry enhancement is possible if the lattice has specific values of angles and radii, $S_2 x S_2 --> D_4, S_4$

*Fischer, et. al., Classification of symmetric toroidal orbifolds, JHEP, 2013

The complete list

Orbifold		Flavor symmetry
$\mathbb{Z}_2\times\mathbb{Z}_2$	(1,1)	$(D_4\times D_4\times D_4\times D_4\times D_4\times D_4)/\mathbb{Z}_2^4$
	(1,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(1,3)	$(D_4\times D_4\times D_4)/\mathbb{Z}_2$
	(1,4)	–
	(2,1)	$(D_4\times D_4\times D_4\times D_4\times D_4)/\mathbb{Z}_2^3$
	(2,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(2,3)	$(D_4\times D_4\times D_4)/\mathbb{Z}_2^2$
	(2,4)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(2,5)	$(D_4\times D_4)$
	(2,6)	–
	(3,1)	$(D_4\times D_4\times D_4)/\mathbb{Z}_2$
	(3,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(3,3)	$(D_4\times D_4)/\mathbb{Z}_2$
	(3,4)	–
	(4,1)	$(D_4\times D_4\times D_4\times D_4)/\mathbb{Z}_2^2$
	(4,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(5,1)	$(D_4\times D_4\times D_4\times D_4)/\mathbb{Z}_2^2$
	(5,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(5,3)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(5,4)	$(D_4\times D_4)$
	(5,5)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(6,1)	$(D_4\times D_4\times D_4\times D_4)/\mathbb{Z}_2^2$
	(6,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(6,3)	D_4
	(7,1)	$(D_4\times D_4)$
	(7,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(8,1)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(9,1)	$(D_4\times D_4\times D_4)/\mathbb{Z}_2$
	(9,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(9,3)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(10,1)	$D_4\times\mathbb{Z}_2$

Orbifold		Flavor symmetry
$\mathbb{Z}_2\times\mathbb{Z}_2$	(10,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
	(11,1)	$(D_4\times D_4\times D_4)/\mathbb{Z}_2$
	(12,1)	$(D_4\times D_4)$
	(12,2)	$\mathbb{Z}_2\times\mathbb{Z}_2$
$\mathbb{Z}_2\times\mathbb{Z}_4$	(1,1)	$(D_4\times D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^3$
	(1,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(1,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(1,4)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(1,5)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(1,6)	$(D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^2$
	(2,1)	$(D_4\times D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^3$
	(2,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(2,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(2,4)	$D_4\times\mathbb{Z}_4$
	(2,5)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(2,6)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(3,1)	$(D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^2$
	(3,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(3,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(3,4)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(3,5)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(3,6)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(4,1)	$(D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2$
	(4,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(4,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(4,4)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(4,5)	$(D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2$
	(5,1)	$(D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^2)$
	(5,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(6,1)	$(D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2$
	(6,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$

Orbifold		Flavor symmetry
$\mathbb{Z}_2\times\mathbb{Z}_4$	(6,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(6,4)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(6,5)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(7,1)	$D_4\times\mathbb{Z}_4$
	(7,2)	$\mathbb{Z}_4\times\mathbb{Z}_2$
	(7,3)	$\mathbb{Z}_4\times\mathbb{Z}_2$
	(8,1)	$(D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^2$
	(8,2)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(8,3)	$\mathbb{Z}_2\times\mathbb{Z}_4$
	(9,1)	$(D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2$
	(9,2)	$\mathbb{Z}_4\times\mathbb{Z}_2$
	(9,3)	$\mathbb{Z}_4\times\mathbb{Z}_2$
	(10,1)	$\mathbb{Z}_4\times\mathbb{Z}_2$
	(10,2)	$\mathbb{Z}_4\times\mathbb{Z}_2$
$\mathbb{Z}_2\times\mathbb{Z}_6$-I	(1,1)	$(D_4\times D_4\times\mathbb{Z}_6)/\mathbb{Z}_2$
	(1,2)	$\mathbb{Z}_2\times\mathbb{Z}_6$
	(2,1)	$(D_4\times D_4\times\mathbb{Z}_6)/\mathbb{Z}_2$
	(2,2)	$\mathbb{Z}_2\times\mathbb{Z}_6$
$\mathbb{Z}_2\times\mathbb{Z}_6$-II	(1,1)	$\mathbb{Z}_2\times\mathbb{Z}_6$
	(2,1)	$\mathbb{Z}_2\times\mathbb{Z}_6$
	(3,1)	$\mathbb{Z}_2\times\mathbb{Z}_6$
	(4,1)	$\mathbb{Z}_2\times\mathbb{Z}_6$
$\mathbb{Z}_3\times\mathbb{Z}_3$	(1,1)	$(\Delta(54)\times\Delta(54)\times\Delta(54))/\mathbb{Z}_3$
	(1,2)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(1,3)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(1,4)	$\Delta(54)\times\Delta(54)$
	(2,1)	$\Delta(54)\times\Delta(54)$
	(2,2)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(2,3)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(2,4)	$\Delta(54)\times\Delta(54)$
	(3,1)	$\Delta(54)\times\mathbb{Z}_3$
	(3,2)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(3,3)	$\Delta(54)\times\Delta(54)$
	(4,1)	$\Delta(54)\times\Delta(54)$
	(4,2)	$\mathbb{Z}_3\times\mathbb{Z}_3$
	(4,3)	$\Delta(54)\times\Delta(54)$
	(5,1)	$\mathbb{Z}_3\times\mathbb{Z}_3$
$\mathbb{Z}_3\times\mathbb{Z}_6$	(1,1)	$\Delta(54)\times\mathbb{Z}_6$
	(1,2)	$\mathbb{Z}_3\times\mathbb{Z}_6$

Orbifold		Flavor symmetry
$\mathbb{Z}_3\times\mathbb{Z}_6$	(2,1)	$\Delta(54)\times\mathbb{Z}_6$
	(2,2)	$\mathbb{Z}_3\times\mathbb{Z}_6$
$\mathbb{Z}_4\times\mathbb{Z}_4$	(1,1)	$(D_4\times D_4\times D_4\times\mathbb{Z}_4\times\mathbb{Z}_4)/\mathbb{Z}_2^3$
	(1,2)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(1,3)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(1,4)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(2,1)	$(D_4\times D_4\times\mathbb{Z}_4^2)/\mathbb{Z}_2^2$
	(2,2)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(2,3)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(2,4)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(3,1)	$(D_4\times\mathbb{Z}_4^2)/\mathbb{Z}_2$
	(3,2)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(4,1)	$(D_4\times D_4\times\mathbb{Z}_4^2)/\mathbb{Z}_2^2$
	(4,2)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(4,3)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(5,1)	$\mathbb{Z}_4\times\mathbb{Z}_4$
	(5,2)	$\mathbb{Z}_4\times\mathbb{Z}_4$
$\mathbb{Z}_6\times\mathbb{Z}_6$	(1,1)	$\mathbb{Z}_6\times\mathbb{Z}_6$

Orbifold		Flavor symmetry
$\mathbb{Z}_3$	(1,1)	$(\Delta(54)\times\Delta(54)\times\Delta(54))/\mathbb{Z}_3^2$
$\mathbb{Z}_4$	(1,1)	$(D_4\times D_4\times D_4\times D_4\times\mathbb{Z}_4)/\mathbb{Z}_2^4$
	(2,1)	$(S_4\times S_2\times S_2)\ltimes(\mathbb{Z}_4^3\times\mathbb{Z}_2^3)$
	(3,1)	$(S_4\times S_4)\ltimes(\mathbb{Z}_4^5\times\mathbb{Z}_2^2)$
$\mathbb{Z}_6$-I	(1,1)	$\Delta(54)$
	(2,1)	$(\Delta(54)\times\mathbb{Z}_6)/\mathbb{Z}_3$
$\mathbb{Z}_6$-II	(1,1)	$\Delta(54)\times[D_4\times D_4/\mathbb{Z}_2]$
	(2,1)	$[\Delta(54)\times\mathbb{Z}_6]/\mathbb{Z}_3\times[D_4\times D_4]/\mathbb{Z}_2^2$
	(3,1)	$[\Delta(54)\times\mathbb{Z}_6]/\mathbb{Z}_3\times[D_4\times D_4]/\mathbb{Z}_2^2$
	(4,1)	$[\Delta(54)\times\mathbb{Z}_6]/\mathbb{Z}_3\times[D_4/\mathbb{Z}_2]$
$\mathbb{Z}_7$	(1,1)	$S_7\ltimes\mathbb{Z}_7^6$
$\mathbb{Z}_8$-I	(1,1)	$(D_4\times D_4\times\mathbb{Z}_8)/\mathbb{Z}_2^2$
	(2,1)	$(D_4\times D_4\times\mathbb{Z}_8)/\mathbb{Z}_2^2$
	(3,1)	$S_4\ltimes(\mathbb{Z}_8\times\mathbb{Z}_4^2\times\mathbb{Z}_2)$
$\mathbb{Z}_8$-II	(1,1)	$(D_4\times D_4\times D_4\times\mathbb{Z}_8)/\mathbb{Z}_2^3$
	(2,1)	$(D_4\times D_4\times\mathbb{Z}_8)/\mathbb{Z}_2^2$
$\mathbb{Z}_{12}$-I	(1,1)	$\Delta(54)$
	(2,1)	$(\Delta(54)\times\mathbb{Z}_{12})/\mathbb{Z}_3$
$\mathbb{Z}_{12}$-II	(1,1)	$(D_4\times D_4)/\mathbb{Z}_2$

Flavor Symmetries in promising models

- Promising heterotic orbifolds models mostly have non-trivial Wilson Lines $A_\alpha \neq 0$.
- Non-trivial WLs break degeneracy of fixed points and hence the S_n part of the flavor symmetry.
- However, flavor symmetry could be completely broken or not in promising models.
- Promising models:
  1. SM gauge group $SU(3) \times SU(2) \times U(1)_y$ times a Hidden sector (see Ricardos's talk).
  2. Non-anomalous $U(1)_y$, compatible with grand unification.
  3. At least one Higgs pair.
  4. Exotics must be vector-like with respect to SM gauge group.

We have Investigated flavor symmetry in promising models. We used `The Orbifolder`*. `The Orbifolder` computes the spectrum, Wilson lines, shifts V_i and compare them to obtain non-equivalent promising models.

* Nilles, Hans Peter and Ramos-Sanchez, Saul and Vaudrevange, Patrick K. S. and Wingerter, Akin

The complete landscape of heterotic $E_8 \times E_8$ orbifolds and their flavor symmetries:

Orbifold		Flavor symmetry with ℓ non-vanishing WLs $\ell = 1$	2	3	4
$\mathbb{Z}_3$	(1,1)	$\Delta(54)^2$ 0	$\Delta(54) \times \mathbb{Z}_3^2$ 0	$\mathbb{Z}_3^2$ 0	
$\mathbb{Z}_4$	(1,1)	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 0	$D_4^2 \times \mathbb{Z}_4$ 0	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0
	(2,1)	$(S_2 \times S_2) \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ 0 $(S_4 \times S_2) \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$S_2 \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ 0 $S_4 \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 27	
	(3,1)	$S_4 \ltimes (\mathbb{Z}_4^4 \times \mathbb{Z}_2)$ 0	$\mathbb{Z}_4^3$ 149		
$\mathbb{Z}_6$-I	(1,1)	$\mathbb{Z}_3^2$ 30			
	(2,1)	$\mathbb{Z}_6 \times \mathbb{Z}_3$ 30			
$\mathbb{Z}_6$-II	(1,1)	$[(D_4 \times D_4)/\mathbb{Z}_2] \times \mathbb{Z}_3^2$ 0 $\Delta(54) \times D_4 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3^2$ 337 $\Delta(54) \times \mathbb{Z}_2^3$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 26	
	(2,1)	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 335 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 14	
	(3,1)	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 333 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 2	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 17	
	(4,1)	$[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2$ 0 $[D_4/\mathbb{Z}_2] \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 312	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2$ 44		
$\mathbb{Z}_7$	(1,1)	$\mathbb{Z}_7^2$ 1			
$\mathbb{Z}_8$-I	(1,1)	$D_4 \times \mathbb{Z}_8$ 38	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 230		
	(2,1)	$D_4 \times \mathbb{Z}_8$ 41	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 204		
	(3,1)	$\mathbb{Z}_8 \times \mathbb{Z}_4$ 389			
$\mathbb{Z}_8$-II	(1,1)	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2$ 396	$D_4 \times \mathbb{Z}_8 \times \mathbb{Z}_2$ 21	$\mathbb{Z}_8 \times \mathbb{Z}_2^3$ 1459	
	(2,1)	$D_4 \times \mathbb{Z}_8$ 227	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 253		

For $\mathbb{Z}_2 \times \mathbb{Z}_2$, including geometries with roto-translations

Orbifold		Flavor symmetry with ℓ of non-vanishing WLs $\ell = 1$	2	3	4	5	6
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	$D_4^5/\mathbb{Z}_2^2$ 0	D_4^4 0	$D_4^3 \times \mathbb{Z}_2^2$ 0	$D_4^2 \times \mathbb{Z}_2^4$ 52	$D_4 \times \mathbb{Z}_2^6$ 152	$\mathbb{Z}_2^8$ 1
	(1,3)	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0				
	(2,1)	$D_4^4/\mathbb{Z}_2$ 0	$D_4^3 \times \mathbb{Z}_2$ 0	$D_4^2 \times \mathbb{Z}_2^3$ 14	$D_4 \times \mathbb{Z}_2^5$ 342	$\mathbb{Z}_2^7$ 13	
	(2,3)	D_4^2 0					
	(2,5)	$D_4 \times \mathbb{Z}_2^2$ 0					
	(3,1)	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 432	$\mathbb{Z}_2^5$ 12			
	(3,3)	$D_4 \times \mathbb{Z}_2$ 0					
	(4,1)	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 0	$\mathbb{Z}_2^6$ 0		
	(5,1)	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 40	$\mathbb{Z}_2^6$ 2		
	(5,4)	$D_4 \times \mathbb{Z}_2^2$ 0					
	(6,1)	D_4^3 0	$D_4^2 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 57	$\mathbb{Z}_2^6$ 344		
	(7,1)	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^4$ 76				
	(9,1)	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 2	$\mathbb{Z}_2^5$ 25			
	(10,1)	$\mathbb{Z}_2^3$ 21					
	(11,1)	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0	$\mathbb{Z}_2^5$ 0			
	(12,1)	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^4$ 3				
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	$\Delta(54)^2 \times \mathbb{Z}_3$ 81	$\Delta(54) \times \mathbb{Z}_3^3$ 696	$\mathbb{Z}_3^5$ 12			
	(1,4)	$\mathbb{Z}_3^4$ 8					
	(2,1)	$\Delta(54) \times \mathbb{Z}_3^2$ 239	$\mathbb{Z}_3^4$ 1219				
	(3,1)	$\mathbb{Z}_3^3$ 6					
	(4,1)	$\mathbb{Z}_3^4$ 127					

- Although in a lot of promising models, the non-abelian discrete symmetry is broken. There are yet models with a remnant non-abelian part.

  1. The $S_4^m \ltimes \mathbb{Z}_n^l$... symmetry does not appear in promising models (without enhancement).
  2. $D_4^m \times \mathbb{Z}_n^l$... appears a lot, $\approx 10^4$ models in $\mathbb{Z}_n \times \mathbb{Z}_m$ orbifolds.
  3. $D_4^m \times \mathbb{Z}_n^l$... appears in $\approx 10^3$ models in $\mathbb{Z}_n$ orbifolds.
  4. $\Delta(54)^m \times \mathbb{Z}_n^l$... appears in $\approx 10^3$ models in $\mathbb{Z}_n \times \mathbb{Z}_m$ orbifolds.
  5. $\Delta(54)^m \times \mathbb{Z}_n^l$... appears in only 2 models in $\mathbb{Z}_n$ orbifolds.

Conclusions

1. Heterotic orbifolds produce abelian and non-abelian symmetries that can be seen as flavor symmetries in the EFT.
2. We have investigated flavor symmetries in all the abelian toroidal orbifolds of the heterotic string.
3. We have found the symmetries in promising or phenomenologically viable heterotic orbifolds.
4. For future work, It would be interesting to study the role of the modular symmetry in this kind of geometries.
5. For future work also, It is necessary to study features of string flavor models and identify predictions.

Thank You!