XXXII RADPYC

High Spins as Lorentz and Spinor Tensors Carrier Spaces of the Lorentz Group and a Road toward Quantization

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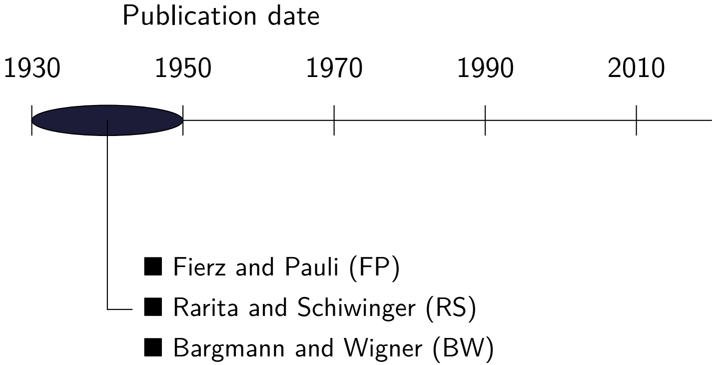
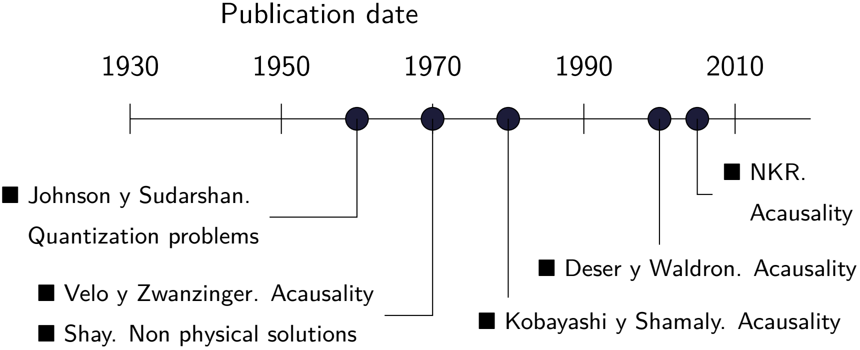
Between 1930-1950, appear the first articles on the description of high spin matter fields.



Since 1960, several authors have adressed the inconsistencies of the high spin theories.



There are mainly three difficulties on classical high spin theories.

Difficulties:

- Acausality.
- Undesirable propagation of degrees of freedom.
- Non physical solutions.

Here I present a new approach which avoids these problems.

Spins described by quantum fields are embedded in finite representation of $SL(2,\mathbb{C})$, which are labeled by two numbers.

Labels for the irreducible representations of $SL(2,\mathbb{C})$

(j_1, j_2)

- Integer
- Half-integer

Spin :

$$|j_1 - j_2|, |j_1 - j_2| + 1, \ldots, |j_1 + j_2|$$

Examples for single and multiple spins

Dirac spinors Ψ_a	$(1/2,0)\oplus(0,1/2)$	Irred. rep.	Spin 1/2
Antisymmetric tensor $F_{[\mu\nu]}$ Symmetric tensor-spinor ψ_{ab} Column-vector	$(1,0)\oplus(0,1)$	Irred. rep.	Spin 1
Four-vector A_μ	$(1/2,1/2)$	Irred. rep.	Spin 0 and 1

The formalism that we developed puts all irreducible representations into Lorentz and/or spinor tensor basis.

(i) Lorentz Tensors $A_{\mu\nu\ldots}$

(ii) Spinor tensors $\Psi_{ab\ldots}$

Lorentz projector $\mathcal{P}_F^{(j_1,j_2)}$

→ We obtain any field which transforms according to the irreducible representation $(j_1, j_2) \oplus (j_2, j_1)$

The Lorentz projectors are constructed by one of the Casimir operators of the Lorentz algebra.

$$\mathcal{P}_F^{(j_1,j_2)} = \prod_{kl \neq j_1 j_2} \left(\frac{F - c_{(j_k j_l)}}{c_{(j_1 j_2)} - c_{(j_k j_l)}} \right)$$

Casimir operator of the Lorentz algebra

Lorentz projectors

(Momentum independent)

Eigenvalues of F

$$F\Psi = c\Psi, \quad \Psi \sim (j_1, j_2)$$
$$c = j_1(j_1+1) + j_2(j_2+1)$$

Example

$$\Psi_\mu \sim (1/2,1/2) \otimes [(1/2,0) \oplus (0,1/2)]$$
$$[(1/2,0) \oplus (0,1/2)] \;\oplus\; [(1,1/2) \oplus (1/2,1)]$$

The Lorentz projectors separate:

$(1/2,0) \oplus (0,1/2)$ from $(1,1/2) \oplus (1/2,1)$

One pure spin	Multiple spins
1/2	1/2, 3/2

We see that we encounter either simple single spin or multiple spins sectors

We can isolate single spin only from two-spin irrep by using the Poincaré projectors.

Poincaré projectors

Casimir operator of the Poincaré algebra

$$\mathcal{P}^{(m,1/2)}_{W^2}(p) = \frac{p^2}{m^2}\frac{W^2 - \epsilon_{3/2}}{\epsilon_{1/2} - \epsilon_{3/2}}$$

$$\left[W^2\right]^{\alpha}{}_{\beta}\,\Psi^{\beta} = \epsilon_s \Psi^{\alpha}$$

$$\epsilon_s = -p^2 s(s+1), \quad s = 1/2, 3/2$$

Napsuciale, Kirchbach and Rodriguez, 2006

Applying this method to ψ_μ allows to separately write three equations, two for spin 1/2 and one for spin 3/2.

Equations for 1/2 in:

$$(1/2,0)\oplus(0,1/2): \quad \left[\mathcal{P}_F^{(1/2,0)}\right]^\alpha_{\ \beta}\left[\mathcal{P}_{W^2}^{(m,1/2)}\right]^\beta_{\ \delta}\Psi^\delta=\Psi^\alpha$$

$$(1/2,1)\oplus(1,1/2): \quad \left[\mathcal{P}_F^{(1/2,1)}\right]^\alpha_{\ \beta}\left[\mathcal{P}_{W^2}^{(m,1/2)}\right]^\beta_{\ \delta}\Psi^\delta=\Psi^\alpha$$

Equation for 3/2 in:

$$(1/2,1)\oplus(1,1/2): \quad \left[\mathcal{P}_F^{(1/2,1)}\right]^\alpha_{\ \beta}\left[\mathcal{P}_{W^2}^{(m,3/2)}\right]^\beta_{\ \delta}\Psi^\delta=\Psi^\alpha$$

The two spin 1/2 equations lead to two particles with different characteristics, as manifest upon gauging.

Spin 1/2 in $(1/2, 0) \oplus (0, 1/2)$

Spin 1/2 in $(1, 1/2) \oplus (1/2, 1)$

Equations minimally coupled with the electromagnetic field

They rewrite as:

$$\left[D^\mu D_\mu + \left(\frac{g}{2}\right)\left(\frac{e}{2}\right)\sigma_{\mu\nu}F^{\mu\nu} + m^2\right]\Psi = 0$$

Generalized Feynman-Gell-Mann equation

$g = 2$

$g = -2/3$

The two spin 1/2 equations lead to two particles with different characteristics, as manifest upon gauging.

Spin 1/2 in $(1/2, 0) \oplus (0, 1/2)$

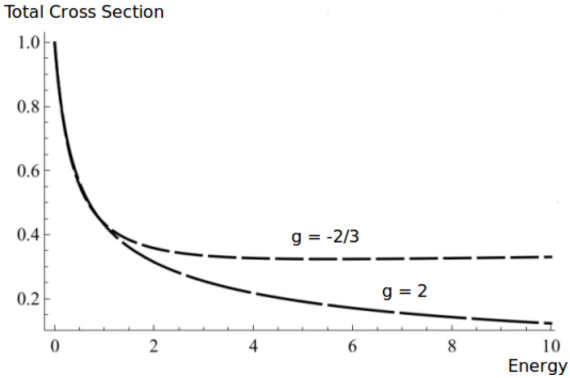
- $g = 2$
- Its equation bi-linearize to the Dirac equation

Spin 1/2 in $(1, 1/2) \oplus (1/2, 1)$

- $g = -2/3$
- Its equation do not bi-linearize to the Dirac equation

(New specie of 1/2 particle)

The two spin 1/2 equations lead to two particles with different characteristics, as manifest upon gauging.



For the general case of any single spin j in $(j,0)\oplus(0,j)$, we write the equation below.

Our free field equation

$$\left(\partial_\mu \partial^\mu \mathcal{P}_F^{(j,0)} - m^2\right)_{\{\dots\}} \Psi^{\{\dots\}} = 0$$

$\Psi^{\{\dots\}}$ → Tensor field (Lorentz/Spinor)

Properties:

- $\mathcal{P}_F^{(j,0)}$ picks up the corresponding space to the representation $(j,0)\oplus(0,j)$.
- It guarantees the mass-shell condition.
- Ψ is a pure spin field, with spin j.

Our formalism has the advantage to avoid the three main difficulties of classical high spin theories.

- **We do not have non-physical solutions**

  The solutions correspond only to pure spin fields

Our formalism has the advantage to avoid the three main difficulties of classical high spin theories.

It propagates the correct number of degrees of freedom

Our equation coupled with the electromagnetic field

$$\left(\Gamma_{\mu\nu} D^\mu D^\nu + m^2\right) \Psi = 0$$

- $\Gamma_{\mu\nu}\partial^\mu\partial^\nu = \mathcal{P}_F \partial^2$
- $\mathcal{P}_F \Gamma_{\mu\nu} = \Gamma_{\mu\nu}$

$$\longrightarrow \mathcal{P}_F \Psi = \Psi$$

$$\downarrow$$

$$\Psi \sim (j,0) \oplus (0,j)$$

Our formalism has the advantage to avoid the three main difficulties of classical high spin theories.

- **Causal propagation of the solutions of the coupled equations**

  Can be shown using the Courant-Hilbert criterion

E. G. Delgado Acosta, V. M. Banda Guzman and M. Kirchbach, 2015

V. M. Banda Guzman and M. Kirchbach, 2016

Applying our formalism to the field $\Psi_{[\mu\nu]}$, we predict a new spin 3/2 particle whose $g = 2/3$.

Our spin-3/2 equation coupled with the electromagnetic field

$$\left[\Gamma_{\mu\nu}^{\left(\frac{3}{2},0\right)}\right]_{[\alpha\beta][\gamma\delta]} D^{\mu} D^{\nu} \left[\Psi^{\left(\frac{3}{2},0\right)}(x)\right]^{[\alpha\beta]} = -m^2 \left[\Psi^{\left(\frac{3}{2},0\right)}(x)\right]^{[\gamma\delta]}$$

$$\left[\Gamma^{\left(\frac{3}{2},0\right)\mu}{}_{\nu}\right]^{[\alpha\beta]}{}_{[\gamma\delta]} = 4 \left[\mathcal{P}_F^{\left(\frac{3}{2},0\right)}\right]^{[\alpha\beta][\sigma\mu]} \left[\mathcal{P}_F^{\left(\frac{3}{2},0\right)}\right]_{[\sigma\nu][\gamma\delta]}$$

Besides applying our formalism to Lorentz tensors, we can equally apply it to spinor-Dirac tensors.

Example: Spin 1 in $\Psi_{a_1 a_2}$

$$\Psi_{a_1 a_2} \sim [(1/2,0) \oplus (0,1/2)] \otimes [(1/2,0) \oplus (0,1/2)]$$
$$= [(1,0) \oplus (0,1)] \oplus \cancel{2(0,0)} \oplus \cancel{2(1/2,1/2)}]$$

$\mathcal{P}_F^{(1,0)}$ picks up only the degrees of freedom of $(1,0) \oplus (0,1)$

Comparing our formalism with the Bargmann-Wigner (BW), we observe that solutions of BW do not transform irreducibly.

BW method

- Simmetric Dirac tensors $\Psi_{b_1 \ldots b_n}$
- Particles with spin $j = n/2$
- Field equations

$$(i\gamma_\mu \partial^\mu - m)^{a_i b_i} \, \Psi_{b_1 \ldots b_i \ldots b_n} = 0$$

Comparing our formalism with the Bargmann-Wigner (BW), we observe that solutions of BW do not transform irreducibly.

Espín 1

Our formalism

$\Psi \sim (1,0) \oplus (0,1)$

Correct number of d.o.f

$+$ irreducibility

BW formalism: $\mathcal{P}_F^{(1/2,1/2)}\Psi \neq 0$

$\Psi \sim (1,0) \oplus (0,1) \oplus (1/2,1/2)$

Correct number of d.o.f,

but not irreducibles

(Implies unphysical properties

and high spin problems)

The mixture of irreducible representations can be avoided by using Weyl spinor fields.

Symmetric Weyl spinor tensor fields:

$\chi_{\alpha\beta\ldots} \sim (j,0)$

$\bar{\eta}^{\dot{\alpha}\dot{\beta}\ldots} \sim (0,j)$

By means of these fields we can describe any spin.
(Laporte and Uhlenbeck,1931. Friedrich Cap and Hermann Donnert, 1954)

So far I have presented a new approach at the level of classical field theory. Now, we would like to move at the quantum level.

Goal: Construction of a suitable Lagrangian as the starting point to elaborate high spin quantum field theories

Conditions:

- □ Scalar action
- □ Quadratic Lagrangian in the fields and its derivatives
- □ Hermitian Lagrangian
- □ **Diagonal Hamiltonian without negative terms**

Idea: Formulate the theory in Weyl-spinor tensor basis

Advantages

- Just by indices simmetrization we can obtain the irreducible representations $(j,0) \oplus (0,j)$, which we use in our fomalism at the classical level.
- Possibility of constructing a family of kinetic terms in such a way to obtain a positive definite diagonal Hamiltonian.

Example: Spin 1 in $(1,0)\oplus(0,1)$ with Weyl-spinor tensors.

$$\begin{aligned}
\mathcal{L} \;=\;& a\partial^\mu\psi^{\alpha\beta}\partial_\mu\psi_{\alpha\beta} + a\partial^\mu\psi^\dagger_{\dot\alpha\dot\beta}\partial_\mu\psi^{\dagger\dot\alpha\dot\beta} \\
&+b\partial_\nu\psi^\dagger_{\dot\alpha\dot\beta}\overline{\sigma}^{\nu\dot\alpha\alpha}\overline{\sigma}^{\mu\dot\beta\beta}\partial_\mu\psi_{\alpha\beta} + b\partial_\mu\psi^\dagger_{\dot\alpha\dot\beta}\overline{\sigma}^{\nu\dot\alpha\alpha}\overline{\sigma}^{\mu\dot\beta\beta}\partial_\nu\psi_{\alpha\beta} \\
&+c\partial_\nu\psi^{\gamma\beta}\sigma^\nu_{\gamma\dot\alpha}\overline{\sigma}^{\mu\dot\alpha\alpha}\partial_\mu\psi_{\alpha\beta} + c\partial_\nu\psi^\dagger_{\dot\alpha\dot\beta}\overline{\sigma}^{\nu\dot\alpha\alpha}\sigma^\mu_{\alpha\dot\gamma}\partial_\mu\psi^{\dagger\dot\gamma\dot\beta} \\
&-m^2\psi^{\alpha\beta}\psi_{\alpha\beta} - m^2\psi^\dagger_{\dot\alpha\dot\beta}\psi^{\dagger\dot\alpha\dot{}}
\end{aligned}$$

a, b and c are real parameters

Work in progess...

Conclusions

- Our second order formalism in the momenta avoid the three main problems of classical high spin theories.

  Acausality

  Undesired propagation of degrees of freedom

  Non-physical solutions

Conclusions

- Combining the projectors$\mathcal{P}_{F}^{(j_1,j_2)}$ and $\mathcal{P}_{W^2}^{(j,m)}$ on the field Ψ_μ, we obtain two equations that describe particles with spin 1/2.

Dirac particle $g = 2$
$(1/2, 0) \oplus (0, 1/2)$ representation

New particle $g = -2/3$
$(1, 1/2) \oplus (1/2, 1)$ representation

Finite Compton cross sections in ultraviolet according to unitarity

Conclusions

- Aplying our formalism to the field $\Psi_{[\mu\nu]}$, we describe a spin $3/2$ particle with g$=2/3$, and thereby distinct from $3/2$ in Ψ_μ with $g=2$.

Therefore, particles with equal spin described by fields in distinct Lorentz irreducible reprsentations have different physical properties.

Conclusions

- We verify that the method of Bargmann-Wigner, although it predicts the right number of degrees of freedom for a spin j, they do not transform irreducibly.

Our formalism	**Bargmann-Wigner formalism**
Spin 1 in $\mathbf{(1,0)\oplus(0,1)}$	**Spin 1 in** $\mathbf{(1,0)\oplus(0,1)\oplus(1/2,1/2)}$

Conclusions

- We initiate the analysis to elaborate high spin quantum fields from the consrtuction of a Lagrangian based on four conditions.

  1. **Scalar action**
  2. **Hermitian Lagrangian**
  3. **Quadratic Lagrangian**
  4. **Diagonal Hamiltonian without negative terms**

- In addition to the four conditions, we use symmetric Weyl tensor fields to construct the Lagrangian.

Conclusions

- We make progress in constructing the Lagrangian for spin 1 particles using second rank symmetric Weyl tensor fields which could possibly guarantee the previous four conditions. We hope we can generalize it to any spin j.